13. Gibrat’s law and the change in artificial land use within and between European cities

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1. INTRODUCTION

In reflecting upon the title of this book *Entropy, Complexity and Spatial Dynamics: A Rebirth of Theory?*, one is reminded of the complex nature of cities, especially when their evolution through time is considered. We return to historic references such as Zipf’s law (Zipf, 1949) and Gibrat’s rule of proportionate growth (Gibrat, 1931). Although both have been explored, we remain no closer to an answer to where cities evolve and why, or to why they evolve the way they do in general. While the focus of Gibrat’s rule of proportionate growth for cities has been population, land use is also important. In addition aggregate measures are often used, but this ignores intracity variations. This study attempts to bridge the gap between historical concepts, in this case Gibrat’s law, and big data by utilising a Europe-wide data set on urban land use. As the data is defined using functional urban areas, it brings into focus recurring issues around city definition, which we know impacts the parameters of those general laws. Such a large data set enables us to test not only the relationship of city size, urban growth and city definition in aggregate terms but to apply a combination of these issues downscaled within the intraurban structure. A rebirth of theory is in fact in need of a better coupling of the intraurban and interurban laws and structures.

> *‘Even if Gibrat’s model remains generic and universal, it can by no means be accepted as a “purely” stochastic process requiring no further explanation. On the contrary, it should be enriched by reference to historical context and trends.’*
> — Pumain (2006a)

1.1 Gibrat’s Law and Urban Growth

Gibrat’s 1931 book *Les Inégalités Éonomiques* (Gibrat, 1931) used several variables (income, population, wealth) to examine why, even though the growth process is considered stochastic, the variables often exhibit skewness or kurtosis. This is despite the central limit theorem (Akhundjanov and Toda, 2019), where a normal distribution would be expected. Gibrat proposed that the proportional rate of growth is independent of size and gives rise to a distribution that is log-normal. More simply a city with population 10 million is as likely to double in size during a given period as a city with population 500 000 (Mansfield, 1962). The consequences of Gibrat and Zipf illustrate that the drivers of agglomeration economies are the same for all cities (Batty, 2008).

Several studies have examined Gibrat for a range of different cities and countries, with mixed results but showing city size and population growth are independent of each other and hence follow Gibrat’s law at least to a certain extent (Robson, 1973; Pumain, 1982a; De Vries, 1984; Moriconi-Ebrard, 1993; Guérin-Pace, 1992; Eeckhout, 2004). Robson (1973) examined UK urban agglomerations >2500 inhabitants between 1801 and 1901 using ten time intervals and found that Gibrat held for the most part, with a slight deviation, as the level of variance in growth rates among smaller cities was higher than among large cities. Pumain (1982a) examined French towns >2500 inhabitants over five time intervals between 1831 and 1975. Gibrat's law could only be verified during periods of slow growth. When annual growth exceeded 2 per cent, an increasing trend of growth rate and city size was found. A temporal autocorrelation of growth rates is also found during high growth periods (Pumain, 1982b).

Over a longer period (100 years), Gibrat's law holds; however, in the short term this becomes weaker. Over the long term Gibrat's law is found to be proportional in means but not in variance (González-Val et al., 2013). A null hypothesis of the city size distribution being log-normal is only rejected for early decades in the twentieth century (González-Val et al., 2013). Although six years is a relatively short period of time some cities have experienced growth >10 per cent over this period. Eeckhout (2004) used a period of ten years to show that cities grow independently of city size and hence follow Gibrat's law. Akhundjanov and Toda (2019) re-examined the 24 data sets found in Gibrat's 1931 book with modern computing and found that 18 of the 24 data sets follow a mixture of Pareto and log-normal distributions, with an exponent at the upper tail of less than 2. The data set on city population is found to be indifferent between a power law or log-normal distribution.

Pumain (2006a) discusses a commonly found deviation from Gibrat: a low and often positive correlation between urban growth rates and city size. Equivalently if cities are classified according to increasing size there will be an increase in the mean value of growth rates. Two explanations are offered for this deviation. First innovations are often adopted earliest in large cities, with large cities then benefiting from initial advantage. The second explanation relates to the increase in speed of transportation, which increases a large city's sphere of influence, stealing market share from smaller surrounding towns and cities.

When testing whether Gibrat's law holds for cities, studies typically consider each city as a whole. This approach fails to recognise the variation in growth within cities, especially the potential effect of the physical pattern of urbanisation or of varying population density levels (Guérois and Pumain, 2008; Schneider and Woodcock, 2008; Jiao, 2015). Differences in growth rates between the core, suburbs and periphery are typically ignored. How cities are defined is therefore important when examining them (Rozenfeld et al., 2011). Cities can be defined using a functional approach (OECD, 2013), as a network such as with the city clustering algorithm (Rozenfeld et al., 2008) or using administrative units such as city boundaries. When defining cities using arbitrary city limits or urban boundaries, issues such as the modified aerial unit problem (Openshaw, 1984) and interactions between closely located cities (Thomas et al., 2018) are encountered.

To understand the urban expansion process a deeper analysis of the internal structure of cities is required. Cities are here defined as a function of distance to the central business district (CBD) using a radial analysis. Similar to other studies (Walker, 2018; Wilson, 2012) the location of the city hall is used as the centre point of the city. This point tends to coincide with the principal residential centre (Griffith and Wong, 2007). Assuming a city has one dominant centre point at its core assumes cities are monocentric. This assumption and the use of distance-based measures is long established in urban geography and economics (Alonso, 1964; Clark, 1951; Fujita, 1989; McDonald, 1989; Von Thünen, 1875). Even within polycentric cities or when important subcentres exist, though, there is often a main centre with the highest population density (Griffith and Wong; 2007). In a study of US metropolitan regions, monocentric was the most prevalent (Arribas-Bel and Sanz-Gracia, 2014). One of the advantages of using a radial analysis is the ability to examine the complex two-dimensional intraurban structure of a city in a one-dimensional space. This approach is compatible with traditional urban economic theory such as the Von Thünen (1875) and Alonso–Mills–Muth models (Alonso, 1964; Mills, 1972; Muth, 1969), which despite their simplifications have the merit of stressing the importance of central accessibility costs on locational decisions. The rescaling methodology used for this study has been found to be compatible with the Alonso model (Delloye et al., 2018) corrected for land use. One of the fundamental determinants of the urban density of a city is the distance to the CBD and the trade-off that occurs between price of land and cost of commuting/accessing the city centre (Brueckner, 1987). Using a distance-based approach enables us to understand urban sprawl and how to mitigate it. We aim to discover which cities are sprawling and how sprawl varies depending on city size and further grouping types.

1.2 Gibrat—from Population to Land Use

Cities are changing in terms both of population and of land use. While studies exist that examine the relationship between population growth and city size, few studies examine the relationship between artificial land use (ALU) growth and city size, and fewer still examine the relationship at the intraurban level. By examining the relationship between growth in ALU—that is, urbanisation—and city size, we test whether ALU growth exhibits Gibrat's law. We add to previous examinations of Gibrat's law for cities by examining the intraurban structure of change. Cities are analysed at various distances to the CBD after those distances are made comparable across cities of different sizes. Cities are also grouped using context or historical characteristics, captured by the coastal/noncoastal location of the city or the year the country joined the European Union (EU). An examination of the evolution of urban areas is important, as this is where the majority of people live (Ioannides and Skouras, 2009). For example the 293 cities used for this study represent 44 per cent of the population of the EU-27. It is important to examine the regularity of cities and seek a statistical explanation for their hierarchy (Pumain, 2006b). Then it is important to examine whether urbanisation patterns are statistically similar for large cities and small cities, since these patterns impact urban sustainability and a given policy might typically need to favour smaller or larger cities.

In Europe between 1990 and 2006, ≈1000 km^2 of land (40 per cent of the total area of the country of Luxembourg) was converted per year to be used for housing, roads and industry. Soil sealing is the covering of soil with impermeable materials such as concrete (Prokop et al., 2011), a process which is rarely reversed. Soil sealing as a result of ALU has several associated negative impacts such as loss of water retention, loss of biodiversity and unsustainable living patterns (Prokop et al., 2011). It is one of the main factors threatening the state of the soil in Europe (Jones et al., 2012). In 2006, 100 000 km^2 or 2.3 per cent of all EU land was sealed, with levels above 5 per cent in the Netherlands, Belgium and Germany. Growing levels of urban expansion are a challenge for cities as they seek to expand sustainably and efficiently. With Europe already highly urbanised, any increase in ALU is likely to occur outside of the city cores.

Poor planning can lead to increasing levels of urban sprawl. Between 1990 and 2015 there were increasing levels of built-up land and decreasing density (Denis, 2020). Such urban sprawl makes provision of utilities and mass transit more difficult. The European Environment

Agency (EEA) has described urban sprawl as the pattern of low-density expansion of large urban areas into surrounding areas, which are mostly agricultural (EEA, 2006). Increasing urban sprawl is the uptake of built-up areas, dispersed over a given landscape with low utilisation intensity in the built-up area (Jaeger and Schwick, 2014). Urban sprawl has several negative effects such as loss of agricultural land, increasing fragmentation, destruction of ecosystems, higher transport costs and increases in greenhouse gas emissions (EEA, 2016).

The urban expansion process is problematic, as it removes land from agriculture, green spaces and nature and is often irreversible. It is the overexpansion of ALU relative to the population that is the issue for cities (Brueckner, 2000). Overexpanding of ALU can also put increasing pressure on existing public services and result in capacity issues. This urban expansion process can take various forms such as sprawling patterns or fragmented patterns. Land conversion from agricultural to artificial (residential or commercial) occurs when a society values artificial land more than agricultural land. Cities where agricultural land is highly productive are typically more compact (Brueckner and Fansler, 1983; Oueslati et al., 2015). For most of Europe, the value of residential land will surpass the value of agricultural land, with exceptions largely due to the topography of land and expensive construction costs. Urban expansion can result from a growing population, rising income and falling commuter costs. Excessive urban expansion may result in several market failures (Brueckner, 2000): failure to account all benefits associated with urban green space; negative externalities associated with excessive commuting and cities that are too large; failure to account fully the cost of all public infrastructure associated with urban expansion (Brueckner, 2000). Sprawling cities will consume more fuel in transportation, more land and more infrastructure materials for water, electricity and roads (O'Meara and Peterson, 1999).

In 2017 building construction and operations accounted for 36 per cent of global energy use and 39 per cent of energy-related carbon dioxide emissions (IEA, 2018). In 2013 the world's urban areas accounted for about 64 per cent of global primary energy use and produced 70 per cent of the planet's carbon dioxide emissions (IEA, 2016a). If current trends continue, combined with the increasing population of cities, by 2050 urban primary energy demand will increase by 70 per cent, accounting for 66 per cent of global demand; carbon emissions will also increase by 50 per cent (IEA, 2016b). Final energy demand in the buildings and transport sector can be reduced by 60 per cent through reduced length and frequency of trips, energy-saving homes and low-carbon fuels. Urban form and density can create the premises for reduced demand for mobility and for greater efficiency of energy use in buildings, including the opportunity to integrate low-carbon district heating and cooling networks with heat generated by low-carbon fuels or waste heat from industrial plants (IEA, 2016b).

The change in ALU may vary depending on city size. How fast are small cities growing compared to larger cities? Smaller cities tend to use more land per capita than larger cities. The fast-growing, newer cities also tend to use more land per capita compared to the older, slower-growing cities (Boyce, 1963). This is why an in-depth examination of Gibrat's law is interesting, to examine this not only at an aggregate level but also at an intraurban level. Controlling for city size using scaling enables us to compare the change in ALU for different groups of cities, that is, small versus large cities. This will be related to issues such as sprawl and urban expansion. The second argument for focusing on Gibrat is where the change in ALU happens and whether that location changes with city size—we can examine where the biggest change in ALU is occurring in relation to the CBD. Examining the internal structure of cities enables us to open and look inside the black box of city land use.

There is a long-established literature that uses scaling laws to compare cities (Batty, 2013; Bettencourt et al., 2007; Louf and Barthelemy, 2014). This chapter utilises a scaling methodology developed by Lemoy and Caruso (2018), who found that the radial ALU profiles of different cities are quite similar if the distance to the city centre is rescaled using the total population to an exponent ½. This rescaling enables us to hold population constant and to compare cities of different population sizes and hence different areas. This analysis examines ALU growth/change, as opposed to population growth. Population growth demands a certain level of expansion; however, when the expansion exceeds the given population growth it becomes a problem. Some cities may be using too much land compared to their size. We know that surface is related to population (Lemoy and Caruso, 2018); it is unclear whether a change in artificial surface is related to population or not, which is the main reason behind examining Gibrat's law.

This chapter examines ALU growth in Europe between 2006 and 2012. A radial analysis is used to calculate the level of artificial land use at several distances to the city centre. Using the radial scaling law of Lemoy and Caruso (2018) to control for population, we examine change in a systematic manner for 300 European cities. The compatibility of Gibrat's law and ALU is investigated first at an aggregate (city) level using ALU growth/change and the population based on the larger urban zone (LUZ) definition of cities. A disaggregated approach is then used to further examine the internal structure of change within cities.

Analysing where the change occurs within cities as a function of distance to the CBD will inform us of how this change is occurring. If city expansion is not homogenous across distance, where are the highest levels of urban expansion occurring? Are these distances the same across the range of city sizes?

2. METHODOLOGY

Cities are examined at both an aggregate and an intraurban level. A radial analysis is introduced to examine the internal structure of cities. Cities are analysed using concentric rings around a single point (co-ordinates of the historic city hall) to represent the CBD. A scaling exponent is then applied to control for city size. The data used in this analysis is from the Urban Atlas data set produced by the Copernicus land monitoring service (Copernicus, 2016). In this section we first describe the data and how ALU is defined then introduce the scaling law used in the analysis.

2.1 Urban Atlas

The data used comes from the EU Copernicus Urban Atlas (Copernicus, 2016), which is available at a five-metre resolution. The boundary of each city corresponds to its functional urban area (FUA). A subset of cities that appear in both the 2006 and 2012 editions are used, yielding 293 cities all located within the EU-27 and the UK. These cities range in population from 62 000 to 11 million (Paris and London). In defining the CBD of the FUA, the location of the historic city hall is used as the point to represent the CBD. As every city has a city hall, this is a method which enables us to perform the radial analysis in a systemic and consistent way (Walker, 2018; Wilson, 2012).

Between the two years of the Urban Atlas, 2006 and 2012, there were some changes to the boundaries of the FUAs. To ensure a common area between the two years, the 2006 FUAs are clipped using the 2012 FUAs and vice versa, which leaves us with their intersection. This ensures we are using the largest possible area that features in both the 2006 and 2012 Urban Atlas. This new common area is then used to calculate the population for each city by employing the 2006 EU GEOSTAT 1 km^2 population grid (Eurostat, 2012).

The 12 categories chosen to represent ALU are those where buildings are dominant, Urban Atlas codes starting with '11' or '12'. These are found to have the least variability between the two years and do not suffer from other issues relating to reclassifying facilities and amenities, such as for construction and urban green areas. For example a construction site in period t can cover a larger area than does the resulting building footprint in period $t+1$, with the remaining area in one of the other nonurbanised categories.

It is worth noting there are some limitations with the Urban Atlas. Reclassification between years occurs in it, and reclassifying land despite no changes occurring can make intertemporal analysis more challenging, as we want to ensure the increase in ALU we are observing is as a result of activity and not because of reclassification. In Munich where the biggest recategorisation of cemeteries occurred, ≈11 hectares were converted from green urban areas to an ALU category. However this represents only 0.00001 per cent of total artificial land for Munich, and we are satisfied our results are not sensitive to these small recategorisations. As one of the goals of this research is reproducibility, the number of edits made to the original data should be kept to a minimum. For this reason no changes have been made to the master Urban Atlas data.

2.2 Scaling

This chapter uses a previously discovered homothetic scaling law to transform ALU (Lemoy and Caruso, 2018). The ALU of a city was found to scale in a homothetic manner with city size measured by total population. More precisely the total artificial area of a city is proportional to its total population, and the radius of the city scales with the square root of its total population. This is the standard relationship between the area and the side length of a surface in two dimensions (a square or disc, for instance). We note that homothetic or isometric scaling uses a fixed factor for all parts of the considered system, in comparison to allometry, which uses different rates of growth (Thompson, 1917; Huxley, 1932) for different parts of the system.

This homothetic scaling of artificial land use can be expressed with mathematical relations. Lemoy and Caruso (2018) found that the radial ALU profiles $s(r)$ of different cities are quite similar if the distance r to the city centre is rescaled to a distance r', given by

$$r' = r \times \sqrt{\frac{N_{\mathrm{London}}}{N}} = r \times k \ , \tag{1}$$

where N is the population of the city being analysed and N_{London} is the population of the largest city in the data set, with the example of London in this case used as a reference.

$$k = \sqrt{\frac{N_{\mathrm{London}}}{N}} \tag{2}$$

is the rescaling factor. For London and Paris, $k \approx 1$. (See appendix for a worked example of scaling.)

The radial analysis of ALU in this chapter uses two different measures: rings and discs. These two measures provide different insights on artificial land use: measures in discs study the share of artificial land within a given distance r from the centre, while measures in rings study artificial land at a given distance r (more precisely between r and $r + \delta$, where δ is the width of the ring).

The surface $V(t,r,i)$ of a disc corresponding to a particular land use class or classes i with radius r at time (year) t is described in equation (3):

$$V\left(t,r,i\right) = \pi r^2 v\left(t,r,i\right), \tag{3}$$

where $v(t,r,i)$ is the share of the disc corresponding to land use class(es) i.

The surface $S(t,r_1,r_2,i)$ of a ring can be seen as the difference between the surfaces of an outer disc of radius r_2 and an inner disc of radius r_1:

$$S\left(t,r_1,r_2,i\right) = V\left(t,r_2,i\right) - V\left(t,r_1,i\right), \tag{4}$$

Equation 5 shows the share of the same ring corresponding to land use class(es) i.

$$s(t,r_1,r_2,i) = \left(\frac{S(t,r_2,i)}{\pi r_1^2 - \pi r_2^2}\right) \tag{5}$$

From these measures, we can now derive two temporal measures of ALU change. Note: Uppercase V and S correspond to the surface area of a disc and ring respectively, whereas lowercase v and s correspond to the share of a surface area. Equation (6) shows relative change in ALU $\Delta\, V(t_1,t_2,r,i)$, that is, how much a disc has changed given its previous level of ALU:

$$\Delta V(t_1,t_2,r,i) = \left(\frac{V(t_2,r,i) - V(t_1,r,i)}{V(t_1,r,i)}\right), \tag{6}$$

where Δ indicates change, V is the surface of a disc, r is the radius, i is the considered land use class(es) and t is time (year).

And $C(t_1,t_2,r,i)$ is the conversion rate, given by

$$C(t_1,t_2,r,i) = \left(\frac{v(t_2,r,i) - v(t_1,r,i)}{(1 - v(t_1,r,i))}\right). \tag{7}$$

Note that we subtract the share of a disc from 1 to calculate the total nonartificialised share of a disc. This conversion metric is more effective at showing change in ALU close to the CBD, where the artificial share of ALU is already high. By replacing v with s we compute these change measures for rings instead of discs. We replace r with r' if we are examining rescaled distances. Table 1 compares rescaled distance to the equivalent actual distance for a number of city populations.

In Figure 1, we map a ratio of core artificial land to peripheral artificial land, computed as

$$\frac{V(t,r_1',i)}{S(t,r_1',r_2',i)}. \tag{8}$$

Table 1 Scaling—Comparison of non-scaled and rescaled distances

	Rescaled distance			
	$r' = 15$ km	$r' = 20$ km	$r' = 40$ km	$r' = 60$ km
Population	Actual distance			
100 000	1	2	4	6
250 000	2	3	6	9
500 000	3	4	8	13
1 000 00	4	6	12	18
2 000 000	6	8	17	25
5 000 000	10	13	27	40

With $r_1' = 20$ km and $r_2' = 40$ km, that is a disc of radius r_1' and a ring of inner radius r_1' and outer radius r_2'. The higher the ratio, the more developed the city is at its core (within $r_1' = 20$ km) relative to the periphery (between $r_1' = 20$ km and $r_2' = 40$ km).

2.3 City Categories

To display the results more effectively, cities are grouped together based on common attributes. Three categories are used: city size, region and share of water. For city size, cities are grouped using an adapted version of the Organisation for Economic Co-operation and Development (OECD) size categories (Dijkstra and Poelman, 2012). We adapt the ranges so there are five categories instead of six: small (50 000–250 000), medium (250 000–500 000), large (500 000–1 million), X-large (1 million–2 million) and XX-large (2 million+). As there are only 13 cities with a population below 100 000, the small and medium categories from the OECD definitions are amalgamated to form a new small category (50 000–250 000). Each subsequent category is a doubling of the previous category, with the exception of the top and bottom categories.

From Table 2 we see 44 per cent of the total EU-27 population lived in these 293 cities, with 19 per cent living in the 24 largest cities, with populations over 2 million. Overall 60 per cent of the sample population live in a city with a population greater than 1 000 000.

To group cities based on their location and economic status, the year in which the country joined the European Union is used to create three categories: those that founded the European Economic Community (EEC) in 1957 (France, Germany, Italy, Luxembourg, Belgium, the Netherlands; labelled EU-1), countries that joined in the intervening years up to 1996 (UK, Ireland, Spain, Portugal, Greece, Austria, Sweden, Finland, Denmark; labelled EU-2) and finally new EU member states who joined between 2004–2013 (Poland, Romania, Czech Republic plus another seven; labelled EU-3). Countries in each of the three groups (EU-1, EU-2, EU-3) share common attributes: the founders' category contains Europe's wealthiest countries, many of which are heavily urbanised; those who joined in the following years were also advanced economies but located around the edge of Europe, such as the UK, Scandinavia and Iberia. The new member states are mostly former communist states and can benefit from the large share of recent EU structural funding.

Figure 1 Ratio of ALU r′*=20 km to ALU* r′*=20–40 km, 2012*

Table 2 Breakdown of cities by city size (2006 population)

City size	Population	No. cities	Share of sample	Share of EU-27 total (2011)
Small	15 669 921	96	0.07	0.03
Medium	30 830 288	84	0.14	0.06
Large	40 688 516	59	0.18	0.08
X-large	41 093 837	29	0.18	0.08
XX-large	93 939 287	24	0.42	0.19
Total	222 221 849	293	1	0.44

Differences in typology such as water or elevation may explain some of the differences in ALU and how a city expands and develops, influencing its urban form (Kasanko et al., 2006). The final categorisation takes this into account by measuring the share of coastal water surrounding the city, using Corine Land Cover (CLC) data for 2006. Following a radial analysis, the share of water within discs at various distances to the CBD is calculated; $r' = 40$ km is used along with a cut-off threshold of 10 per cent share of water, a value high enough to select only cities with a large body of water such as a sea, ocean or lake. Using a rescaled distance helps to capture only those cities where the share of water limits urban development given their size.

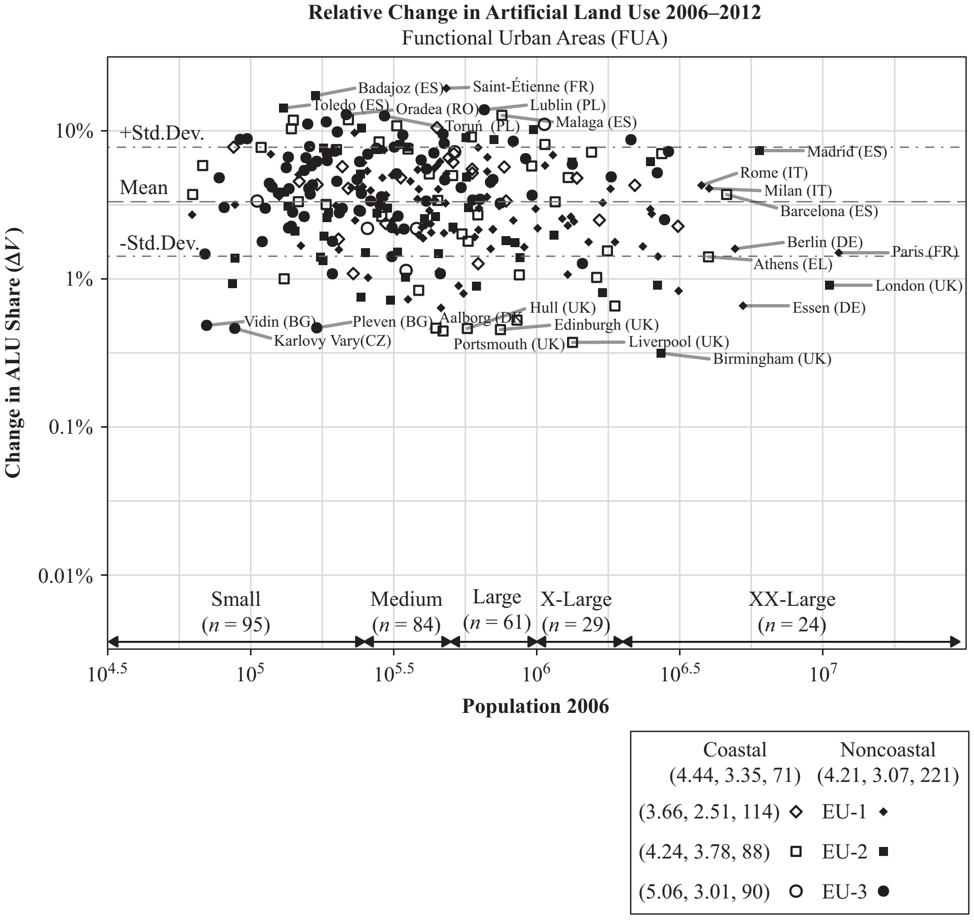
3. RESULTS

The results are divided into two sections. The first section examines the relationship between ALU and city size using different city definitions. The second section analyses the internal structure of cities with the addition of city categories.

3.1 Gibrat's Law for Land Use

Figure 2 shows the relationship between population and ALU relative change (ΔV) at the FUA level. We can see ΔV is constant across all populations. Most observations are within + or—one standard deviation of the mean, highlighting a narrow range. The largest cities tend to be at the lower end, but still within one standard deviation of the mean. The largest cities, Paris and London, are both below the mean, around minus one standard deviation. The third largest city, Madrid, is behaving rather differently to Paris and London but still within the range. Interestingly there appears to be a cluster of former industrial powerhouse cities of Essen, Birmingham and Liverpool to the bottom right. This below-average level of ALU growth may be a consequence of the decline in certain industry and manufacturing sectors in Europe such as textiles (50 per cent decrease in production 1995–2015; Eurostat, 2019).

Utilising radial discs, the change in ALU (ΔV) is calculated at different distances to the CBD: $r' = 20$ km and $r' = 40$ km. In Figures 3 and 4 the log of ΔV between 2006 and 2012 is plotted against the log of population for 2006 to test whether ΔV satisfies Gibrat's law at different distances to the CBD. In Figure 3 we examine the internal relative change in ALU share for $r' = 20$ km. This figure measures total ALU within a distance of $r' = 20$ km to the



Note: Axes are log scaled

Figure 2 Relative change in ALU (ΔV)—FUA

CBD for 2006 and 2012. The relative change is then computed for each city. Compared to Figure 2, we can see that the mean change is lower (≈1 per cent compared to ≈5 per cent in the aggregate measure). The change in ALU for cities is also more dispersed than before. There are more cities outside one standard deviation of the mean. This is reflected in the distance between the two standard deviations being larger. Paris has a lower level of change at $r' = 20$ km but is still within one standard deviation. The change for London has decreased even further. The low levels for these cities can be explained by them already having a high level of ALU at these distances. There is limited land availability, as most available land plots are already urbanised. The levels of relative change decreased as a result of examining an area smaller than the FUA.

Comparing Figure 3 to Figure 4 we can see the cities are more tightly clustered. This suggests that at r'=40 km cities are experiencing similar levels of ALU growth/change. There is a group of cities—Cardiff, Belfast and Edinburgh—outside one standard deviation of the

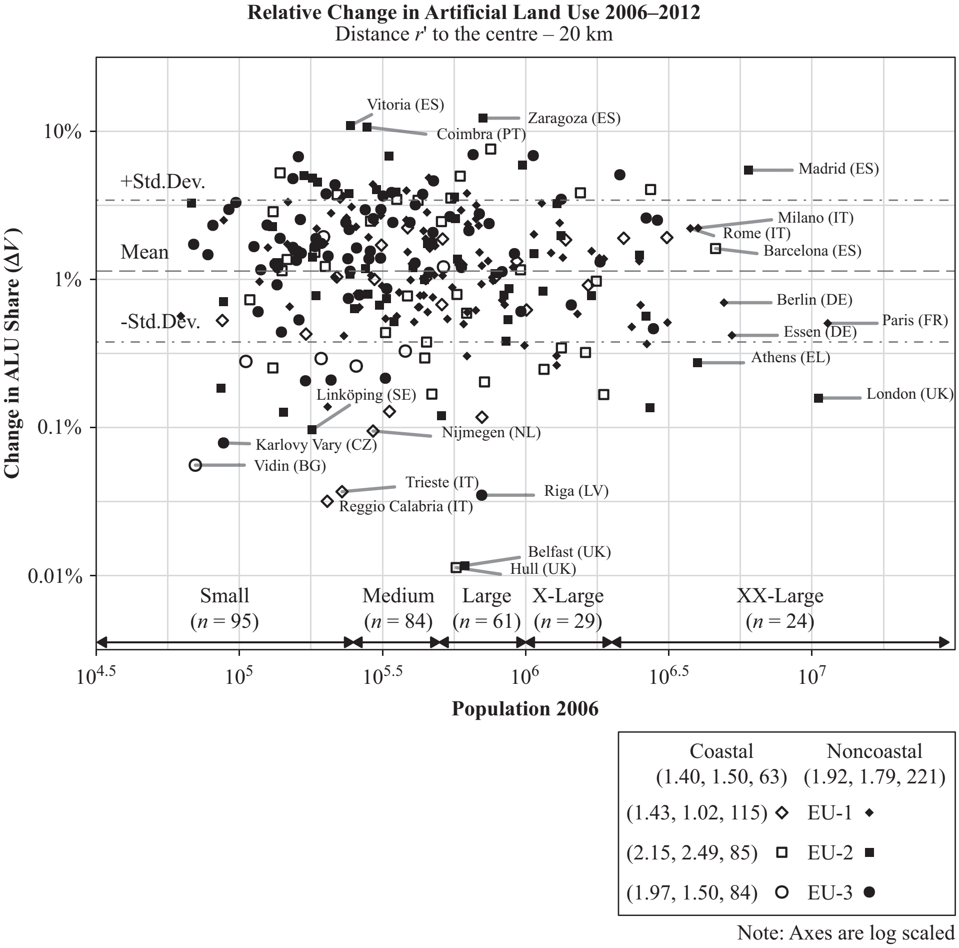
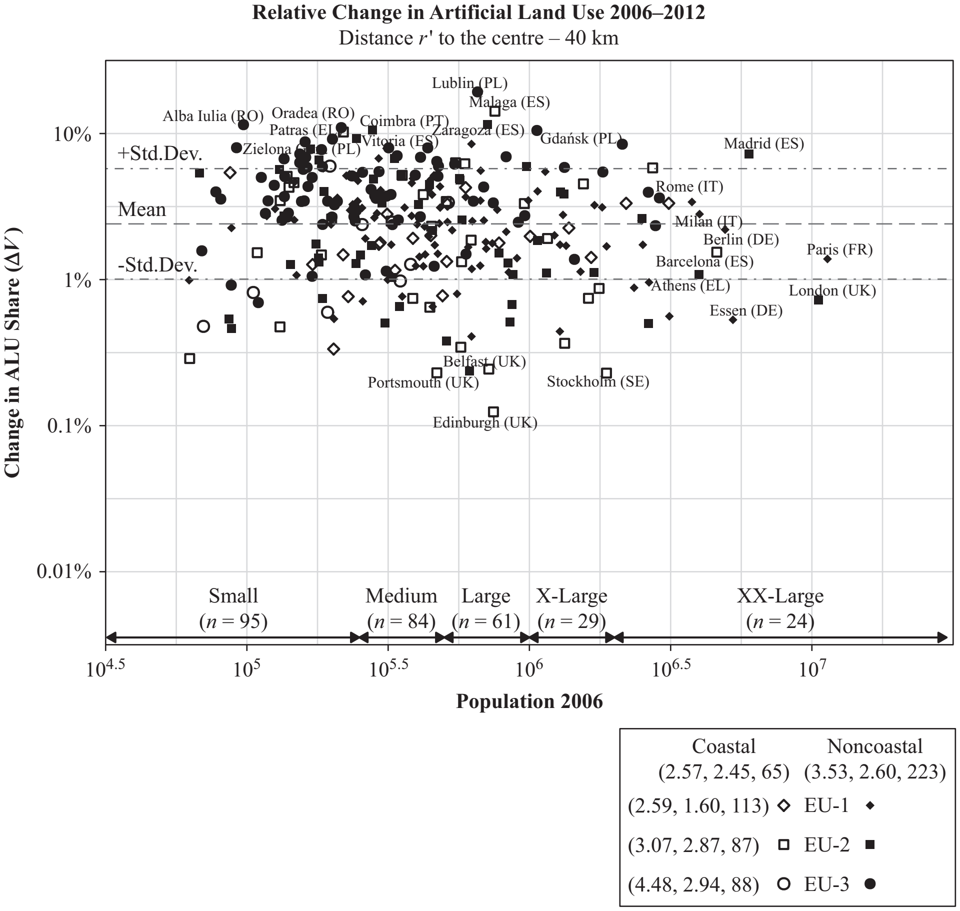


Figure 3 *Relative change in ALU (ΔV)—r′=20 km*

mean. Given these are all in the UK there may be some economic reason behind this. The mean change is also higher, with smaller distances between standard deviations, reflecting the fact that cities are more closely clustered. In the $r' = 20$ km and $r' = 40$ km graphs, while the mean value of EU-3 is higher, the standard deviation is also higher, suggesting there is high within-group variability. There are similar results for noncoastal cities. Cities such as Lublin, Madrid and Malaga are constantly around 10 per cent ΔV in all three graphs, highlighting a large change has occurred there. The lowest change appears to have occurred among EU-1 and EU-2 cities.

The debate around Gibrat's law is that it is based upon how we define cities. We show that the rescaling method is robust to the way in which we draw that definition. At a narrow definition of a city $r' = 20$ km or at a wider definition $r' = 40$ km, ΔV and population appear to meet Gibrat's law. The results highlight the strength of Gibrat's law for ALU growth, as it is



Note: Axes are log scaled

Figure 4 Relative change in ALU (ΔV)—r′=40 km

robust irrespective of city definition ($r' = 20$ km, $r' = 40$ km or FUA). Each of the three graphs yields a similar pattern. Even the use of the FUA definition of cities satisfies Gibrat's law. City definition based on radial distance to the CBD has highlighted a number of cities at the upper and lower ends that were not highlighted when using the FUA definition. These cities warrant further investigation to understand the low changes in ALU.

Although Figures 3 and 4 are characteristic of Gibrat's law, $r' = 40$ km corresponds to the law more strongly. As there is a greater variation in the change for $r' = 20$ km, the pattern is more dispersed. Compared to the aggregate measure of ALU change for cities, examining the internal structure of cities has shown that the change is not homogeneous across all distances to the CBD. Relative change in ALU is lower at distances closer to the CBD, but a larger part of the available (nonartificial) land is converted to artificial land uses. We turn now to a deeper study of these intraurban variations.

3.2 Internal Structure of Cities

Figure 5 below shows the relative change in ALU between 2006 and 2012. A rolling mean with a $r' = 2$ km window is used to reduce the noise in the graph because of the high level of variability in ALU change across rings within a city. The relative change in ALU increases with the distance to the centre. Mean ALU change increases linearly from 0 per cent in the centre until a distance of $r' = 30$ km, after which the increase is constant, around 6 per cent. The graph highlights that ALU increase is higher in the outer suburbs and periphery. There is more variation at rescaled distance $r' = 10$ km than at $r' = 60$ km.

Cities at $r' = 40$ km are changing at different levels. Some are exhibiting greater levels of urban sprawl than others. There appears to be a turning point at $r' = 20$ km beyond which city groups begin to diverge. The noncoastal EU-3 have the highest levels of ALU change beyond $r' = 20$ km with a growth rate greater than the 75th percentile on average. The ALU change for EU-3 coastal cities varies; this can be explained by the low number of cities ($n = 6$) in this category. The lowest levels of growth were experienced in EU-1 coastal cities; the difference

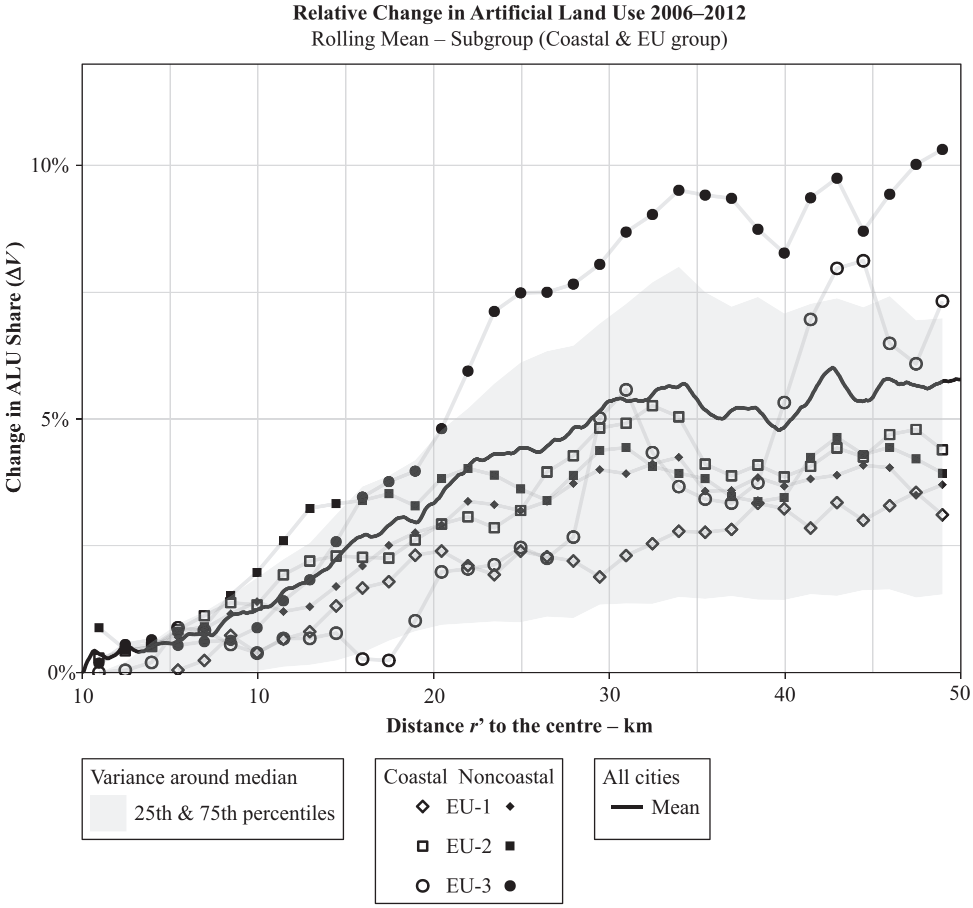


Figure 5 Relative change in ALU (2006–2012) by EU group and coastal status

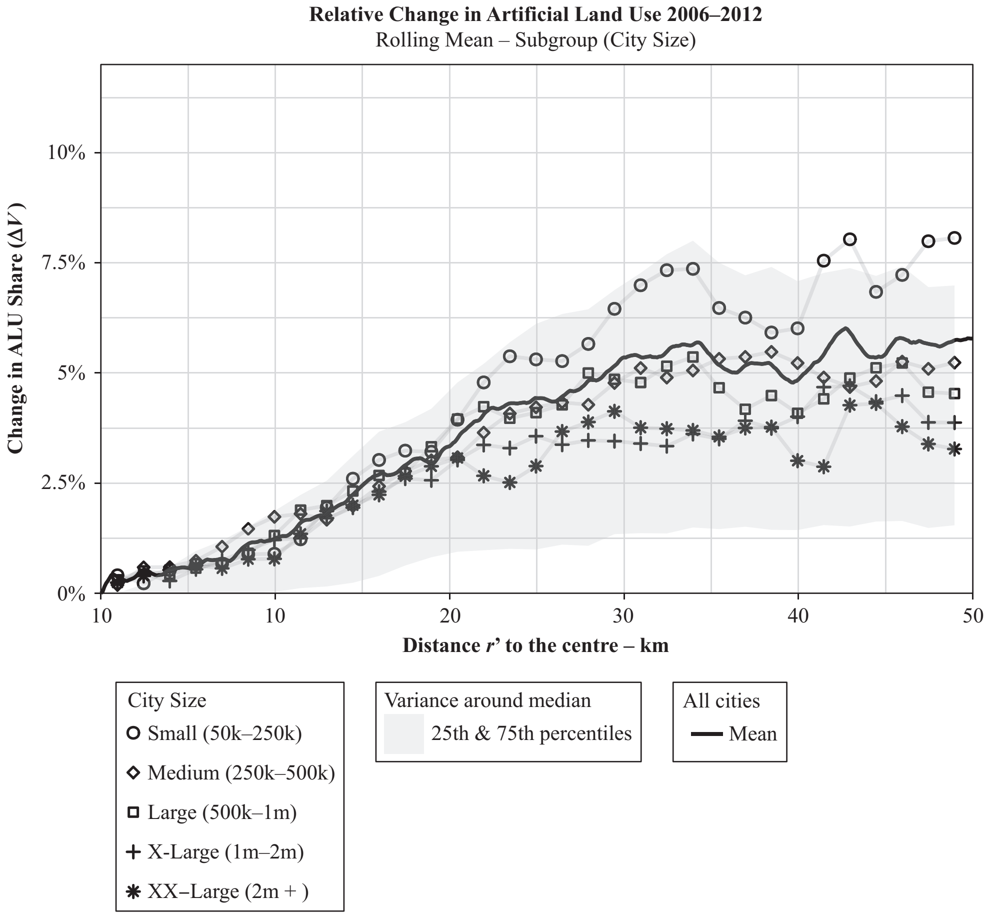


Figure 6 Relative change in ALU (2006–2012) grouped by city size

between EU-1 coastal and noncoastal cities highlights the impact of topography on ALU change. For all city categories a change in the rate of growth occurs at $r' = 20$ km.

In Figure 6 cities are grouped by city size category. At distances $r' = 0$–20 km cities on average have similar levels of ALU change. Beyond a distance of $r' = 20$ km there is a divergence in the rate of change, with small/medium cities growing more on average than XX-large cities. Focusing on $r' = 40$ km we see a clear sorting of the city size categories from smallest to largest. There is a slight deviation from Gibrat's law, with small/medium cities growing more on average than larger cities. To examine this finding further the conversion rate of nonurbanised land to ALU is examined. Larger cities may already have higher levels of ALU than smaller cities, which would reduce the quantity of land available for development.

Figure 7 shows the moving average of the conversion rate of nonurbanised land. Although the relative change in ALU increases with distance to the centre, the change as a share of nonurbanised land decreases (rather linearly) with distance to the CBD. At distances less than $r' = 12$ km there is high variation between the city size categories; beyond this distance, however, again there is a sorting by city size category. Beyond $r' = 12$ km small/medium cities

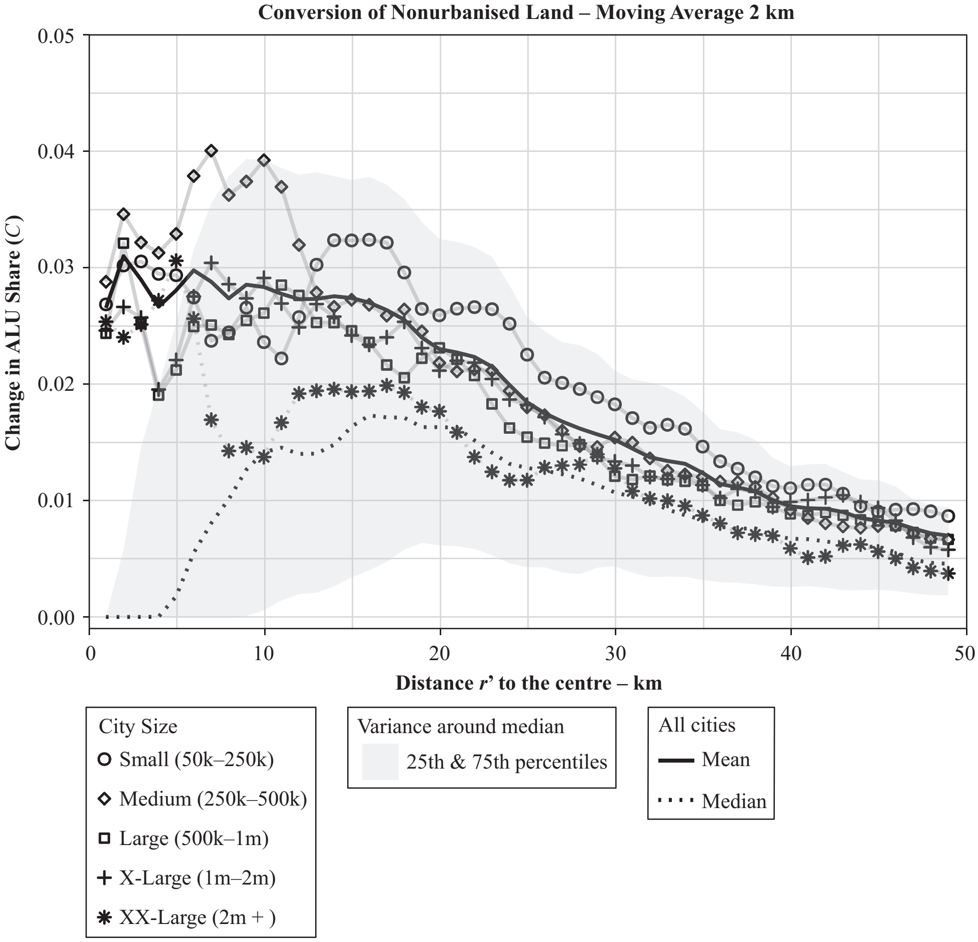


Figure 7 Conversion of nonurbanised land to ALU

are on average experiencing greater levels of urban expansion. This suggests that these smaller cities are sprawling more than XX-large cities such as Paris or London. The large difference between the mean and median value for all cities highlights the wide variation in the conversion rate between cities. For over 50 per cent of cities the conversion rate is 0 per cent at $r' < 5$ km. Although redevelopment of existing urbanised land may occur, these curves show that the potential to develop nonurbanised land may be low.

4. CONCLUSION

In this chapter we have examined whether Gibrat's law holds in relation to urban land use change across 293 cities in the EU and UK. In conducting such an exercise we have attempted to examine a concept such as Gibrat's law through the use of a data-rich source, the Urban Atlas. Our results have shown that a kind of Gibrat's law holds for urban land use change.

However, there is a small deviation as larger cities grow slightly less than smaller ones. The aggregate measures of ALU growth/change (ΔV) are shown to follow Gibrat and are robust to changes in the way we define a city. Results are similar whether the aggregate FUA, $r' = 20$ km (reflecting the city core) or $r' = 40$ km (considering the suburbs) is used. Using a tight definition of a city, as in the $r' = 20$ km case, we see greater variability in the level of ALU change. This may reflect the differences across cities in demand for land in the city core.

There is, however, a need to focus on the internal structure and internal change of ALU across cities. Utilising the scaling law found by Lemoy and Caruso (2018) has enabled us to examine the internal structure of cities despite cities having various populations and extents. Cities are more or less growing in a consistent way in their cores; however, in the peripheries there are clearly different growth rates. Grouping cities based on size and examining the intracity growth rates, there are clear differences between city size groups, with smaller cities having higher ALU growth rates. These differences, however, only become apparent beyond $r' = 20$ km. New EU member states (EU-3) grew at a faster rate than older, more established member states (EU-1 and EU-2). The reasons behind these differences warrant further investigation.

The overall trend of the internal structure of cities shows increasing levels of ALU at all distances. These increasing levels of urban expansion point to increasing levels of urban sprawl, with more sprawl occurring in smaller cities. This raises important questions around the sustainability of cities, as this evidence points to increasing urban sprawl and stagnant growth in urban centres across cities of all sizes. It also bears theoretical implications for the nature of sprawl and its scaling.

Turning to the conversion of nonurbanised land, there is a differentiation between large and small cities around the intensity of the change and where it occurs. Large cities appear to have a greater level of conversion at the core compared to smaller cities; beyond the core smaller cities have greater levels of conversion. Due to the demand for land in larger cities, there would appear to be a greater incentive to utilise any nonurbanised land. For smaller cities this is perhaps not as large an issue, as land in the periphery can be easily sourced and the actual distance to the CBD remains relatively small.

The results highlight issues relating to how we define a city and more specifically its outer limit. Using a narrow versus a wider definition, we are able to see the level of variability in ALU change. A recursive definition for cities is required to move away from subjectivity defining city boundaries/limits. Changes in the fractal dimension of built-up areas may also offer further insights into the boundary used for a city (Frankhauser; 1998; Tannier et al., 2011; Tannier and Thomas, 2013). Scaling has the potential to be the solution here, enabling us to determine a city's extent through the use of data.

In previous years, due to computing limitations or a lack of data, it was not possible to examine concepts such as Gibrat's law at a detailed level of disaggregation. Computing power has also improved considerably, as evidenced in the downward trajectory in the price per GB of RAM ($2384 in 2000, $23 in 2010 and $8 in 2017) (McCallum, 2020). We are currently experiencing a data revolution, with data sets such as the Urban Atlas and the Global Human Settlement Layer becoming more common and more detailed. Further studies should benefit from new data sources to examine socioeconomic characteristics such as income and/or property prices at an intracity level. As we have shown, aggregate 'city'-based measures hide a large amount of heterogeneity that occurs within cities. Examining these data at an intracity level may be what is required to expand our understanding of the complexity of cities, particularly in relation to how they grow, and make concepts such as

Zipf's and Gibrat's laws more relevant than ever. The growth in open-source data, methods and computing power combined could see a rediscovery and empirical re-examination of published theories from yesteryear.

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TECHNICAL APPENDIX

1. Land Use Categories

Artificial land use is examined to measure the increasing/decreasing levels of soil sealing across European cities. Out of the 20 land use categories for 2006 and 24 land use categories in 2012, 12 categories are combined to calculate a measure of artificial land use (ALU); land use classes 1–12 form Table A.1. When we refer to ALU, it is these 12 categories to which we are referring.

2. Scaling

Example 1: Worked example of scaling
Rescaling Liverpool, UK, using its population (1 371 238) and the reference city of London, UK, population (11 312 174). Equation (9) shows how the rescaling factor k is calculated:

$$k = \sqrt{\frac{Pop_{\text{London}}}{Pop_{\text{Liverpool}}}} = \sqrt{\frac{11\,312\,174}{1\,371\,238}} \simeq 2.872 \tag{9}$$

Table A.1 Urban Atlas land use classes

#	Category
1	Continuous urban fabric (S.L.: >80%)
2	Discontinuous dense urban fabric (S.L.: 50%–80%)
3	Discontinuous medium-density urban fabric (S.L.: 30%–50%)
4	Discontinuous low-density urban fabric (S.L.: 10%–30%)
5	Discontinuous very low-density urban fabric (S.L.: <10%)
6	Isolated structures
7	Industrial, commercial, public, military and private units
8	Fast transit roads and associated land
9	Other roads and associated land
10	Railways and associated land
11	Port areas
12	Airports
13	Mineral extraction and dump sites
14	Construction sites
15	Land without current use
16	Green urban areas
17	Sports and leisure facilities
18	Arable land (annual crops)
19	Forests
20	Water

Here k is used as the rescaling factor to rescale distances for Liverpool. Equation (10) illustrates this for Liverpool using a distance of 1.2 km.

$$r' = r \times k = 1.2 \times 2.872 = r' = 3.456 \text{ km} \tag{10}$$

Result: After rescaling, 1.2 km Liverpool has a similar land use profile to 3.4 km London. At 10 km, Liverpool has a similar profile to 29 km London.

3. Geoprocessing

The data from the Urban Atlas is first rasterised into a 20-metre resolution grid. Each raster cell is given the land use of the polygon at the centre of cell. With the CBD as the centre point, concentric rings with a fixed width $\delta = 100\sqrt{2} \approx 141$ m are created until the outer edge of the FUA is reached. It is then possible to examine the share of ALU at various distances to the CBD. In addition to rings, ALU shares are calculated using discs. As opposed to rings, discs consider the entire area of the concentric circle with radius r, not just the area of the difference between a circle with radius r and a circle with radius $r+\delta$.