RHV, 2023, No 23, 259-285 eISSN 0719-4242
DOI: https://doi.org/10.22370/rhv2023iss23pp259-285
Articles Section

A Framework for Inductive Reasoning in Model-Based Science

Un marco para el razonamiento inductivo en la ciencia basada en modelos

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Abstract

This paper argues that the linguistic approach to analyzing induction, according to which induction is a type of inference or argument composed of statements or propositions, is unsuitable to account for scientific reasoning. Consequently, a novel approach to induction in model-based science is suggested. First, in order to show their adherence to the linguistic treatment of induction, two strategies are reviewed: (i) Carnap and Reichenbach's attempts to justify induction and (ii) Norton's recent material theory of induction. Second, three reasons are provided to support the claim that the linguistic treatment of induction is insufficient in accounting for model-based reasoning in science. Finally, a framework focused on models—rather than statements or propositions—is suggested to address induction in science. William Whewell's theory of induction is briefly outlined as an example of a non-propositional treatment of induction that is consistent with model-based scientific practice.

Keywords: induction, inductive reasoning, scientific inference, material theory of induction, models, Whewell's induction.

Resumen

Este artículo argumenta que el enfoque lingüístico para analizar la inducción, según el cual la inducción es un tipo de inferencia o argumento compuesto de enunciados o proposiciones, no es adecuado para dar cuenta del razonamiento científico. En consecuencia, se sugiere un nuevo enfoque para la inducción en la ciencia basada en modelos. En primer lugar, con el fin de mostrar su adhesión al tratamiento lingüístico de la inducción, se revisan dos estrategias: (i) los intentos de Carnap y Reichenbach de justificar la inducción y (ii) la reciente teoría

Received: 14/12/2023. Final version: 28/12/2023
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material de la inducción de Norton. En segundo lugar, se proporcionan tres razones para apoyar la afirmación de que el tratamiento lingüístico de la inducción es insuficiente para dar cuenta del razonamiento basado en modelos en la ciencia. Por último, se sugiere un marco centrado en modelos, en lugar de enunciados o proposiciones, para abordar la inducción en la ciencia. La teoría de la inducción de William Whewell se esboza brevemente como un ejemplo de un tratamiento no proposicional de la inducción que es consistente con una práctica científica basada en modelos.

Palabras claves: inducción, razonamiento inductivo, inferencia científica, teoría de material de Norton, modelos, inducción de Whewell.

1. Introduction

Even though induction has been considered the reasoning that underlies methods of confirmation in science, it has been deemed an invalid argument or a non-truth-preserving inference. This is because, as is well known, inductive conclusions go beyond the premises that support them, and, therefore, they are uncertain or based only on their likelihood (see Barker, 1957; Salmon, 1967; Wright, 1965).

The debate on the lack of validity of inductive arguments had a significant impact on the philosophy of science in the twentieth century. One result was the division of philosophers into two broad groups: deductivists and justificationists. While the former rejected induction as the reasoning behind beliefs[1], the latter accepted it as a type of argument or inference that, although uncertain, is indispensable for comprehending the empirical basis of science and defended its application in scientific practice (Carnap, 1945; 1950ab; Reichenbach, 1957; 1949).

After the shift towards the model-based perspective in philosophy of science (Bailer-Jones, 1999), the concern about induction has been replaced by other subjects that were seen as more crucial from this perspective, such as the study of models in science as well as their role in scientific representation. Despite some exceptions[2], philosophers in the model-based perspective seem less interested in induction than their predecessors in the first half of the twentieth century. Nonetheless, scientists have been employing inductive reasoning to extend and enrich scientific knowledge in order to obtain better representations of the world.

Since induction has continued to play a fundamental role in scientific activity, it is legitimate to ask why it has yet to receive sufficient attention from philosophers interested in model-based science, specifically, from philosophers within semantic and pragmatic approaches.

[1] Popper and his followers.

[2] See Raisis (1999); Redhead (1980); and Beirlaen (2017).

A possible reason for the shortage of interest among contemporary philosophers on induction after the neo-positivist period in the twentieth could be the unresolved nature of what Williams called the 'tragic puzzle of induction' (Williams, 1948, p. 227), which refers to the problem of justifying the beliefs based on past experiences.

On the one hand, justificationists thought that inductive beliefs should be considered "valid" because they have been informed by events in the past. However, this response does not guarantee the regularity of the phenomena investigated but only supports our expectations about the phenomena in the future based on their past behavior. Thus, the justification of induction evolved from the search for a logical solution to the search for the criteria for establishing beliefs based on logically imperfect inductive inferences.

On the other hand, deductivists considered that despite empirical beliefs having no justification, they could be accepted provisionally as hypothesis or conjectures if they remained unharmed by attempts to falsify them.

Recently, a third group of philosophers has advocated for a material theory of induction, stating that beliefs supported by induction can only be justified locally, which means that their justification relies on inferences drawn from specific material facts.

However, these alternatives have not gained general acceptance or consensus among philosophers of science either because they incur either circularity or regression to infinity by attempting to provide a valid basis for inferring about one set of facts from another.

The prejudice against the context of discovery in the early twentieth-century philosophy of science could be another motivation for the contemporary philosophers' disdain for induction. Since induction is a logically invalid argument, it has been argued that a logical analysis of the scientific invention practices is unattainable.

It seems that the rejection of analyzing the context of discovery by early twentieth century philosophers comes from the same bias that resulted in induction being considered an irrational reasoning. The term linguistic bias is used here to refer to this tendency in the treatment of induction, which treats it as a type of inference or argument consisting of statements or propositions.

The linguistic bias is prior to the classical conception. It goes back to nineteenth-century inductive theories and Richard Whatelys' suggestion to treat induction as an abbreviated syllogism or enthymeme. This idea had a lasting impact over time. According to Whately, the omitted premise in an enthymeme corresponds to the principle of the uniformity of nature. John Stuart Mill, who was strongly influenced by Whately's ideas, played a central role in the consolidation of this approach to account for induction, which has persisted almost unchanged until today (McCaskey, 2020).

This article addresses one of the two reasons for the lack of interest in induction among contemporary philosophers mentioned above. The main goal is to show that the supposed irrationality of our empirical beliefs comes from a misperception that results from an inadequate approach to inductive reasoning that consists of taking induction as an inference or argument.

Two strategies are reviewed which address induction within the linguistic approach.[3] First, the attempts to justify induction by the criteria of validation and vindication by Rudolf Carnap (1945; 1947; 1950ab) and Hans Reichenbach (1949; 1957) are outlined. Second, the material theory of induction by John Norton (2003; 2005; 2014; 2021) is reviewed. According to the latter, inductive inferences rely not on reasoning based on universal or formal rules but on what Norton terms the 'material postulate of the induction' (2003, p. 650).

It is argued that, although inductive conclusions in science will be justified either by formal criteria or by considering the material dimension of inductive statements that support them, the main problem is that the treatment of induction within a linguistic framework, i.e., treating induction as an inference or argument consisting of statements or propositions, leads us to the same outcomes: the impossibility of justifying the rationality of inductive methods in science. Therefore, replacing the linguistic treatment of induction with a model-based framework is suggested. In addition, William Whewell's theory of induction is proposed as an example of a non-propositional treatment of induction.

2. The traditional analysis of induction

The analysis of induction in the philosophy of science has typically been driven by the traditional conception of theories, according to which scientific theories are defined as classes or systems of related statements. While the statements that provide empirical basis for scientific theories or hypotheses are viewed as being supported by some inductive support.

While they are somewhat related, it is crucial to distinguish between two issues that are often confused about the traditional treatment of induction. On one hand, scientific theories or hypotheses are established based on given evidence. The question arises: 1. On what basis does a scientist confirm a hypothesis or theory? On the other hand, regarding the scientific justification for using induction in science, the question is: 2. On what basis do philosophers consider that inductive reasoning employed by scientists is justified?

Generally, the answer to question 1 has been that scientists confirm a scientific hypothesis or theory when they have empirical evidence to back it up. However, as is well known, the support given by the evidence to scientific theories or hypotheses is not a deductive one, which means that it is not based on a set of inference rules that warrant the truth from the premises

[3] Strategies interested in accounting for rationality using inductive methods are addressed; therefore, strategies that deny induction, such as those of deductivists, are excluded.

to the conclusion. Instead, the support provided by the evidence is generally statistical or probabilistic. Concerning 2, traditionally, the metascientific justification of induction has been carried out by justifying scientists' preceding statistical or probabilistic process (1).

The following two approaches focused on the second issue mentioned above: identifying logical or pragmatic mechanisms to validate or vindicate the inductive procedures by scientists. In doing so, as it will be shown, they conceived of inductive reasoning in science as an inference involving sentences or propositions, that is, as a linguistic procedure. Consequently, these proposals faced several difficulties rooted in the linguistic treatment that characterized them.

2.1 Rudolf Carnap's validation based on inductive logic[4]

In the classical or enunciative conception of theories, the empirical meaning criterion was commonly defined in connection with verification methods. According to this, the significance of a statement is determined by its cognitive meaning, that is, by its potential to be tested, either through logical rules or by empirical evidence. A statement whose truth or falsity could not be anticipated using this criterion was deemed a pseudo-statement outside scientific language.

As a result, the universal statements of empirical sciences—such as scientific laws— were considered as standing beyond the logical analysis. Also, a significant proportion of scientific statements that contain terms like atoms, quarks, bosons, and libido, cannot be directly observed, with which they are relegated to stay beyond the metascientific analysis scope. These issues, among others, contributed to the eventual abandonment of the empirical criterion of meaning initially embraced by logical positivists and empiricists.

Carnap's solution to the limitations imposed by the criterion of empirical meaning involved proposing a dual-language system for introducing scientific concepts. This system includes an observational language L_O and a theoretical language L_T, where L represents the entire language of the system. In Carnap's system, each language in L holds a particular vocabulary, V_O and V_T, respectively, which consists of the specific primary constants for each case, such as the logical and non-logical (or descriptive) constants, as well as individual variables, formula formation rules, and logical connectives.

Carnap defined a theory as an uninterpreted calculus that comprises a finite number of postulates that are formulated within L_T (where T means the conjunction of the mentioned postulates and rules of deduction), such as T = L_T postulates + rules of deduction (Carnap, 1950a). He provided the correspondence rules, known as C-rules, for interpreting the terms

[4] This article refers to Carnap's semantic works on probability, omitting the deductivism that characterizes his early years.

and theoretical statements of T. These rules allow the derivation of L_O's terms from the L_T's terms, and vice versa, the L_O's terms from the postulates of T, thus providing a partial and indirect interpretation of the V_T's terms.

Correspondence rules can be understood as inference rules that relate the theoretical terms, which refer to physical measures (e.g., mass, temperature, and force), with the observational terms and statements. Essentially, these rules allow indirect derivation of conclusions in L_O, such as prediction statements concerning observable events, reports of observational results, and determining the probability or degree of confirmation of a conclusion (Carnap, 1950a).

Carnap thought that inductive logic was a branch of semantics (1945, p. 73). However, he maintained the term 'inference' to indicate the support provided by the evidence. Induction was not for him "merely a transition from one sentence to another (viz., "from the evidence or premise *e* to the hypothesis or conclusion *h*) but the determination of the degree of confirmation c (h, e)." (1945, pp. 83-84).

The degree of confirmation c* is based on one of two types of probability commonly used in science, epistemic probability, which Carnap defined as an a priori and analytical concept of probability. Precisely, the degree of confirmation c* pretends to show the degree to which a hypothesis *h* is supported by the positive instances *e* that support it. The above is expressed as c (h, e) = r (Carnap, 1945; 1947; 1950b). With it, Carnap strived to provide a logical reconstruction of the process by which scientific hypotheses or theories are accepted by assigning probabilistic values from data derived from experience.

The measure of the degree of confirmation c* was employed for Carnap to display in semantic terms the degree of probabilistic support provided by [the premises representing] the evidence to [the conclusion representing] the hypothesis or scientific theory. With the above, Carnap aims to overcome the syntactic limitations that require presenting a scientific hypothesis or theory as a necessary consequence of the empirical supporting evidence. Instead, he formulated a probability statement detailing the degree of confirmation c* by which the evidence *e* partially involves the scientific hypothesis or theory *h.* In this sense, Carnapian confirmation is a logical function that provides a probabilistic criterion for determining the degree of subjective belief engaged in accepting a theory or hypotheses.

The evidential support for a scientific hypothesis or theory can be interpreted as states or possible worlds in the Carnapian confirmation theory. In the Carnap system, possible worlds are considered as "state descriptions," which represent the combination of all atomic statements (consisting of a predicate and *n* individual constants) within a finite language L_N on the domain of N individuals. These state descriptions express the properties and relationships in L_N, as well as their negations. The notion of state description in L_N can also be replaced by a model N, where N is the cardinality of the domain of the individuals in L (Carnap, 1950b).

Interpreting Tarski's truth results, Carnap proposed relativizing it to a language and positing the L-truth notion, which is a rare combination between the semantic notion of truth and syntactic concepts such as L-implication and L-equivalence (Carnap, 1997, p. 64). Carnap defined L-truth as tautological: "A sentence S_i is L-true . . . $=_{Df}$ S_i holds in every state-description." (Carnap, 1997, p. 9-10).

While statements representing evidence are implicit in some possible states or worlds, others contradict them. This contingency, specific to the empirical sciences, is determined by estimating the extent to which *h* is implicitly or logically contained in *e*. For instance, according to Carnap, 'c (h, e) = 3/4' indicates that *h* is not entirely given by *e* but that the assumption of *h* is supported in degree 3/4 by the observational evidence expressed in *e.* (1945, p. 72). The contingent truth was conceived as "almost-L-truth" and interpreted by his confirmation value.

Carnap's efforts were drawn to provide the theorems to obtain a comparative measure of probabilistic or statistical systems of inference, with the caveat that these can only be applied to languages with monadic predicates, that is, systems that express only properties, not relations (1945, p. 81). As Kemeny (1951) points out, most of his theorems and postulates on probability are correct. However, Carnap's proposal faced several difficulties, some of which he acknowledged. These issues are closely related to his reliance on a linguistic framework to understand induction. For the time being, two difficulties are highlighted that were related to his commitment to a linguistic treatment of induction.

As mentioned above, Carnap's inductive logic only considers languages with unary predicates, meaning that it only deals with statements that denote properties and not relations. The extension to other types of predicates, such as binaries and ternaries, is conditional on a "future development" of deductive logic to address issues related to "the number of structures in a given finite language system" (Carnap, 1945, p. 82). But, for now this is an important limitation of Carnap's linguistic induction treatment.

Another difficulty was that Carnap's inductive logic refers only to particular statements since they are the only ones that can be confirmed (Kemeny, 1953). A sentence or proposition in a universal logical form is unconfirmable in Carnap's program (Carnap, 1945, pp. 88-90). Thus, Carnap's response to addressing this question is to abandon hopes of confirming generalizations and concentrate on particular "instance confirmations" (Barker, 1957, p. 88).

However, even if a specific part of science can be expressed using statistical statements, universal statements are essential for scientific theories. Although the Carnapian system does not explicitly contain general statements, it does presuppose them. As argued later, a non-linguistic or semantic approach to induction would overcome all these difficulties.

Carnap was aware that his inductive logic, designed to rationally reconstruct induction in science, was insufficient to validate theory's set of scientific beliefs and that this was a more profound and challenging issue (Carnap, 1945, p. 95).

2.2 Reichenbach's vindication: Induction as the best bet in science

The situation was not better for accounting to inductive reasoning from a pragmatic viewpoint. While Reichenbach accepted that inductive reasoning was an invalid inference, he defended the vindication of induction as the human procedure that ensures better success in handling empirical data in scientific research.

From Reichenbach perspective, although scientific laws cannot be directly verified because of their generality, they can be tested by confirming their predictions (Reichenbach, 1933, p. 407).[5]

Reichenbach's proposal can be briefly outlined in three stages, each showing his adherence to the linguistic treatment of induction.

First, Reichenbach placed the analysis of induction in the context of justification. According to him, hypotheses or theories are justified based on the facts that support them. In this sense, he founded the empirical basis of science on induction by enumeration, according to which the observation of *n* members of a class allows us to conclude about the members of the whole class.[6]

Second, Reichenbach believed that the relative probability frequency was adequate for approaching inductive logic in science. According to this framework, the probability of a hypothesis is determined by two frequencies: "the frequency of the events of the narrower class considered and the frequency of the events of the wider class to which the probability is referred" (Reichenbach, 1957, p. 301). The aim is to determine the limit of the relative frequency of an attribute in an infinite sequence. He asserted that an inductive conclusion is a statement that describes a fact, that is, the probability in terms of the relative frequency of the occurrence of an event.

Thus, for Reichenbach, it only makes sense to use inductive inference in order to assign probabilities to theories and accept the most plausible one. However, the transition from assigning probabilities to one or more theories to approving the theory with the highest probability is not logically justified.

The lack of justification comes from the implicit assumption of regularity, or more precisely, from the idea that the most plausible theory will behave the same way in the future as it did in the past. But, as Hume already pointed out, such an assumption cannot be justified either

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[5] Reichenbach considered the relationship between universal laws and future propositions crucial for expanding our empirical knowledge.

[6] Reichenbach argued that all types of induction can be reduced to induction by enumeration, as first suggested by Hume and later confirmed by the axiomatization of the calculus of probabilities (Reichenbach, 1957, p. 389; 1968, p. 242).

a priori or a posteriori. However, according to Reichenbach (1968), this problem only arises if we incorrectly assume that knowledge must be demonstratively true. In contrast, he asserts that inductive proofs do not necessarily lead to true but probable conclusions.

Third, Reichenbach thought that accepting a theory depends on whether it represents -or not- the most optimal option. He believed that assuming a hypothetical inductive principle as the principle of uniformity of nature, the probability of successful inductive conclusions increased significantly compared to conclusions drawn through any other method. Therefore, Reichenbach thought that the most rational thing to do was to accept induction rather than any other method. If other methods were to succeed consistently, they would constitute a principle of uniformity of nature that could only be vindicated by induction. In a nutshell, only induction can explain the rationality that underlies conclusions based on observational data. The reason is that inductive conclusions are the most reasonable bets, so they are preferable. [7]

To summarize, from Reichenbach's metascientific perspective, scientific theories or hypotheses are universal statements that represent generalizations of finite sequences observed and expressed through singular statements. Whereas, from a scientific point of view, Reichenbach understood theories or hypotheses as confirmed by probabilistic support, which consists of continuous values ranging from 0 to 1.[8] Reichenbach justified the acceptance of theories or hypotheses according to their probability as the best way to approach the truth in scientific research because it primarily safeguards scientific predictions.

Reichenbach's proposal faced essential criticism. Generally, it was thought that inferences based on frequentist probability were inadequate because they did not fulfill their purpose of providing authentic explanations of the occurrence of the events but only indicated a statistical measure of their occurrence.

Even if the limit of the relative frequency eludes the margins of error to the extent that finite observational portions within infinite sequences increase, as Reichenbach said, the limit of the estimate to account for its approximation to the truth is uncertain. In other words, we do not know how many attempts will be necessary to obtain a correct or probably correct estimate for a true conclusion "in the long run," so there is no guarantee at all that our conclusion is the best bet (Lenz, 1958, p. 100; Eberhardt & Glymour, 2011, p. 362). It is not helpful to be told that if we continue using Reichenbach's frequentist induction indefinitely, we will eventually approximate the truth. We are interested in determining whether it is reasonable

[7] Reichenbach compared inductive inferences in science to casting a net into the vast sea. The fisherman is uncertain whether he will catch a fish, but casting the net is his best bet. Thus, he has no choice but to take a chance and gamble on it (Reichenbach, 1968, pp. 245-246). Hence, inductive conclusions are not demonstratively true, but they represent the best wagers for improving our beliefs according to the available empirical evidence.

[8] Reichenbach established the existence of an isomorphic 'coordination' between the structure of events and the mathematical structure of the probability statement (Eberhardt & Glymour, 2011, p. 361). Nonetheless, he did not clarify what he meant by the 'events' structure' or reality.

to accept this specific estimate here and now (see Barker, 1957). Thus, the conclusions of Reichenbach's frequentist theory cannot be justified except for their being the highest wagers that can be achieved through continuous and endless application of the inductive method.

Reichenbach's adherence to what has been called in this paper the linguistic approach to induction, in which the treatment of induction runs as a type of inference consisting of statements or propositions, is quite clear. However, if there is any doubt, we can look at Reichenbach's rule of induction, in which he raises the controversial notion of "posit."

To account for what he calls "primitive induction", that is, the induction that takes place before one can even assign probability values to statements expressing the limit of frequency relative to an event, Reichenbach uses the notion of "blind posit." A "blind posit" is a statement with no initial weight but that is considered the best available functional assumption. Once the elements of the judgment are available and go through evaluation, they become "appraised posits' (Reichenbach, 1949, p. 445-446).

Aside from criticizing Reichenbach's assertion that empirical statements can be treated "as if" they were true (Reichenbach, 1968, p. 246), Bertrand Russell (2009) argued that frequentist probability is a statistical notion that does not provide a practical criterion for adopting a doxastic attitude toward scientific theories. It just provides mathematical data on the limit of their relative frequency. Moreover, Russell, along with other authors, criticized Reichenbach's introduction of the notion of 'posit,' claiming it was an attempt to take advantage of everything and to avoid labeling probability statements as true since they could be wrong (Lenz, 1958; Eberhardt & Glymour, 2011).

3. Material analysis of induction: A local non-formal approach

Inductive reasoning is typically rated based on its adherence to a formal scheme in the classical conception of theories. Consequently, a deductive model was chosen, intentionally or not, as the main framework for approaching scientific practice reasoning. However, a lively debate is currently taking place in order to abandon the formal approximation and instead embrace a material and local induction framework. From a contemporary perspective, this debate has John Norton as its most famous exponent. However, it must be said that it began with Isaac Levi's works on the local context of justification of theories (1973) and J. Bogdan (1976), among others. The following discusses a brief review of Norton's material theory of induction and some criticisms it has received.

Recently, John Norton has criticized formal treatments of induction as inadequate for explaining ampliative inferences in science (Norton, 2003; 2005; 2014; 2021). Instead, he proposes a material theory of induction. Two ideas -or slogans- are central to his proposal: "All induction is local" and "No universal rules of induction" (Norton, 2021, preface v).

Norton categorizes traditional induction treatments as either qualitative or quantitative. The first is based on either inductive generalization, supported by evidence, or hypothetical induction, where the generalization implies the evidence. While the latter works by assigning a numerical measure that indicates the degree of inductive support given by the evidence to the generalization (Norton, 2003; 2021).

In Norton's view, traditional approaches to induction presuppose that all the inductive inferences follow a formal schema. The typical form for this schema is 'some As are B', so 'all As are B.' The above is, in some cases, interpreted in statistical or probabilistic terms. Norton argues that this interpretation has generated the expectation that inductive schemes blend in some way with the principles of deductive inference and that the scheme can be applied universally.

Philosophers have been searching in vain for a formal scheme that can be applied universally to provide inductive support for their theories. However, as Norton realized, no universal scheme can be applied to all scientific ampliative inferences. Inductive strategies are only suitable for specific types of factual matters. For instance, Bayes's theorem can provide inductive support to the evidence from the DNA sequencing of suspects in a crime. But it cannot be used to account for experiments that require intervention and controlled trials or to explain the anomaly of Mercury's perihelion based on the theory of relativity. Instead, confirmatory strategies and severe testing are better options for these latter cases. Therefore, as Norton concludes, the current approaches to induction in the philosophy of science fail to provide a clear and consistent account of induction (Norton, 2021, p. 4).

The above led Norton to abandon formal approaches and focus instead on the 'background factual conditions' as a secure anchor for induction (Norton, 2005, pp. 25-31; 2003). Norton's proposal fuses induction so profoundly into the facts that there seems to be no epistemological distinction between facts and the inductive rules used to make inferences from them. In Norton's view, an inductive inference, such as,

> The proposition "If A, then B" is both a factual proposition and also a warrant that authorises a deductive inference from A to B. The material theory asserts that, ultimately, this dual role for factual propositions is the only way that inductive inferences are warranted. (2021, p. 7).

An adequate treatment of inductive inferences in science requires justifying the facts that serve as its warrant instead of justifying a specific type of induction (Norton, 2014, p. 672). Following Norton's example, the inductive inference from 'some samples of bismuth melt at 271°C' to the generalization 'all samples of bismuth melt at 271°C' is warranted by the fact that, usually, samples of the same element maintain uniformity in their physical properties.

In Norton's bismuth case, the inductive inference is guaranteed by a chemical fact: despite some exceptions, such as allotropes, the molecules of the same element have the same properties. According to Norton, bismuth melting at 271°C is an inherent property of bismuth. Therefore, the statement about the physical properties Bismuths shares constitutes both a statement of fact and a warrant for its inference.

It is well known that statements about facts are contingent. In some cases, they may be true, and in others, they may be false. The general statements about bismuth properties do not escape from this contingency. As a result, Norton contends that universality in inductive inferences is not feasible at all (Norton, 2014, p. 674).

The assertion about bismuth properties is also, at the same time, a factual statement that justifies an inductive inference concerning the melting point of this element under certain conditions. As a contingent statement, the assertion about the properties of bismuth is not necessary; therefore, a property as truth cannot be inferred from it. That said, the question arises: how is the inference about bismuth's melting temperature justified?

According to Norton, relying on contingent truths that support the general statements that validate the more general inductive inferences is our only option. In a nutshell, we must trust those truths that depend on the facts referred to in the general statements.

However, as was mentioned before, the facts Norton presents in his material theory are not epistemologically distinct from the inductive inferences substantiating them. Hence, "we learn the warranting fact by further inductive inferences, which in turn have their own distinct warranting facts; and so on." (Norton, 2014, p. 676).

In the bismuth case, the inference from 'I observed a sample of bismuth with a melting point of 271°C' to the general statement 'most bismuths share this property' is supported by a material fact: typically, bismuth samples had been uniform physical properties. This fact can be expressed by the statement: 'With some exceptions, bismuths have uniform physical properties.'

This latter, the justifying general statement, is -at the same time- justified by other inferences, such as prior knowledge about the uniformity of the atomic composition of physical elements in general, which is derived from microscopic observations of the atoms of each class of material elements. The above depends, in turn, on knowledge of what is expected of the same type of physical element, and so on. This cycle perpetuates itself, with each subsequent inference relying on the last.

Norton's justification operates under the fallacy known as regression to infinity. Nevertheless, he claims that, unlike formal treatments, regression to infinity in his proposal is not "harmful" since it aligns with how science effectively employs induction (Norton, 2014, p. 677). From his viewpoint, scientific statements rely on prior inferences from the same or different domains. Consequently, Norton argues that the interconnection of different sciences supports regression to infinity to ensure the truth of the general statements that provide the basis for subsequent inductive inferences.

As was mentioned above, inductive inferences do not have a rule or scheme that can be applied universally; instead, the inductive strength in Norton's material theory comes from supporting other scientific propositions that are based on other inductive inferences that come from the same field of inquiry. Therefore, the inductive inferences are always relative to the facts they deal with. For instance, the inference based on the regularity of the properties of bismuth cannot be applied to wax because the material composition of wax can differ between samples. Hence, the slogan 'all induction is local'.

Some philosophers have criticized Norton's proposal for leading back to Humean skepticism: the regression to infinity never ends, so Norton's proposal still leaves us with no guarantee for our knowledge (Sober, 1988; Kelly, 2010). So, the infinite regression tree must end at some point to prevent this skepticism from persisting. (Worrall, 2010).

Peter Achinstein (2010) criticized Norton's denial of the universal approach to induction. He said Norton's local material theory could easily coexist with a formal theory like Mill's inductive theory. Norton's theory holds that any inference that arrives at a true conclusion is locally correct, regardless of whether it is formally correct or not. Thus, inferences that are materially correct in Norton's view could be explained by formal correctness.

On the other hand, facts that behave as a warrant for inductive inferences are held in "a regular manner, which authorizes the inferences" (Norton, 2021, p. 8). Thus, Norton points out that biological predicates like "Mortal" or "Having a blood system" regularly appear in living beings. This regularity allows us to assert the truth of the statement "All living beings are mortal", as well as the inference to statements like "Whoever writes these lines is mortal".

Norton's assertion seems to be plausible in some way. If we establish the premise that every living being is mortal, then we can deduce that any living being is mortal. However, the point is that in the linguistic approach of induction, there is an unsolvable issue in establishing the truth of general statements such as 'Every living thing is mortal' or 'If it is a living thing, then it is mortal', because these statements may be false. As Popper said, we can only know their falsity, not their truth. For instance, the individual who makes it false could be a living thing, but their mortality would be false. Yes, it is true that all living beings that we have encountered thus far have been mortal. However, it may not necessarily be true because we do not know every living being, now or in the future.

The main issue is that there is no assurance given by the regularity of our knowledge based on the sum of facts observed. The reason is that we cannot fully determine whether the facts will always be "hospitable" (in Norton's terms) with this empirical knowledge. Therefore, Norton's material theory seems flawed for the same reason that Norton critiques other inductive theories: considering induction from a linguistic standpoint, which leads us to a formal approach to interpreting it. Contrary to Norton's belief, the main obstacle to fully comprehending inductive strategies in science is not the formal dimension but rather the adherence to a linguistic framework, as is explained in the next section.

4. Why is the linguistic approach to induction inadequate?

Except for certain attempts, such as Carnap's endeavor to construct an inductive logic based on semantic rules, inductive reasoning was never regarded as the epistemological basis for interpreting theories in the classical conception. On the contrary, most philosophers dedicated to induction have opined that a purely semantic account of confirmation is inadequate (Sprenger, 2011, p. 236).

The classical conception of theories gave an essential role to the formal treatment of scientific hypotheses or theories at the expense of their material ones. Most of the time, induction was viewed as a connection that originates within the arguments rather than as an extralogical connection between the empirical posits and the facts.

Despite having a particular affinity towards semantic analysis, Carnap (1959) clearly distinguished between the material and formal language modes, viewing the latter as epistemologically superior. Carnap believed that material mode gave rise to confusion and pseudo-problems. On his part, Carl Hempel considered that "semantics does not enable us to decide whether the theoretical terms in a given system T' do or do not have semantical, factual, or ontological reference..." (1965, p. 217). Those who argued that it is possible to determine the material adequacy of scientific statements from reference to a given term in the metalanguage were accused of lack of clarity by Hempel.

In addition, it was believed that if a scientific hypothesis or theory was tested and verified, its implied entities were real. The preceding meant that truth might be formally examined as a relationship between the premises and their conclusions. As a result, the truth was taken away from its 'natural habitat,' the semantic one, and moved into a foreign domain, the syntactic one. The above is a common mistake in the classical conception of theories.

As was seen, Carnap's attempt to account for induction was unsuccessful because it required the combination of intensional semantics and the logical relation of implication. While intensional semantics focuses on the meaning of scientific terms and statements -rather than their extension or reference- the logical relation of consequence plays a central role in deriving observational statements from scientific predictions or laws. The result of this process was assigning a corresponding measure to the degree of confirmation c*.

Carnap's attempt to depict induction as a logical relationship between premises and their conclusions comes from his commitment to the linguistic approach to treat inductive inferences, which understands hypotheses and theories as comprising two languages: theoretical and observational. As a result, Carnap misunderstood induction as a form of inferential reasoning.

In Carnap's view, the derivability relation between premises and conclusions was crucial for the degree of confirmation. Because of that, he argued that inductive logic must be a priori and analytical, meaning deductive, even though this derivability was indirectly supported by the correspondence between the theory -or formal calculus- and the domain or universe to which it refers, i.e., in a semantic way. Reichenbach's work, *mutatis mutandis*, makes a similar mistake.

In the next section, it will be asserted that the basis of induction should be sought in the semantic and pragmatic domain of the interpretation and representation process instead of the syntactic or linguistic domain of scientific theories.

Norton and other local induction theorists deserve credit for highlighting the material dimension and bring induction back into the contemporary debate. However, his proposal is also limited by his commitment to the linguistic treatment of induction. His and other attempts to establish a logical or epistemological foundation for induction have been limited to a linguistic perspective, that is, an enunciative or propositional perspective to refer to inductive reasoning.

The preceding was clearly seen by Hacking (1975, p. 134), who noticed a pattern in the classic induction treatments: In all of them, it is repeated, among other things, that induction is a relationship between sentences or propositions and has a global or universal application. The second point was already criticized by Norton and the authors within the local approach to induction. However, the first has gone unnoticed among philosophers of science. That means no one has asked whether induction is necessarily a relationship between sentences or propositions.

These lines aim to underline the importance of the first point made by Hacking, namely that traditional explanations of induction have understood it under a linguistic framework, that is, as an argument or inference containing propositions or statements, and that this approach is inadequate to deal with induction in scientific practice.

As is well known, the logical justification of inductive conclusions depends on what Hume called the principle of uniformity of nature, which is, in epistemic terms, the belief that nature behaves in the same way at every time and every place. This principle cannot be proven a priori because it is possible to conceive a different course of nature without contradiction; for instance, we can think that the sun will not rise tomorrow. Neither could be a matter of a posteriori justification as this would either incur in a circularity since it requires a principle based on the presumption that the future will be like the past because such a pattern has been experienced previously or lead to infinite regress by appealing to observation to affirm inductive inferences which involve, implicitly or explicitly, a generalization of higher inductive principles *ad infinitum*.

Resolving this puzzle requires renouncing the conception of induction as an argument or inference, that is, as a relation between sentences or propositions. The question then arises: If induction in science does not operate argumentatively or propositionally, what does this reasoning consist of? To answer this question, we must examine scientific practices. If accounting for induction means answering how knowledge is justified by experience, it will require more than the linguistic or enunciative approach because it forces us to look for the "validity" of induction as a certain kind of inference or argument.

Scientists approach experimental scenarios through models, namely, iconic models, mathematical models, models of scale, and so on. Scientists go to the world with representations of that world, not with statements or propositions as such. A comprehensive understanding of the nature of induction requires approaching it as a relationship between the objects or events in the world and their scientific interpretations or representations. This means approaching induction as a relationship between models in their semantic and pragmatic meanings.

Understanding induction as an inference or an argument presupposes using statements or propositions to represent the items involved in an inductive reasoning and understanding their relationship through -valid- inference rules. As was mentioned before, treating induction as an argument or inference has been a common mistake among philosophers of science. However, induction was not always considered linguistically.

In the nineteenth century, Richard Whately defined induction as "A kind of argument which infers, respecting a whole class, what has been ascertained respecting one or more individuals of that class." (Whately, 1827, p. 344). Whately (1827, p. 211) distinguished between two aspects of induction: one as reasoning and the other as an investigative process. Regarding the first, Whately maintained that logic does not distinguish between types of reasoning but rather between types of arguments: those in which the conclusion is deduced from the premises and those in which a premise is missing but the conclusion can still be inferred from the available premises. Whately considered the latter category, called enthymemes, to be inductive arguments until the missing premise is made clear and the argument is transformed into a deductive one.

Any set of statements, some of which are premises and others their conclusions, can be turned into a syllogistic form, even if some are not explicitly stated. Thus, Whately contended that,

> Induction, therefore, so far as it is a two-sense argument, may, of course, be stated syllogistically; but so far as it is a process of inquiry with a view to obtaining the Premises of that argument, it is, of course, out of the province of logic (1827, p. 210).

The interpretation of scientific statements was out of the logicians' scope and put under the scientist's domain, who, owing to their expertise, was best equipped to determine whether the argument aligned with empirical evidence (Whately, 1827, p. 211). As far as Whately was concerned, there was only one kind of reasoning: deductive. As noted earlier, induction is a deductive argument featuring an unspecified general premise. Therefore, since material adequacy in scientific practice is unsuitable for philosophical inquiry, the logician's task was to turn an inductive inference into a deductive argument and, in doing so, into a valid one.

Whately's definition of induction as a formal argument was reproduced, albeit with some modifications, by John Stuart Mill, who maintained that induction is not an argument, i.e., a syllogism, but rather an inference that proceeds from one particular (observed cases) to another particular (similar cases in the present or future) (Mill, 1843).

The reasoning from one particular to another was considered primitive because both animals and humans share it. When an animal or a child gets burned by touching fire, they learn immediately that contact with fire will cause pain in the future and, therefore, become afraid of it. However, Mill argued that neither children nor animals require the general premise "every fire burns" to infer and respond appropriately (Mill, 1843, pp. 168–182).

Unlike Whately, Mill believed that the syllogism was not the complete inductive process but only a second stage. Inductive inference, Mill maintained, does not rely on the major premise, or require the general proposition to derive a conclusion. These elements only serve as a notation or formula that outlines the inference and allows the application of derivation rules as a means of ensuring its consistency (Mill, 1843, p. 188).

Mill was consistent with his radical empiricism when he asserted that general propositions can only be used as a notation for particular-to-particular inferences based on facts. Mill did not reject syllogism as a way to derive a conclusion from a general premise; instead, he considered it a result of the process of inferences from particulars to particulars.

Consequently, Mill's induction methods changed the concept of induction from a mere syllogism featuring elided premises, as Whately proposed, to a form of epistemological inference that proceeds from one particular case to another. Mill justified these inferences based on their formal correctness or consistency. However, Whately and Mill's induction was not free of opposition from his contemporaries.

Part of this resistance came from William Whewell, who opposed Mill's conception of induction.[9] Unlike Mill, Whewell (1840) did not understand induction as a form of reasoning that can be conveyed through inferences or arguments. Instead, Whewell believed that induction involved attributing fundamental ideas and concepts to objects or events in the world. Whewell referred to this process of reasoning as 'superinduction,' which involves observing facts through a new idea or conception that allows considering it under the same common thread. According to Whewell, induction has its cognitive foundation in the general process of superinduction of conceptions to the world.

Returning to our analysis, the mistake of treating induction as linguistic reasoning was not only adopted by the classical philosophers but also by contemporaries such as Norton. Analyzing induction in linguistic terms has led most philosophers to a paradoxical situation: to demand that scientific arguments be evaluated under a deductive framework while also requiring that they provide new and accurate information about the world.

[9] See Foster (2011), McCaskey (2020), Snyder (1997), and Achinstein (2010).

The following section succinctly suggests how Whewell's notion of induction can be an alternative for accounting for induction from a non-linguistic perspective, that is, as a reasoning that it does not take place in an enunciative or propositional way. In this context, a novel approximation for induction is suggested, in which induction is understood as a common way of reasoning in science. This reasoning, again, is not an argument or inference but rather a logical relationship viewed from a semantic standpoint and an extralogical link from a pragmatic perspective.

More precisely, with the help of Whewell's notion of induction, from a semantic point of view, it is proposed to understand induction as reasoning that connects material facts with the structures (or interpretations) that satisfy formally expressed theories. Whereas, from a pragmatic perspective, induction can be considered the reasoning that underlies scientists' validation strategies based on models.

5. Towards an analysis of induction based on the notion of model

Since the 1950s, the notion of model has become increasingly relevant in the philosophy of science. This result was initially linked to two divergent views that were precursors to what is known today as model-based science.

One of these views was concerned with the formal aspects of theories and denied that theories could be identified with statements or propositions. Instead, they defined theories in terms of the class of their models (McKinsey *et al.*, 1953; Suppes 1960). While the other focused on scientific practice, specifically on the dynamic aspects of scientific practice, such as the application of models in experimental tests and scientific discoveries, among others (Braithwaite, 1954; Hesse, 1953; Hutten, 1954).

The two conceptions that emerged from these two views are widely known as the semantic conception (Balzer *et al.*, 1987; van Fraassen, 1989; Giere, 1988; Da Costa & French, 2003; Bueno & French, 2018) and the pragmatic conception of theories (Morgan & Morrison, 1999; Cartwright *et al.*, 1995, p. 138; Suárez & Cartwright, 2008). They are commonly viewed as rival paradigms trying to establish their notion of model in the philosophy of science (Bailer-Jones, 1999). However, they could also be outlined as complementary approaches that provide valuable theoretical frameworks for studying scientific models and theories (see Winther, 2021).

Both the semantic and pragmatic conceptions of theories are frameworks based on models. From them, two notions of models are currently available in philosophy of science that can account for induction from a model-based science perspective. By emphasizing the role of models in scientific activity, these conceptions have enabled interdisciplinary investigations of scientific models, resolving several difficulties that arise from the traditionally linguistic viewpoint (Da Costa & French, 2003).

This paper argues that the analysis of induction through the lens of model-based conceptions of science may offer promising results for the philosophy of science. To achieve this, it is necessary, as a preliminary step, to distinguish between theories perceived as classes of models rather than linguistic elements such as propositions or sentences.

Except for Giere, among the original adherents of the semantic conception the non-linguistic perspective prevailed, according to which the relationship between non-linguistic entities like theories and models was essential for comprehending scientific activity. Regarding this view, theories can be defined by state spaces (van Fraassen 1989), by set-theoretical predicates (Suppes, 1960; Balzer *et al.*, 1987), or by partial structures (Bueno & French, 2018), and so on. Whereas models are defined—not exclusively—by the Tarskian concept of formal structure in most cases, and the relationship between theories and models is expressed in terms of their interpretation, which is, in terms of their satisfiability or truth. [10]

The authors of the semantic conception have tried to understand the relationship between models and the empirical systems to which they refer, but they have yet to succeed. Generally, this issue has been considered within the semantic approach as a representation problem (Frigg & Nguyen, 2020), but a new and novel analysis of induction, as is suggested here, could be also the key to solving this problem (XXX 2023).

Aside from the semantic conception, there are two main directions to understanding models from the pragmatic conception. In one of these, models play the role of mediators for theories; in the other, models are entirely independent of the desiderata of theories. Both directions are better known as the Pragmatic view of theories and the Pragmatic view of models, respectively (Winther, 2021, pp. 31-41). Despite this, all these views emphasize models' non-formal components and understand scientific practice as primarily one based on models. [11]

The next step is to understand semantic models, i.e., the structures that satisfy theories, as metascientific representations of models used in scientific practice, along with understanding scientific models as the pragmatic link between one theory -or model- and the objects and events in the world.[12] This is very close to what was advocated by the forerunners of the

[10] An interpretation (or a structure) is a model of -or for- a system Γ, and the statements that follow from the interpretation (or structure) are the truths of the system. The theory of truth concerned is the one described by Alfred Tarski, i.e., a notion of truth relative to a language L, whose predicate "is true" belongs to the meta-language rather than to the object language and which can only be accounted for in an adequately formalized language (Tarski, 2006).

[11] Most current pragmatists emphasize the importance of models over theory (Morgan & Morrison, 1999; Cartwright *et al.*, 1995, p. 138; Suárez & Cartwright, 2008). These proposals approach the concept of model from the perspective of its practical application in scientific practice instead of its formal dependence on theories.

[12] As can be seen, this is a theory-driven perspective of models. It is believed that there is always a trade-off between models and theories, either from the theory that models come from or another that the model poses.

pragmatic conception, who, from the beginning, saw models as the link between theories and the experimental aspect of science, as E.H. Hutten once suggested: "Theories are explained and tested in terms of models." (1954, p. 289). [13]

The question is: How can these two notions of models be related to induction in science? As we introduce in the later section, from Whewell's inductive theory, induction could be seen as reasoning that relates non-propositional entities as models in these two meanings: models like structures (or interpretations) from a semantic point of view and scientific models such as mathematical models, scale models, phenomenological models, and so on, like mediators between models of the theory and the target or real world, from a pragmatic point of view.

Whoever reads this might ask: Why should we look to a nineteenth-century philosopher to account for a contemporary issue? The short answer to this question is this: because Whewell's induction consists methodologically in the assignment of relations and properties to the objects and events of the world, by means of which the multiplicity of the objects of experience can be seen under a bond of unity. Whewell's induction is closer to model-based scientific practice than the traditional notion.

Whewell's induction can be brought into contemporaneity by reinterpreting three of Whewellians' central notions from a semantic and pragmatic view. These Whewellian notions are (i) the explication of an idea, (ii) colligation of facts, and (iii) consilience of inductions.

In Whewell's philosophy, fundamental ideas and conceptions are general terms or predicates (like space, number, cause, circle, force, etc.) from which we observe the facts but are not in the facts themselves. For Whewell, one of the most critical issues was determining "how the conception shall be understood and defined in order that the proposition may be true." (Whewell, 1847, p. 12).

Roughly spoken, (i) the explication of an idea or conception is the process by which the meaning of the fundamental ideas or conceptions furnished by our mind and applied to the facts is made explicit and unfolded by the formulation of the axioms as well as by the definition of such idea or conception (Whewell, 1858, p. 30). This process can be reinterpreted in semantic terms, for instance, by the definition of a set-theoretical predicate that fixes the mathematical structure of the theory, that is, the classes of the objects, relations, and highlight items that satisfy the predicate, along with the axioms that feature the theory. Notice that so far, formal identification of a theory in these terms has been conducted in the informal set theory, that is, in a non-propositional way.[14]

[13] Pragmatist authors have endeavored to establish the notion of agents and the many roles that models play in scientific practice at the center of the debate about scientific models. Functions such as explanation, exploration, and representation, among others, can be understood as inductive practices only if the concept of induction is placed in an appropriate conceptual framework, i.e., an interpretive or representational.

[14] Axioms are indeed propositions, but they do not play a direct role in the inductive process of verifying or confirming theories from this perspective, as will be seen shortly.

(ii) Colligation of facts is the process in which, by an act of the intellect, an idea or appropriate conception (In formal terms, the definition mentioned in (i)) is introduced in order to establish a connection between phenomena and represent the diversity of observed facts (Whewell, 1840, p. xxxix). The result of this process is a proposition (Axiom in (i)), a hypothesis that must be true in the fundamental idea or conception (in the definition or, in semantic terms, in the mathematical structure mentioned in (i)).

Colligation of facts is essentially a discovery process, and, as Whewell said, there are no rules for its success; instead, a peculiar sagacity from the discoverers is required (Whewell, 1840, xxxviii).[15] Once colligation is made, it is trialed in order to verify whether the facts have the same connection in the colligation as in reality (Whewell, 1847, p. 44). Until here, we are just in possession of a hypothesis that explicitly expresses the assumption made in the colligation, but this must enable us to predict the "phenomena that have not yet been observed." (Whewell, 1847, p. 62), which leads to assessment process of colligations in scientific practice.

(iii) Verification by prediction, coherence, and consilience. The latter is the most relevant for our discussion because the other two are included in it somehow. Consilience of inductions is when two or more colligations coincide in a theory and, in doing so, predict what is unobserved, which means a correct understanding of the nature alphabet and a proof of the theory's truth (Whewell, 1847, p. 65). This could be reinterpreted in terms of how philosophers of science have raised the validation criteria of models in scientific practice.

According to Downes, philosophers evaluated epistemically the goodness of models used by scientists as well as their objectives for using these models, in terms of, at least, five strategies, including (i) their accuracy in describing phenomena (accurate description); (ii) their confirmation and the level of fit (confirmation and fit); (iii) their capacity to provide credible explanations (goodness of explanation); (iv) their ability to adhere to the preferences or desiderata of the model (trade-offs); or (v) their robustness (robustness) (Downes, 2021). [1]

The epistemic evaluation promoted in the literature as possible strategies for supporting scientific models can be rethought as the plural inductive procedure stated by Whewell and

[15] In this sense, Whewell claim: "In most cases, if we could truly analyze the operation of the thoughts of those who make, or who endeavour to make discoveries in science, we should find that many more suppositions pass through their minds than those which are expressed in words; many a possible combination of conceptions is formed and soon rejected. There is a constant invention and activity, a perpetual creating and selecting power at work, of which the last results only are exhibited to us. Trains of hypotheses are called up and pass rapidly in review; and the judgment makes its choice from the varied group. It would, however, be a great mistake to suppose that the hypotheses, among which our choice thus lies, are constructed by an enumeration of obvious cases, or by a wanton alteration of relations which occur in some first hypothesis." (Whewell, 1847, p. 43).

reformulated based on a semantic and pragmatic conception of inductive reasoning in model-based science. This is a perspective of induction that accommodates better the scientific practice of confirmation than one based on sentences or propositions like traditional ones.

It could be argued that the novel and radical way of understanding induction proposed here could make the idea of confirmation as we know it somewhat obsolete. However, instead of being a problem, this new perspective could provide a solution. The traditional notion of confirmation has recently been criticized for its lack of adequacy in accounting for recent advancements in modern physics. For instance, philosophers have had to reconsider their beliefs about orthodox confirmation models in science, as theories like chaos theory and linear dynamics do not comply with the fusion between prediction and observation that science demands (Redhead, 1980; Koperski, 1998).

The reliance on the relationship between theoretical elements to establish the acceptance of a theory by researchers rather than on empirical observations, due in part to the factual impossibility of conducting experiments to confirm them (in traditional terms), is not limited to modern physics. Theories from anthropology, history, and other scientific fields are complex to confirm or reject by viewing predicted outcomes and observational data. This situation has given rise to other proposals for traditional confirmation theory (Dawid, 2016; Dardashti & Hartman, 2019), but they must abandon completely the linguistic approach to understand induction.

Furthermore, conventional confirmation theories have been criticized for being largely disconnected from the actual behavior of scientists during the complex and intricate process of testing models in scientific practice. An analysis of the current academic literature reveals that philosophers have no clear understanding of the validity of models, or, in Stephen Downes' (2021) words, about the features that make good or bad a model. Given the diversity of opinions surrounding this subject, consensus on the basic minimum criteria for classifying a model as 'valid' seems to be a distant goal.

The above is, of course, only a suggestion of how we could understand induction from these two notions of model. The main aim is to call attention to the fact that induction has always been misunderstood by considering it as an inference or argument consisting of statements or propositions. Scientific induction is better understood when placed in the context in which it occurs, i.e., in scientific practice. Scientists go to the world with models, not statements or propositions. We have two notions of models in philosophy of science and several interesting proposals about them. The semantic and pragmatic notions of model.

6. Conclusion

The traditional analysis of induction was examined through two proposals that represent the classical conception of scientific theories: Hans Reichenbach's vindication and Rudolf Carnap's inductive logic. On the other hand, John Norton's material theory of induction was also reviewed to represent current debates on induction. The purpose of the above was to demonstrate that the study of induction in linguistic terms, that is, in terms of statements or propositions, has hindered the ability to explain induction in science adequately.

The reasons why the linguistic approach is considered inadequate to deal with induction are summarized in the following three ideas. First, the linguistic approach of induction is anchored to a specific way or perspective of understanding scientific theories, namely, the enunciative or standard view of theories. Second, the previous perspective forces one to understand induction as an inference or argument. That is, it leads to consider that confirmation of theories in science proceeds using statements or propositions. Finally, this view of science does not correspond to actual scientific practice: When scientists seek to confirm or test their hypotheses or theories, they do not go into the world with statements or propositions but with scientific models.

Thanks to the shift towards model-based science perspective in the philosophy of science, scientific theories can be explored in ways that are beyond linguistic or enunciative treatment. This shift has allowed the interpretation and representation of scientific theories through semantics and pragmatics tools. The above has refined the metascientific analysis of science by emphasizing the evaluation of models instead of statements or propositions within scientific theories or hypotheses, as was common in the classical conception. It is suggested that the model-based science perspective should be extended to studying inductive reasoning in science.

After reviewing the literature on the epistemic strategies for justifying the suitability of models in science, it seems clear that philosophers who have addressed this issue have focused on aspects of interpretation and representation of the models rather than on the formal validity of the inferences or arguments within the theories. These strategies could be reframed as inductive practices that articulate scientific practices and, at the same time, allow for a semantic formalization, as suggested here. William Whewell's induction could be helpful to this purpose since it is a non-propositional account of induction that can be reframed in interpretational and representational terms.

References

Achinstein, P. (2010). The War on Induction: Whewell Takes on Newton and Mill (Norton Takes on Everyone). *Philosophy of Science, 77*(5), 728-739. https://doi.org/10.1086/656540

Bailer-Jones, D. (1999). Tracing the Development of Models in the Philosophy of Science. In L. Magnani, N. J. Nersessian, & P. Thagard (Eds.), *Model-Based Reasoning in Scientific Discovery* (1st ed., pp. 23-40). Springer. https://doi.org/10.1007/978-1-4615-4813-3

Balzer, W., Moulines, C. U., & Sneed J. (1987). *An Arquitectonic for Science: The Structuralist Program.* Springer Dordrecht. https://doi.org/10.1007/978-94-009-3765-9

Barker, S. (1957). *Induction and Hypothesis: A Study of the Logic of Confirmation*. Cornell University Press. https://doi.org/10.7591/9781501741173

Bogdan, R. (1976). *Local induction*. Stanford University.

Braithwaite, R. (1954). The nature of theoretical concepts and the role of models in an advanced science. *Revue lnternationale de Philosophie, 8*(1-2), 34-40.

Bueno, O., & French, S. (2018). *Applying Mathematics. Immersion, Inference, Interpretation*. Oxford University Press. https://doi.org/10.1093/oso/9780198815044.001.0001

Carnap, R. (1945). On Inductive Logic. *Philosophy of Science, 12*(2), 72-97. https://doi.org/10.1086/286851

Carnap, R. (1947). On the Application of Inductive Logic. *Philosophy and Phenomenological Research, 8*(1), 133-148. https://doi.org/10.2307/2102920

Carnap, R. (1950a). Empiricism, Semantics and Ontology. *Revue Internationale de Philosophie*, *4*(11), 20-40. https://doi.org/10.1515/9781400838684-018

Carnap, R. (1950b). *Logical Foundations of Probability*. University of Chicago Press.

Carnap, R. (1956). The Methodological Character of Theoretical Terms. In H. Feigl & M. Scriven (Eds.), *Minnesota Studies in the Philosophy of Science 1* (1st ed., pp. 38-76). University of Minnesota Press.

Carnap, R. (1959). Psychology in Physical Language. In J. A. J. Ayer (Ed.), *Logical Positivism* (1st ed., pp. 165-198). The Free Press.

Cartwright, N., Shomar, T., & Suárez, M. (1995). The tool box of science: Tools for the building of models with a superconductivity example. In W. E. Herfel, W. Krajewski, I. Niiniluoto & R. Wójcicki (Eds.), *Theories and Models in Scientific Processes* (pp. 137-49). Rodopi.

Da Costa, N. C. A., & French, S. (2003). *Science and partial truth: a unitary approach to models and scientific reasoning*. Oxford University Press.

Dardashti R., & Hartmann S. (2019). Assessing Scientific Theories The Bayesian Approach. In R. Dardashti, R. Dawid, & K. Thébault (Eds.), *Why Trust a Theory? Epistemology of Fundamental Physics* (1st ed., pp. 67-83). Cambridge University Press.

Dawid, R. (2016). Modelling Non-empirical Confirmation. In E. Ippoliti, F. Sterpetti, & T. Nickles (Eds.), *Models and Inferences in Science. Studies in Applied Philosophy, Epistemology and Rational Ethics, vol 25*. Springer Nature. https://doi.org/10.1007/978-3-319-28163-6_11

Dethier, C. (2018). William Whewell's Semantic Account of Induction. *HOPOS: The Journal of the International Society for the History of Philosophy of Science, 8*(1), 141-56.

Downes, S. (2021). *Models and Modeling in the Sciences*. Routledge. https://doi.org/10.4324/9781315647456

Eberhardt, F., & Glymour, C. (2011). Hans Reichenbach's Probability Logic. In D. M. Gabbay, S. Hartmann, & J. Woods (Eds.), *Handbook of the History of Logic* (1st ed., pp. 357-89). Elsevier. https://doi.org/10.1016/B978-0-444-52936-7.50010-0

Foster, M. (2011). The debate between Whewell and Mill on the nature of scientific induction. In D. M. Gabbay, S. Hartmann, & J. Woods (Eds.), *Handbook of the History of Logic*. (1st ed., pp.10-93). Elsevier. https://doi.org/10.1016/B978-0-444-52936-7.50003-3

Frigg, R., & Nguyen J. (2020). Scientific Representation. *The Stanford Encyclopedia of Philosophy*. https://plato.stanford.edu/archives/win2021/entries/scientific-representation/

Giere, R. N. (1988). *Explaining science: a cognitive approach*. University of Chicago Press. https://doi.org/10.7208/9780226292038

Hacking, I. (1975). *The emergence of probability. A philosophical study of early ideas about probability, induction, and statistical inference*. Cambridge University Press.

Hempel, C. (1970). On the 'Standard Conception' of Scientific Theories. In M. Radner *et al.* (Eds.), *Minnesota Studies in the Philosophy of Science IV* (pp.142-163). University of Minnesota Press.

Hempel, C. (1965). *Scientific Explanation. Essays in the Philosophy of Science.* The Free Press.

Hesse, M. (1953). Models in physics. *British Journal for the Philosophy of Science, 4*(15), 198-214.

Hutten, E. H. (1954). The role of models in physics. *British Journal for the Philosophy of Science, 4*(16), 284-301.

Jeffrey, R. (1975). Carnap's Empiricism. In G. Maxwell & R. Anderson Jr. (Eds.) *Induction, Probability and Confirmation. Minnesota Studies in the Philosophy of Science 6* (1st ed., pp. 37-49). University of Minnesota Press.

Kelly, T. (2010). Hume, Norton, and Induction without Rules. *Philosophy of Science, 77*(5), 754-764. https://doi.org/10.1086/656548

Kemeny, J. (1951). Review of Logical Foundations of Probability, by R. Carnap. *The Journal of Symbolic Logic*, *16*(3), 205-207.

Kemeny, J. (1953). A Logical Measure Function. *Journal of Symbolic Logic*, *18*(4), 289-308.

Koperski, J. (1998). Models, Confirmation, and Chaos. *Philosophy of Science*, *65*(2), 624-648. https://doi.org/10.1086/392663

Lenz, J. (1958). Problems for the Practicalist's Justification of Induction. *Philosophical Studies*, *9*(1-2), 4-8.

Levi, I. (1973). *Gambling with Truth*. Knopf.

McCaskey, J. (2020). Reviving Material Theories of Induction. *Studies in History and Philosophy of Science Part A, 83*, 1-7. https://doi.org/10.1016/j.shpsa.2019.12.006

McKinsey, J., Sugar, A., & Suppes, P. (1953). Axiomatic Foundations of Classical Particle Mechanics. *Indiana University Mathematics Journal, 2*(2), 253-72.

Mill, J. S. (1843). *A Sistem of Logic: Ratiocinative and Inductive*. John W. Parker Ed.

Morgan, M., Morrison M. (1999). *Models as Mediators: Perspectives on Natural and Social Science (Ideas in Context).* Cambridge University Press. https://doi.org/10.1017/CBO9780511660108

Moulines, C.U. (2002). Introduction: Structuralism as a Program for Modelling Theoretical Science. *Synthese, 130*(1), 1-11. https://doi.org/10.1023/A:1013892808077

Norton, J. (2003). A Material Theory of Induction. *Philosophy of Science, 70*(4), 647-670. https://doi.org/10.1086/378858

Norton, J. (2005). A little survey of induction. In P. Achinstein (Ed.), *Scientific Evidence: Philosophical Theories and Applications* (1st ed., pp. 9-34). Johns Hopkins University Press.

Norton, J. (2014). A Material Dissolution of the Problem of Induction. *Synthese*, *191*(4), 671-690. https://doi.org/10.1007/s11229-013-0356-3

Norton, J. (2021). *The Material Theory of Induction*. University of Calgary Press. https://doi.org/10.2307/j.ctv25wxcb5

Raisis, V. (1999). Expansion and Justification of Models: The Exemplary Case of Galileo Galilei. In L. Magnani, N. J. Nersessian, P. Thagard (Eds.), *Model-Based Reasoning in Scientific Discovery* (1st ed., pp.149-164). Springer. https://doi.org/10.1007/978-1-4615-4813-3_10

Redhead, M. (1980). Models in Physics. *The British Journal for the Philosophy of Science, 31*(2), 145-163.

Reichenbach, H. (1933). Carnap's Logical Structure of the World [1933c]. In M. Reichenbach & R. S. Cohen (Com.), *Hans Reichenbach Selected Writings 1909-1953*, (pp. 405.408). Springer Dordrecht. https://doi.org/10.1007/978-94-009-9761-5_42

Reichenbach, H. (1949). *The Theory of Probability: An Inquiry into the Logical and Mathematical Foundations of the Calculus of Probability*. University of California Press.

Reichenbach, H. (1957). *Experience and prediction: an analysis of the foundations and the structure of knowledge*. University of Notre Dame Press.

Reichenbach, H. (1968). *The Rise of Scientific Philosophy*. University of California Press.

Russell, B. (2009). *Human knowledge: its scope and limits*. Routledge. https://doi.org/10.4324/9780203875353

Salmon, W. (1967). Carnap's Inductive Logic. *The Journal of Philosophy, 64*(21), 725-739.

Salmon, W. (1974). The pragmatic justification of induction. In R. Swinburne (Ed.), *The Justification of Induction* (1st ed., pp.85-97). Oxford University Press.

Snyder, L. (1997). The Mill-Whewell Debate: Much Ado about Induction. *Perspectives on Science, 5*(2), 159-98. https://doi.org/10.1162/posc_a_00523

Sober, E. (1988). *Reconstructing the Past: Parsimony, Evolution, and Inference*. MIT Press.

Suárez, M., & Cartwright, N. (2008). Theories: Tools versus Models. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics, 39*(1), 62-81. https://doi.org/10.1016/j.shpsb.2007.05.004

Suppes, P. (1960). A Comparison of The Meaning and Uses of Models In Mathematics and The Empirical Sciences. *Synthese*, *12*, 287-301. https://doi.org/10.1007/BF00485107

Sprenger, J. (2011). Hempel and the Paradoxes of Confirmation. In D. M. Gabbay, S. Hartmann, & J. Woods (Eds.), *Handbook of the History of Logic 10*, (1st ed., pp.235-63) Elsevier. https://doi.org/10.1016/B978-0-444-52936-7.50007-0

Tarski, A. (2006). The Concept of Truth in Formalized Languages. In J. Corcoran (Ed.), *Logic, Semantics, Metamathematics. Papers from 1923 to 1938 by Alfred Tarski*, (1st ed, pp. 152-278). Hackett Publishing Company.

Van Fraassen, B. (1989). *Laws and Symmetry*. Oxford University Press. https://doi.org/10.1093/0198248601.001.0001

Whately, R. (1827). *Elements of Logic*. J. Mawman.

Whewell, W. (1840). *The Philosophy of the Inductive Science, Founded Upon Their History, vol. 1.* John W. Parker West Strand.

Whewell, W. (1847). *The Philosophy of the Inductive Science, Founded Upon Their History, vol. 2.* John W. Parker West Strand.

Whewell, W. (1858). *Novum Organon Renovatum. Being The Second Part of the Philosophy of the Inductive Science.* John W. Parker and Son, West Strand.Williams, D. (1948). Induction and the Future. *Mind, LVII*(226), 226-229. https://doi.org/10.1093/mind/LVII.226.226

Winther, R. (2021). The Structure of Scientific Theories. *The Stanford Encyclopedia of Philosophy*. https://plato.stanford.edu/archives/win2016/entries/structure-scientific-theories/

Worrall, J. (2010). For Universal Rules, Against Induction. *Philosophy of Science, 77*(5), 740-753. https://doi.org/10.1086/656823

Wright, G. (1965). *The Logical Problem of Induction.* Oxford Basil Blakcwell.