An infectious disease model considering the age of infection

Development of a model based on integro-differential equations to improve the prediction of disease dynamics at change points.

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Motivation

- Age of infection: Time that has passed since becoming infected
- Include this into our model by using integro-differential equations (IDE)
- Allows better prediction of infection dynamics at change points such as implementation of non-pharmaceutical interventions

Methods

- IDE model is generalization of model based on ordinary differential equations (ODE)
- Solving IDE model requires other methods than ODE model
- Discretization of equations is based on idea in [1] using a non-standard numerical scheme

Model

$$S'(t) = -\frac{S(t)}{N - D(t)} \int_{-\infty}^{t} \phi \Big(\rho_C(t-x)\xi_C(t-x)\sigma_E^C(x)\Big[\mu_C^I\,\gamma_C^I(t-x) + \mu_C^R\,\gamma_C^R(t-x)\Big] + \rho_I(t-x)\xi_I(t-x)\sigma_C^I(x)\Big[\mu_I^H\,\gamma_I^H(t-x) + \mu_I^R\,\gamma_I^R(t-x)\Big]\Big)\, dx = -S(t)\varphi(t)$$

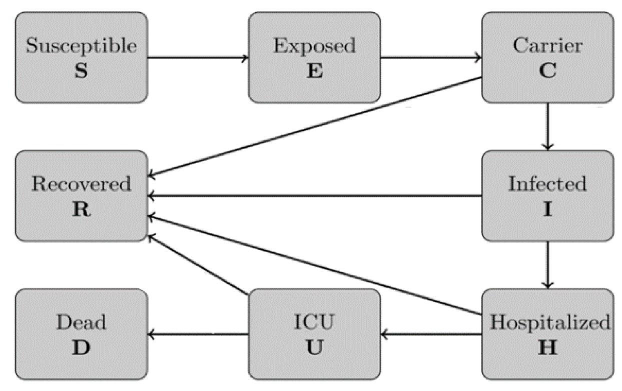

Figure 1: Overview of compartments of model [2].

$\sigma_E^C(x)$: Number of individuals going from E to C at time x

$\gamma_C^I(\tau)$: Mean fraction of individuals that will go from C to I at some point and are still in C at time τ after entering C

μ_C^I : Probability of going from C to I

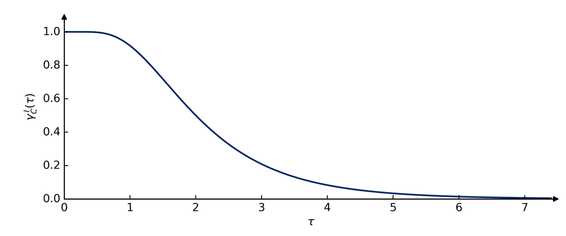

Figure 2: Example for $\gamma_C^I(\tau)$.

Discretization

$$S'(t) = -S(t)\varphi(t)$$

$$\Downarrow$$

$$\frac{S(t_{n+1}) - S(t_n)}{\Delta t} = -S(t_{n+1})\varphi(t_n)$$

$$\Downarrow$$

$$S(t_{n+1}) = \frac{S(t_n)}{1 + \Delta t\, \varphi(t_n)}$$

$$\varphi(t) = \frac{1}{N - D(t)} \int_{-\infty}^{t} \phi \Big(\rho_C(t-x)\xi_C(t-x)\sigma_E^C(x)\Big[\mu_C^I\,\gamma_C^I(t-x) + \left(1-\mu_C^I\right)\gamma_C^R(t-x)\Big] + \rho_I(t-x)\xi_I(t-x)\sigma_C^I(x)\Big[\mu_I^H\,\gamma_I^H(t-x) + \left(1-\mu_I^H\right)\gamma_I^R(t-x)\Big]\Big)\, dx$$

$$\Downarrow$$

$$\varphi(t_{n+1}) \approx \frac{\Delta t}{N - D(t_{n+1})} \sum_{i=a}^{n} \phi \Big(\rho_C(t_{n+1-i})\xi_C(t_{n+1-i})\sigma_E^C(t_{i+1})\Big[\mu_C^I\gamma_C^I(t_{n+1-i}) + (1-\mu_C^I)\gamma_C^R(t_{n+1-i})\Big] + \rho_I(t_{n+1-i})\xi_I(t_{n+1-i})\sigma_C^I(t_{i+1})\Big[\mu_I^H\gamma_I^H(t_{n+1-i}) + (1-\mu_I^H)\gamma_I^R(t_{n+1-i})\Big]\Big)$$

First results

- Validation of our model: Special case of IDE model compared with ODE model shows similar results

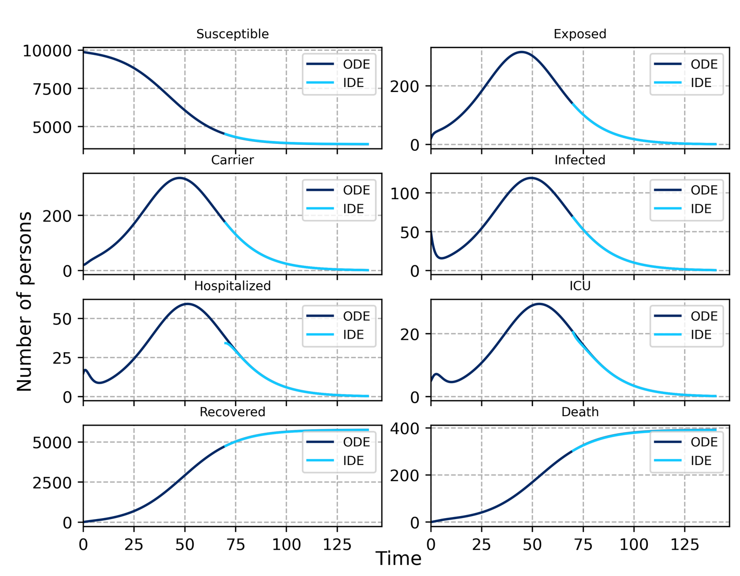

Figure 3: Comparison of ODE model and IDE model showing individuals in different compartments that are representing infection states.

Outlook

- Make initialization more robust with respect to starting conditions
- Investigate effect of non-pharmaceutical interventions

References

[1] Eleonora Messina, Mario Pezzella, and Antonia Vecchio. "A non-standard numerical scheme for an age-of-infection epidemic model". In: *Journal of Computational Dynamics* 9.2 (2022), p. 239.

[2] Martin J. Kühn et al. "Assessment of effective mitigation and prediction of the spread of SARS-CoV-2 in Germany using demographic information and spatial resolution". In: *Mathematical Biosciences* 339 (Sept. 2021), p. 108648.