https://doi.org/10.46813/2023-143-131

COMPACT CORRUGATED HORN ANTENNA INITIAL DESIGN FOR MICROWAVE DIAGNOSTICS IN URAGAN-2M STELLARATOR

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In this paper, a well-known design procedure is proposed for the design of wideband constant-beam width conical corrugated horn antennas, with minimum design and construction complexity. The inputs to the procedure are the operating frequency band, the required minimum beam width in the entire frequency band, and the frequency in which the maximum gain is desired to occur. Based on these values, the procedure gives a relatively good design with a relative bandwidth of up to 2.1:1. Based on the proposed procedure, a corrugated horn antenna with a constant beam width over the frequencies of 10 to 14 GHz was designed and simulated using commercial software. This paper presents initial design for the development of quasi-optical corrugated conical horn for reflectometry diagnostics for the Uragan-2M stellarator plasma experiments.

PACS:52.25.Mq; 52.25.Os; 52.35.Hr; 52.35.Mw; 52.55.Hc

INTRODUCTION

Corrugated feed horns can be considered as Gaussian Beam launchers with high efficiency provided that the HE11 hybrid mode propagate inside the horn. The gaussian beam propagation theory can be used to characterize this kind of horns in a very simple way. With the gaussian representation of feeds some commercial optical software, not specially developed for mm-wave application, can be used for predicting the performances of mm-wave telescopes where the diffraction effects are not negligible, without using time-consuming calculation methods like Physical Optics. The wave or Helmholtz equation (1)

$$(\nabla^2 + k^2)\psi = 0 \tag{1}$$

is the starting point to derive the gaussian beam theory. Introducing the so-called paraxial approximation on the quasi-plane wave solution of the equation (1) a paraxial equation is found and solution are the Gaussian Modes. In Section 1 the well-known fundamental gaussian beam representation has been presented following standard methods [1–8]. The application of the theory to the corrugated feed horns has been extensively reported in many articles; some practical formulas have been derived, special devoted to the calculation of basic horn parameters like the edge taper, the angular resolution, the phase center location and the beam pattern shape at the far-field (Section 2). After that in Section 3 a primer for low-density reflectometer antenna is presented.

1. GAUSSIAN BEAM PROPAGATION AT THE CORRUGATED FEEDHORNS

The wave equation for an electric field component, *E(x, y, z)*, of an electromagnetic wave with wavelength $\lambda$is represented by equation (2):

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} + \left(\frac{2\pi}{\lambda}\right)^2 E = 0 \tag{2}$$

provided that the wave is propagating through the uniform medium and assuming a time dependence $e^{-i\omega t}$. Solution of this equation are the plane wave if the field amplitudes are independent on the position. Considering a more general solution (3) with the amplitude dependent on spatial cylindrical coordinates (r, ϕ, z),

$$E(r,\phi,z) = E_0(r,\phi,z)e^{-ikz} \tag{3}$$

and assuming that the variation of the field amplitude is small along the propagation direction (*z*-direction) in a distance comparable the wavelength λ, and small to the variation on the orthogonal plane (r, ϕ) the paraxial wave equation (4) has the form:

$$\frac{\partial^2 E_0}{\partial r^2} + \frac{1}{r}\frac{\partial E_0}{\partial r} + \frac{1}{r}\frac{\partial^2 E_0}{\partial \phi^2} - 2ik\frac{\partial E_0}{\partial z} = 0. \tag{4}$$

Solutions is called gaussian beam modes. Under the cylindrical symmetry, the solution of eq. (4) will be (5):

$$E_0(r,z) = A(z)e^{-\frac{ikr^2}{2q(z)}}, \tag{5}$$

where $A(z)$ and $q(z)$ are two complex functions of z only. $A(z)$ and $q(z)$ are related one to each other by (6):

$$\frac{dA}{A} = -\frac{dq}{q}, \tag{6}$$

$A(z)$ and $q(z)$ is related to two fundamental parameters of the gaussian beam: the radius of curvature of beam *R* and the beam radius *w* which is the value of the radius at which the field is 1/*e* of the on-axis field. The solution from (4), (5) is possible to write in more convenient form (7):

$$E(r,z) = \left(\frac{w_0}{w}\right)e^{-\frac{r^2}{w^2} - ikz - \frac{i\pi r^2}{\lambda R} + i\phi_0}, \tag{7}$$

where

$$R = z + \frac{1}{z}\left(\frac{kw_0^2}{2}\right)^2; \tag{8}$$

$$w = w_0\left[1 + \left(\frac{2z}{kw_0^2}\right)^2\right]^{\frac{1}{2}}; \tag{9}$$

$$\tan\phi_0 = \frac{2z}{kw_0^2} \tag{10}$$

the parameter w_0 is beam waist (a parameter that fully characterize the beam) and the beam radius at *z=0*, where the curvature of the beam is $R=\infty$ and the wave front is a plane.

To represent the feedhorns as a gaussian launchers only the Beam Waist (value and position on space) has to be determined. This parameter can be determined by the geometry of the corrugated horn. However, some properties on the far-field beam shape can be used calculate the value of w_0 as well. One can apply the inversed formulas (8), (9) to calculate w_0 and z for the corrugated feedhorn for HE11 mode. The aperture field of a balanced horn is characterized by the J_0 Bessel function with the first null at $r=a$. The gaussian fit of this function is well represented by the fundamental mode with (11):

$$w_a = 0.644 \cdot a \tag{11}$$

at the horn aperture plane. Introducing parameter s by (12):

$$s = \frac{a^2}{\lambda L} \ . \tag{12}$$

The curvature radius of the beam at the aperture is set by cone apex to the aperture plane; thus, we can substitute these the horn length L from the quantities on equations (9) and (8), obtaining more practical form (13), (14):

$$w_0 = \frac{0.644 \cdot a}{\left[1+1.69763 \cdot s^2\right]^{\frac{1}{2}}}, \tag{13}$$

$$z = \frac{L}{1+\left[\frac{0.58906}{s^2}\right]}. \tag{14}$$

In fact, s is a phase error at the aperture parameter. It can be calculated by considering the wave front tangent to the aperture plane and originated at the cone apex. For small angle horns the wave path at the horn side is longer than the path at the horn axis by a quantity Δ (15):

$$\Delta = R - L = R - R \cdot \cos(\theta_f) = a \cdot \frac{1-\cos(\theta_f)}{\sin(\theta_f)} = a \cdot \tan\left(\frac{\theta_f}{2}\right) \simeq a \cdot \frac{a}{2L} \ . \tag{15}$$

The corresponding phase difference could be derived from (16):

$$\delta\phi = k \cdot \Delta = \frac{2\pi}{\lambda} \cdot \frac{a^2}{2L} = \frac{\pi \cdot a^2}{\lambda L} = \pi \cdot s \ , \tag{16}$$

where θ_f is flare angle of the horn, z in eq. (13) is the distance between the phase center and the horn aperture. For an open-ended corrugated cylindrical waveguide s=0 and thus the phase center is at the aperture z=0 and the waist of the beam at the aperture, $w_0=w_a=0.644a$. From the eq. (7) the power $P(r_i,z) = |E(r_i,z)|^2$ could be presented as a ratio at different radii (17):

$$\frac{|E(r_1,z)|^2}{|E(r_2,z)|^2} = e^{2\left\{\frac{r_2^2-r_1^2}{w^2}\right\}} \tag{17}$$

for $r_2 = 0$ and $r_1 = r_{ET}$, where ET is edge Taper level (18):

$$ET = \frac{P(r,z)}{P(0,z)} = e^{-2\left(\frac{r_{ET}}{w}\right)^2}. \tag{18}$$

The ET parameter has to match feedhorn with the diagnostic port vacuum window diameter. This directly controls the angular resolution (Full Width High Maximum (FWHM)) and the spillover radiation. The ET in dB could be written (19):

$$ET(\mathrm{dB}) = 10 \cdot \log(ET). \tag{19}$$

Keeping the ET constant means that the ratio r_{ET}/w is constant as function of z. At the far-field, $z\to\infty$, the angle at the ET can be seen is (20):

$$\theta_{ET} = \lim_{z\to\infty}\left[\arctan\left(\frac{r_{ET}}{z}\right)\right] = \lim_{z\to\infty}\left[\arctan\left(\frac{r_{ET}}{w}\cdot\frac{w}{z}\right)\right] = \arctan\left(B \cdot \frac{\lambda}{\pi w_0}\right), \tag{20}$$

where B is specific ratio governed by (21):

$$B = \frac{r_{ET}}{w} = \sqrt{-\frac{1}{2}\ln\ ET} = \sqrt{-0.11513 \cdot ET(\mathrm{dB})}\ . \tag{21}$$

The far-field angle at which the field fails to $1/e$ (-8.686dB) can be easily calculated substituting $r_{ET}=w$ or B=1 (22):

$$\theta_0 = \arctan\left(\frac{\lambda}{\pi w_0}\right). \tag{22}$$

Also, the FWHM can be easily calculated by setting ET = -3dB and then B=0.5877, Full Width High Maximum is (23)

$$\mathrm{FWHM} = 2 \cdot \arctan\left(0.5877 \cdot \frac{\lambda}{\pi w_0}\right). \tag{23}$$

The normalized far field pattern function of θ, which is the angle of boresight direction, calculated from eq. (17). Taking in account that at the far-field $z/w=(\pi w_0)/\lambda$ with $z=\infty$ and $t=\tan\theta=r/z$ in the terms of t (24) or in the terms of θ (25):

$$P(t) = e^{-\frac{t^2}{2\left(\frac{\lambda}{2\pi w_0}\right)^2}}\ , \tag{24}$$

$$P(\theta) = e^{-\frac{(\tan\,\theta)^2}{2\left(\frac{\lambda}{2\pi w_0}\right)^2}}\ . \tag{25}$$

To obtain the normalized field pattern in dB let introduce a simple substitution $\sigma = \frac{\lambda}{2\pi w_0}$(26):

$$P^{\mathrm{dB}}(\theta) = -\frac{10}{2\ln\,10} \cdot \frac{(\tan\,\theta)^2}{\sigma^2} = -2.1715 \cdot \frac{(\tan\,\theta)^2}{\sigma^2}\ . \tag{26}$$

2. DESIGN PROCEDURE

For the narrow-band antenna one the operational (minimum, center, maximum, output) frequencies for the feed-horn must be chosen to be as (27, 28):

$$f_{max} \le 1.4 f_{min} f_c = \sqrt{f_{min} f_{max}}\ , \tag{27}$$

$$f_c \le f_o \le 1.05 f_c\ . \tag{28}$$

The fundamental mode in a circular waveguide is the TE11 mode, which has a cut-off wavenumber (29)

$$k = \frac{2\pi}{\lambda} = \frac{1.841}{\text{radius of circular waveguide}}\ . \tag{29}$$

Therefore, the input radius, a_i of a corrugated horn must satisfy the inequality (30)

$$\frac{2\pi f_{min}}{c} a_i \ge 1.84118 \tag{30}$$

and is often chosen to be such that (31)

$$k_c a_i = \frac{2\pi}{\lambda_c} a_i = 3\text{, i.e., } a_i = \frac{3\lambda_c}{2\pi}\ . \tag{31}$$

Simple equations for the far-fields of corrugated-conical horns have been presented and discussed, demonstrating a practical approximation method.

The general drawing of the corrugated conical feed-horn is presented at the Fig. 1.

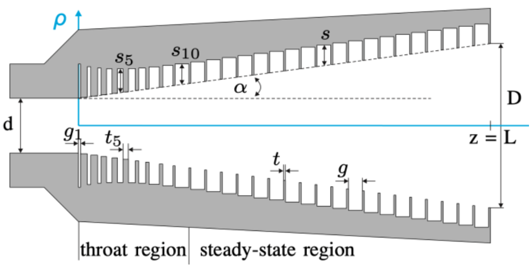

Fig. 1. General view of the corrugated conical horn antenna; the picture notations are adopted to the corrugated parameters stated in the Table: d=2a_i, D=2a_o, $\alpha=\theta_f$, g+t=p, s=d, length of the throat region (mode convertor) is N_{MC} slots

The calculation of the variable-depth-slot mode converter can be done as follows (please refer to the Figs. 1, 2): first let us summarize several feedhorn parameters which will be used in the calculations.

Corrugated-horn parameters

Quantity	Symbol
Input radius	a_i
Output radius	a_o
Length	L
Total number of slots	N
Number of slots in the mode converter	N_{MC}
Slot pitch	$p = L/N$
Slot width	w
Slot pitch-to-width ratio	$\delta = w/p$
Width of the slot teeth	$(p - w) = (1 - \delta)p$
Depth of the j th slot	d_j where $1 \le j \le N$

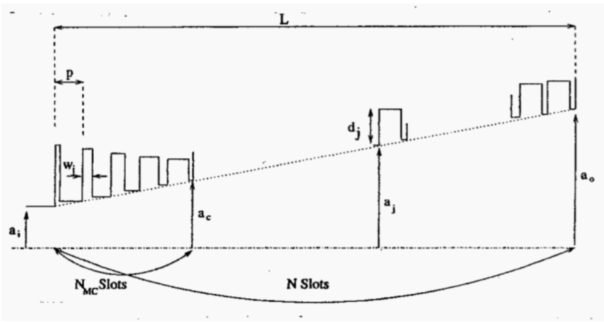

Fig. 2. Simplified drawing of the mode converter (see Fig. 1) geometry for the corrugated conical horn antenna

The length of the horn L is usually set by the application, but around, $6\lambda_c$ to $12\lambda_c$, is usually required, although some applications may need a horn $15\lambda_c$ to $20\lambda_c$ long.

For deriving the mode convertor region depth d_j profile (32) of the feedhorn the correction factor of κ (33) is introduced to tune the linear corrugated surface profile $a(z)$ (34). Assuming that the principal frequency range is the lowest part of the band.

$$d_j = \kappa \frac{3\lambda}{4} \; ; \qquad (32)$$

$$\kappa = \exp\left[\frac{1}{5.955\left(k_c a_j\right)^{1.079}}\right]; \qquad (33)$$

$$a(z) = a_i + (a_o - a_i)\frac{z}{L}. \qquad (34)$$

When $1 \le j \le N_{MC} + 1$ then the slot depth of the j-th slot is (35):

$$d_j = \left\{\sigma - \frac{j-1}{N_{MC}}\left(\sigma - \frac{1}{4}\exp\left[\frac{1}{2.114\left(k_c a_j\right)^{1.134}}\right]\right)\right\}\lambda_c, \qquad (35)$$

where $\sigma (0.4 \le \sigma \le 0.5)$ is a percentage factor for the first slot depth of the mode converter. When $N_{MC}+2 \le j \le N$, then the slot depth of the j-th slot is (36):

$$d_j = \frac{\lambda_c}{4}\exp\left[\frac{1}{2.114\left(k_c a_j\right)^{1.134}}\right] - \left(\frac{j - N_{MC} - 1}{N - N_{MC} - 1}\right)\left\{\frac{\lambda_c}{4}\exp\left[\frac{1}{2.114(k_c a_o)^{1.134}}\right] - \frac{\lambda_o}{4}\exp\left[\frac{1}{2.114(k_o a_o)^{1.134}}\right]\right\}. \qquad (36)$$

3. FEDHORN PARAMETERS FOR URAGANS PROPOSED MICROWAVE DIAGNOSTICS

As an example, consider a standard Kα-band operation from a typical low-density reflectometer. We assume the designed antenna will be used for reflectometer system for Uragan-2M plasma production experiment [9–13]. The expected electron plasma density is close to $n_e = 2\cdot10^{12}$ cm^{-3}. Thus, the operational range for O-mode subsystem [14, 15] with a requirement to receive reflected signals in the 10.7…12.75 GHz frequency band. The half-subtended angle from the focus of the antenna to the vacuum window is 20°, and a -15 dB edge taper on the feed pattern is specified at this angle.

So, following our procedures, we have: f_{min}=10.7 GHz; f_{max}=14.5 GHz ($f_{max}\approx1.36\,f_{min}$); in this case we obtain $f_c=(f_{min}\cdot f_{max})^{0.5}$ = 12.46 GHz and f_0=1.02 f_c = 12.71 GHz. The input radius is chosen to be; $a_i = 3\lambda_c/2\pi$ =11.49 mm and $a_0 \approx 1.95\,\lambda_c$=46.93 mm. The pitch is chosen to be $p=\lambda_c/8 \approx 3$ mm, with a pitch-to-width ratio $\delta = 0.8$, (i.e., the slots will be 2.4 mm wide, and the teeth will be 0.6 mm wide). As, $f_{max}\approx1.36\,f_{min}$ we choose a variable-depth-slot mode converter, with σ=0.42. The length of the feedhorn is chosen to be L=6 λ_c=60 p≈180 mm, while we select a linear profile. The obtained radiation pattern is presented on the Fig. 3.

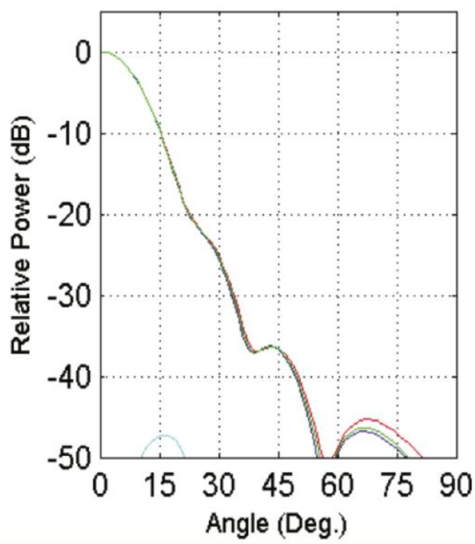

Fig. 3. The radiation pattern of a corrugated horn at Kα-band: 12.46 GHz

CONCLUSIONS

In this report a first order treatment of corrugated feedhorns with gaussian beam has been investigated. Simple equations for the initial design for the far-fields of corrugated-conical horns have been presented and discussed, demonstrating a practical approximation method using aperture field augmented with quadratic phase. Basic formulas, based on fundamental gaussian mode, have been derived. These formulas can be used to characterize corrugated feedhorns starting from (a) geometrical parameters of the corrugated horns, like length and aperture diameter; (b) far field beam pattern shape.

From the geometrical parameter the beam waist and the location of the phase center below the horn aperture as well can be obtained. From the beam pattern shape, unfortunately, only the beam waist can be recovered (eq. (25)). However, since the first approach is applicable only for pure hybrid mode corrugated horn, it is not suitable to calculate the gaussian beam parameters for multi-mode horns, like profiled or dual-profiled horns. On the contrary, the second approach can be used, in principle, for any kind of horns using the variety of commercial software like CST Microwave Studio and HFSS, TICRA CHAMP, MICIAN μWave Wizard.

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Article received 20.02.2023

РОЗРОБКА МОДЕРНІЗОВАНОЇ ХВИЛЕВОДНОЇ СИСТЕМИ ДЛЯ МІКРОХВИЛЬОВОЇ ДІАГНОСТИКИ СТЕЛАРАТОРА УРАГАН-2М

Р.О. Павліченко, Н.І. Павліченко

Пропонується відома методика проектування широкосмугових конічних гофрованих рупорних антен з постійною шириною діаграми спрямованості, яка має мінімальну складність проектування і побудови. Вхідними даними для процедури є робоча смуга частот, необхідна мінімальна ширина променю у всій смузі частот і частота, на якій бажано досягти максимального коефіцієнта підсилення. На основі цих значень процедура дає відносно хорошу конструкцію з відносною шириною смуги пропускання до 2,1:1. На основі запропонованої методики було спроектовано і змодельовано за допомогою комерційного програмного забезпечення гофровану рупорну антену з постійною шириною смуги пропускання у діапазоні частот від 10 до 14 ГГц. У статті представлено початковий проект розробки квазіоптичної гофрованої конічної рупорної антени для рефлектометричної діагностики для плазмових експериментів стеларатора Ураган-2М.