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CONVERSION OF THE ENERGY OF LOW-FREQUENCY OSCILLATIONS INTO THE ENERGY OF HIGH-FREQUENCY OSCILLATIONS IN MAGNETOACTIVE PLASMA

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It is shown that in a plasma located in an external magnetic field (magnetically active plasma) it is possible to convert the energy of low-frequency oscillations into the energy of high-frequency oscillations. Such a transformation is possible due to the fact that in such a plasma it is possible to create conditions for nonreciprocal coupling between high-frequency waves.

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INTRODUCTION

Non-reciprocal mediums in many of their properties, differ significantly from the usual reciprocal mediums. Such properties of media and such systems, in which nonreciprocity arises, arise practically in all areas of physics. This special property of systems and media is well studied and widely used in electrodynamics and optics (see, for example, [1-5]). Below we will be interested in that special property of nonreciprocal systems, the use of which makes it possible to convert the energy of low-frequency oscillations into the energy of high-frequency oscillations. In our previous works [6-9], it was shown that the non-reciprocal coupling of high-frequency oscillatory systems allows them to draw energy from a low-frequency source. In particular, the work [8] experimentally shows the possibility of excitation of high-frequency oscillations of non-mutually coupled circuits by a low-frequency source, the frequency of which is forty times lower than the frequency of high-frequency exited circuits. Note that the non-reciprocal coupling of high-frequency oscillations can be created artificially (as in [8]). In addition, it can exist naturally in media that have the property of nonreciprocity. There are many such media. Well-known examples are ferrites and plasmas in an external magnetic field. In the present work, it is shown that conditions can indeed be created in a magnetoactive plasma under which electromagnetic waves do not mutually interact with each other. Such connection between these high-frequency waves makes it possible to excite them using low-frequency sources (for example, using low-frequency longitudinal plasma oscillations).

Below, in the second section, the problem is formulated, and the system of equations is written out. In the third section, a system of truncated equations is obtained, which allows analytical methods to find a solution to the original equations. Conditions for the excitation of high-frequency waves using the energy of low-frequency longitudinal plasma oscillations are obtained. In particular, there found increment of parametric instability, which is proportional to the amplitude of longitudinal plasma oscillations. The fourth section describes the nonlinear dynamics of the three-wave interaction for the case when the interaction matrix elements for high-frequency waves are proportional to the first power of their wave vectors. In conclusion, the main results of the work are formulated.

1. STATEMENT OF THE PROBLEM AND BASIC EQUATIONS

The system of initial equations are the Maxwell equations for fields and the equations of hydrodynamics for plasma:

$$rot\vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t}\,;\; div\vec{E} = 4\pi en\,;$$

$$rot\vec{H} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j}\,;\quad \vec{j} = en\vec{v}\,;$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\vec{\nabla}\right)\vec{v} = \frac{e}{m}\vec{E} + \frac{e}{mc}\left[\vec{v}\times\vec{H}\right]\;\frac{\partial n}{\partial t} + div\left(n\vec{v}\right) = 0\,. \quad (1)$$

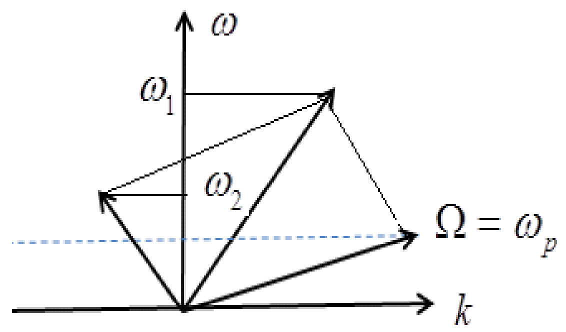

Fig. 1. Dispersion diagram of interacting waves. The interaction occurs with the participation of longitudinal plasma waves. Case $k_1 > 0;\, k_2 < 0$

To simplify the form of general formulas below, we will proceed from the diagrams of interacting waves, which are presented in Figs. 1, 2. It follows from them that we will consider the interaction of three waves. Two of them are transverse (ω_1, ω_2), and the third is longitudinal, i.e. type interaction is $t \to t + l$ considered. We will also consider a spatially one-dimensional case, i.e., the entire process of interaction will depend on only one spatial coordinate z. The whole system is placed in an external magnetic field H_0, which is directed along the z axis. In this case, the system of equations (1) for transverse waves will take the form:

$$\frac{\partial E_x}{\partial z} = -\frac{1}{c}\frac{\partial H_y}{\partial t};\ \frac{\partial E_y}{\partial z} = \frac{1}{c}\frac{\partial H_x}{\partial t};$$
$$\frac{\partial H_x}{\partial z} = \frac{1}{c}\frac{\partial E_y}{\partial t} + \frac{1}{c}4\pi nev_y;$$
$$-\frac{\partial H_y}{\partial z} = \frac{1}{c}\frac{\partial E_x}{\partial t} + \frac{1}{c}4\pi nev_x; \qquad (2)$$
$$\frac{\partial v_x}{\partial t} = \frac{e}{m}E_x + \omega_H v_y - v_z\left(\frac{e}{mc}H_y + \frac{\partial v_x}{\partial z}\right);$$
$$\frac{\partial v_y}{\partial t} = \frac{e}{m}E_y - \omega_H v_x + v_z\left(\frac{e}{mc}H_x - \frac{\partial v_y}{\partial z}\right).$$

Let us write out the equations for low-frequency longitudinal waves:

$$\frac{\partial v_z}{\partial t} = \frac{e}{m}E_z - v_z\frac{\partial v_z}{\partial z} + \frac{e}{mc}\left(v_x H_y - v_y H_x\right);\ \frac{\partial E_z}{\partial z} = 4\pi en;$$
$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}nv = 0. \qquad (3)$$

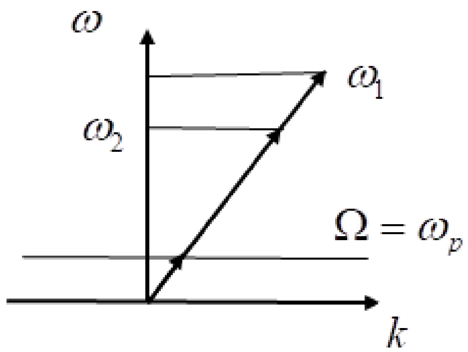

Fig. 2. Dispersion diagram of the same trio of waves as in Fig. 1. Distinction: high-frequency waves are directed in one direction: $k_1 > 0;\ k_2 > 0$

Let a low-frequency longitudinal wave has been excited in the plasma. The electric field of this wave can be written as:

$$E_z = E\exp\left(i\Phi\right) + k.c. \qquad (4)$$

Here $\Phi \equiv (\Omega t - \kappa z)$.

The dimensionless velocity of plasma particles under the action of such a field is $v_z = V_0\exp(i\Phi) + k.c.$, $V_0 = -i(eE / mc\Omega)$. The main result we are interested in can be obtained by assuming that this speed is given. It is not perturbation. Then we substitute this speed into the system of equations that describe the high-frequency dynamics of fields and particles, i.e. into the system of equations (2). In addition, we will assume that the interaction process occurs in a spatially limited volume (in a resonator). Therefore, the dynamics of the process under study depends only on time ($E_x \sim E_y \sim exp(i\omega t - ik_z z)$; $\partial/\partial z \to -ik_z$). As a result, after simple but cumbersome calculations, the following system of nonlinear equations with varying coefficients can be obtained to determine the dynamics of high-frequency fields and high-frequency velocities:

$$\ddot{E}_{j,\pm} + (k_j c/\omega_1)^2 E_{j,\pm} = \mu^2\left[\dot{v}_\pm\left(1+n\right) + \dot{n}v_\pm\right];$$
$$\dot{v}_{j,\pm} = \left(E_{j,\pm} \pm i\omega_H v_{j,\pm}\right) - k_{j,z}v_z\left(E_{j,\pm} - i\omega_j v_{j,\pm}\right). \qquad (5)$$

Here

$E_\pm = \left(E_x \pm iE_y\right)$; $H_\pm = \left(H_x \pm iH_y\right)$; $\dot{E}_{j,\pm} = dE_{j,\pm}/d\tau$;

$\tau = \omega_1 t$; $v_\pm = \left(v_x \pm iv_y\right)$; $j = \{1,2\}$;

$E_\pm \to \left(eE_\pm / mc\omega_1\right)$; $n = \tilde{n}/n_0$; $\mu^2 = \left(\omega_p/\omega_1\right)^2$; ω_p – plasma frequency. System (5) is closed since the longitudinal velocity of particles (low-frequency velocity) is given (see (3) and (4)).

2. ANALYTICAL SOLUTION

The system of equations (5) is a system of four second-order ordinary differential equations for fields (two equations for the first and second high-frequency waves, as well as two equations for different polarizations), as well as four first-order ordinary differential equations for determining high-frequency particle velocities plasma. Thus, the system of equations (5) is equivalent to the system of twelve ordinary differential equations of the first order. Note that each polarization can be considered independently. The system (5) does not contain terms that describe the interaction of these polarizations. Note that such terms appear when it is necessary to take into account the dynamics of low-frequency waves. The system of equations (5) does not contain equations describing the low-frequency dynamics of fields and velocities. Despite this simplification, it is still quite complex. Solutions of system (5) will be sought in the form:

$$E_\pm = A_1\exp\left(i\varphi_1\right) + A_2\exp\left(i\varphi_2\right) + k.c.;$$
$$v_\pm = a_1\exp\left(i\varphi_1\right) + a_2\exp\left(i\varphi_2\right) + k.c.;$$
$$v_z = a_3\exp(i\Phi) + k.c., \qquad (6)$$

where $\varphi_{1,2} = \omega_{1,2}t - k_{1,2}z$, $a_3 = V_0 = const$ – is given.

The factors in front of the high-frequency exponents in the linear approximation are constants and slow functions of time when nonlinear terms are taken into account:

$$A_{1,2} = A_{1,2}(t);\ \dot{A}_{1,2} << \omega_{1,2}A_{1,2};$$
$$a_{1,2} = a_{1,2}(t);\ \dot{a}_{1,2} << \omega_{1,2}a_{1,2}.$$

The first terms in formulas (6) describe (in a complex form) a wave that propagates along the axis, and the second terms describe a wave that propagates towards it ($k_2 < 0$, see Fig. 1) or in the same direction ($k_2 > 0$, see Fig. 2). We will also assume that the frequencies and wave vectors of the interacting waves satisfy the synchronism conditions:

$$\omega_1 = \omega_2 + \Omega ; \quad k_1 = k_2 + \kappa . \quad (7)$$

In the linear approximation, the connection between the field amplitudes and velocities is described by the formula

$$A_j = a_j \cdot i \cdot \left(\omega_j \pm \omega_H\right) . \quad (8)$$

Nonlinear terms determine the slow dynamics of the amplitudes:

$$\dot{A}_1 = i\mu a_3\left(A_2 + i\omega_H a_2\right); \quad \dot{A}_2 = i\mu a_3^*\left(A_1 + i\omega_H a_1\right)/\omega_2 ;$$
$$\dot{a}_1 = -k_2 a_3\left(A_2 - ia_2\right); \; \dot{a}_2 = -k_1 a_1^*\left(A_1 - ia_1\right). \quad (9)$$

When obtaining (9), we took into account relations (7).

Let us substitute expressions (8) into the second equation of system (9):

$$\dot{a}_1 = \pm k_1 a_3 a_2 \omega_H ; \quad \dot{a}_2 = \mp k_2 a_3^* a_1 \omega_H . \quad (10)$$

Equations (10) are equivalent to the pendulum equation:

$$\ddot{a}_1 + \left|a_3\right|^2\left(k_1 \cdot k_2\right)\omega_H^2 a_1 = 0 . \quad (11)$$

It follows from this equation that if high-frequency waves are unidirectional (see Fig. 2), then only oscillatory dynamics of high-frequency waves occurs. The amplitude of these oscillations is determined by the initial conditions. If the interacting high-frequency waves propagate in opposite directions (see Fig. 1), then equation (11) describes the dynamics of an unstable pendulum. The amplitude of such a pendulum grows exponentially. The instability increment is

$$\Gamma = \left(\frac{e \cdot |E|}{m \cdot c \cdot \Omega}\right) \cdot \omega_H \cdot \sqrt{k_1 |k_2|} . \quad (12)$$

Let us substitute (8) into the first two equations of system (9). We obtain the following equations for determining the amplitudes of slowly changing high-frequency fields:

$$\dot{A}_1 = i\mu a_3 A_2 \omega_2 / \left(\omega_2 - \omega_H\right); \dot{A}_2 = i\mu a_3^* A_1 / \left(1 - \omega_H\right) . \quad (13)$$

System (13) corresponds to the linear oscillator equation:

$$\ddot{A}_1 + \left[\mu^2 \left|a_3\right|^2 / \left(1 - \omega_H\right)^2\right] A_1 = 0 . \quad (14)$$

It follows from Eq. (14) that the slow dynamics of high-frequency fields due to non-linear terms leads only to slow oscillatory dynamics of these fields. It is only due to the nonlinear dynamics of particles (see (12)) the amplitude of the fields can also increase.

3. GENERAL MODEL OF THREE-WAVE INTERACTION IN THE PRESENCE OF NON-RECIPROCAL COUPLING BETWEEN HIGH-FREQUENCY WAVES

Let's see what the linear dependence of the matrix coupling elements on the wave vectors of interacting high-frequency waves leads to. In the general case, the system of equations that describes such a three-wave interaction will look like:

$$\dot{a}_1 = k_1 V_1 a_2 a_3 ; \quad \dot{a}_2 = k_2 V_2 a_1 a_3^* ; \quad \dot{a}_3 = V_3 a_1 a_2^* . \quad (15)$$

Multiply the left and right parts of each equation in system (15) by. We multiply each equation of the complex conjugate system by. We add the resulting equations. The left side of the resulting equation will be the derivative of the total energy of the interacting waves. Introducing the notation adopted in [10], the resulting equation will take the form

$$\frac{d}{d\tau}\sum_{j=1}^{3}\omega_j N_j = \begin{bmatrix}\omega_2\left(k_1V_1 + k_2V_2\right) + \\ +\omega_3\left(k_1V_1 + V_3\right)\end{bmatrix} = 0 . \quad (16)$$

Here $N_j = \left|a_j\right|^2$ – number of quanta in j-wave.

Multiplying each equation of system (15) by $\left(\vec{k}_j \cdot a_j^*\right)$, and the complex conjugate system by $\left(\vec{k}_j \cdot a_j\right)$, we obtain a relation expressing the momentum conservation law:

$$\frac{d}{d\tau}\sum \vec{k}_j N_j = \begin{bmatrix}\vec{k}_2\left(k_1V_1 + k_2V_2\right) + \\ +\vec{k}_3\left(k_1V_1 + V_3\right)\end{bmatrix} = 0 . \quad (17)$$

In the one-dimensional case, from equations (16) and (17) we obtain the following relations between the matrix elements of system (15):

$$V_3 = k_1V_1 ; \; k_1V_1 = -k_2V_2 . \quad (18)$$

Let us substitute (18) into the system of equations (15).

From the resulting system of equations (taking into account the complex conjugate system), we can obtain the following integrals:

$$N_1 - N_2 = const ; N_1 + N_3 = const . \quad (19)$$

Integrals (19) are obtained for the case when interacting waves propagate in opposite directions (see Fig. 1). Integrals (19) indicate that high-frequency waves can simultaneously increase their amplitude (the first integral of system (19)). The second integral shows that the energy of high-frequency waves is drawn from the energy of low-frequency waves. If high-frequency waves propagate in one direction (see Fig. 2), then their energy does not change ($N_1 + N_2 = const$).

CONCLUSIONS

Thus, in a magnetoactive plasma there is a range of parameters in which conditions can be created for converting the energy of low-frequency oscillations into the energy of high-frequency oscillations. Let us note that after works [6-9] it might seem that in all nonreciprocal media such conditions can be created.

The results of the work show that finding such conditions is a rather difficult task, and it is not always obvious that such conditions can be found. It should also be noted that the reason for the non-reciprocal coupling between high-frequency waves is the particle dynamics. More specifically, these are nonlinear terms due to the magnetic Lorentz force, as well as nonlinear terms associated with hydrodynamics. It is important to note that in the absence of an external magnetic field, these two nonlinear terms annihilate each other.

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ПЕРЕТВОРЕННЯ ЕНЕРГІЇ НИЗЬКОЧАСТОТНИХ КОЛИВАНЬ НА ЕНЕРГІЮ ВИСОКОЧАСТОТНИХ КОЛИВАНЬ У МАГНІТОАКТИВНІЙ ПЛАЗМІ

В.О. Буц

Показано, що в плазмі, яка знаходиться у зовнішньому магнітному полі (магнітоактивна плазма), є можливість перетворювати енергію низькочастотних коливань в енергію високочастотних коливань. Таке перетворення можливе завдяки тому, що в такій плазмі можна створити умови для невзаємного зв'язку між високочастотними хвилями.