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Gradient Tracking for Coalitional Aggregation of Wind Power

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Abstract— **In this work we study coalition formation for a set of independent wind power producers. The wind power producers bid a contract in a day-ahead market, and they wish to determine the optimal contract that maximizes their expected profit. To cope with the volatility of the wind, the producers can form coalitions and aggregate their power production. We consider a communication topology and we assume that each wind power producer gets information about the wind powers realisation in the network through the contracts bidden by its neighbours. To determine the optimal contract for each coalition, we use a data learning approach based on gradient tracking. We prove that, for each coalition, the producers converge to the optimal contract for such a coalition. From the optimal contract we obtain the profit of each coalition which represents the coalitions' values of the resulting coalitional game. Then, we design a stabilizing allocation mechanism based on the Shapley value.**

I. INTRODUCTION

The energy generated by the wind can be very uncertain due to its volatility. In this work, we study the coalition formation for a set of independent wind power producers. We show that aggregate production may reduce the risk deriving from the volatility of the wind. We assume that the producers that belong to a coalition aggregate their wind power production and they determine the optimal contract that maximizes the expected profit of such a coalition. Coalitional games for the aggregation of wind power have been already studied by Baeyens et al. [1], [2], [3]. The main difference with respect to these works is that we obtain the optimal contract by taking into account a communication topology between the producers. We assume that each producer gets information about the contracts bidden by its neighbours. We then apply a distributed consensus mechanism based on gradient tracking, and we prove that the producers converge to the optimal contract in a distributed fashion.

In the literature different versions of the gradient tracking algorithm have been studied. In [4], [5], [6] the authors apply the idea of Newton-Raphson direction to track the gradient. In [7] directed networks with lossy communication are analysed. In [8], [9] strongly convex cost functions are studied. In this paper we prove that our profit function is concave and that it has a Lipschitz continuous gradient. Furthermore, we consider an undirected graph and we adapt the gradient tracking algorithm by weighting the neighbours of each producer according to the direction of their gradients.

We analyse the aggregation of the wind power generated by independent producers from the perspective of transferable utilities coalitional games [10]. The value of the coalition is given by the maximum average of the expected profits of the producers in the coalition with respect the contract that they offer jointly. We apply the Shapley value as an allocation mechanism to distribute the profit among the members of the coalition.

The paper is organized as follows. In Section II we explain the Coalitional Game. In Section III we introduce the problem and model. In Section IV we present the gradient tracking algorithm. In Section V we provide the main results. In Section VI we illustrate our results by a numerical example. Finally, in Section VII we provide conclusions and future directions.

II. COALITIONAL GAME

Coalitional games with transferable utilities are represented by the couple $\langle W,v\rangle$, where $W=\{1,2,\ldots,N\}$ is the set of players, which in this case corresponds to the set of wind power producers. A coalition is any subset $S\subseteq W$, and the cardinality of a coalition is denoted by $|S|$. The set W is also called the grand coalition. Let us denote by 2^W the set of all possible coalitions of W. On the other hand, $v: 2^W \longrightarrow \mathbb{R}$ is the *value function* that assigns a real value to each coalition $S\subseteq W$. The computation of the value of the coalition, denoted by $v(S)$, is explained in the following section. The main challenge in coalitional games is to determine the way in which the value of the coalition is allocated among the members of the coalition in a fair way. Let us denote by $x\in\mathbb{R}^N$ the vector that determines the profit allocation, where each entrance x_i is the amount assigned to player $i\in S$. We assume that the allocation vector is determined by an external entity. Let us recall the following definitions which will be used in the analysis of the game.

Definition 1: The *Core of the game* $\langle W,v\rangle$ is the set of allocations that satisfy *efficiency*, *individual rationality* and *stability with respect to subcoalitions*. It can be formally defined as follows:

$$\mathscr{C}(v)=\left\{x\in\mathbb{R}|\sum_{i\in W}x_i=v(W),\quad \sum_{i\in S}x_i\geq v(S),\quad \forall S\subseteq W\right\}. \tag{1}$$

Definition 2: A coalitional game $\langle W,v\rangle$ is *supperadditive* if for any pair of disjoint coalitions S, T the following is satisfied: $v(S)+v(T)\leq v(S\cup T),\quad \forall S,T\subset W,\quad S\cap T=\emptyset$.

Definition 3: A map $\alpha: 2^W\longrightarrow[0,1]$ is *balanced* if for all $i\in W$, we have $\sum_{S\in 2^W}\alpha(S)\mathbf{1}_{i\in S}=1$.

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Definition 4: A game $\langle W, v\rangle$ is a *balanced game* if for any balanced map α, it holds $\sum_{S\in 2^W} \alpha(S)v(S) \leq v(W)$.

The following results highlight some properties on the coalitional game $\langle W, v\rangle$. These results are borrowed and adapted from [2].

Lemma 1: The coalitional game $\langle W, v\rangle$ is superadditive.

Proof: The proof follows from Lemma 22 in [2]. ■

A consequence of the superadditivity of the game is that the producers increase their expected profit by forming a coalition and jointly offer a contract. The larger the coalition the higher the expected profit. However, this property is not sufficient to guarantee the existence of a nonempty core. The following result states that the game is balanced. Based on the Bondareva-Shapley theorem, it implies that the core of the game is not empty and that there exists a stable payoff allocation for the grand coalition.

Lemma 2: Let $\alpha : 2^W \longrightarrow [0,1]$ be an arbitrary balanced map. The coalitional game $\langle W, v\rangle$ is balanced.

Proof: The proof follows from Lemma 24 in [2]. ■

To allocate the expected profit we apply the well known allocation mechanism based on the *Shapley value*. The allocation to player $i \in W$, is then computed as follows:

$$x_i = \sum_{S\subset W/\{i\}} \frac{|S|!(N-|S|-1)!}{N!}[v(S\cup\{i\}) - v(S)]. \quad (2)$$

Note that the Shapley value involves all the permutations of the N players. It can be computationally very expensive as we increase the number of players. There exist different algorithms to approximate the Shapley value and make its computation more efficient [11], [12]. However, the study of these algorithms is outside of the scope of this research.

III. PROBLEM STATEMENT AND MODEL

Let us consider a network of N wind power producers. The topology of the network is represented by an undirected and weighted graph $G(W,E)$, where $W = \{1,2,\ldots,N\}$ is the set of nodes which correspond to the wind power producers. The set $E \subseteq W \times W$ denotes the edges connecting the nodes. For each $(i, j) \in E$ we assign a non-negative weight a_{ij}, $i, j \in W$. The weights a_{ij} satisfy the following assumption.

Assumption 1: The weights a_{ij} satisfy $\sum_{j\in W} a_{ij} = 1$ for all $i \in W$, and $\sum_{i\in W} a_{ij} = 1$ for all $j \in W$. Furthermore, $a_{ij} = 0$ if $(i, j) \notin E$, and $a_{ii} > 0$ for all $i \in W$.

To take into account the volatility of the wind, the power generated by each producer $i \in W$ at time $t \in \mathbb{R}$ is modeled by a scalar valued stochastic process $w_i(t) \in [0, \mathscr{W}_i]$, where $\mathscr{W}_i$ denotes the capacity of the wind-farm of producer i. To overcome the uncertainty of the wind, we assume that the wind power producers can form coalitions and aggregate their power outputs. Let us denote by $S \subseteq W$ a generic coalition of wind power producers. The aggregated power of coalition S at time t is given by $w_S(t) = \sum_{i\in S} w_i(t)$, where $w_S(t) \in [0, \mathscr{W}_S]$ is also a scalar valued stochastic process, and $\mathscr{W}_S = \sum_{i\in S} \mathscr{W}_i$. We assume that at the beginning of every time interval τ, coalition S bids a contract denoted by $C^\tau(S)$. We then have a two-settlement market system, consisting of the ex-ante forward market and the ex-post mechanism to penalize any contract deviations from the real wind power produced by the coalition. Let $\Pi(C^\tau(S), w_S, q, \lambda)$ be the profit of a coalition $S \subseteq W$, for an offered contract $C^\tau(S)$ in the time interval τ. Let us denote by t_0^τ and t_f^τ the first and last time instants of interval τ. The profit is computed as follows:

$$\Pi(C^\tau(S), w_S, q, \lambda) = \int_{t_0^\tau}^{t_f^\tau} pC^\tau(S) - q[C^\tau(S) - w_S(r)]^+ \\ -\lambda[w_S(r) - C^\tau(S)]^+ dr, \quad (3)$$

where $p \in \mathbb{R}_+$ is the settlement price paid in the day-ahead forward market, and $(q, \lambda) \in \mathbb{R}_+^2$ are the imbalance prices. If there is a negative deviation from the contract offered ex-ante, namely $C^\tau(S) - w_S(r) < 0$, the imbalance price λ has to be paid ex-post. If there is a positive deviation from the contract offered ex-ante, namely $C^\tau(S) - w_S(r) > 0$, the imbalance price q has to be paid ex-post. The imbalance prices (q, λ) are also affected by volatility, and we assume that they are statistically independent from the wind power $w_S(t)$. We model these prices as random variables with expected value denoted by μ_q and μ_λ, where $\mu_\lambda \leq p \leq \mu_q$. In addition, $x^+ = max\{x, 0\}$, $x \in \mathbb{R}$.

Let us denote by $J(C^\tau(S)) = \mathbb{E}[\Pi(C^\tau(S), w_S, q, \lambda)]$ the expected profit of coalition S, for an offered contract $C^\tau(S)$ in the time interval τ. The wind power producers aim to obtain the optimal contract such that their expected profit in every time interval τ is maximized. We assume that each wind power producer $i \in W$ has different information about the wind power generated in the past by coalition $S \subseteq W$, and as a result each producer has a different probability distribution associated with it. Let us denote by $f_{i,S}(\mathrm{w}, t)$ the probability distribution function of producer i for the wind power generated by coalition S at time t. The expected profit from the perspective of producer i for coalition S for a contract $C^\tau(S)$, denoted by $J_{i,S}(C^\tau(S))$, is given by:

$$J_{i,S}(C^\tau(S)) = \mathbb{E}_{i,S}[\Pi(C^\tau(S), w_S, q, \lambda)] \\ = \int_{t_0^\tau}^{t_f^\tau} \Big[pC^\tau(S) - \mu_q \int_0^\infty [C^\tau(S) - \mathrm{w}]^+ f_{i,S}(\mathrm{w}, t) d\mathrm{w} \\ -\mu_\lambda \int_0^\infty [\mathrm{w} - C^\tau(S)]^+ f_{i,S}(\mathrm{w}, t) d\mathrm{w} \Big] dt. \quad (4)$$

The optimization problem to solve is then as follows:

$$\max_{C^\tau(S)} \frac{1}{|S|} \sum_{i\in S} J_{i,S}(C^\tau(S)), \text{ s.t. } C^\tau(S) \in \mathbb{R}^+, \quad (5)$$

where each expected profit function $J_{i,S} : \mathbb{R}_+ \longrightarrow \mathbb{R}$ is known only by producer i, for all $i \in W$. The solution of (5) gives the value of the coalition $v(S)$. To solve problem (5) for each coalition $S \subseteq W$ we apply a *gradient tracking method.*

IV. GRADIENT TRACKING

We assume that each wind power producer only gets information about the contracts bidden by its neighbours. As it is explained later, this assumption is necessary to ensure convergence among the producers. Based on this information each producer $i \in W$ estimates a contract for coalition S in each time interval τ, denoted by $C_i^\tau(S)$. The gradient tracking algorithm consists in the implementation of a distributed consensus-based mechanism to track the gradient of the expected profit function of a coalition S, such that for each time interval τ in which the contract is bidden the members of the coalition converge to the optimal contract $C^{\tau *}(S)$. To understand this mechanism, first let us introduce the gradient method. The idea behind this method is to find the optimal contract $C^{\tau *}(S)$ that maximizes the expected profit $J(C^{\tau}(S))$ for coalition S in the time interval τ by learning from the contracts of the neighbours of each producer, and following the direction of the gradient of the expected profit function around the contract $C^{\tau}(S)$. We assume that the wind power producers in the network share information with their neighbours about their contracts at discrete time intervals. Every time the producers share this information a new iteration of the algorithm is performed, which is denoted by k. The value of k is reinitialized at zero at the beginning of every time interval τ, namely at t_0^{τ}. At the end of each time interval τ the producers aim to converge to the optimal contract of the coalition, which is given by:

$$C^{\tau *}(S) = arg \max_{C^{\tau}(S)\in\mathbb{R}^{+}} \frac{1}{|S|}\sum_{i\in S} J_{i,S}(C^{\tau}(S)). \tag{6}$$

In the centralized gradient method, a new value of the contract for coalition $S \subseteq W$ is obtained as follows:

$$C^{k+1}(S) = C^{k}(S) + \gamma \sum_{h\in S} \nabla J_{h,S}(C^{k}(S)), \tag{7}$$

where $\nabla J_{h,S}(C^{k}(S))$ is the gradient of the profit function of producer h at $C^{k}(S)$. Let us denote by W_i the set of neighbours of producer i. In the distributed consensus mechanism, at each iteration k producer $i \in W$ obtains its own contract for coalition S and takes the weighted average of the contracts obtained by its neighbours for this coalition, denoted by $C_j^k(S)$ for all $j \in W_i$. Hence, (7) can be rewritten as follows:

$$C_i^{k+1}(S) = \sum_{j\in W_i} a_{ij} C_j^k(S) + \gamma \sum_{h\in S} \nabla J_{h,S}(C_h^k(S)). \tag{8}$$

Note that in (8) the term $\sum_{j\in W_i} a_{ij} C_j^k(S)$ enforces convergence among the wind powers producers in $S \subseteq W$. On the other hand, the gradient term $\sum_{h\in S} \nabla J_{h,S}(C_h^k(S))$ represents global information for the coalition S. However, each producer $i \in S$ only has access to local information from its neighbours. In the gradient tracking method the gradient term is tracked by a local ascent direction, which is updated at every iteration k through a *dynamic average consensus iteration*. Let us denote by $y_i^k(S)$ the local direction of the wind power producer $i \in W$ at iteration k. Thus, the value of the contract for producer i at the next iteration is updated as follows:

$$C_i^{k+1}(S) = \sum_{j\in W_i} a_{ij} C_j^k(S) + \gamma y_i^k(S). \tag{9}$$

In addition, the dynamic average consensus to update the local direction and track the gradient is computed as follows:

$$y_i^{k+1}(S) = \sum_{j\in W_i} a_{ij} y_j^k(S) + \left(\nabla J_{i,S}(C_i^{k+1}(S)) - \nabla J_{i,S}(C_i^k(S))\right). \tag{10}$$

At every iteration k, each producer has its own belief about the direction of the expected cost function for coalition S, as well as the information about the direction of the expected costs obtained by its neighbours. Let us separate the set of neighbours of producer i in two disjoint subsets. Subset $W_i(k)^{+}$ represents the set of neighbours of producer i that have a local direction $y_j^k(S)$, $j \in W_i$, greater than the local direction of producer i at iteration k. Similarly, the subset $W_i(k)^{-}$ represents the set of neighbours of producer i that have a local direction $y_j^k(S)$, $j \in W_i$, lower than or equal to the local direction of producer i at iteration k. We assume that, to reach consensus, at each iteration k each wind power producer i assigns a different weight θ_i^k to its neighbours depending on which subset they belong to. Let $\theta_i^k = \frac{|W_i^{-}(k)|}{|W_i|}$ be the fraction of neighbours in the subset $W_i^{-}(k)$. Equation (10) can be rewritten as follows:

$$\begin{aligned} y_i^{k+1}(S) =& \theta_i^k \sum_{j\in W_i^{-}(k)} a_{ij} y_j^k(S) + (1-\theta_i^k) \sum_{j\in W_i^{+}(k)} a_{ij} y_j^k(S) \\ &+ \left(\nabla J_{i,S}(C_i^{k+1}(S)) - \nabla J_{i,S}(C_i^k(S))\right). \end{aligned} \tag{11}$$

The gradient tracking method applied in this research from the perspective of a generic producer i for a coalition $S \subseteq W$ is explicitly explained in Algorithm 1.

Algorithm 1 Gradient tracking

Input: Number of players N, topology of the network, prices p, expected value of imbalance prices (μ_q, μ_λ), step size γ, probability distribution function of the wind
Output: Contract $C_i^{\tau}(S)$, and gradient tracker $y_i^{\tau}(S)$, $i \in S$
Initialization: $C_i^0(S)$ and $y_i^0(S) = \nabla J_{i,S}(C_i^0(S))$

```
for every k = 0, 1, ... do
    Gather C_j^k(S) from neighbours j ∈ W_i
    Gather y_j^k(S) from neighbours j ∈ W_i
    Obtain the sets W_i^-(k) and W_i^+(k)
    Compute θ_i^k
    Update C_i^{k+1}(S) according to (9)
    Update y_i^{k+1}(S) according to (11)
end for
```

It is worth mentioning that in order to apply Algorithm 1 the expected profit function $J_{i,S}(\cdot)$ has to be concave.

V. MAIN RESULTS

In this section we analyse the properties of the expected profit function $J_{i,S}(\cdot)$, and we show the convergence of Algorithm 1. Since the wind power producers share information about their contracts at discrete time, let us discretize the profit function (3) with respect to time as follows:

$$\begin{aligned} \Pi(C_i^k(S), w_S, q, \lambda) = \sum_{m=t_0^{\tau}}^{m=t_f^{\tau}} \Big[& pC_i^k(S) - q[C_i^k(S) - w_S(m)]^{+} \\ & - \lambda [w_S(m) - C_i^k(S)]^{+} \Big]. \end{aligned} \tag{12}$$

Let us denote by $f_{i,S}(\mathrm{w},m)$ the probability distribution function estimated by producer i for coalition S at time m. The corresponding expected profit is then given by:

$$J_{i,S}(C_i^k(S)) = \mathbb{E}_{i,S}[\Pi(C_i^k(S), w_S, q, \lambda)] = \sum_{m=t_0^\tau}^{m=t_f^\tau} \Big[pC_i^k(S) \\ -\mu_q \left(\int_0^{C_i^k(S)} \Big(C_i^k(S) f_{i,S}(\mathrm{w},m) - \mathrm{w} f_{i,S}(\mathrm{w},m) \Big) d\mathrm{w} \right) \\ -\mu_\lambda \left(\int_{C_i^k(S)}^{\infty} \Big(\mathrm{w} f_{i,S}(\mathrm{w},m) - C_i^k(S) f_{i,S}(\mathrm{w},m) \Big) d\mathrm{w} \right) \Big]. \quad (13)$$

Lemma 3: The expected profit function of producer i, $J_{i,S}(C_i^k(S)) = \mathbb{E}_{i,S}[\Pi(C_i^k(S), w_S, q, \lambda)]$ is concave.

Proof: The first derivative of $J_{i,S}(C_i^k(S))$ with respect to $C_i^k(S)$ is:

$$\frac{d}{dC_i^k(S)} J_{i,S}(C_i^k(S)) = \sum_{m=t_0^\tau}^{m=t_f^\tau} \Bigg[p - \mu_q \int_0^{C_i^k(S)} f_{i,S}(\mathrm{w},m) d\mathrm{w} \\ +\mu_\lambda \left(1 - \int_0^{C_i^k(S)} f_{i,S}(\mathrm{w},m) d\mathrm{w} \right) \Bigg]. \quad (14)$$

By computing the second derivative we obtain:

$$\frac{d^2}{dC_i^k(S)^2} J_{i,S}(C_i^k(S)) = \sum_{m=t_0^\tau}^{m=t_f^\tau} -(\mu_q + \mu_\lambda) f_{i,S}(C_i^k(S), m) \leq 0. \quad (15)$$

Hence the expected profit function $J_{i,S}(C_i^k(S))$ has a maximum and the function is concave. ■

Note that the expected profit is a scalar-value function that takes values in $\mathbb{R}_+$. Then, the gradient corresponds to the derivative of the function, namely:

$$\nabla J_{i,s}(C_i^k(S)) = \frac{d}{dC_i^k(S)} J_{i,S}(C_i^k(S)). \quad (16)$$

Lemma 4: The expected profit function of producer i, $J_{i,S}(C_i^k(S))$ has a Lipschitz continuous gradient with constant $L > 0$, such that:

$$|\nabla J_{i,S}(C_i^k(S)) - \nabla J_{i,S}(C_i^\ell(S))| \leq L|C_i^k(S) - C_i^\ell(S)|, \quad (17) \\ \forall C_i^k(S), C_i^\ell(S) \in \mathbb{R}_+,$$

where the gradient $\nabla J_{i,S}(\cdot)$ is defined as in (16).

Proof: From (13) we have that for any coalition $S \subseteq W$ the expected profit function $J_{i,S}(\cdot)$ is continuous in the interval $[0, \mathscr{W}_S]$. Furthermore, as we can see in the proof of Lemma 3 the first and second derivative of (13) exist, they are continuous and well defined in the interval $[0, \mathscr{W}_S]$. Then, the derivative of the expected profit function is differentiable. Let $C_i^k(S), C_i^r(S), C_i^\ell(S) \in \mathbb{R}_+$, such that $C_i^k(S) \leq C_i^r(S) \leq C_i^\ell(S)$. Hence, by applying the mean value theorem we have that $\frac{\nabla J_{i,S}(C_i^k(S)) - \nabla J_{i,S}(C_i^\ell(S))}{C_i^k(S) - C_i^\ell(S)} = \nabla^2 J_{i,S}(C_i^r(S)) \implies \left| \frac{\nabla J_{i,S}(C_i^k(S)) - \nabla J_{i,S}(C_i^\ell(S))}{C_i^k(S) - C_i^\ell(S)} \right| = \left| \nabla^2 J_{i,S}(C_i^r(S)) \right|$. Note that from the second derivative of the expected profit (15) we obtain $\left| \nabla^2 J_{i,S}(C_i^r(S)) \right| = \left| \sum_{m=t_0^\tau}^{m=t_f^\tau} [-(\mu_q + \mu_\lambda) f_{i,S}(C_i^r(S), m)] \right| = \sum_{m=t_0^\tau}^{m=t_f^\tau} (\mu_q + \mu_\lambda) f_{i,S}(C_i^r(S), m) \leq \Delta\tau(\mu_q + \mu_\lambda)$, where $\Delta\tau = t_f^\tau - t_f^0$. Then we have that $\left| \frac{\nabla J_{i,S}(C_i^k(S)) - \nabla J_{i,S}(C_i^\ell(S))}{C_i^k(S) - C_i^\ell(S)} \right| \leq \Delta\tau(\mu_q + \mu_\lambda) \implies \left| \nabla J_{i,S}(C_i^k(S)) - \nabla J_{i,S}(C_i^\ell(S)) \right| \leq L \left| C_i^k(S) - C_i^\ell(S) \right|$, with constant $L = \Delta\tau(\mu_q + \mu_\lambda) > 0$. Hence, the gradient $\nabla J_{i,S}(C_i^k(S))$ is Lipschitz continuous. ■

As a consequence of Lemma 3 and Lemma 4, since $J_{i,S}(C_i^k(S))$ is concave we can conclude that the average of the expected profit $\frac{1}{|S|} \sum_{i \in S} J_{i,S}(C_i^k(S))$ is also concave and that it has a unique optimal value.

Before showing the convergence of Algorithm 1 to the optimal contract $C^{\tau *}(S)$ let us analyse the average of the local solutions $C_i^k(S)$, and the average of the trackers $y_i^k(k)$, for all $i \in W$. Let us denote by $\bar{C}^k(S)$ the average of the local solutions at iteration k and by $\bar{y}^k(S)$ the average tracker:

$$\bar{C}^k(S) := \frac{1}{N} \sum_{i \in W} C_i^k(S), \quad (18)$$

$$\bar{y}^k(S) := \frac{1}{N} \sum_{i \in W} y_i^k(S). \quad (19)$$

From (9) and (18), and since from Assumption 1 $\sum_{i \in W} a_{ij} = 1$, the average of the contract for coalition S at iteration $k+1$ is given by:

$$\bar{C}^{k+1}(S) = \frac{1}{N} \sum_{i \in W} C_i^{k+1}(S) = \frac{1}{N} \sum_{i \in W} \left[\sum_{j \in W_i} a_{ij} C_j^k(S) + \gamma y_i^k(S) \right] \\ = \frac{1}{N} \sum_{j \in W_i} \left(C_j^k(S) \sum_{i \in W} a_{ij} \right) + \frac{1}{N} \gamma \sum_{i \in W} y_i^k(S) = \bar{C}^k(S) + \gamma \bar{y}^k(S). \quad (20)$$

Let us now analyse (19) for the special case $\theta_i^k = 1$. Namely when all the producers have a local direction lower than or equal to producer $i \in W$, that is when $W_i^- = W_i$. This analysis for the special case is used later on in the proof of Theorem 1. From (11), when $\theta_i^k = 1$ we have:

$$\bar{y}^{k+1}(S) = \frac{1}{N} \sum_{i \in W} y_i^{k+1}(S) \\ = \frac{1}{N} \sum_{i \in W} \left[\sum_{j \in W} a_{ij} y_j^k(S) + \Big(\nabla J_{i,S}(C_i^{k+1}(S)) - \nabla J_{i,S}(C_i^k(S)) \Big) \right] \\ = \bar{y}^k(S) + \frac{1}{N} \sum_{i \in W} \Big(\nabla J_{i,S}(C_i^{k+1}(S)) - \nabla J_{i,S}(C_i^k(S)) \Big). \quad (21)$$

From equation (21), and knowing from Algorithm 1 that the initialization of the tracking is $y_i^0(S) = \nabla J_{i,S}(C_i^0(S))$, for the special case $\theta_i^k = 1$ we obtain that:

$$\bar{y}^{k+1}(S) - \frac{1}{N} \sum_{i \in W} \nabla J_{i,S}(C_i^{k+1}(S)) = \bar{y}^k(S) - \frac{1}{N} \sum_{i \in W} \nabla J_{i,S}(C_i^k(S)) \\ = \bar{y}^{k-1}(S) + \frac{1}{N} \sum_{i \in W} \Big(\nabla J_{i,S}(C_i^k(S)) - \nabla J_{i,S}(C_i^{k-1}(S)) \Big) \\ - \frac{1}{N} \sum_{i \in W} \nabla J_{i,S}(C_i^k(S)) = \bar{y}^{k-1}(S) - \nabla J_{i,S}(C_i^{k-1}(S)) = \dots \\ = \bar{y}^0(S) - \frac{1}{N} \sum_{i \in W} \nabla J_{i,S}(C_i^0(S)) = 0. \quad (22)$$

We now have all the necessary information to prove that the result from Algorithm 1 converges to the optimal contract, as it is stated in the following theorem.

Theorem 1: The local solutions of the individual contracts for a coalition S, $C_i^k(S)$ for all $i \in W$, obtained from Algorithm 1 converge asymptotically to the optimal contract $C^{\tau *}(S)$, namely $\lim_{k \longrightarrow \infty} |C_i^k(S) - C^{\tau *}(S)| = 0, \quad \forall i \in S$.

Proof: We know that $J_{i,S}(C_i^k(S))$ is a concave function. Therefore, its gradient is monotonically decreasing with respect to $C_i^k(S)$. Let us analyse the gradient of the expected profit $\nabla J_{i,S}(C_i^k(S))$. From (15) we have:

$$\left|\nabla J_{i,S}(C_i^k(S))\right| = \left|\sum_{m=t_0^\tau}^{m=t_f^\tau} \left[p - \mu_q \int_0^{C_i^k(S)} f_{i,S}(\mathrm{w},m)d\mathrm{w} \right.\right.$$
$$\left.\left. + \mu_\lambda \left(1 - \int_0^{C_i^k(S)} f_{i,S}(\mathrm{w},m)d\mathrm{w}\right)\right]\right| \leq \sum_{m=t_0^\tau}^{m=t_f^\tau} (p + \mu_\lambda).$$

In Algorithm 1 we track the gradient by following its local direction. For a small enough step size γ, the gradient $\nabla J_{i,S}(C_i^k(S))$ is monotonic with respect to the iterations. Hence, the average of the gradient $\frac{1}{N}\sum_{i\in W} \nabla J_{i,S}(C_i^k(S))$ is also monotonic and bounded, and we can ensure that it converges. Furthermore, since the expected cost $J_{i,S}(C_i^k(S))$ is concave, for a small enough step size γ, the absolute value of the gradient $|\nabla J_{i,S}(C_i^k(S))|$ decreases with the iterations. This can be captured by the following expression:

$$\frac{1}{N}\sum_{i\in W} \lim_{k \longrightarrow \infty} \left|\nabla J_{i,S}(C_i^{k+1}(S))\right| = \frac{1}{N}\sum_{i\in W} \lim_{k \longrightarrow \infty} \left|\nabla J_{i,S}(C_i^{k}(S))\right| = 0. \tag{23}$$

From the previous equality it is clear that in the limit the expected profit functions of all producers have the same direction of the gradient. Therefore, $W_i^- = W_i$, and $\lim_{k \longrightarrow \infty} \theta_i^k = 1$. From (22) we know that when $\theta_i^k = 1$, the average of the gradient tracking $\bar{y}_S^k = \frac{1}{N}\sum_{i\in W} \nabla J_{i,S}(C_i^k(S))$. In addition, from (23) we have that $\lim_{k \longrightarrow \infty} \frac{1}{N}\sum_{i\in W} |\nabla J_{i,S}(C_i^k(S))| = 0$. Hence, in the limit the average of the optimal contract given by (20) converges. Namely, $\lim_{k \longrightarrow \infty} \bar{C}(S)^{k+1} = \lim_{k \longrightarrow \infty} \bar{C}^k(S) + \gamma \bar{y}_i^k(S) = \lim_{k \longrightarrow \infty} \bar{C}^k(S) + \gamma \frac{1}{N}\sum_{i\in W} \nabla J_{i,S}(C_i^k(S)) = \lim_{k \longrightarrow \infty} \bar{C}(S)^k$.

On the other hand, from Lemma 3 and Lemma 4 we know that the optimal contract $C^{\tau *}(S)$ is unique. In addition, at the optimal value the gradient of the average of the expected profit function (5) is equal to zero, then we have $\frac{1}{N}\sum_{i\in W} \nabla J_{i,S}(C^{\tau *}(S)) = \lim_{k \longrightarrow \infty} \frac{1}{N}\sum_{i\in W} \nabla J_{i,S}(C_i^k(S)) \Longrightarrow \lim_{k \longrightarrow \infty} \bar{C}^k(S) = C^{\tau *}(S)$.

By definition of $\bar{C}^k(S)$, and since the gradient is equal to zero only at the unique optimal contract $C^{\tau *}(S)$, we obtain that $\lim_{k \longrightarrow \infty} \bar{C}^k(S) = \lim_{k \longrightarrow \infty} \frac{1}{N}\sum_{i\in W} C_i^k(S) = C^{\tau *}(S) \implies \lim_{k \longrightarrow \infty} \left|\frac{1}{N}\sum_{i\in W} C_i^k(S) - C^{\tau *}(S)\right| = 0$. We know that for all $i \in W$ the contracts $C_i^k(S)$ converge to the same value in the limit. Then, we have that $\frac{1}{N}\sum_{i\in W} \lim_{k \longrightarrow \infty} \left|C_i^k(S) - C^{\tau *}(S)\right| = 0 \Longrightarrow \lim_{k \longrightarrow \infty} \left|C_i^k(S) - C^{\tau *}(S)\right| = 0$. Therefore, $\lim_{k \longrightarrow \infty} C_i^k(S) = C^{\tau *}(S)$. ■

VI. NUMERICAL ANALYSIS

In this section we apply the model and results obtained in the previous sections to an example of five wind power producers, namely $W = \{1,2,3,4,5\}$. We generate randomly the stochastic weighted network with five nodes. The communication topology of this network is depicted in Figure 1.

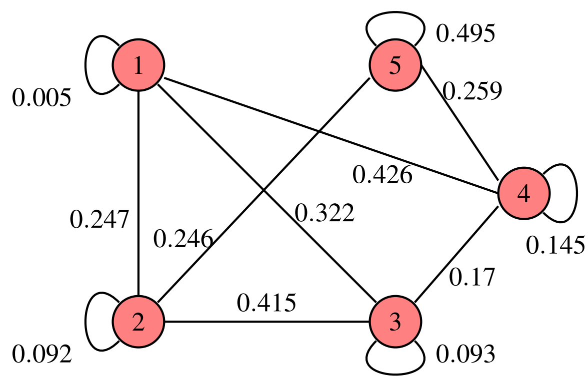

Fig. 1. Communication topology of five wind power producers.

We consider that the settlement price is $p = 7$€, and the expected value of the imbalanced prices μ_q and μ_λ are 10 and 5€, respectively. We assume that after sharing the wind information, each producer approximates the wind power model by a normal distribution. The parameters estimated by the producers for each coalition, based on the size of the coalition, are determined according to Table I. The values of the mean lie inside the interval $[\underline{\mu}, \overline{\mu}]$. Similarly the values of the standard deviation lie inside the interval $[\underline{\sigma}, \overline{\sigma}]$.

TABLE I
WIND DISTRIBUTIONS FOR EACH COALITION $S \subseteq W$.

Coalition's size	$\underline{\mu}$	$\overline{\mu}$	$\underline{\sigma}$	$\overline{\sigma}$
1	5	15	1	3
2	10	30	1	3
3	20	40	1	3
4	30	50	1	3
5	40	65	1	3

After applying Algorithm 1 with a step size $\gamma = 0.1$, we observe that for each coalition, the producers converge to the optimal contract (Figure 2). Furthermore, in Figure 3 it is possible to see how the gradient tracking of the expected profit of each producer $i \in W$ converges to zero.

The allocations to the players are obtained by applying the Shapley value (2). In Figure 4 we can observe the benefit of aggregation. It depicts the allocation x_i to each individual wind power producer in the grand coalition. The blue bar represents the value of the individual player $v(\{i\})$ obtained based on its individual expected profit. The red bar represents the additional amount gained by the producer when it belongs to the grand coalition, namely $x_i - v(\{i\})$.

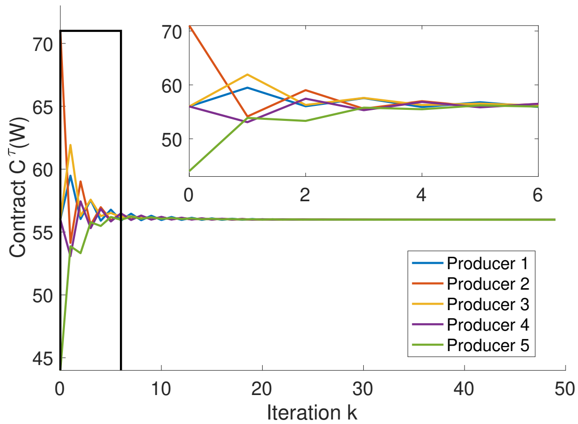

Fig. 2. Convergence to the optimal contract $C^{\tau*}(W)$

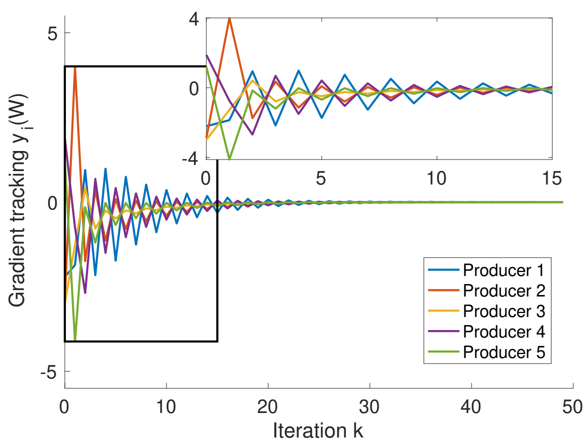

Fig. 3. Convergence of the gradient tracking $y_i(W)$ to zero

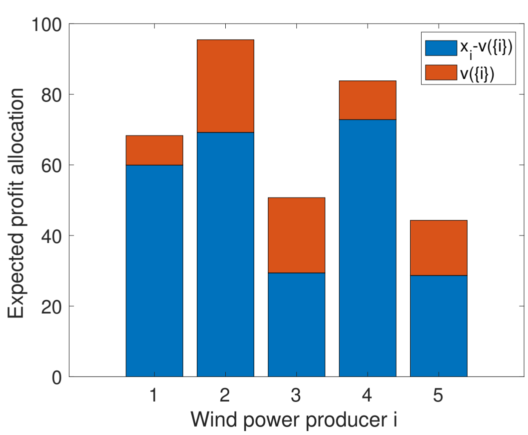

Fig. 4. Benefit of aggregation represented by the allocation x_i. The blue bar corresponds to the value of the individual player $v(\{i\})$, and the red bar the amount gained by the player in the coalition $x_i - v(\{i\})$.

VII. CONCLUSIONS

In this work we analyse the coalition formation of a set of independent wind power producers. To reduce the risk deriving from the volatility of the wind we assume that the producers bid a contract jointly in a day-ahead market, and that they wish to determine the optimal contract that maximizes their expected profit. To determine the optimal contract for each coalition we use a data learning approach that considers a distributed consensus-based mechanism, called gradient tracking. To develop this mechanism we take into account the communication topology of the producers and we assume that each wind power producer gets information about the contracts bidden by its neighbours. We prove that, for each coalition, the producers converge to the optimal contract for such a coalition. From the optimal contract we obtain the profit of each coalition, which represents the coalitions' values of the resulting coalitional game. To distribute the profit among the members of the coalition we make use of the Shapley value as an allocation mechanism. Future work involves the study of an approximation mechanism for the computation of the Shapley value, the analysis of a dynamic coalitional game as well as a dynamic allocation, and the application of a time varying step size for the implementation of the gradient tracking algorithm.

REFERENCES

[1] E. Baeyens, E. Bitar, P. P. Khargonekar, and K. Poolla, "Wind energy aggregation: A coalitional game approach", 50th IEEE CDC, pp. 3000–3007, 2011.

[2] E. Baeyens, E. Y. Bitar, P. P. Khargonekar, and K. Poolla, "Coalitional aggregation of wind power", IEEE Trans on Power Syst., vol. 28, no. 4, pp. 3774–3784, 2013.

[3] P. Chakraborty, E. Baeyens, P. Khargonekar, and K. Poolla, "A cooperative game for the realized profit of an aggregation of renewable energy producers", 55th 50th IEEE CDC, pp. 5805–5812, 2016.

[4] F. Zanella, D. Varagnolo, A. Cenedese, G. Pillonetto, and L. Schenato, "Newton-Raphson consensus for distributed convex optimization", IEEE CDC-ECC. pp. 5917–5922, 2011.

[5] F. Zanella, D. Varagnolo, A. Cenedese, G. Pillonetto, and L. Schenato, "Asynchronous Newton-Raphson Consensus for Distributed Convex Optimization", IFAC Workshop on Distributed Estimation and Control in Networked Systems, 2012.

[6] D. Varagnolo, F. Zanella, A. Cenedese, G. Pillonetto, and L. Schenato, "Newton-Raphson consensus for distributed convex optimization", IEEE Trans. Automat. Contr., vol.61, no.4, pp. 994–1009, 2016.

[7] R. Carli, G. Notarstefano, L. Schenato, and D. Varagnolo, "Analysis of Newton-Raphson Consensus for multi-agent convex optimization under asynchronous and lossy communications", IEEE CDC, pp. 418–424, 2015.

[8] J. Xu, S. Zhu, Y. C. Soh, and L. Xie, "Convergence of asynchronous distributed gradient methods over stochastic networks", IEEE Transactions on Automatic Control. vol. 63, no. 2, pp. 434–448, 2018.

[9] G. Notarstefano, I. Notarnicola and A. Camisa, "Distributed Optimization for Smart Cyber-Physical Networks", Found. Trends Syst. Control, 2019.

[10] J. Von Neumann and O. Morgenstern, "Theory of Games and Economic Behavior", Princeton Univ. Press, 210, pp. 459-466.

[11] R. Stanojevic, N. Laoutaris, and P. Rodriguez, "On economic heavy hitters: shapley value analysis of 95th-percentile pricing". In Proceedings of the 10th ACM SIGCOMM conference on Internet measurement, pp. 75–80, 2010.

[12] P. H. Le, T. Nguyen, and Tolga Bektas, "Efficient computation of the Shapley value for large-scale linear production games." Ann. Oper. Res., vol. 287, pp. 761-781, 2018.