

Pair decay width of the Hoyle state and carbon production in stars

Peter von Neumann-Cosel¹, Maksym Chernykh¹, Hans Feldmeier^{2,3},
Thomas Neff² and Achim Richter^{1,4}

¹ Institut für Kernphysik, Technische Universität Darmstadt, D-64289 Darmstadt, Germany

² Gesellschaft für Schwerionenforschung (GSI), D-64291 Darmstadt, Germany

³ Yukawa Institute for Theoretical Physics, Kyoto, Japan

⁴ ECT*, Villa Tambosi, I-38123 Villazzano (Trento), Italy

E-mail: vnc@ikp.tu-darmstadt.de

Abstract. The pair decay width of the first excited 0^+ state in ^{12}C (the Hoyle state) is deduced from a novel analysis of the world data on inelastic electron scattering covering a wide momentum transfer range, thereby resolving previous discrepancies. The extracted value $\Gamma_\pi = (62.3 \pm 2.0) \mu\text{eV}$ is independently confirmed by new data at low momentum transfers measured at the S-DALINAC and reduces the uncertainty of the literature values by more than a factor of three. A precise knowledge of Γ_π is mandatory for quantitative studies of some key issues in the modeling of supernovae and of asymptotic giant branch stars, the most likely site of the slow-neutron nucleosynthesis process.

1. Introduction

The production of the most abundant stable carbon isotope ^{12}C in the center of stars proceeds through the fusion of three α particles (the triple- α process). Its reaction rate is of critical significance for a variety of key problems of nuclear astrophysics [1] like the elemental abundance in the universe [2], the size of the iron core in massive stars determining the properties of supernova explosions [3, 4], the dynamics of asymptotic giant branch (AGB) stars [5], the site of the main slow-neutron capture process (s -process) of heavy elements [6], or the weak s -process in massive stars [7].

Under most astrophysical conditions the reaction exclusively proceeds through a scattering resonance of three α -particles that represents an excited state in ^{12}C with quantum numbers $J^\pi = 0^+$ at an excitation energy $E_x = 7.654$ MeV (the so-called Hoyle state [8]), which then decays to the stable ground state. The reaction rate of the resonant triple- α process is proportional to the radiative decay width Γ_{rad} of the Hoyle state. Although this state is experimentally studied for more than 50 years, at present Γ_{rad} is known with an uncertainty of about $\pm 12\%$ only. However, an accuracy of about $\pm 5\%$ is requested for applications in astrophysics [3, 5].

Experimentally the radiative width cannot be accessed directly but is determined as a product of quantities

$$\Gamma_{rad} = \Gamma_\gamma + \Gamma_\pi = \frac{\Gamma_\gamma + \Gamma_\pi}{\Gamma} \cdot \frac{\Gamma}{\Gamma_\pi} \cdot \Gamma_\pi \quad (1)$$

measured in different experiments. Here, $\Gamma = \Gamma_\alpha + \Gamma_\gamma + \Gamma_\pi$ is the total decay width taking into account α , γ and e^\pm decays of the Hoyle state. Whereas the first quantity on the right-hand side of Eq. (1) has been precisely determined with an uncertainty of $\pm 2.7\%$ (see Ref. [9] and refs. therein), the branching ratio Γ_π/Γ is known with an error of $\pm 9.2\%$ only (see Ref. [10] and refs. therein). However, a new measurement of Γ_π/Γ with an expected precision of about $\pm 5\%$ is currently underway [11].

The pair decay width Γ_π in Eq. (1) can be determined by inelastic electron scattering [12]. Two values were extracted from measurements at low momentum transfers q with a so-called 'model-independent' analysis [13, 14] in the distorted wave Born approximation (DWBA) explained in detail below. They agree well with each other and an averaged value $60.5 \pm 3.9 \mu\text{eV}$ is quoted in Ref. [15]. Alternatively, a Fourier-Bessel analysis [16] of the transition form factor including data over a wide momentum transfer range has recently been done by Crannell *et al.* [17]. The extracted value of $\Gamma_\pi = 52.0 \pm 1.4 \mu\text{eV}$ has an error of 2.7% only, but clearly deviates from the low- q results.

2. New experiment at low momentum transfer

To resolve this discrepancy we have performed a new high-resolution electron scattering experiment at low momentum transfer and reinvestigated both approaches. A novel model-independent ansatz was developed to analyze the global form factor. This provides not only a precise determination of the pair width Γ_π but also an important test of current models for the structure of the Hoyle state [18], which represents a prime example of an α -cluster state predicted to possess the features of a low-density gas of α particles resembling a Bose-Einstein condensate [19]. As shown in Ref. [18], state-of-the-art models describe the (e, e') form factor quite well over a broad momentum transfer range but predict a monopole matrix element (directly related to Γ_π) about 20% too large (cf. Table I in [18]). The present very precise results confirm this discrepancy and provide new insight into its origin.

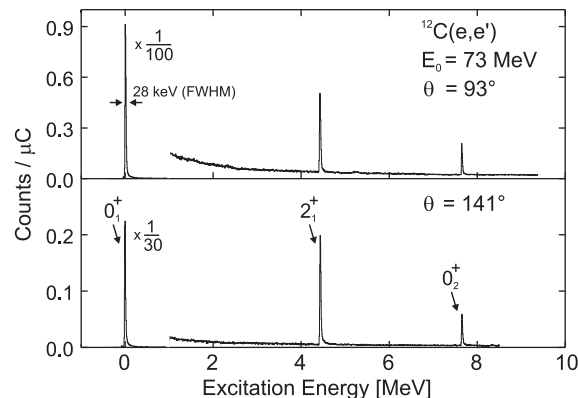


Figure 1. Spectra of the $^{12}\text{C}(e,e')$ reaction measured at a beam energy $E_0 = 73 \text{ MeV}$ and scattering angles $\theta = 93^\circ$ (top) and $\theta = 141^\circ$ (bottom).

The experiment was carried out at the high energy-resolution magnetic spectrometer [20] of the Darmstadt superconducting electron linear accelerator S-DALINAC. Excitation energy spectra were taken at initial electron energies between 29 MeV and 78 MeV and scattering angles from 69° to 141° with beam currents of about $1 \mu\text{A}$. The momentum transfer range for the excitation of the Hoyle state varied thus between 0.21 fm^{-1} and 0.66 fm^{-1} . A self-supporting carbon target (98.9% ^{12}C) with an areal density of 6.4 mg/cm^2 was used. In dispersion-matching mode an energy resolution $\Delta E \approx 28 \text{ keV}$ (full width at half maximum, FWHM) was achieved.

Examples of spectra at a beam energy of 73 MeV are presented in Fig. 1. The peaks correspond to the elastic line, the 2_1^+ state and the Hoyle state in ^{12}C .

As the form factor is the Fourier transform of the density, it corresponds to a cross section calculated in plane wave Born approximation (PWBA). The measured cross section, however, corresponds to the distorted wave Born approximation (DWBA). Therefore, we deduce the square of the experimental form factor - the differential cross section divided by the Mott cross section - at the measured momentum transfer q_i and electron beam energy E_{0i} by

$$|F_{\text{tr}}^{\text{exp}}(q_i)|^2 = \frac{4\pi}{Z^2} B(C0, q_i, E_{0i}) \frac{B^{\text{PWBA}}(q_i)}{B^{\text{DWBA}}(q_i, E_{0i})}, \quad (2)$$

where Z is the number of protons and $B(C0, q_i, E_{0i})$ denotes the reduced transition probability, while $B^{\text{PWBA}}(q_i)$ and $B^{\text{DWBA}}(q_i, E_{0i})$ are reduced transition probabilities calculated within plane wave and distorted wave Born approximation, respectively. This so-called PWBA transformation allows to relate cross sections measured at different q_i and E_{0i} directly to the form factor or corresponding transition density. The data of the different experiments should then collapse into a single line for $|F_{\text{tr}}^{\text{exp}}(q_i)|^2$. The factors $B^{\text{PWBA}}(q_i)/B^{\text{DWBA}}(q_i, E_{0i})$ in Eq. (2) are computed with the code of Heisenberg and Blok [21] in an iterative procedure starting with the transition density $\rho_{\text{tr}}(r)$ taken from the models discussed in Ref. [18], e.g. the Fermionic Molecular Dynamics (FMD) model [22].

The form factor $F_{\text{tr}}(q)$ and density $\rho_{\text{tr}}(r)$ of a monopole transition are related by

$$F_{\text{tr}}(q) = \frac{4\pi}{Z} \int_0^\infty \rho_{\text{tr}}(r) j_0(qr) r^2 dr = \frac{1}{Z} \sum_{\lambda=1}^\infty \frac{(-1)^\lambda}{(2\lambda+1)!} q^{2\lambda} \langle r^{2\lambda} \rangle_{\text{tr}}, \quad (3)$$

where $\rho_{\text{tr}}(r) = \langle 0_1^+ | \hat{\rho}(\vec{r}) | 0_2^+ \rangle$ is the matrix element of the charge density operator $\hat{\rho}(\vec{r})$ between the ground state and the Hoyle state.

3. Pair decay width from global fit

In a first step we combine our new results with the previous world data set [17] and perform a model-independent analysis of the $0_1^+ \rightarrow 0_2^+$ transition form factor by a global fit to all data using the model-independent ansatz

$$F_{\text{tr}}(q) = \frac{1}{Z} e^{-\frac{1}{2}(bq)^2} \sum_{n=1}^{n_{\text{max}}} c_n (bq)^{2n} \quad (4)$$

which respects the condition $F_{\text{tr}}(q=0) = 0$ and the fact that $j_0(qr)$ contains only even powers of q . The fit parameters are b and c_n , where b plays the role of a length scale. The transition density $\rho_{\text{tr}}(r)$ corresponding to the ansatz (4) is given by

$$\rho_{\text{tr}}(r) = \frac{1}{b^3} e^{-\frac{1}{2}\left(\frac{r}{b}\right)^2} \sum_{n=0}^{n_{\text{max}}} d_n \left(\frac{r}{b}\right)^{2n}, \quad (5)$$

where the relation between the coefficients c_n and d_n can be calculated by the inverse transformation of Eq. (3).

Due to the Coulomb distortion of the in- and outgoing scattering states the cross section remains finite at momentum transfers where the form factor goes through zero and hence the uncertainty in deducing $|F_{\text{tr}}^{\text{exp}}(q)|$ according to the PWBA transformation Eq. (2) is large around $q = 2.1 \text{ fm}^{-1}$, see Fig. 2. Therefore only data $|F_{\text{tr}}^{\text{exp}}(q)|^2 > 10^{-6}$ are considered. Variation of the maximum power of the polynomial in Eq. (4) shows that $n_{\text{max}} = 5$ is sufficient for the present analysis.

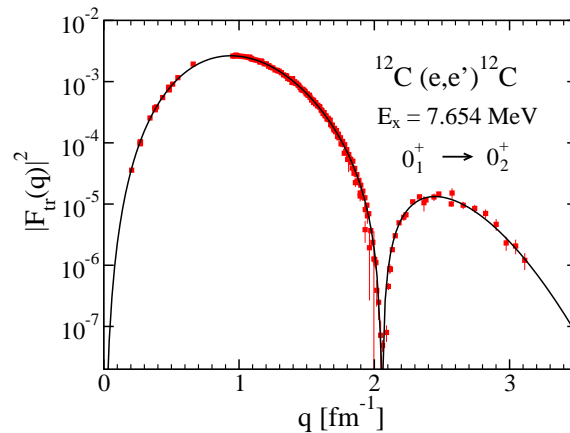


Figure 2. Squared form factor for the transition from the ground state in ^{12}C to the Hoyle state extracted by the model-independent analysis (solid line) described in the text in comparison to the experimental data [14, 17] transformed according to Eq. (2).

Figure 2 shows the resulting global fit. It describes the data well over the whole q range of measured momentum transfers including the minimum region, where data were excluded from the fit. The following transition radii are extracted: $\langle r^2 \rangle_{\text{tr}} = (5.47 \pm 0.09) \text{ fm}^2$ and $\langle r^4 \rangle_{\text{tr}} = (115 \pm 8) \text{ fm}^4$. The uncertainties are estimated by varying the data base because they are dominated by systematic errors in the data sets. The accuracy is essentially limited by the experimental uncertainties at $q > 1.7 \text{ fm}^{-1}$ where the cross sections are small.

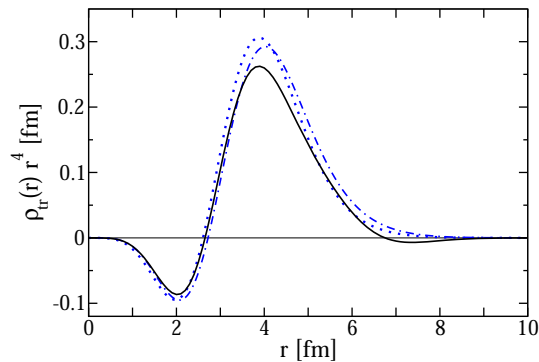


Figure 3. Transition charge density corresponding to the form factor displayed in Fig. 2 multiplied by r^4 (full line) in comparison to the theoretical predictions [18] of FMD (dotted line) and an α particle gas (dashed-dotted line).

In Fig. 3 we display $\rho_{\text{tr}}(r)r^4$ to show that $\langle r^2 \rangle_{\text{tr}}$, which is 4π times the integral over this function, is very sensitive to contributions beyond $r = 4 \text{ fm}$ where the ground state density is very small but the density of the Hoyle state is not, see Ref. [18]. The transition densities obtained in the FMD and BEC (gas of α -particles) models are somewhat too large beyond 4 fm indicating that the calculated charge density of the Hoyle state is overestimated at large distance. Although the deviation does not seem to be big the transition radius $\langle r^2 \rangle_{\text{tr}}^{\text{calc}}$ is for both models about 20% too large because negative and positive contributions cancel up to $r \approx 3.5 \text{ fm}$ and only the region beyond 3.5 fm matters.

Crannell *et al.* [17] used the same data, except for the seven new data points, and performed a Fourier Bessel (FB) analysis [16]. Figure 3 illustrates that beyond their adopted cut-off radius

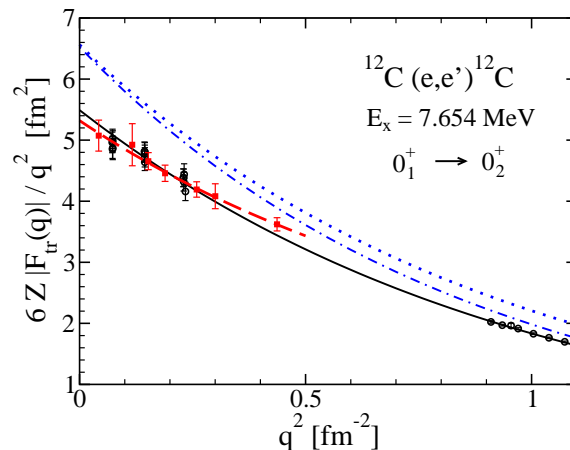


Figure 4. Experimental form factors at low q corrected by Eq. (2) compared to different model approaches. Extrapolation to zero momentum transfer provides the monopole matrix element. Square symbols and dashed line: new data and polynomial fit with Eq. (6). Circles and full line: old data and global fit with Eq. (4). Dashed-dotted line: FMD model, dotted line: α -cluster model [18].

$R_c \approx 8.5$ fm the transition density vanishes. They concluded however a significantly smaller Γ_π . To understand this discrepancy we take our fit as an approximation of the experimental form factor and read off the FB expansion coefficients at $q_\nu = \nu\pi/R_c$ for $\nu = 1$ up to $\nu_{max} = 8$. The resulting transition density has $\langle r^2 \rangle_{tr}^{FB} = 4.99$ fm² or $\Gamma_\pi^{FB} = 51$ μ eV in accordance with the result of [17]. In order to reproduce our own result within a FB analysis we have to go up to at least $\nu_{max} = 10$ which corresponds to $q_{max} = 3.7$ fm⁻¹ where no data exist. In general one should keep in mind that due to the oscillatory behavior of the Bessel functions a FB expansion can only reproduce the tail region well, if ν_{max} and q_{max} are sufficiently large.

4. Pair decay width from low q expansion

The Taylor expansion of the form factor around $q = 0$ offers itself a model-independent way to deduce the transition radius $\langle r^2 \rangle_{tr}$ which is the leading coefficient in the expansion

$$-6Z \frac{F_{tr}(q)}{q^2} = \langle r^2 \rangle_{tr}^{low\ q} - \frac{q^2}{20} \langle r^4 \rangle_{tr}^{low\ q} + \dots \quad (6)$$

which is used to fit the low- q data measured at the S-DALINAC. The quantities $\langle r^2 \rangle_{tr}^{low\ q}$ and $\langle r^4 \rangle_{tr}^{low\ q}$ are taken as free parameters. Higher powers up to q^8 are included in the fit such that the last term in Eq. (6) contributes less than 1% of the first term. Because of the limited number of data points we approximated the higher transition matrix elements by $(\langle r^2 \rangle_{tr}^{low\ q})^n \cdot x_n$ with $x_n = \langle r^{2n} \rangle_{tr} / \langle r^2 \rangle_{tr}^n$. These ratios were taken from the results of the global fit to all data.

The dashed curve in Fig. 4 represents the fit to the data resulting in $\langle r^2 \rangle_{tr}^{low\ q} = (5.29 \pm 0.14)$ fm² and $\langle r^4 \rangle_{tr}^{low\ q} = (96 \pm 10)$ fm⁴. As can be seen from Fig. 4 an extraction of the transition radius by extrapolation to $q = 0$ considering only low- q data is very sensitive to the experimental errors. The global fit to all data (full line) reproduces the new low- q data. The slope of the polynomial fit (dashed line) is apparently not steep enough to match the data at higher momentum transfer around $q^2 = 1$ fm⁻². The inherent uncertainty when considering low- q data only has also been pointed out by Sick for an extraction of the proton charge radius from elastic electron scattering [23]. Although the value of Γ_π obtained with our new data at

low q agrees within error bars with the global fit it is clear from Fig. 4 that the fit of all available data with a suitable ansatz is more reliable.

The deviations of the model transition densities at large radii (cf. Fig. 3) lead to $\langle r^2 \rangle_{\text{tr}}$ values exceeding the experimental results. Moreover, Fig. 4 demonstrates that also the slopes at $q = 0$ (proportional to $\langle r^4 \rangle_{\text{tr}}$) are too steep indicating the need for an improved description of the tails of the model wave functions.

5. Summary

To summarize, the pair decay width of the Hoyle state has been extracted from a new $^{12}\text{C}(e,e')$ experiment at low momentum transfers and independently with a novel model-independent global fit of the world data. The latter, shown to be less prone to systematic errors, provides $\Gamma_\pi = (62.3 \pm 2.0) \mu\text{eV}$ in accord with values deduced 40 years ago but with much reduced uncertainty. Combined with an improved determination of the pair width branching ratio presently underway, the astrophysical reaction rate of the triple- α process is then known with a precision sufficient to quantitatively constrain the modeling of a variety of key problems in nuclear astrophysics. The result for the pair width deviates from a more recent Fourier Bessel (FB) analysis. The origin of this discrepancy is explained by shortcomings of the FB and the lack of data beyond $q = 3.1 \text{ fm}^{-1}$. Furthermore, the study of the monopole transition to the Hoyle state by electron scattering at low momentum transfer provides important constraints on models by a unique test of the nuclear wave function at large distances where one expects α -cluster structures. A full account of the present work can be found in Ref. [24].

Acknowledgments

This work has been supported by the DFG under contract SFB 634.

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