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Determining The Source in Complete Parabolic Equations

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The problem of determining the source has been analyzed during the last years in different areas of ap- plied mathematics and has received considerable attention in many current research, as it has applications in fields such as driving of heat, crack identification, electromagnetic theory, geophysical prospecting, the detection of contaminants and detection of tumor cells, among others.

Source identification from noisy measurements is often a problem ill-posed in the sense of Hadamard since the solution does not depend continuously of the data. Among the most important tools used in the literature to determine a source can be found: the potential logarithmic method [11], the projective method [10], Green's functions [2], dual reciprocity boundary element methods [1], the dual reciprocity method [14], the fundamental solution method [3].

Regarding applications in transport processes or parabolic differential equations, there are not many articles published for the general case [15], most of the articles available in the literature focused on the heat equation. The sources in this equation are retrieved using different methods, techniques and strategies, see for example [4]. There are many articles that analyze particular cases with simplifications or restrictions in mathematical models, source type, boundary conditions, or chosen domain, such as [4, 9, 17]. The most commonly used methods in these cases are: the boundary element method [4], the fundamental solution method [17], the Ritz-Galerkin method [12], the method of finite differences [19], the meshless method [18], the conditional stability method [16] and the shooting method [9]. Some authors studied different strategies for estimating the source, with different characteristics, in the fractional diffusion equation, see for example [22].

On the other hand, regularization methods [6] they play an important role in stabilizing estimated solutions. The most used are the iterative regularization method [5], the simplified Tikhonov regularization method [20], the modified regularization method [15, 18], Fourier truncation [20] and the mollification method [21]. Regarding the determination of the source of a parabolic equation, in [13] the author considers a convection-diffusion equation, meanwhile in [19, 23] only diffusion is considered. Recently, the problem of finding the source term in a complete parabolic equation was solved by the quasi-reversibility method, see [8]. This method can be used to solve the inverse problem of determining the source in nonlinear parabolic equations [7].

There are numerous situations that can be described as a source estimation problem and that highlight the importance of its analysis and the techniques used to address and resolve it. An application that concerns urbanized cities is the detection of contaminants in groundwater layers. It can also be applied to the estimation of metabolic heat in a biological tissue using the Pennes model, where any abnormality within the body can lead directly to variations in temperature and heat flux at the surface.

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