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SYNTHESIS OF ADAPTIVE CRITICAL CONTROL METHODS, IDENTIFICATION AND MANAGEMENT

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The main problem of dynamic plant control is the variability of the parameters of the control object, and we are talking even about the nature of external disturbances, the lack of information about their statistical or stochastic characteristics. An approach to the development of an adaptive controller based on the recurrent least squares method is proposed.

Introduction

When solving control problems, it is assumed that the structure and parameters of the object are a priori known and unchanged on the accepted control interval. In real life, things are different. The object of control and management is subject to the influence of many controlled and uncontrolled disturbances, and it is simply unjustified to talk about their stochastic nature, and even more so about specific laws of interference distribution. Object parameters are generally unknown and may change in unpredictable ways over time. Thus, the problem of control in general and critical control in particular must be solved under conditions of significant and a priori current uncertainty.

In computer engineering and control problems, a situation often arises when a closed control system, which is under the influence of an external disturbing signal w (external, setting signal, interference, signals from other objects, variations in environmental parameters, etc.), must maintain the characteristics of the object control v (object output signal, control error, etc.) within some a priori specified boundaries so that

$$\left| v\left(t, w\right) \right| \le \varepsilon \, \forall \, t \in R, \tag{1}$$

where t is continuous or discrete time. In the event that the violation of inequality (1) is in principle unacceptable, for example, leads to catastrophic consequences, the control law that ensures strict maintenance of (1) is called critical, and the control system that implements it is called critical [1, 2].

Some features
of the synthesis of adaptive control systems

To date, the problem of managing technical objects under conditions of uncertainty is one of the central problems of modern control theory. An a dequate mathematical apparatus for solving this problem is the theory of adaptive control systems, and the widespread use of microprocessor technology has led to the development of discrete adaptive control systems [3, 4, 8].

The application of traditional methods of automatic control theory in practice is complicated by the fact that many essential properties of controlled objects and the conditions for their operation are not known in advance. At the same time, knowledge of these properties and conditions is necessary for the synthesis of the control law. Consequently, the control system in the process of work must accumulate the missing information and, on its basis, constantly improve the control law. This property is defining for adaptive systems.

At the same time, the application of the principles of adaptation in comparison with non-adaptive control will ensure high quality of control under conditions of unpredictable changes in the characteristics of an object subject to uncontrolled interference, increase the reliability of the control system, ensure its invariance to various types of objects, and reduce the time for development and adjustment of systems. All this will make it possible to make the transition from individual methods of designing control systems to mass ones, which is caused by the need for a sharp increase in the number of systems being developed and comes down to the widespread use of standard design tools.

Significant functionality that provides effective control of multidimensional non-stationary objects disturbed by noise of unknown nature, allows solving the problems of modeling and controlling complex objects, adapting the synthesized models to real data, finding optimal control laws and implementing them using simpler controllers. In principle, any task of automatic control turns into an adaptive control task if a number of characteristics of the object and the conditions for its operation are unknown, however, even those tasks that "fit" into the framework of traditional theory can be solved much easier using an adaptive approach due to the absence of a formalization stage a priori information.

The process of synthesizing an adaptive system consists of a number of stages, while as a priori information, as a rule, a structural description of the object model, specified up to parameters, is taken. In a more complex case, when the structure of the model is assumed to be unknown, one has to solve the problem of structural identification or operate with many possible different structures.

At the first stage of synthesis, the structure of the control device (regulator) is selected, which is largely determined by the adopted control objectives. The second stage is connected with the formation of a set of adjustable parameters. This is followed by the choice of the adaptation algorithm and, finally, at the fourth stage, the rationale for its choice is carried out. Since an adaptive system is generally non-linear and non-stationary, such a justification may present certain difficulties. Unfortunately, most of the practical work on adaptive systems does not have a rigorous theoretical basis. This direction still requires its further development, the creation of new adaptation algorithms, new design and research methods. It should be taken into account that most of the adaptation algorithms provide convergence to optimal modes with an infinite control interval. Therefore, there is an acute problem of synthesizing accelerated algorithms that provide high quality control over small intervals, and at the same time simple enough to ensure their implementation on microprocessor means.

Any adaptive control system contains at least two loops. The inner loop is the actual control loop, the outer loop is the adaptation loop. It is possible, of course, to introduce additional circuits, which expands the functionality of the system, but significantly complicates its study.

The key property of adaptive systems is the ability to track the drift of object parameters. It should be noted that the problems of control of non-stationary objects are the exclusive prerogative of the theory of adaptive systems. In the case of slow drift, outdated information is discounted using finite memory algorithms. Here, however, a rather complicated problem of establishing a compromise between the filtering and tracking properties of algorithms arises, which is far from being solved by now. In the case of a fast drift, one has to resort to special procedures associated with one form or another of the drift parameterization.

To date, a number of relatively independent trends in the theory of adaptive systems have been formed. Here, first of all, it is necessary to single out adaptive controllers with a minimum variance, adaptive controllers with a generalized minimum variance, adaptive systems with the required placement of zeros and poles (fundamental work, systems with adaptive predictors.

In all these approaches, it is assumed that the perturbations acting in the system are of a stochastic nature, and, as a rule, this is white noise with zero mathematical expectation and limited variance. In practical situations, the statistical assumptions are often far-fetched, and therefore it seems much more realistic to assume only that the noise or its differences in amplitude are limited. Under these conditions, the use of identification methods based on quadratic criteria and, above all, the recurrent least squares method is clearly inefficient. The difficulties that arise can be partially overcome within the framework of adaptive robust control systems, in which, nevertheless, certain statistical prerequisites are "hidden" anyway.

In this regard, it seems appropriate to develop adaptive critical methods for monitoring, identifying and managing dynamic objects operating under conditions of significant uncertainty about the characteristics of the object and the environment based on the combination of the principles of the theory of adaptive and critical control.

Modified Least Squares with Critical Properties

Consider a dynamic plant operating in a closed control system $S_D(P,C)$, described by the difference equation

$$A(q)y(k)=q^{-d}B(q)u(k)+w(k), \tag{2}$$

where polynomials $A(q)\in R[q,n]$ with $a_0=1$, $B(q)\in R[q,m]$;

d – net delay time;

y, u and w – output, control and disturbing signals, respectively.

With respect to perturbations, it is assumed that $w\in D(0,\delta)$, i.e. the boundedness of their first differences.

If the object parameters are a priori known and unchanged, the critical control problem can be solved using the controller

$$\Delta F(q)B(q)u(k)=-E(q)y(k), \tag{3}$$

where the polynomials $F(q)\in R[q,d-1]$ with $f_0=1$ and $E(q)\in R[q,n)$ are given by the equations

$$AF+q^{-d}E=C, \tag{4}$$

$$AQ+q^{-d}BP=C. \tag{5}$$

In the event that the object parameters are unknown, one can use one or another identification method, and then apply the control law (3), in which the true values of the object parameters are replaced by their estimates. This is the essence of the adaptive approach to the design of control systems for objects operating under conditions of uncertainty. As a rule, certain modifications of the recurrent least squares method or projection algorithms, one way or another related to quadratic criteria, are used as identification procedures. The use of criteria other than quadratic, for example, modular, although it leads to robust procedures, the statistical meaning of the identification problem is nevertheless preserved. Naturally, such identification algorithms cannot be used in critical control systems.

In this regard, there is a need to synthesize adaptive identification algorithms that are not associated with any statistical prerequisites, have a high convergence rate, computational simplicity, and are suitable for real-time operation in the circuit of a critical control system for a dynamic object.

Consider the polynomial

$$G(q) = 1 - \Delta A(q), \tag{6}$$

where $G(q) = g_1 q^{-1} + g_2 q^{-2} + ... + q^{-n-1}$

and rewrite the object equation (2) in the form

$$y(k) = \Theta^T \psi(k-1) + \Delta w(k), \tag{7}$$

where $\Theta = \left(g_1, g_2, ..., g_{n+1}, b_0, b_1, ..., b_m\right)^T$

$$\psi(k-1) =$$

$$= \left(y(k-1), y(k-2), ..., y(k-n-1), \Delta u(k-d), \Delta u(k-d-1), ..., \Delta u(k-d-m)\right)^T;$$

$$\Delta u(k) = u(k) - u(k-1);$$

$$\Delta w(k) = w(k) - w(k-1).$$

Note also that the symbol "lid" $\hat{x}$ in what follows will denote the estimate of the unknown parameter x.

RLSM and conditions for its convergence

RLSM is perhaps the most widely used in solving problems of identification, forecasting and adaptive control [9].

There are various approaches to obtaining RLSM, among which the most common is the construction of RLSM from the usual LSM based on the block representation of the observation matrix and the use of the matrix inversion lemma.

Modification of the minimized square functional (introduction of weight parameters, exponential smoothing mechanism, etc.) leads to the corresponding modification of the RLSM.

However, RLSM can be obtained directly by minimizing some quadratic functional. Since criteria (Lyapunov function) of the form $\tilde{\Theta}^T P^{-1}\tilde{\Theta}$, where $\tilde{\Theta}$ is the vector of identification errors, P^{-1} – is a positive definite matrix, fare widely used to analyze the convergence of recurrent algorithms, it is advisable to use such a criterion to obtain the RLSM algorithm itself.

This approach was applied in [5–7]. As shown in [5], the minimization of the functional

$$I(\Theta) = \left(\Theta - \hat{\Theta}(k-1)\right)^T P(k-1)\left(\Theta - \hat{\Theta}(k-1) + \gamma\left(y(k) - \Theta^T\psi(k-1)\right)^2\right) \tag{8}$$

leads to the following algorithm:

$$\hat{\Theta}(k) = \hat{\Theta}(k-1) + \gamma(k)P(k)\psi(k-1)\left(y(k) - \hat{\Theta}^T(k-1)\psi(k-1)\right); \tag{9}$$

$$P^{-1}(k) = P^{-1}(k-1) + \gamma(k)\psi(k-1)\psi^T(k-1). \tag{10}$$

Applying the matrix inversion lemma to the last relation gives

$$\begin{aligned} P(k) &= \left[P^{-1}(k-1) + \gamma(k)\psi(k-1)\psi^T(k-1)\right]^{-1} = \\ &= P(k-1) - \gamma(k)\frac{P(k-1)\psi(k-1)\psi^T(k-1)P(k-1)}{1 + \gamma(k)\psi^T(k-1)P(k-1)\psi(k-1)}. \end{aligned} \tag{11}$$

Substituting (11) into (8), we obtain a formula for the recursive calculation of the estimate

$$\begin{aligned} &\hat{\Theta}(k) = \hat{\Theta}(k-1) + \\ &+ \gamma(k)\frac{P(k-1)\psi(k-1)}{1 + \gamma(k)\psi^T(k-1)P(k-1)\psi(k-1)}\left(y(k) - \hat{\Theta}^T(k-1)\psi(k-1)\right). \end{aligned} \tag{12}$$

In this case, the modified RLSM algorithm is described by relations (10), (11), from which the usual RLSM follows at $\gamma(k) = 1$. By choosing different values of the parameter $\gamma(k)$, we obtain various modifications of the RLSM.

Let us determine the requirements that the parameter $\gamma(k)$ must satisfy in order to ensure the convergence of the algorithm.

Consider a Lyapunov function of the form

$$V(k) = \tilde{\Theta}^T(k)P^{-1}(k)\tilde{\Theta}(k). \tag{13}$$

Subtracting from both parts of (8) Θ and denoting $\tilde{\Theta}(i)=\hat{\Theta}(i)-\Theta$, we obtain a recursive expression for $\tilde{\Theta}(k)$, whose substitution into (13) gives

$$\begin{aligned} V(k) &= \left(\tilde{\Theta}(k-1)-\gamma(k)P(k)\psi(k-1)e(k)\right)^T \times \\ &\quad \times P^{-1}(k)\left(\tilde{\Theta}(k-1)-\gamma(k)P(k)\psi(k-1)e(k)\right)= \\ &= \tilde{\Theta}^T(k-1)P^{-1}(k)\tilde{\Theta}(k-1)-\gamma(k)\tilde{\Theta}^T(k-1)\psi(k-1)e(k)+ \\ &\quad +\gamma^2(k)\psi^T(k-1)P(k)\psi(k-1)e^2(k). \end{aligned} \tag{14}$$

Taking into account that $e(k)=\tilde{\Theta}^T(k-1)\psi(k-1)+w(k)$ and the matrix $P(k)$ is also related $P(k-1)$ to relation (11), $P^{-1}(k)$ with $P^{-1}(k-1)$is related to relation, we obtain

$$\begin{aligned} V(k) &= \tilde{\Theta}^T(k-1)P^{-1}(k)\tilde{\Theta}(k-1)+\gamma(k)\tilde{\Theta}^T(k-1)\psi(k-1))^2- \\ &\quad -2\gamma(k)\left(\tilde{\Theta}^T(k-1)\psi(k-1)\right)e(k)+\gamma^2(k)\psi^T(k-1)\times \\ &\quad \times\left(P(k-1)-\gamma(k)\frac{P(k-1)\psi(k-1)\psi^T(k-1)P(k-1)}{1-\gamma(k)\psi^T(k-1)P(k-1)\psi(k-1)}\right)\psi(k-1)e^2(k)= \\ &= V(k-1)+\gamma(k)w^2(k)-\gamma(k)e^2(k)+ \\ &\quad +\gamma^2(k)\frac{\psi^T(k-1)P(k-1)\psi(k-1)e^2(k)}{1+\gamma(k)\psi^T(k-1)P(k-1)\psi(k-1)}= \\ &= V(k-1)+\gamma(k)w^2(k)-\gamma(k)\frac{e^2(k)}{1+\gamma(k)\psi^T(k-1)P(k-1)\psi(k-1)}. \end{aligned}$$

If $|w(k)|\le\delta$, then

$$V(k)\le V(k-1)+\gamma(k)\delta^2-\gamma(k)\frac{e^2(k)}{1+\gamma(k)\Psi^T(k-1)P(k-1)\Psi(k-1)}$$

and for the convergence of the algorithm, i.e. to fulfill the condition

$$\Delta V(k)=V(k)-V(k-1)\le 0 \tag{15}$$

inequality must hold

$$\gamma(k)\delta^2-\gamma(k)\frac{e^2(k)}{1+\gamma(k)\psi^T(k-1)P(k-1)\psi(k-1)}\le 0. \tag{16}$$

In this case, the Lyapunov function $V(k)$ is non-negative and bounded. It limits $V(0)=\tilde{\Theta}^T(0)P^{-1}(-1)\tilde{\Theta}(0)$ her. And since $V(k)\geq 0$, the sequence $V(k)$ converges.

Thus, from (15) and (16) it follows

$$\lim_{k\to\infty}\frac{\delta^2\left(1+\gamma(k)\psi^T(k-1)P(k-1)\psi(k-1)\right)-e^2(k)}{1+\gamma(k)\psi^T(k-1)P(k-1)\psi(k-1)}=0;$$
$$\delta^2\left(1+\gamma(k)\psi^T(k-1)P(k-1)\psi(k-1)\right)-e^2(k)\leq 0. \tag{17}$$

Thus, the establishment of the convergence of modifications of the RMLS with different choice of parameter $\gamma(k)$ is reduced to checking the fulfillment of inequality (16).

Conclusions

The article deals with the synthesis of an adaptive controller under the action of a limited interference. A procedure based on LMNC is proposed, which is characterized by sufficient computational simplicity. Conditions for the convergence of the proposed procedure are determined.

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