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On the endogenous determination of the degree of meritocracy in large cooperatives

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HIGHLIGHTS

- Low-wealth cooperatives choose degrees of meritocracy below the optimal.
- High-wealth cooperatives choose degrees of meritocracy above the optimal.
- Total labor and output are directly proportional to the degree of meritocracy.

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ABSTRACT

We consider a cooperative formed by a large number of workers differentiated by their initial endowment of wealth, which is both primary input (labor) and consumption (leisure). The cooperative is characterized by its wealth distribution, and produces a consumption good from labor, which allocates among workers according to a convex combination of the Proportional and the Egalitarian rule. In the first stage, workers decide this combination by simple majority. In a second stage, they choose how much labor to provide to the cooperative. We find that when in the cooperative's wealth distribution, the median wealth is lower (higher) than the average, the degree of meritocracy chosen by workers is lower (higher) than that of the optimum, and coincides with it when both statistics coincide. This choice has similar consequences on the cooperative's labor–output, since it increases with respect to the degree of meritocracy.

1. Introduction

Private and public firms' workplaces and departments, the household and the neighborhood domain and even mere cooperative enterprises are cases of economic organizations where the technology is publicly owned by the workers. In such cases, surplus distribution is achieved by a sharing rule which maps efforts into surplus shares for each member of the cooperative. Among the sharing rules conceived and studied in the literature (Sen, 1966; Moulin, 1987; Kang, 1988; Roemer and Silvestre, 1993), the Proportional and the Egalitarian rule emerge as the most natural ones. However, one question that arises is, which sharing rule would be chosen by the cooperative s members? Different approaches to this issue have been tackled in the literature by Corchón and Puy (1998), Barberá et al. (2015) and Beviá and Corchón (2018). A common feature of these papers is that the cooperative s workers have quasilinear preferences, such that the efficient contribution of labor from each worker is independent of the other workers contributions. As a consequence, the position of the median voter with respect to the average labor contribution determines the share rule chosen in the cooperative.

The aim of this paper is to study how interdependency among the labor contributions of the cooperative s workers affects the choice of the share rule. To do so, we consider a large cooperative with small identical workers characterized by Cobb–Douglas preferences, and differentiated by their endowment of wealth. The wealth is both consumed by workers and/or provided to the cooperative as primary input. The cooperative produces output from this input by means of a returns-to-scale-parameterized technology. Individuals, in the first stage, choose the degree of meritocracy, that is, the weight of the Proportional rule in a sharing rule that results from the convex combination of the former with the Egalitarian rule, by means of a simple majority voting equilibrium. In the second stage, workers choose the level of labor,

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which determines the amount of good produced. Across the paper, a cooperative is characterized by its wealth distribution, and the cooperative s equilibrium degree of meritocracy can be expressed as a function of the average and the median wealth. This is a major point for discussion, since the relative position of these statistics characterize how wealthy a cooperative is. For instance, when the average wealth is lower than the median, more than half of the cooperative s workers are wealthier than the average one. In such a cooperative high wealth predominates. Otherwise, when average wealth is higher than the median, low wealth predominates in the cooperative. Let us call the former a high-wealth cooperative and the latter, a low-wealth cooperative. As a consequence, our main result asserts that, when high (low) wealth predominates in the cooperative, the equilibrium degree of meritocracy chosen by workers is higher (lower) than the efficient one, and equals it when the average equals the median wealth. In turn, since in the model total labor and output depend positively on the degree of meritocracy, our result implies that high-wealth (low-wealth) cooperatives provide labor, or produce output, above (below) the optimal level.

The structure of the paper is as follows: a second section that describes the model; a third section that states the efficient outcome, the equilibrium and the main result; and section four, devoted to comments.

2. Model

There is a cooperative formed by a continuum of workers normalized to one. Workers distinguish themselves by their endowment w of wealth, which distributes according to the distribution function $F \in \mathcal{F}$, where $\mathcal{F}$ is the family of distribution functions defined in the support $\Omega \subset R^+$, so that $min\Omega = 1$, that is, the lower level of wealth in the cooperative is normalized to one. Let A and m be the average and the median wealth, respectively. As was pointed out in the Introduction, the predominance of lower or higher wealth in the cooperative is related to the relative position of these statistics. For instance, when $m < A$, more than half of the workers are less wealthy than the average one; in such a case low wealth predominates in the cooperative. When $m > A$ the opposite occurs, and high wealth predominates in the cooperative.

Each worker has the same Cobb–Douglas utility function with respect to the amounts C and $l \in (0, w]$ of per-capita consumption and labor, respectively. This assumption also implies that a cooperative's worker is identified by his/her level w of wealth.

$$u(C, l) = C^{\alpha}(w - l)^{1-\alpha}, \tag{1}$$

where $\alpha \in (0, 1)$ represents consumption intensity. Moreover, labor is used to produce the amount Y of per-capita consumption good according to the publicly-owned production function

$$Y = L^{\gamma} \tag{2}$$

where

$$L = \int_{\Omega} l dF(w), \tag{3}$$

is the total per-capita amount of labor and $\gamma \in (0, 1]$ represents the (not increasing) returns to scale parameter which, given our technology, also represents the elasticity of output with respect to labor.

The consumption of an individual is determined by the sharing rule which is a convex combination of the Egalitarian and the Proportional share rule. Kang (1988) proves that this rule completely characterizes the fair distribution rule for more than two workers. Hence, in our size-one population cooperative, it can be written as

$$C = Y\left[1 - \rho + \rho\frac{l}{L}\right], \tag{4}$$

where $\rho \in [0, 1]$ is the degree of meritocracy, that is, the weight attached to the relative contribution of each individual to production.

3. Efficiency and equilibrium

According to Beviá and Corchón (2018), Nash equilibrium is compatible with efficiency whenever $\partial C/\partial l = \partial Y/\partial l$. Since in our large-number-of-workers' cooperative, each worker's decision about labor contribution is negligible, we can obtain the efficient degree of meritocracy by equalizing the partial derivatives of Eqs. (2) and (4). As a result, the efficient degree of meritocracy is that which equals the production function's returns to scale parameter, that is $\rho = \gamma$. This result is compatible with Sen (1966), for the identical individual case; and with Beviá and Corchón (2016), for a dynamic framework with large numbers of workers.

To determine the equilibrium degree of meritocracy we consider a two-stage problem where, in the first stage, workers choose the degree of meritocracy by simple majority voting and, in the second stage, each worker chooses the amount of labor that maximizes his/her utility, given the degree of meritocracy chosen in the first stage, and taking L (and Y) as given. As in Beviá and Corchón (2016), the fact that the worker takes L as given is justified by the very large number of workers forming the cooperative. Hence, starting from the second stage, let us plug Eq. (4) into Eq. (1) to express the utility function as

$$u(l) = Y^{\alpha}\left[1 - \rho + \rho\frac{l}{L}\right]^{\alpha}[w - l]^{1-\alpha}, \tag{5}$$

the solution of this second-stage problem is:

$$l(w, \rho) = \alpha\left(w - \frac{(1-\alpha)}{\alpha}\frac{(1-\rho)}{\rho}L\right). \tag{6}$$

To determine the value $L(\rho)$ of total per-capita amount of labor in equilibrium by integrating Eq. (6) in the whole Ω, we have to assume that every worker in the cooperative contributes with a positive amount of labor, that is $l(w, \rho) > 0, \forall w \in \Omega$. Considering Eq. (6), we realize that such an assumption implies a minimum degree of meritocracy for which all individuals in the cooperative have incentives to provide labor. The following Proposition tackles this point.

Proposition 1. *$l(w, \rho) > 0, \forall w \in \Omega$ whenever $\rho > \frac{(1-\alpha)(A-1)}{(1-\alpha)A+\alpha} \equiv \rho_0$.*

Proof. Taking into account Eq. (3) and assuming that $l(w, \rho) > 0, \forall w \in \Omega$, the value $L(\rho)$ of total per-capita amount of labor in equilibrium is given by

$L(\rho) = \alpha\int_{\Omega}\left(w - \frac{(1-\alpha)}{\alpha}\frac{(1-\rho)}{\rho}L(\rho)\right)dF(w)$, which can be written explicitly as

$$L(\rho) = \frac{\alpha\rho}{1-\alpha(1-\rho)}A. \tag{7}$$

Finally, taking into account Eqs. (6) and (7) the worker amount of labor in equilibrium can be written as:

$$l(w, \rho) = \alpha\left(w - \frac{(1-\alpha)(1-\rho)}{1-\alpha(1-\rho)}A\right). \tag{8}$$

Eq. (8) shows that $\forall w \in \Omega, l(w, \rho)$ is an increasing function of both the wealth and the degree of meritocracy. Hence, exploiting Eq. (8), we can assess the lower degree of meritocracy, ρ_0, for which the less wealthy worker in the cooperative has incentives to provide labor, that is, ρ_0, so that $l(1, \rho_0) \geq 0$. This value is given by

$$\rho_0 \equiv \frac{(1-\alpha)(A-1)}{\alpha + (1-\alpha)A}. \tag{9}$$

Note that $\rho_0 \in (0, 1)$. Finally, since $l(w, \rho) \geq l(1, \rho), \forall w \in \Omega$, and $\rho \in (0, 1]$, the result follows. ■

From the proof of Proposition 1 let us remark that, on the one hand, Eq. (8) shows the dependency of worker-w's labor contribution with respect to the rest of the workers' labor contributions, a feature that represents the major difference of our model with respect to that of Corchón and Puy (1998) and Beviá and Corchón (2018). On the other hand, $L(\rho)$ is increasing and concave with respect to the degree of meritocracy, and $L(1) = \alpha A$. In words, the higher the degree of meritocracy, the higher the total amount of labor and the higher the amount of consumption good produced in the cooperative. In what follows we assume that $\rho \in [\rho_0, 1] \subset (0, 1]$. Moreover, provided that the efficient degree of meritocracy equals the returns to scale parameter, we also assume that $\rho_0 < \gamma$, in order to preserve interior solutions in the determination of the equilibrium.

Therefore, to assess the degree of meritocracy in the first stage, let us plug Eqs. (7) and (8) into the utility function (5) and dropping the constant $(1-\alpha)^{1-\alpha}(\alpha A)^{\alpha\gamma}A^{-\alpha}$, the worker's indirect utility function can be written as:

$$V(w, \rho) = \frac{\rho^{\gamma\alpha}[w + \alpha(1-\rho)(A-w)]}{[1-\alpha(1-\rho)]^{1-\alpha+\alpha\gamma}}, \quad w \in \Omega \tag{10}$$

It can be seen, form Eq. (10), that

$$\frac{\partial V(w, \rho)}{\partial w} = \rho^{\alpha\gamma}[1-\alpha(1-\rho)]^{\alpha(1-\gamma)} > 0.$$

In words, IUF is monotonic increasing in Ω. Which implies that, $\forall w > w'$ and $\rho \in [\rho_0, 1], V(w, \rho) > V(w', \rho)$. On the other hand,

$$\frac{\partial V(w, \rho)}{\partial \rho} = K(\rho)[H(w, \rho) - J(w, \rho)], \rho \in [\rho_0, 1], w \in \Omega, \tag{11}$$

where:

$$K(\rho) = \alpha\rho^{\gamma\alpha}[1-\alpha(1-\rho)]^{\alpha-\gamma\alpha-2},$$
$$H(w, \rho) = (1-\alpha)[\alpha A + (1-\alpha)w]\left(\frac{\gamma}{\rho} - 1\right),$$
$$J(w, \rho) = (A-w)\left[(1-\alpha)(1+\alpha\gamma) + \alpha^2\rho\right].$$

It is fairly easy to see that, (i) $K(\rho) > 0$ and increasing in $[\rho_0, 1]$; (ii) $H(\cdot, \rho)$ is decreasing and convex in $[\rho_0, 1]$, so that $H(\cdot, \rho) \gtreqless 0$ iff $\rho \lesseqgtr \gamma$; (iii) $J(w, \rho)$ is linear increasing (decreasing) in $[\rho_0, 1]$ when $A > w$ $(A < w)$; and $J(w, \rho) \gtreqless 0$ iff $w \lesseqgtr A, \forall \rho \in [\rho_0, 1]$. Moreover,

$$\frac{\partial V^2(w, \rho)}{\partial \rho^2} = \frac{\partial K(\rho)}{\partial \rho}[H(w, \rho) - J(w, \rho)] + K(\rho)\left[\frac{\partial H(w, \rho)}{\partial \rho} - \frac{\partial J(w, \rho)}{\partial \rho}\right]. \tag{12}$$

where

$$\frac{\partial H(w, \rho)}{\partial \rho} = -(1-\alpha)[\alpha A + (1-\alpha)w]\frac{\gamma}{\rho^2} < 0,$$
$$\frac{\partial J(w, \rho)}{\partial \rho} = \alpha^2(A-w) \gtreqless 0 \text{ iff } A \gtreqless w. \tag{13}$$

These preliminaries allow us to state the following Proposition about the most preferred degree of meritocracy of each worker.

Proposition 2. *Let $\hat{\rho}(w) > 0$ be so that $H(w, \hat{\rho}(w)) = J(w, \hat{\rho}(w))$ and $\frac{\partial V^2(w, \hat{\rho}(w))}{\partial \rho^2} < 0$. The worker-w's most preferred degree of meritocracy is given by $\rho: [\rho_0, 1] \times \Omega \to [\rho_0, 1]$, so that*

$$\rho(\gamma, w) = \begin{cases} \max\{\hat{\rho}(w), \rho_0\}, & 1 \leq w < A, \\ \gamma, & w = A, \\ \min\{\hat{\rho}(w), 1\}, & w > A. \end{cases}$$

Proof. Let us divide the Proof into three cases:

(2.1) $1 \leq w < A$. In this case, since $H(w, \rho)$ is decreasing and $J(w, \rho)$ is positive and linear increasing in $[\rho_0, 1] \times R^+$, there is a unique $\hat{\rho}(w) \in (0, \gamma)$ that, taking into account Eqs. (12) and (13), is a maximum of $V(w, \rho)$. Hence, if $\hat{\rho}(w) < \rho_0$, $V(w, \rho)$ reaches a peak in ρ_0, and when $\hat{\rho}(w) > \rho_0$, $V(w, \rho)$ reaches a peak in $\hat{\rho}(w)$. Thus $\rho(\gamma, w) = \max\{\hat{\rho}(w), \rho_0\} < \gamma$.

(2.2) $A = w$. In this case $J(A, \rho) = 0$. Thus, according to Eq. (11), $\frac{\partial V(A,\rho)}{\partial \rho} = K(\rho)H(A, \rho)$, and the first order condition holds for $\hat{\rho}(w) = \gamma$. In turn, it is fairly easy to see, from Eqs. (12) and (13), that this value is a maximum of $V(w, \rho)$. Hence, $\rho(\gamma, A) = \gamma \in (\rho_0, 1)$.

(2.3) $A < w$. In this case $H(w, \rho) - J(w, \rho) = 0$ may be a quadratic equation with two positive roots, that can be written as $a(w)\rho^2 + b(w)\rho + c(w) = 0$, where

$$a(w) \equiv \alpha^2(w - A),$$
$$b(w) \equiv (1-\alpha)[\alpha(1+\gamma)w - (1+\alpha+\gamma\alpha)A], \tag{14}$$
$$c(w) \equiv \gamma(1-\alpha)[\alpha A + (1-\alpha)w],$$

Whose roots are $\overline{\rho}(w) = \left(-b(w) \pm \sqrt{b^2(w) - 4a(w)c(w)}\right)/2a(w)$. Hence, this case have to be subdivided in two subcases, according to $D(w) \equiv b^2(w) - 4a(w)c(w)$ is negative or not.

(2.3.1) When $D(w) \geq 0, \overline{\rho}(w)$ are real. Thus we have to determine which root is maximum and which one is minimum. For that purpose, notice that $a(w) > 0$ and $c(w) > 0, \forall w > A$, and $-b(w)$ is linear decreasing in w, so that, $-b(w) \geq 0$ for $w \in (A, (1 + 1/\alpha(1+\gamma))A]$, since $da(w)/dw > 0, db(w)/dw > 0, dc(w)/dw > 0$, and $b(w) \leq 0$. In addition, $d^2D(w)/dw^2 = 2(db(w)/dw)^2$, since $d^2a(w)/dw^2 = 0, d^2b(w)/dw^2 = 0, d^2c(w)/dw^2 = 0$. Hence $D(w)$ is a convex parabola decreasing in $w \in (A, (1 + 1/\alpha(1+\gamma))A]$, such that $D((1 + 1/\alpha(1+\gamma))A) = -4\gamma(1-\alpha)(1+\alpha\gamma)A^2(1+\gamma)^{-2} < 0$. This means that the first root of $D(w)$ is given by

$$\tilde{w} = \frac{A\left[(1-\alpha(1-\gamma))(1+\alpha(1-\gamma)+\gamma) - 2\sqrt{(1-\alpha(1-\gamma))\gamma}\right]}{\alpha(1-\alpha)(1-\gamma)^2}. \tag{15}$$

Thus, we can assert that for $w \in (A, \tilde{w}]$, the first (the small) and the second (the large) positive and real roots of Equation $H(w, \rho) - J(w, \rho) = 0$ are, respectively

$$\overline{\rho}_1(w) = \left(-b(w) - \sqrt{D(w)}\right)/2a(w), \tag{16}$$
$$\overline{\rho}_2(w) = \left(-b(w) + \sqrt{D(w)}\right)/2a(w). \tag{17}$$

To determine which of them is maximum and which is minimum, let us exploit Eqs. (12)–(14), to write the second derivative of $V(w, \rho)$ evaluated at $\overline{\rho}_i(w), i = 1, 2$, as

$$\frac{\partial V^2(w, \overline{\rho}_i(w))}{\partial \rho^2} = K(\rho)\frac{a(w)}{\overline{\rho}_i(w)^2}\left[\overline{\rho}_i(w)^2 - \frac{c(w)}{a(w)}\right]. \tag{18}$$

And plugging respectively Eqs. (16) and (17) in Eq. (18) we hold

$$\frac{\partial V^2(w, \overline{\rho}_1(w))}{\partial \rho^2} = -K(\rho)\frac{\sqrt{D(w)}}{\overline{\rho}_1(w)} < 0;$$
$$\frac{\partial V^2(w, \overline{\rho}_2(w))}{\partial \rho^2} = K(\rho)\frac{\sqrt{D(w)}}{\overline{\rho}_2(w)} > 0.$$

In words, $\overline{\rho}_1(w)$ is a maximum and $\overline{\rho}_2(w)$ is a minimum of $V(w,\rho)$. Hence, by definition, $\hat{\rho}(w) = \overline{\rho}_1(w)$. Finally, assuming that $\overline{\rho}_2(w) \geq 1$, to guarantee the singlepeakedness of the IUF in $[\rho_0, 1]$ (Lemma 1 deals with this detail), if $\hat{\rho}(w) < 1$, $V(w,\rho)$ reaches a peak in $\hat{\rho}(w)$, and when $\hat{\rho}(w) > 1$, $V(w,\rho)$reaches a peak in 1. Hence, $\rho(\gamma, w) = \min\left\{\hat{\rho}(w), 1\right\} > \gamma$.

(2.3.2) When $D(w) < 0$, $\overline{\rho}(w)$ are complex. This is because to $H(w,\rho) > J(w,\rho), \forall \rho \in [\rho_0, 1]$, and $\rho(\gamma, w) = 1$. ■

Proposition 2 plots the shape of the different workers' IUF, in regards to the position of her/his wealth with respect to the average wealth. When a worker's wealth is lower than or equal to the average, its IUF exhibits singlepeakedness (cases 2.1 and 2.2). On the other hand, when a worker's wealth is higher than the average there are two cases: the second one, case (2.3.2), makes the worker's IUF singlepeaked, with a peak in $\rho(\gamma, w) = 1$; and the first case (2.3.1), where the first order condition on $V(w,\rho)$ has two positive roots. This case may give rise to a doublepeaked IUF in $[\rho_0, 1]$, for some w, whenever $\overline{\rho}_2(w) < 1$. The following Lemma states the condition for which $\overline{\rho}_2(w) \geq 1$, and thus, the condition for which $V(w,\rho)$ is singlepeaked $\forall w \in \Omega$ in $[\rho_0, 1]$.

Lemma 1. $\overline{\rho}_2(w) \geq 1$ *iff* $\gamma \geq \frac{\sqrt{(1-\alpha)^2+4\alpha^2}-\left(1-\alpha+2\alpha^2\right)}{2(1-\alpha)}$.

Proof. Since $-b(w)$, $D(w)$ and $1/a(w)$ are positive and decreasing in $w \in (A, \tilde{w}]$, Eq. (17) allows us to assert that $\overline{\rho}_2(w)$ is also real, decreasing and positive function $\forall w \in (A, \tilde{w}]$. Hence, $\overline{\rho}_2(w)$ reaches its minimum whenever $w = \tilde{w}$. In such a case $D(\tilde{w}) = 0$, and $\overline{\rho}_2(\tilde{w}) = \overline{\rho}_1(\tilde{w})$. Let $\overline{\rho}_2(\tilde{w})$ be this minimum, which is obtained substituting Eq. (15) in Eq. (17), so that $\overline{\rho}_2(\tilde{w}) \equiv \gamma + \sqrt{\gamma(\alpha\gamma + 1 - \alpha)}/\alpha$. Since $\forall w \in (A, \tilde{w}]$, $\overline{\rho}_2(w) \geq \overline{\rho}_2(\tilde{w})$, the condition for which $\overline{\rho}_2(w) \geq 1$, is the same that guaranties that $\overline{\rho}_2(\tilde{w}) \geq 1 \forall w \in (A, \tilde{w}]$. Hence $\gamma + \sqrt{\gamma(\alpha\gamma + 1 - \alpha)}/\alpha \geq 1$ whenever $\alpha\gamma^2 + (1-\alpha)\gamma \geq \alpha^2(\gamma - 1)^2$. But this condition can be written as a function of the parameters as $g(\gamma, \alpha) \geq 0$, where $g(\gamma,\alpha) \equiv \alpha(1-\alpha)\gamma^2 + \left(1 - \alpha + 2\alpha^2\right)\gamma - \alpha^2$. Notice that $g(\gamma, \cdot)$ is a convex parabola, increasing for $\alpha \in [0, 1]$ and non-negative for $\gamma(\alpha) \leq \gamma < 1$, where $\gamma(\alpha)$ is the positive root of $g(\gamma, \alpha)$, given by

$$\gamma(\alpha) \equiv \frac{\sqrt{(1-\alpha)^2 + 4\alpha^2} - \left(1 - \alpha + 2\alpha^2\right)}{2(1-\alpha)}. \tag{19}$$

And the result follows. ■

Corollary 1. *A sufficient condition for* $\overline{\rho}_2(w) \geq 1$ *is* $\gamma \geq 1/2$.

Proof. It is easy to check from Eq. (19) that $\gamma(\alpha) < 1/2$. Hence, when $\frac{1}{2} \leq \alpha < 1$, the statement of Lemma 1 holds and $\overline{\rho}_2(w) \geq 1 \forall w \in (A, \tilde{w}]$. ■

As seen, the condition for which $V(w,\rho)$ is singlepeaked $\forall w \in \Omega$ in $[\rho_0, 1]$ implies a very reasonable set of parameters and, in particular, it is fulfilled whenever the returns to scale parameter is equal to or higher than 1/2. In what follows, and to guarantee the singlepeakedness of all workers' IUF, let us assume that $\gamma \geq \max\{\rho_0, \gamma(\alpha)\}$.[1] Finally, it should be noted that $\rho(\gamma, w)$ of Proposition 2 is continuous in Ω but not differentiable in $w = A$. This can be seen by analyzing $\hat{\rho}(w)$ on each side of $w = A$. On the right hand side of $w = A$, $\hat{\rho}(w) = \overline{\rho}_1(w)$ of Eq. (16), and it can be checked, by applying l'Hôpital's rule, that $\lim_{w\to A^-} \overline{\rho}_1(w) = \gamma$. On the other hand, when $1 \leq w < A$, taking into account Eq. (14), $a(w) < 0$, $b(w) < 0$ and $c(w) > 0$, so $D(w) > 0$; and, using Eq. (16) and extracting the negative sign from the numerator to keep the denominator of $\overline{\rho}_1(w)$ positive, it can be written as $\overline{\rho}_1(w) = \left(b(w) + \sqrt{D(w)}\right)/(-2a(w))$. In such a case, it can be checked as well that, by applying l'Hôpital's rule, $\lim_{w\to A^+} \overline{\rho}_1(w) = \gamma$. The following Proposition concerns the relation between the each worker's most preferred degree of meritocracy and her/his level of wealth.

Proposition 3. *$\rho(\gamma, w)$ is monotonically non-decreasing in Ω.*

Proof. On the one hand, in the borders of the range of $\rho(\gamma, w)$, the increasing monotonicity of $V(w,\rho)$ with respect to w, allows to assert that if $\exists w' \in (1, A]$ such that $\rho\left(\gamma, w'\right) = \rho_0 \Rightarrow \rho(\gamma, w) = \rho_0 \forall w < w'$; and if $\exists w' > A$ such that $\rho\left(\gamma, w'\right) = 1 \Rightarrow \rho(\gamma, w) = 1 \forall w > w'$. On the other hand, in the interior of the range of $\rho(\gamma, w)$, for $w \in \Omega - \{\gamma\}$, Proposition 2 claims that $\hat{\rho}(w)$ is the maximum of $V(w,\rho)$, which fulfills the first and the second order condition for $V(w,\rho) \forall w \in \Omega$, that is:

$$H\left(w, \hat{\rho}(w)\right) - J\left(w, \hat{\rho}(w)\right) = 0,$$

$$\frac{\partial H\left(w, \hat{\rho}(w)\right)}{\partial \rho} - \frac{\partial J\left(w, \hat{\rho}(w)\right)}{\partial \rho} < 0.$$

Hence, implicitly deriving from the first order condition, taking into account the sign of the second order condition and the fact that

$$\frac{\partial H(w,\rho)}{\partial w} - \frac{\partial J(w,\rho)}{\partial w} = (1-\alpha)^2\gamma/\rho + \alpha(1-\alpha)(1+\gamma) + \alpha^2\rho > 0,$$

we hold that

$$\frac{\partial \hat{\rho}(w)}{\partial w} = -\frac{\frac{\partial H(w,\hat{\rho}(w))}{\partial w} - \frac{\partial J(w,\hat{\rho}(w))}{\partial w}}{\frac{\partial H(w,\hat{\rho}(w))}{\partial \rho} - \frac{\partial J(w,\hat{\rho}(w))}{\partial \rho}} > 0.$$

Hence, the result follows. ■

Proposition 3 states that the worker's preferred degree of meritocracy is ordered so that, $\forall w, w' \in \Omega$, $\rho(\gamma, w) \geq \rho\left(\gamma, w'\right)$ iff $\forall w > w'$. The following Proposition is devoted to stating the majority-voting equilibrium degree of meritocracy.

Proposition 4. *The degree of meritocracy under majority-voting equilibrium, $\rho(\gamma, m)$, depends on the relative position of wealth distribution statistics so that, when $m \lesseqgtr A$, $\rho(\gamma, m) \lesseqgtr \gamma$.*

Proof. Taking into account Propositions 2 and 3, we can apply the median voter criterion to characterize the simple majority voting equilibrium. Hence, in such a case, the median voter's indirect utility function, $V(m,\rho)$, is held by substituting the median wealth, m, for w in Eq. (10). Finally, the result is held by applying the different cases held in Proposition 2 for A and m. ■

Proposition 4 shows that the relative position of the equilibrium degree of meritocracy with respect to the efficient one depends on the relative position of wealth distribution statistics, such that, when the median worker's wealth is lower (higher) than the average worker's wealth, the equilibrium degree of meritocracy is lower (higher) than the optimal one, and equals it when the median worker's wealth is just the average. Given our interpretation concerning the relative position of these statistics in the wealth distribution, we conclude that: low-wealth (high-wealth) cooperatives choose degrees of meritocracy below (above) the efficient one. This result is in line with Beviá and Corchón (2018)[2]; who, assuming independence among workers' labor contributions, find

[1] $\rho_0 \leq \gamma$, has already been assumed.

[2] This result is a version of that which Corchón and Puy (1998) find for the Proportional and the Equal Benefit rule. In that paper they do not consider the Egalitarian rule because, unlike with the former share rules, it is not both efficient and individually rational.

that when the labor contribution of the median worker is higher (lower) than the average, the outcome of majority voting is the Proportional rule (Egalitarian rule). Moreover, since in our model, the provision of labor, and thus, output production, is related to the cooperative's wealth, low-wealth (high-wealth) cooperatives tend to work and produce below (above) the optimal level.

4. Conclusion

We have addressed the endogenous determination of the degree of meritocracy in a cooperative that allocates its output by means of a convex combination of the Proportional and the Egalitarian sharing rules. The determination of the degree of meritocracy is made by a two stage equilibrium where in the first stage, workers choose the degree of meritocracy by vote and, in the second, they choose the amount of labor they contribute to the cooperative to produce the output. Since we consider interdependency among workers' labor contributions, we have to lower bound the degree of meritocracy to encourage all individuals in the cooperative to provide positive amounts of labor. Although the cooperative's workers are identical in their tastes, they distinguish themselves by their endowment of wealth. This allows us to characterize a cooperative by its wealth distribution. Therefore, low-wealth (high-wealth) cooperatives are ones in which the median wealth is lower (higher) than the average wealth. Our main conclusion is that low-wealth (high-wealth) cooperatives choose degrees of meritocracy below (above) the optimal. However, efficiency can be reached in equilibrium whenever the median and average wealth are equal. Finally, since the higher the degree of meritocracy the higher the workers' labor contribution, the former choice has consequences on the cooperative's output, such that low-wealth (high-wealth) cooperatives tend to provide labor, or produce output, below (above) their optimal level.

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