Agent-based Modeling And Market Microstructure

Liu, Buhong

Awarding institution:
King's College London

Agent-based Modeling And Market Microstructure

Buhong Liu

Supervisor: Dr. Maria Polukarov
Prof. Carmine Ventre
Dr. Lingbo Li

The Department of Informatics
King's College London

This dissertation is submitted for the degree of
Doctor of Philosophy

January 2024

Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements.

The following publications are associated with the candidate's PhD work at King's College London:

1. Liu, B., Polukarov, M., Ventre, C., Li, L. and Kanthan, L., 2021, April. Call Markets with Adaptive Clearing Intervals. In 20th International Conference on Autonomous Agents and Multiagent Systems (AAMAS-21).

2. Liu, B., Polukarov, M., Ventre, C., Li, L. and Kanthan, L., 2021, October. Agent-based Markets: Equilibrium Strategies and Robustness. In Proceedings of the 2nd ACM International Conference on AI in Finance (ICAIF-21).

3. Liu, B., Polukarov, M., Ventre, C., Li, L., Kanthan, L., Wu, F. and Basios, M., 2021, October. The spoofing resistance of frequent call markets. In Proceedings of the 21st International Conference on Autonomous Agents and Multiagent Systems (AAMAS-22).

4. Liu, B., Polukarov, M., Ventre, C., Li, L., Kanthan, L., Wu, F. and Basios, M., 2021, October. Analysis on the Impact of Iceberg Orders in Financial Markets. In Proceedings of the 4th Games, Agents, and Incentives Workshop (GAIW-22).

Buhong Liu
January 2024

Acknowledgements

I would first like to thank my supervisors: Dr Maria Polukarov, Prof Carmine Ventre and Dr Lingbo Li, for their invaluable instruction, patient guidance and generous support during my PhD study.

I also owe a great debt of gratitude to my sponsor, TurinTech AI, for their generous financial aid during these four years. I wish to thank the administration team of TurinTech AI, including Dr Leslie Kanthan, Dr Fan Wu, Dr Michail Basios and Dr Lingbo Li, the research team, the engineering team and the business team. I had a very happy time when I participated in the research project at TurinTech AI and learned a lot from all of them.

I could not have undertaken this journey without the support from my friends in the UK. I would like to extend my sincere thanks to Dr Chen Zhang, Dr Fan Fang, Dr Wei Zhao, Miss Yuxi Huan, Miss Sophia Shi, Miss Xinran Tao, Mr Mingyang Wang and all members KCL Finance Hub of the department of Informatics, all members of KCL Mandarin Debate Society, all students of MSc Computational Finance at KCL between 2018 and 2022.

Finally, I want to thank my loving parents and grandmother for their unwavering support.

Abstract

In most modern financial markets, traders express their preferences for assets by making orders. These orders are either executed if a counterparty is willing to match them or collected in a priority queue, called a limit order book. Such markets are said to adopt an order-driven trading mechanism. A key question in this domain is to model and analyze the strategic behavior of market participants, in response to different definitions of the trading mechanism (e.g., the priority queue changed from the continuous double auctions to the frequent call market). The objective is to design financial markets where pernicious behavior is minimized.

The complex dynamics of market activities are typically studied via *agent-based modeling* (ABM) methods, enriched by *Empirical Game-Theoretic Analysis* (EGTA) to compute equilibria amongst market players and highlight the market behavior (also known as market microstructure) at equilibrium. This thesis contributes to this research area by evaluating the robustness of this approach and providing results to compare existing trading mechanisms and propose more advanced designs.

In Chapter 4, we investigate the equilibrium strategy profiles, including their induced market performance, and their robustness to different simulation parameters. For two mainstream trading mechanisms, continuous double auctions (CDAs) and frequent call markets (FCMs), we find that EGTA is needed for solving the game as pure strategies are not a good approximation of the equilibrium. Moreover, EGTA gives generally sound and robust solutions regarding different market and model setups, with the notable exception of agents' risk attitudes. We also consider heterogeneous EGTA, a more realistic generalization of EGTA whereby traders can modify their strategies during the simulation, and show that fixed strategies lead to sufficiently good analyses, especially taking the computation cost into consideration.

After verifying the reliability of the ABM and EGTA methods, we follow this research methodology to study the impact of two widely adopted and potentially malicious trading strategies: spoofing and submission of iceberg orders. In Chapter 5, we study the effects of spoofing attacks on CDA and FCM markets. We let one spoofer (agent playing the spoofing strategy) play with other strategic agents and demonstrate that while spoofing may be profitable in both market models, it has less impact on FCMs than on CDAs. We also

explore several FCM mechanism designs to help curb this type of market manipulation even further. In Chapter 6, we study the impact of iceberg orders on the price and order flow dynamics in financial markets. We find that the volume of submitted orders significantly affects the strategy choice of the other agents and the market performance. In general, when agents observe a large volume order, they tend to speculate instead of providing liquidity. In terms of market performance, both efficiency and liquidity will be harmed. We show that while playing the iceberg-order strategy can alleviate the problem caused by the high-volume orders, submitting a large enough order and attracting speculators is better than taking the risk of having fewer trades executed with iceberg orders.

We conclude from Chapters 5 and 6 that FCMs have some exciting features when compared with CDAs and focus on the design of trading mechanisms in Chapter 7. We verify that CDAs constitute fertile soil for predatory behavior and toxic order flows and that FCMs address the latency arbitrage opportunities built in those markets. This chapter studies the extent to which adaptive rules to define the length of the clearing intervals — that might move in sync with the market fundamentals — affect the performance of frequent call markets. We show that matching orders in accordance with these rules can increase efficiency and selfish traders' surplus in a variety of market conditions. In so doing, our work paves the way for a deeper understanding of the flexibility granted by adaptive call markets.

Table of contents

List of figures

List of tables

Nomenclature

Acronyms / Abbreviations

ABM Agent-based Modelling

CDA Continuous Double Auction

CVACM Combined Adaptive Call Market

CVACM Cumulative-Volume-driven Adaptive Call Market

DCM Deterministic Call Market

EGTA Empirical Game-theoretic Analysis

EVACM Extreme-Volume-driven Adaptive Call Market

FCM Frequent Call Market

RCM Random Call Market

SACM Stability-driven Adaptive Call Market

Chapter 1

Introduction

Nowadays, most modern financial markets adopt the so-called order-driven mechanism, where traders express their trading motivations by submitting selling or bid orders with a particular price and quantity. The market will match these orders and either execute or collect them in a priority queue following the matching rule. The outstanding orders are recorded in a limit-order book. We have seen much research on the market microstructure. Market microstructure is "the area of finance that is concerned with the process by which investors' latent demands are ultimately translated into transactions" (Madhavan, 2000). and studies "the process and outcomes of exchanging assets under the explicit trading rule" (O'hara, 1997). In detail, researchers are more interested in the following market microstructure topics:

- price formation: to model how the price of the asset is determined considering the role in the market (for example, the dealer or the market maker), the inventory, and the asymmetric information (Hendershott et al., 2011; Rust et al., 2018).
- market structure and design: to study the stylized facts including the volatility and the statistical properties of the price dynamics and the connections between these properties and the market rules, for example, the auction type, the order type accepted, the order matching priority, and others (Baur and Schulze, 2009; Cason and Friedman, 1996; Cont, 2007; Cont et al., 2014).
- transparency, and other applications: transparency describes all involved information during the trading process provided to participants and is involved with regulation policies Madhavan (2000).

In this thesis, we mainly investigate the market design and its impact on strategic participants. We start with critically evaluating the most popular trading mechanism, the continuous-time double auctions (CDAs). In a CDA market, the order submissions and withdrawals are processed serially (Budish et al., 2015). Due to the property, CDA markets are believed to lead to the latency arms race problem because traders can enjoy huge benefits from having just a tiny access-time advantage over the others (Sparrow, 2012). In other words, CDAs create the perfect environment for the proliferation of high-frequency traders, seen by many as an unhealthy feature of modern financial markets, that leads to flash crashes and, more generally, highly volatile markets (Kirilenko et al., 2017). A way to address this problem is by switching from continuous-time clearing mechanisms to a discrete-time clearing model called batch auction or call market. Specifically, in a call market, there is a clearing interval: orders received during this time interval are accumulated and processed in batch at the end of the interval only, rather than serially in the order of arrival. In modern financial markets, there are thousands of trades within a tiny time period. Thus, the research on call markets nowadays lies mainly in the high clearing frequency scenario, a.k.a. frequent call markets. Frequent call markets have been shown to successfully address arbitrage problems caused by latency arms race (Budish et al., 2015; Wah and Wellman, 2013), which encouraged their substance use, especially in the determination of the open price of some exchanges (such as the London Stock Exchange). Naturally, call markets motivate a further design problem concerning the length of the clearing interval.

The second question is about transparency. In order-driven markets, the market data reveals the universal beliefs on the market and is a key information source for traders. If the market data, including the best ask and bid price and their sizes, are manipulated, other participants could be misled and suffer from potential loss. It is believed that market manipulation can harm market liquidity (Comerton-Forde and Putniņš, 2014; Cumming et al., 2020), reduce market efficiency (Pirrong, 1995), and raise spreads and volatility (Williams and Skrzypacz, 2020). Spoofing is a simple and popular market manipulation method that can be defined as "strategically placing and canceling orders in order to move prices and trade later in the opposite direction" (Williams and Skrzypacz, 2020). Spoofing is illegal in many countries (Lee et al., 2013; Lin et al., 2019). For example, the Dodd-Frank Wall Street Reform and Consumer Protection Act made spoofing illegal in 2010 in the U.S. However, spoofing is still common in financial markets; for example, JP Morgan Chase was fined by U.S. regulators in 2020 for price manipulation in precious metal and treasury bill markets (Williams and Skrzypacz, 2020). The impact of spoofing on participants and markets is an important research question in the context of market mechanism design.

The third question is about the response of participants in the CDA market. In these order-driven financial markets, some market information, usually the price and accumulated volumes of the best limit orders, is open to the market participants so that traders have information to reach trading decisions and submit their orders. These orders provide liquidity to the market. However, transparency is not always desirable for the market or its participants. When a trader seeks to execute an order for a large amount of the asset, opportunistic traders could be tempted to hurt the effective execution of the trade (Frey and Sandås, 2017) and profit (an activity known as front running). On the other hand, such large orders could leak information about the true value of the asset to the market, and other participants would adjust their strategies and stop the execution of the large trade at the pre-specified price (Esser and Mönch, 2007). To address these concerns, some exchanges adopt a particular order type, called *iceberg order*. An iceberg order is similar to a normal limit order, except for the revealed volume. A limit order is comprised of a price and a volume, with the meaning that the originator would like to buy a certain quantity (specified by the volume) up to a certain limit (defined by the price). When a trader submits an iceberg order, the price, the volume, and the peak size will be specified, with the peak size always smaller than the volume. After the iceberg order arrives to the limit order book, the peak size instead of the total volume is displayed whilst the remaining volume is hidden. If the order is executed for the peak size, another peak size is automatically displayed in the order book with a new time stamp, and the hidden part is reduced by the corresponding number of shares (Esser and Mönch, 2007). Some also argue that a trader using iceberg orders faces a loss in priority, which means the hidden orders lose time priority to displayed orders at the same price when she hides her intentions, as most electronic limit order books penalize the usage of hidden liquidity (Bessembinder et al., 2009). Thus, the impact of iceberg orders on the market performance and the choice of trading strategies of the traders are meaningful questions.

Before we attempt to answer these questions, we select appropriate tools carefully for our investigation. Existing market microstructure theory models the interaction between agents in a 1- or 2-period game-theoretic setting, with an emphasis on information asymmetry and adverse selection. For the study of markets over a longer term, quantitative modeling is more popular as the game dynamics become too complex to analyze over long horizons. An intuitive quantitative method is to consider the limit order book as a queuing system. The state of the order book is changed by actions or so-called order book events, such as placing limit orders, market orders, and cancellations. These frameworks use a top-down approach as they model the market directly, and thus may lack some micro-level foundations. Specifically, the motivations underlying each transaction are unknown. These issues can be addressed with the use of advances in computational capability and algorithms that provide new, alternative tools to understand financial markets. Agent-based modeling, a bottom-up approach developed in the late 1940s, generates the market from the perspective of traders (a.k.a. agents) involved. There are principally two benefits, amongst others, coming from the application of ABM methods. First, mathematical models often make several assumptions like the market efficiency hypothesis and market clearing, which are seldom true in real markets. All these assumptions are not required in ABM models. Second, with particular strategies assigned to agents, we may know why the market changes instead of only knowing how it changes. To this end, empirical game-theoretic analysis (EGTA) constitutes a powerful tool to account for the strategic response of agents to market rules and each other's actions. EGTA allows us to find equilibrium states of these agent-based markets when strategic and utility-maximizing traders follow an approximate form of Nash equilibrium and has been applied in numerous empirical studies of market interactions (Brinkman and Wellman, 2017; Wah et al., 2016b). The detailed description of the EGTA method can be seen in Chapter 2.3.4.

The thesis is organized as follows: Chapter 2 comprehensively surveys related literature on the modeling of market microstructure from stochastic models to agent-based models. The advantages and disadvantages of different approaches are critically discussed. This chapter also describes the process of the EGTA method and illustrates the research gaps between related literature and this thesis. Chapter 3 describes the preliminaries to make the thesis self-contained. In Chapter 4, we investigate the equilibria of agent-based simulations of two mainstream trading mechanisms, continuous double auctions (CDAs) and frequent call markets (FCMs) via EGTA. We find under certain conditions, pure strategies could be a good approximation of the equilibrium. However, in general cases, the EGTA is needed for solving mixed strategies equilibrium, and we verify the soundness and robustness of the EGTA and show that both heterogeneous and homogeneous ways of applying EGTA lead to sufficiently good analyses. In Chapter 5, we study the effects of spoofing attacks on CDA and FCM markets using Agent-Based Modeling. We find that in widely-adopted CDA markets, spoofing will decrease market efficiency and order volume of the market, having further negative effects on market liquidity. However, spoofing is not as profitable in the frequent call markets. We investigate the underlying reasons and find that the key is the volume of traded feigning orders, which is also equivalent to the speed of accessing updated level 1 market data. When we address the problem by introducing a random auction ending time, the spoofing can be curbed. In Chapter 6, we present an agent-based model to study the impact of iceberg orders on the price and order flow dynamics in financial markets. We find that in CDA markets, the order volume has significant effects on the agents' strategy choice and market performance and the iceberg orders are helpful for avoiding speculating and the decrease in market efficiency. We also show that FCMs are much more robust to orders of large size and suffer almost no impact from iceberg orders. We conclude from Chapters 5 and 6 that FCMs have some exciting features when compared with CDAs and focus on the design of trading mechanisms in Chapter 7.

Chapter 2

Literature Review

Market microstructure studies ”the process and outcomes of exchanging assets under explicit trading rule” (O’hara, 1997). Within related topics, the price formation and participants’ behavior attract the most interest because the price is key information used for further analysis.

2.1 Stochastic Models

We take modern electrical financial markets as an example. Usually, traders submit instructions to the exchange including their trading wills (buy or sell), the quantity of the asset they would like to trade, and the accepted price. The price describes the maximum amount they would like to pay for the asset if the instruction is a bid order or the minimum amount they would like to take to sell the asset if the instruction is an ask order. Thus, such instructions are called limit orders. The exchange also maintains a record of submitted limit orders if they are not executed upon arrival so that they can be matched to incoming limit orders. Such a record is called the limit order book. As a result, every time a limit order arrives in the market, if it is executed, an executed price will be recorded; if not, it will be added to the limit order book. In the static status, the limit prices of all bid orders in the limit order book must be smaller than the limit prices of all ask orders; otherwise, it means someone is willing to pay more for the asset than another asks, and the transaction will be triggered. We call the highest bidding price in a static limit order book the best bid price and call the lowest ask price the best ask price. It is easy to see that the gap between the best bidding price and the asking price is the smallest among all bid-ask gaps and we call the smallest gap the bid-ask spread. When we are more interested in the general price level of the limit order book and do not distinguish the difference between the bid orders and the ask orders, we use the average value of the best bid price and the best ask price as the measurement, and it is called the mid-price. Overall, when the financial market runs normally, we can observe the dynamics of the best bid price, the best ask price, the mid-price, and the bid-ask spread, and all of them are important components of the market microstructure.

A straight way is to apply a process to describe the dynamics. The dynamics of the limit order book are updated because of the arrival of new orders. Thus, the book can be considered as a system of queues, and each component of the queue is determined by an event, which can be modeled by a point process. Cont et al. (Cont et al., 2010) use a multi-dimensional process $X_i(t)$ to represent the state of the limit order book where $X_i(t)$ is the volume of limit order at a price i. The limit order book is described by a multi-dimensional, continuous-time Markovian process. We can see similar work in (Bacry et al., 2016) where the Hawkes processes were applied to describe the limit order book events.

Reduced-form diffusion approximations are also popular approaches to model the limit order book dynamics. Cont et al. (Cont and De Larrard, 2012) adopt the reduced-form representation of a limit order book, using four components including the bid price, the asking price, the size of the bid queue, and the size of the ask queue. They then use a Poisson point process to model the arrival of market orders and limit orders, and the four-component state process would be updated according to the arrival of new orders. They fit the reduced-form diffusion model to real data to estimate the parameters. Similarly, Yang and Zhu (Yang and Zhu, 2016) used the reduced-form diffusion model to describe the movement of both the best bid order and the best ask order. Their two-dimensional diffusion process contains the price and the quantity. They show how the drift effect, correlation, and volatility of both processes depend on the imbalance of the quantities of both processes.

The advantages of stochastic models lie in the following aspects. First, it is easy to investigate the statistical features of limit order books and reproduce the dynamic properties. Cont and Larrard (Cont and De Larrard, 2013) propose a Markovian model of a limit order market, which captures some salient features of the dynamics of market orders and limit orders and enables a wide range of properties of the price process to be computed analytically. They observed that, if one is primarily interested in the price dynamics, it is sufficient to focus on the dynamics of the (best) bid and ask queues. Secondly, detailed behavioral assumptions about market participants or introducing unobservable parameters describing participants' preferences are not required so that the stochastic models are easily employed and applicable to a wider range of markets. However, the price process is often not Markovian or shows some good statistical properties that could be described by a stochastic process perfectly. In addition, the limit order book events tend to be not independent or stationary, but dependent on the state of the limit order book. Finally, it lacks the micro perspective. In other words, the underlying incentives of participants who submit limit orders are completely ignored, which makes the model a black box.

2.2 Game-theoretic Models

In the microeconomics view, the price is determined by the demand curve and supply curve: the intersection point of both curves represents the equilibrium price and equilibrium quantity. It is intuitive to apply game theory tools to find the series of equilibrium prices when the demand and the supply have a perfect match. We assume that traders observe the information from the market, estimate the actions of other participants, and make their own decisions based on this information and their estimations. That is a typical dynamic game. Much research on price formation is based on game theoretical equilibrium analysis, modeling the strategic interaction between a small number of agents (informed/non-informed traders, market makers, etc.) in a 1 or 2-period game theoretical setting and attempt to find the equilibrium price which is the market price where the quantity of supplied goods is equal to the quantity of demanded goods.

Parlour introduces a one-tick dynamics model for a limit order market in his work (Parlour, 1998). There are several assumptions in this market: first, traders have different valuations of the same risky asset, and secondly, they arrive in the market randomly, faced with the choice of either executing an immediate market order or opting for a potentially better price with the associated risk of non-execution by submitting a limit order. The likelihood of order execution is endogenously determined by factors such as the current state of the order book and their expectations regarding the number of orders that will arrive throughout the day. The final assumption is that traders are aware that their decisions impact the incentives for future traders, and they consider this in their choice between submitting market orders and limit orders. This dynamic creates a scenario where a trader entering the market evaluates both sides of the order book to formulate their optimal order strategy. Parlour's approach explicitly links the state of the limit order book to order submission strategies. As a result, it identifies the typical characteristics of a continuous double auction with price contingent orders under full information conditions, and the equilibrium prices are computed by solving this game.

In another research study, Rosu presents a continuous-time model for price formation in a market that operates based on a limit-order book (Roşu, 2009). This model focuses on strategic liquidity traders who arrive randomly in the market and make dynamic decisions between employing limit and market orders. These traders carefully weigh the execution price against the waiting costs associated with their choices. The trading process, conducted through a double auction, is viewed as a dynamic stochastic game, where each participant's strategies are represented by the prices and directions of their orders. The equilibrium in this context revolves around mixed bidding strategies, wherein each player has no incentive to alter their approach, considering the strategies of others. Notably, when these mixed strategies are conditioned on the game only, the equilibrium is called a Markov equilibrium. Rosu's work establishes the existence of a Markov equilibrium in which the bid and ask prices are determined solely by the number of buy and sell orders in the order book. Furthermore, this equilibrium can be analytically characterized in closed form in various meaningful scenarios. The model yields empirically supported implications regarding the shape of the limit-order book and the dynamics of prices and trades. In particular, it demonstrates that buy and sell orders can cluster away from the bid-ask spread, resulting in a distinctive hump-shaped limit-order book. Additionally, following a market buy order, both the ask and bid prices increase, with the asking price experiencing a greater increase, leading to a widening of the bid-ask spread.

Parlour's and Rosu's method provides a straightforward methodology to model the double-auction-style market and investigate its features. However, there are some strict constraints in doing equilibrium analysis. Due to the complexity limitation, we cannot put many agents or time steps in this model. Also, the probability of order submission depends on order books and cannot be adjusted across the whole period. All agents follow the same strategy and cannot be distinguished from others, which fails to represent multiple types of agents in real markets. To address the limitations of the game-theoretic models, we turn to the agent-based modeling method.

2.3 Agent-based Modeling

Agent-based modeling is a bottom-up approach developed in the late 1940s, providing an approach from the perspective of involving agents.

2.3.1 Design of Artificial Market

The primary concern is the design of the market structure. Lettau's work in 1997 serves as an exemplary model for future research and establishes a valuable benchmark (Lettau, 1997). Within his constructed market, two types of assets are actively traded: a risk-free bond offering zero interest and a risky asset. The risky asset generates random dividends in the subsequent period, following a normal distribution pattern. Agents are confronted with choosing between these two assets, effectively transforming the issue into one where agents must determine their optimal allocation of the risky asset. Importantly, the asset prices are externally determined, allowing Lettau to focus on examining agents' behavior in this context.

However, Lettau's market provides a view of only one side of trading because agents can buy assets from the market but don't sell. Also, the setting of externally-determined price makes the one-directional interaction between the agent and the market because the action of agents has almost no effect on the movement of the market price. So, we would need a setting to ensure the full interactions between buyers and sellers.

The Santa Fe Stock Market model stands out as one of the most daring ventures in artificial market research (Arthur et al., 1996). This innovative model aims to combine a precisely defined economic framework for market trading mechanisms with inductive learning through a classifier-based system. Within this framework, agents with constant absolute risk aversion (CARA) utility are designed to make decisions regarding their asset allocation for a single trading period. They must choose between a risk-free bond and a risky stock that yields stochastic dividends. The bond is perpetually available and offers a consistent interest rate denoted as r. The dividends adhere to a clearly defined stochastic process. In this market, a well-defined linear homogeneous rational expectations equilibrium (REE) emerges, where all traders share a common understanding of how to forecast future dividends and the relationship between prices and the fundamental dividend. Agents form their individual expectations by utilizing a classifier system that endeavors to discern the pertinent state of the economy. These expectations, in turn, influence the price and dividend forecasts, which are integral to the demand function.

The basic features of the Santa Fe Artificial Stock Market are listed below(LeBaron, 2002, 2006; Palmer et al., 1999):

- Discrete-time market: $t = 0, 1, 2, \cdots$
- N traders and an auctioneer play in the same market
- Traders are identical except each trader individually forms expectations over time through inductive learning
- Each trader has the same initial wealth W0 in the initial time period
- There are two assets to be traded in the market: a risk-free asset and a risky stock
  - Risk-free asset pays a constant known 1-period net return rate r
  - Risky stock pays an uncertain dividend d_{t+1} at the end of each period and has an uncertain one-period net return rate R_t over each holding period
  - Stock dividend d_t is generated by a random process unknown to the traders

The Santa Fe model is suitable for the study of the market microstructure in the long term because risk-free interests and risky dividends are considered. However, When the ABM method is used to investigate the financial markets with much higher trading frequency, the net returns from the risk-free asset and dividends from the risky asset can be removed because the trading period in such research is too short to generate significant returns and dividends. For the same reason, there is no need to distinguish the difference between the risk-free assets and the cash. As a result, in recent market microstructure research using the ABM method (Brinkman and Wellman, 2017; Wah et al., 2016b; Wang and Wellman, 2017; Wellman et al., 2013), the artificial market is relaxed to be a single-asset market with discrete or continuous time. There exists a fundamental values dynamics that is partially public to all involved traders and represented by a mean-reverting stochastic process shown in (4.1).

2.3.2 Design of Agents

There exists a common assumption that the theory of Rational Expectation (Muth, 1961) doesn't apply to the design of agents. According to this theory, agents are assumed to know the model and have complete information, perfect rationality, and common expectations. The theory of Rational Expectations is unrealistic because its fundamental assumptions are too strong in most cases and it is impossible for all agents to behave in the same optimal way. Considering the complexity of the stock market, a more realistic bounded rationality setting was founded by Palmer et al. (Palmer et al., 1994). Under the bounded rationality assumption, agents are independent and adaptive based on their own observation of the environment.

There are different measurements of the gains of agents. In the Santa Fe model (LeBaron, 2002), agents are all wealth-seekers, and they will have the identical utility of wealth function $U(W)$ exhibiting constant absolute risk aversion, where W is the accumulative wealth in the end. Brinkman et al. (Brinkman and Wellman, 2017) consider the surplus of agents from their transactions. To compute the surplus, they assign the individual valuation to each agent and the surplus is the difference between the valuation and the executed price. The gain of each agent is the sum of the accumulative surplus across the trading period.

The design of agents' strategies plays an important role in ABM, and we have seen abundant strategies from previous research. Gode and Sunder are the first to put forward the concept of Zero-Intelligence (ZI) agents(Gode and Sunder, 1993). A ZI agent lacks cognitive abilities, does not pursue or maximize profits, and does not engage in observation, memory, or learning. Its trading decisions are entirely random, guided solely by the constraint of a budget limit. Specifically, a buyer refrains from bidding above the cost, while a seller avoids offering below the cost. They compare the performance between markets with well-trained human traders only and with ZI traders only. In experimental markets featuring human traders, they observe the convergence toward the equilibrium price at which the demand matches the supply within a few rounds. In another market, the budget-constrained traders also exhibit a calmer price series close to equilibrium. Market efficiency tests corroborate these findings, indicating that constrained traders achieve asset allocations with over 97% efficiency in most cases, a level comparable to that achieved by human traders. They also test the ZI agents without the budget constraint. Conversely, random computer traders without budget constraints exhibit entirely random behavior, resulting in a lack of convergence and highly volatile transaction prices.

The experimental results of ZI agents claim that ZI agents with budget constraints have similar behavior with humans and it is good enough to employ budget-constrained ZI agents in agent=based models. We can see both support and criticism of the design of ZI agents. Othman expands the utilization of ZI agents within the context of prediction markets (Othman, 2008). Prediction markets are places where people can trade assets whose payments are determined by the outcomes of unknown future events. Othman integrates two modeling frameworks: an intuitive private-value model of prediction markets initially proposed by Manski (Manski, 2006) and a model of Zero-Intelligence agents originally employed by Gode and Sunder (Gode and Sunder, 1993) in their study of double auctions. Othman develops a prediction market model that does not rely on any "higher-order" human characteristics such as utility maximization, trend following, or learning. The model demonstrates that the resulting prices closely align with the findings of empirical studies and concludes that the pricing behavior observed in prediction markets can be replicated without the necessity for "expert-level" intelligence or reasoning. Cliff and Bruten make their criticisms (Cliff and Bruten, 1998) and argue that, despite the irrationality of Zero-Intelligence (ZI) agents, Gode and Sunder's framework essentially guarantees convergence to prices that are closely aligned with efficiency. In other words, the setting is designed to ensure a favorable outcome for ZI agents. In response to this critique, the concept of 'zero-intelligence-plus' (ZIP) traders is introduced. Similar to ZI traders, ZIP agents make stochastic bids. However, in contrast to ZI traders, ZIP traders incorporate a basic form of machine learning into their decision-making processes. In their experiments, the performance of ZIP traders is found to be significantly closer to human data compared to the performance of Gode and Sunder's ZI traders.

The Santa Fe model adopts strategic trading behavior rather than random actions (LeBaron, 2002). According to their design, each trader updates its set of forecasting of the price movement of the risky asset with a certain probability in each period using a genetic algorithm (GA). In other words, agents always update their strategies and play the optimal strategy only. Brinkman et al. (Brinkman and Wellman, 2017) adopt a brand new design of trading strategies. They construct a strategy pool for all agents and each agent will choose one from the pool. The component of the strategy pool is described by a range that illustrates the expected surplus from a transaction. Suppose $\alpha_{\min}$ is the lower bound of the range and $\alpha_{\max}$ is the upper bound. The price of the limit order represents the action this agent plans to submit and the price is drawn from the uniform distribution parameterized by its valuation and the range of expected surplus. Detailed description can be seen in Chapter 4.3.2.

The design of the ZI agent's family is unrealistic, although they perform closely to humans, according to Gode and Sunder. However, they have only 12 traders in their experiments and all of them are graduate students of business. In other words, their experiments are too simple to approximate the complex financial markets filled with professional traders. We plan to investigate the market microstructure when the market is in an equilibrium in which all agents have no incentive to alter their strategies. Thus, we decided to adopt a similar design as Brinkman et al. However, more trading strategies should be added to the strategy pool to simulate the complex financial markets.

2.3.3 Simulations

An important question of employing the artificial market and ABM is whether it looks like a real market. Or, in statistics, does it have similar statistical features with real markets? In addition to the work of Gode et al. and Othman, many other researchers attempt to investigate this problem. Maslov (Maslov, 2000) implements a model where traders face the decision of either trading at the market price or placing a limit order. The selection between these options is entirely random, without any strategic considerations. The execution price of a limit order is determined by simply offsetting the most recent market price by a random amount. Through numerical simulations of this model, it becomes apparent that despite its minimalist rules, the price pattern generated exhibits realistic features including "fat" tails in the distribution of price fluctuations, etc. Raberto et al. examine an agent-based artificial financial market, where diverse agents trade a single asset (Raberto et al., 2001). Initially, agents possess a finite amount of cash and a predetermined portfolio of assets. There are no cash inflows or outflows throughout the whole trading period. Similarly to the ZIP agents, agents in this market make random buy and sell decisions, constrained by their available resources, subject to their observation of the clustering and the volatility in previous periods. The proposed model successfully reproduces the leptokurtic shape of the probability density of log price returns, which means the density of the distribution of log returns looks like a bell with a sharp peak and fat tails. It also captures the clustering of volatility. Consiglio et al. propose an artificial market where multiple risky assets are traded and examine that the similarity between the artificial markets and the real markets still holds in multi-asset markets in terms of the non-normal distribution of price changes and temporal patterns (Consiglio* et al., 2005).

2.3.4 Empirical Game-theoretic Analysis Method

Although the above research has shown the effectiveness of ABM, they are conducted in a simplified setting where agents either take random actions or play the optimal action parameterized by market information only. As we discussed in the design of agents, playing the optimal strategy doesn't guarantee equilibrium. When we apply traditional game theory analysis, we need to know the payoffs for every strategy profile. It works only when we are solving small games, or larger games with an entirely analytic specification. If we face complex game models, they may not have any analytic form and are generally referred to as simulation-based games (Vorobeychik et al., 2008). Wellman et al. (Wellman, 2006) raise the standard procedure for analyzing simulation-based games, which is known as the *empirical game-theoretic analysis* (EGTA). When performing the EGTA method, all agents will be assigned to roles, which are groups of symmetric agents. The symmetric agents, or agents in the same role, must share the same strategy pool, and switching the agents in the same role does not affect the game. A game is referred to as a role symmetric if there exists a unique role for every agent, and the role-symmetric mixed-strategy equilibrium is defined by the equilibrium of a role-symmetric game where each agent within a role chooses a strategy from the same distribution independently (Brinkman, 2018). They have shown that every finite role-symmetric game has at least one role-symmetric mixed strategy Nash equilibrium (Brinkman, 2018; Wellman, 2006).

The outline of the EGTA can be summarized as the following steps:

- Assign agents to symmetric roles, i.e., find mutually exclusive groups to cover all agents and ensure all agents within a group have the same access to strategies. For example, assign agents to the buyer group and the seller group.

- Select finite samples of the strategies to convert the game to the normal form if the size of the strategy set is too large. The strategy selection techniques have been discussed in (Schvartzman and Wellman, 2009) while the profile search techniques can been seen in (Jordan et al., 2010; Wellman et al., 2013).

- Evaluate the payoffs of selected profiles. Run the simulation only once if the payoffs are deterministic or a sufficient number of simulations to have a low variance estimate if the payoffs are variables.

- Construct a role-symmetric normal-form game using evaluated profiles and use numeric methods to find the solutions (Daskalakis et al., 2009).

- In each simulation, the regret value defined by the maximum gains an agent can have by unilaterally deviating from other strategies will be computed. The regret value is also a measurement of plausibility because if the gains from turning to other strategies are small, agents will tend to stick to the current strategy. The accumulative regret values of all agents are the criteria for sorting the outcomes. We choose the strategy profile with the smallest regret value as the best approximation of the role-symmetric mixed-strategy Nash equilibrium. Suppose the number of players is still too large. In that case, an aggregation technique called deviation-preserving reduction (DPR) (Wiedenbeck and Wellman, 2012) can be applied to reduce the size of required players to approximate games by only evaluating a proportion of total profiles.

We have seen a few research on financial market microstructure via ABM methods and EGTA technique. Wang and Wellman (Wang and Wellman, 2017) adopt an ABM model to examine the possibility of manipulating prices in financial markets through spoofing and how to detect in a CDA market. Wah et al. (Wah et al., 2017) investigate the allocative efficiency of market making in a CDA market. Brinkman and Wellman (Brinkman and Wellman, 2017) study the optimal clearing intervals for the frequent call markets.

The benefits of applying ABM methods are concluded in the following aspects. First, mathematical models are developed based on several assumptions like market efficiency hypothesis, market clearing, and rationality, which are unrealistic in real markets. All these assumptions are not required in ABM models. Second, with the strategies assigned to agents, we may know why the market changes instead of only knowing how it changes. However, the shortcomings of the ABM approach cannot be ignored. There are too many degrees of freedom in the design of the environment and agents and a small difference in the experiment design can make huge gaps among outcomes, which makes it difficult to re-produce the findings. The computation is also costly if the size of agents is too large or the length of the trading period is too long.

2.4 Research Gaps

In this thesis, we aim to investigate whether some changes to the market mechanism are helpful in achieving certain objectives, such as high efficiency, avoiding spoofing, and maintaining stability. After carefully reviewing related literature, we adopt the ABM and EGTA methods as the research tools because we are interested in the market microstructure within a short scope and expect to observe the behavior of strategic agents as the response. Although we surveyed several successful research using ABM and EGTA as discussed above, the examination of the robustness and details of performing the EGTA method is missing. We are the first to verify that the EGTA produces the same qualitative results when the environment settings are different. We are also the first to study the difference between the heterogeneous EGTA and the homogeneous EGTA. The possible effects caused by risk aversion are missing in previous research. Furthermore, we examine the microstructure of markets in different types. We also extend previous research on the mechanism design from the study of optimal clearing intervals to other adaptive designs.

Chapter 3

Preliminaries

This chapter introduces the fundamental background of the market types and limit orders to make the thesis self-contained.

3.1 Order-driven Markets

A market is a place for people to exchange the goods they have. In financial markets, the people holding cash seek opportunities for undervalued financial assets, and the people holding assets look for a higher price. An immediate question is how we can make the demand and supply match. There existed a role played by professional financial institutions, whose duty was to buy assets from the people who were willing to sell and sell assets to the people who were willing to buy. They are called market makers and market makers are still playing an important role in financial markets to provide liquidity. The advantages of the market makers are obvious: buyers and sellers do not need to find the deals by themselves. They do not need to search for counterparties and negotiate on the price and quantity of deals. Much time and workload have been saved and counterparty risk is also avoided. However, they must accept the price provided by market makers if they need to do the trade. Market makers are almost guaranteed to make profits from their service because they provide liquidity, but it may cause a decrease in social welfare. With the application of computing technology, market information can be gathered, recorded, and published to the public without much delay. Then, the auction-style, order-driven markets have become the majority. Another question occurs to the management institution: can we design a good auction-style market? To answer this question, we need to define what is a good market and analyze the features of a good market. For participants in the markets, their questions are quite simple: how can we make more profits through trading in different types of markets?

3.1.1 Market Types

A financial market is a market where people buy or sell financial assets such as stocks and options. Therefore, a set of rules and procedures are developed for the market to serve the trading. Traditional financial markets are quote-driven markets. In these markets, a dealer, which is also called a market maker, provides both buy and sell orders to other participants by setting bid and ask quotes. Recently, with the development of electronic communications networks (ECNs), the order-driven trading mechanism has become the majority and adopted by the biggest exchanges like NYSE, NASDAQ, and LSE. In an order-driven market, there are no centralized market makers. All participants can submit buy and/or sell orders containing the quantity and price; at the same time, all outstanding orders will be recorded in an order book.

3.1.2 Double Auction-style Market

In an order-driven market, sellers and buyers have conflicting purposes in transactions. Buyers are interested in purchasing underlying assets with the lowest price, which is like an English auction, while the sellers desire to exchange for as much money as possible, which is like a Dutch auction. Such a double auction mechanism is widely used in financial markets (Gil-Bazo et al., 2007). When new orders arrive, they will be compared with the outstanding orders in the order book. If they match the best outstanding opposite orders, a trade happens, and executed orders will be removed from the book. Otherwise, the arrived orders will be added to the book. The orders which make trade happen immediately are called market orders while the orders added to the book are called limit orders. The difference lies in whether the price is specified or not. The aim of submitting a market order is to seek the trading opportunity immediately. Thus, the market order will be matched to the best outstanding order on the opposite side automatically, and no price is specified in the market order. For example, if someone submits a market bid order with the quantity of 2 shares, the best ask orders in the limit order book will be matched and the executed price is the best ask price. Suppose the best ask order has only one share while the market order requires two shares; there will be the second transaction that the second best ask order (with the second lowest ask price) will be executed. Thus, there will be two separate transactions whose executed prices are the lowest asking price and the second lowest asking price, respectively. Outstanding orders in the book can be canceled or modified before new market orders match them. The order book is essentially a table where the records from the bottom to the top are sorted from the lowest to the highest price.

Apart from the auction type, the priority rule is also an important component of a trading mechanism. Usually, price priority is basic, and time is a secondary priority (Hasbrouck, 2007). In other words, the market will examine if the price will trigger a transaction first upon the arrival of new orders. For instance, the best outstanding bidding price is £100, and there are three ask orders in the order of time. The price of these three ask orders are £101, £99 and £99. The exchange will examine the price first. As the price of the first ask order is higher than £100, the maximum amount a buyer would like to pay, it will not be executed. The prices of the second and the third ask orders are the same and below the best bidding price; both orders can be traded. On this occasion, the market will apply the time priority and only the second ask order will be executed because it arrives earlier than the third one. Although continuous limit order books are often regarded as essentially equivalent to the competitive market ideal of running in perfectly continuous time (Kominers et al., 2017), some attempts to change rules are made to avoid unexpected losses for participants.

For example, the best three bids are 100 shares at £100, 100 shares at £98, and 200 shares at £50, and a market sell order with 250 shares arrives. In a typical double auction market, the first 100 shares will be executed at £100, the second 100 shares will be executed at £98, and the rest will be traded at only £50. The procedure is called “walking the book". To avoid unexpected loss, a market order can be forbidden to walk the book. Only the best outstanding opposite orders will be matched, and the rest of the market orders will be regarded as a limit order at the execution price and added to the book. In the example above, 100 shares of the market sell order will be executed at £100 and the rest will turn to a limit sell order with 150 shares at £100 and be recorded in the book.

Another possible modification is the validity of orders. The time-in-force (TIF) specifies how long the order is to be considered actively (Hasbrouck, 2007). Some markets permit immediate-or-cancel (IOC) orders. IOC orders will not appear in the book because if they are traded, there will be no visible trace, and if they are not traded, they will be canceled. Assume the best bid price is £100 in the market. If a market sell IOC order or a limit sell IOC order at a price lower than 100 is submitted, it will be executed at £100 immediately. While if a limit sell IOC order at a price higher than £100, it will be canceled and will not appear in the book. Thus, other traders cannot observe the trace of an IOC order.

The order book contains all the current outstanding orders and is the most important information source for traders. In some cases, the whole book is open to the participants. However, if a trader seeks to hide his or her trading interests, he or she will not succeed when submitting orders in a traditional order-driven market. Therefore, some markets adopt rules allowing hidden and/or reserve orders. Hidden orders will not be revealed to other traders. For reserve orders, they are invisibility partially (Hasbrouck, 2007).

3.1.3 Frequent Call Market

Another important market type in this thesis is the frequent call market. The biggest difference between the continuous double-auction market and the frequent call market lies in the processing of orders. In a frequent call market, there exists a small time interval during which the market collects new orders only and this interval is called the clearing interval. Only at the end of the clearing interval, the market will sort all outstanding orders and collected orders during the interval and generate the demand-supply curve to see if any overlaps exist. For example, suppose till the end of the current clearing interval, there are three bid orders at the price of £100, £100, and £90, and three ask orders at the price of £85, £100, and £90. The market will sort the bid orders from the highest bidding price to the lowest bidding price and sort the ask orders from the lowest asking price to the highest asking price to generate the demand-supply curve. We can see two bid-ask price pairs with overlaps, which are (£100, £85) and (£100, £90). The executed price could be determined by the mid-price of the bid-ask price pair with the smallest gap, which is £95 in this case. Executed orders will be removed from the limit order book and others will added back to the limit order book. The new cycle starts, and the market will collect the new orders until the end of the new round of clearing interval.

Chapter 4

Agent-based Markets: Equilibrium Strategies and Robustness

Agent-based modeling (ABM) is broadly adopted to study the market microstructure empirically. Researchers set up market mechanisms such as the matching priority and behavior rules for participating traders, for example, if traders are allowed to hide their submitted orders. Researchers model the environment and strategic agents and observe the simulation results. However, these results can qualitatively change if trader incentives are ignored; for example, traders have incentives to hide part of their orders to avoid negative beliefs towards the trend of the movement of the asset's price. Thus, an equilibrium analysis is key to ABM. Empirical game-theoretic analysis (EGTA) is widely adopted to compute the equilibria of these agent-based markets. In this chapter, we investigate the equilibrium strategy profiles, including their induced market performance, and their robustness to different simulation parameters. For two mainstream trading mechanisms, continuous double auctions and call markets, we find that EGTA is needed for solving the game, as pure strategies are not a good approximation of the equilibrium. Moreover, EGTA gives generally sound and robust solutions regarding different market and model setups, with the notable exception of agents' risk attitude. We also consider heterogeneous EGTA, a more realistic generalization of EGTA whereby traders can modify their strategies during the simulation, and show that fixed strategies lead to sufficiently good analyses, especially taking the computation cost into consideration.

4.1 Introduction

In this chapter, we use EGTA to investigate the underlying details of mixed strategy equilibria and the corresponding market performance. We especially examine the soundness and robustness of the EGTA method in studying the behavior of strategic agents in financial markets. To the best of our knowledge, this is the first study to assess the quality of EGTA methods. Notably, we consider completely unexplored dimensions in EGTA, such as the risk attitude of traders and strategies that are heterogeneous over the simulation horizon, and measure their consequences on the equilibrium strategy states (a.k.a. profiles). Ultimately, we want to establish under which circumstances the qualitative conclusions we reach via ABM enriched by EGTA can be trusted to hold beyond the simulation environment used. To make our findings more compelling, our experiments are conducted on two popular trading mechanisms: continuous-double auctions (CDAs) markets and frequent call markets (FCMs).

The chapter is organized as follows: Section 4.2 overviews the literature on this topic. Section 4.3 describes the research design and experimental setting. The main empirical results and their analysis are shown in Section 4.4. The last section summarises our findings.

4.2 Related Work

The key feature of the agent-based market design is the trading mechanism. Lettau (Lettau, 1997) construct a simple market where two kinds of assets are being traded: a risk-free bond paying zero interest and a risky asset. However, while making a useful benchmark for financial market studies, Lettau's market provides a view of only one side of trading, as agents can only buy assets from the market. To address this issue, the Santa Fe Stock Market (Arthur et al., 1996) was proposed, where agents can buy and sell risk-free and risky assets in a discrete-time fashion. The Santa Fe Market is a well-known artificial market that inspired much follow-up research and modifications (Ehrentreich, 2003; LeBaron, 2002; Palmer et al., 1994).

Another essential issue in the focus of a market designer is the auction style. Most markets adopt the continuous double auction (CDA) model where order submissions and withdrawals are processed serially. Budish et al. Budish et al. (2015) consider an alternative model termed frequent call markets, where orders received during the clearing interval are accumulated and processed in a batch at the end of the day, demonstrating their advantages over the CDA markets. Wah et al. Wah et al. (2016a) construct a single-asset agent-based market to compare these two market types empirically. This market model has been adopted in several follow-up studies Brinkman and Wellman (2017); Liu et al. (2021a).

The design of agents has started from the work by Gode and Sunder (Gode and Sunder, 1993). They put forward Zero-Intelligence (ZI) agents who have no intelligence and make decisions randomly. Later works introduce bounded rationality to the study of ZI agents (Cliff and Bruten, 1998; Palmer et al., 1994). In particular, Brinkman et al. Brinkman and Wellman (2017) assigns the agents a strategy space where each strategy contains the minimum and the maximum shaded surplus from the trade.

To evaluate the market performance, it is essential to know the equilibrium state of the market. However, while game theory provides general tools to compute equilibrium agent strategies, it is difficult to find close-form solutions to financial market models. To this end, Wellman Wellman (2006) develop the framework for empirical game-theoretic analysis (EGTA), which is a simulation-based way to find an approximate equilibrium. EGTA method requires a large number of systematic simulations. The complex game is induced to a normal-from game by sampling strategy profiles and corresponding payoffs from the simulation results. EGTA method has been extensively applied in research on auctions and markets Schvartzman and Wellman (2009); Wah et al. (2016b, 2017); Wang and Wellman (2017).

4.3 Experimental Setup

In this work, we explore the equilibrium strategies of agent-based markets solved by EGTA. Specifically, we study what the equilibrium strategy profile looks like and if there exists a simple approximation of the equilibrium, for example, using a pure-strategy profile as the approximation to avoid complex and time-consuming computation using EGTA. Furthermore, we examine the quality and robustness of the equilibrium profiles computed by EGTA by comparing the strategy profiles computed in different simulations with the same experimental setup. To simply the experiments and be consistent with previous research, we employ a parameterized single-asset financial market model, which is inspired by the agent-based market models raised by Wah et al. (Wah et al., 2016a) and Brinkman et al. (Brinkman and Wellman, 2017).

4.3.1 Market Models

There are N agents in the market, who are allowed to submit limit orders following their assigned strategies. Each agent plays the role of a buyer or a seller randomly each time it enters the market. The size of the limit order is set to be 1 unit. Before placing new orders, agents will cancel all existing orders. Agents can only observe current level 1 of the limit order book of the market.

We assume there exists a fundamental value dynamics of the asset, and introduce the following mean-reverting stochastic process to simulate it:

$$f_t = r\bar{f} + (1-r)f_{t-1} + s_t \qquad f_0 = \bar{f} \qquad s_t \sim N(0, \sigma_s^2) \tag{4.1}$$

where f_t is the fundamental value at time t and $r \in (0,1)$ is the reversion rate. It is easy to see that the values of r and σ_s determine the range of the average shift of fundamental value. So higher values of r and lower values of σ_s^2 model more stable assets, like some equities, whilst a smaller value of r and larger value of σ_s^2 encapsulate rather volatile markets, such as cryptocurrencies.

At each time t, the fundamental value alters, and then all agents will generate their own valuations. Subsequently, the agents will generate their own valuations. We set the individual estimation of the fundamental value equal to $\lambda_{i,t} + f_t$, where $\lambda_{i,t}$ denotes the bias of agent i at time t. Thus, for the Agent i, its valuation of the asset at time t is defined by $v_{i,t}$ where $v_{i,t} = \lambda_{i,t} + f_t$. The closer this value is to 0, the better the ability of the agent to collect and analyze information and valuation. In our experiments, $\lambda_{i,t}$ is independently generated from a normal distribution $N(0, \sigma^2_{\text{bias}})$ for every t. For Agent$_i$, its surplus in time t is defined by the difference between its valuation and the executed price if its orders are executed. For example, if its valuation is 500 and its bid order at a price of 450 is successful, then its surplus from this transaction is 50 (500-450). However, if its valuation is 500 and its outstanding ask order is triggered at a price of 470 by the counterparty, then its surplus from this transaction is -30 (470-500). Agent's surplus at time t is the sum of surplus from all its transactions that happened between $t-1$ and t, and its accumulative surplus is the sum of the surplus over all the rounds of the simulation.

We are interested in both CDA markets and frequent call markets. The specialized setup for frequent call markets is inspired by Liu et al. (Liu et al., 2021a); there is an interval I during which the fundamental value does not change, and the calling frequency is measured by the ratio between the length of the clearing interval and the length of I. It is easy to see that a higher ratio indicates a lower frequency, and the higher the calling frequency, the more the market is like a CDA market.

4.3.2 Strategy Space

If an agent places a limit order with an exact valuation, we call it the telling-truth bidding strategy. We follow the strategy space design of Brinkman et al. Brinkman and Wellman (2017) and consider the situation where agents require an extra bonus from a transaction. Thus, we assign the agents with an expected surplus range $[\alpha_{\min}, \alpha_{\max}]$, representing the minimum acceptable surplus $\alpha_{\min}$ and the maximum expected surplus $\alpha_{\max}$ from trading. If an agent enters the market as a seller, the asking price is uniformly distributed in $[v+\alpha_{\min}, v+\alpha_{\max}]$ where v is its valuation at a given time. Similarly, if the agent plays the role of a buyer, the bidding price is generated from the range $[v-\alpha_{\max}, v-\alpha_{\min}]$ uniformly. It is easy to see that if the agent requires more shaded surplus, the probability of being matched will decrease. The next question before we use EGTA is how many different pure-strategy profiles should be considered for each simulation. Suppose there are m different strategies and N role-symmetric agents; then a possible profile could be N agents play m_1, and no agent plays others, or $N-1$ agents play m_1, one agent plays m_2, and no agent play others. To cover all possible profiles, it is easy to show that we need to simulate $C^m_{N+m-1} = \frac{(N+m-1)!}{m!(N-1)!}$ cases. Thus, to decrease the computation cost, we choose a limited strategy space denoted by S1 to S4, as shown in Table 4.1. To control the computation cost, we select a smaller strategy space in this work containing S1, S2, S3, and S4. If the agent plays strategy S3, it will submit a bid order at a price generated from the uniform distribution $U[v_{i,t} - 100, v_{i,t} - 20]$ or submit an ask order at a price generated from the uniform distribution $U[v_{i,t} + 20, v_{i,t} + 100]$. Obviously, strategy S1 is exactly the telling-truth strategy because the order price is its exact valuation and the latter strategies are more greedy than the former ones.

Table 4.1 Strategy space

Strategy	S1	S2	S3	S4
$\alpha_{\min}$	0	0	0	20
$\alpha_{\max}$	0	50	100	100

4.3.3 Environment Parameters

In our comparative analysis, the markets and the agents share the same parameters except for the strategies they employ. Some common parameters including $\bar{f}, \sigma_s^2, \sigma_{\text{bias}}^2$ are fixed through all the experiments. We consider the thickness of the market, assigning 40 agents to a thin market and 80 agents to a thick market. As from above, agents are allowed to trade only one unit of the asset at each time. The length of the simulation horizon T is 500. The asset price is a discrete integer ranging from 1 to 1000. We also consider the stability of the fundamental value which is controlled by the reversion rate r. In our experiments, the reversion value r has three options: 0.8, 0.5, and 0.2. A higher reversion rate makes more stable fundamental dynamics. We run 100 simulations for each game and take the average value as an approximation. The values of the most common parameters are listed in Table 4.2.

Table 4.2 Environment parameters

$\bar{f}$	r	σ_s^2	σ_{bias}^2	T
500	{0.8,0.5,0.2}	100	50	500

4.4 Experiments And Analysis

The key metric of the market we are concerned with here is market efficiency, which is measured by the ratio between the realized and the potential total surplus, which is calculated by supposing all involved orders are matched at the base of maximum surplus rather than how they are processed in reality. We also consider the total surplus obtained by the agents through trading in a certain type of market. The last metric is the average trading volume, which measures the activeness of the market. In what follows, we regard the average market efficiency, average total surplus, and average trading volume simply as *Efficiency, Surplus* and *Volume*.

4.4.1 Explanation for Mixed Equilibrium Strategies

Our investigation starts with the standard scenario where we use EGTA to calculate the equilibrium strategies in both CDA and frequent call markets filled with 20 agents. The results are listed in Table 4.3 (in cases with multiple equilibria, we choose the equilibrium with the smallest regret value, which is described in Section 2.3.4). The values of S1, S2, S3, and S4 are the probabilities of each strategy in the mixed-strategy Nash-equilibrium profiles. For example, we can see that the FCM with reversion value 0.2 achieves the highest efficiency of 0.95, and in this market, the mixed-strategy Nash-equilibrium profile is that playing S1 with the probability of 0.999 and playing S2 with the probability of 0.001.

Table 4.3 Equilibrium strategy profiles

Market Type	r	S1	S2	S3	S4	Efficiency
CDA	0.8	0.620	0.380	0.000	0.000	0.824
CDA	0.5	0.925	0.075	0.000	0.000	0.844
CDA	0.2	0.161	0.835	0.003	0.000	0.835
FCM	0.8	0.999	0.001	0.000	0.000	0.950
FCM	0.5	1.000	0.000	0.000	0.000	0.948
FCM	0.2	1.000	0.000	0.000	0.000	0.947

On the one hand, the efficiency of frequent call markets is always significantly greater than that of CDA markets sharing the same environment parameters, which supports the conclusion of previous research (Brinkman and Wellman, 2017; Liu et al., 2021a) that frequent call markets increase the market efficiency when the length of clearing interval is properly selected.

On the other hand, we find that in all equilibrium strategy profiles, strategies S1 and S2 account for most of the probability mass. To figure out the potential underlying reason

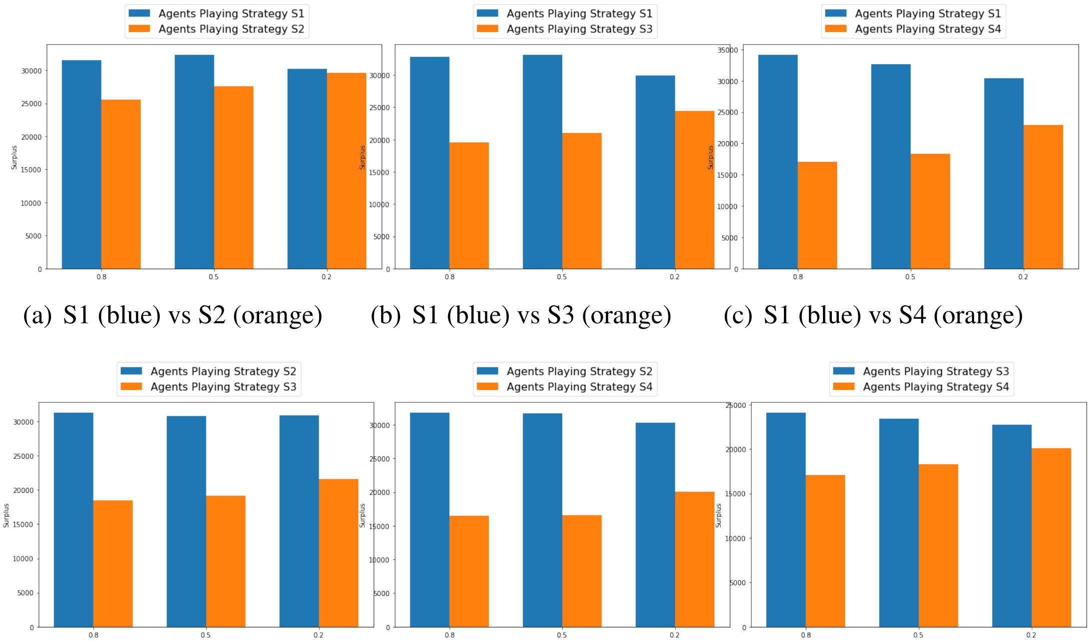
Table 4.4 Performance of CDA markets where agents play pure strategies only

r	Strategy	Surplus	Efficiency	Volume
0.8	S1	113218	0.818	3221
	S2	59598	0.831	1218
	S3	43560	0.626	950
	S4	38745	0.561	830
0.5	S1	115785	0.821	3141
	S2	58799	0.819	1233
	S3	42095	0.609	934
	S4	38466	0.542	821
0.2	S1	115368	0.820	3157
	S2	59778	0.822	1218
	S3	46797	0.639	900
	S4	40304	0.560	763

for this, we re-simulate these markets with different combinations of parameters where all agents can only play a pure strategy. We find that the number of agents and the length of the full horizon are immaterial to our findings. We select the experiment results of CDA markets with the same r and T but different strategies and reversion rates and show the results in Table 4.4. The telling-truth strategy (S1) makes the highest accumulative surplus and trading volume. Strategy S2 maintains similar efficiency to S1; however, both the accumulative surplus and the trading volume decrease sharply. The requirement of extra surplus makes up for the loss caused by the decreased volume. However, the other two greedy strategies (S3 & S4) lead to poor performance in all market metrics.

To verify whether strategies S1 and S2 dominate S3 and S4, we divide the agents into two equal groups, each group playing a different strategy, accordingly. We compare the average accumulative surplus of each group to see if one group dominates the other. The comparison results are shown in Figure 4.1. We observe that conservative strategies always outperform greedy strategies. This explains why the telling-truth strategy accounts for a large probability in the mixed strategy equilibrium. However, with the reversion rate decreasing, the advantages of conservative strategies become smaller and smaller because the surplus gaps between conservative and greedy strategies are decreasing along with the decreasing of the reversion rate. Another finding is that the differences between the group playing S1 and the group playing S2 are rather small; in particular, when the reversion rate is 0.2, there is almost no difference between these two groups. This may explain the empirical result that strategy S2 takes up the second largest probability in most mixed strategy equilibria and even climbs to the top in CDA markets with a reversion rate of 0.2.

Taking the equilibrium strategy profiles of FCMs into consideration, we conjecture that playing the telling-truth strategy could lead to a pure strategy Nash equilibrium, and it also



(a) S1 (blue) vs S2 (orange) (b) S1 (blue) vs S3 (orange) (c) S1 (blue) vs S4 (orange)

(d) S2 (blue) vs S3 (orange) (e) S2 (blue) vs S4 (orange) (f) S3 (blue) vs S4 (orange)

Fig. 4.1 Comparing pure strategies

could be a dominant-strategy equilibrium under some conditions (see Section 4.4.3). We leave this open for further studies.

4.4.2 Introducing Risk Aversion

In this section, we consider the risk aversion level of the agents by introducing corresponding utility functions. The payoff in the game is thus defined as the accumulative utility instead of the surplus. Given a utility function $U(x)$, there are several measures of the risk aversion. In our study, we focus on the coefficient of absolute risk aversion (Arrow, 1965; Levy, 1994; Pratt, 1978) defined in Equation (4.2):

$$A(x) = -\frac{U''(x)}{U'(x)}. \tag{4.2}$$

If $A(x)$ is constant to x, it means that the risk aversion is constant with respect to the total surplus x. $A(x)$ can also be increasing or decreasing, which means the increased risk aversion or the decreased risk aversion with respect to the growth in the total surplus, respectively.

It is easy to show that the utility functions $U_c(x), U_i(x), U_d(x)$ defined in Equations (4.3)-(4.5) below have constant absolute risk aversion (CARA), increasing absolute risk aversion

(IARA) and decreasing absolute risk aversion (DARA) respectively.

$$U_c(x) = e^x, \tag{4.3}$$

$$U_l(x) = x + x^2, \tag{4.4}$$

$$U_d(x) = \log(x). \tag{4.5}$$

Table 4.5 Equilibrium of CDAs given utility functions

Risk Aversion	Utility Function	CDA
CARA	$U_c(x) = e^x$	(0.270, 0.730, 0.000, 0.000)
IARA	$U_l(x) = x + x^2$	(0.460, 0.540, 0.000, 0.000)
DARA	$U_d(x) = \log(x)$	(0.357, 0.634, 0.009, 0.000)
-	$U(x) = x$	(0.620, 0.380, 0.000, 0.000)

Table 4.6 Equilibrium of FCMs given utility functions

Risk Aversion	Utility Function	FCM
CARA	$U_c(x) = e^x$	(0.768, 0.232, 0.000, 0.000)
IARA	$U_l(x) = x + x^2$	(1.000, 0.000, 0.000, 0.000)
DARA	$U_d(x) = \log(x)$	(0.810, 0.189, 0.000, 0.000)
-	$U(x) = x$	(0.999, 0.001, 0.000, 0.000)

From the results shown in Table 4.5 and Table 4.6, we see that introducing different utility functions makes the corresponding equilibrium strategy profiles vary a lot in both CDA markets and FCMs. It turns out that we cannot take a pure strategy as an approximation of the equilibrium when we consider the risk aversion of the agents because most of the equilibrium profiles are mixed strategies. More generally, we conclude that EGTA results are not robust to the traders' risk attitude.

4.4.3 Quality Analysis of EGTA

In this section, we examine the quality of the EGTA method in the following two aspects.

First, we check the reliability, i.e., whether the strategy profiles computed via EGTA are better than pure strategy profiles. We apply a similar comparison method as in Section 4.4.1: we divide the agents into two equal groups, one group playing the equilibrium strategies shown in Table 4.3 and the other group playing a pure strategy from our pre-set strategy space shown in Table 4.1. If one group has increased surplus without sacrificing the other group's surplus, we say that the equilibrium strategy profile loses; otherwise, it wins. We

Table 4.7 Market performance comparison, $r = 0.8, \sigma_S^2 = 100, N = 5$

Market	Profile	Winning Rate (vs S1, vs S2, vs S3, vs S4)
CDA, 0.8	(0.620, 0.380, 0.000, 0.000)	(58%, 79%, 98%, 100%)
CDA, 0.5	(0.925, 0.075, 0.000, 0.000)	(76%, 84%, 100%, 100%)
CDA, 0.2	(0.161, 0.835, 0.003, 0.000)	(61%, 55%, 97%, 100%)
FCM, 0.8	(0.999, 0.001, 0.000, 0.000)	(NA, 92%, 100%, 100%)
FCM, 0.5	(1.000, 0.000, 0.000, 0.000)	(NA, 95%, 100%, 100%)
FCM, 0.2	(1.000, 0.000, 0.000, 0.000)	(NA, 89%, 100%, 100%)

run 100 simulations and calculate the winning rate. The closer the winning rate is to 100%, the more sound the equilibrium strategy profile via EGTA is. The results are shown in Table 4.7.

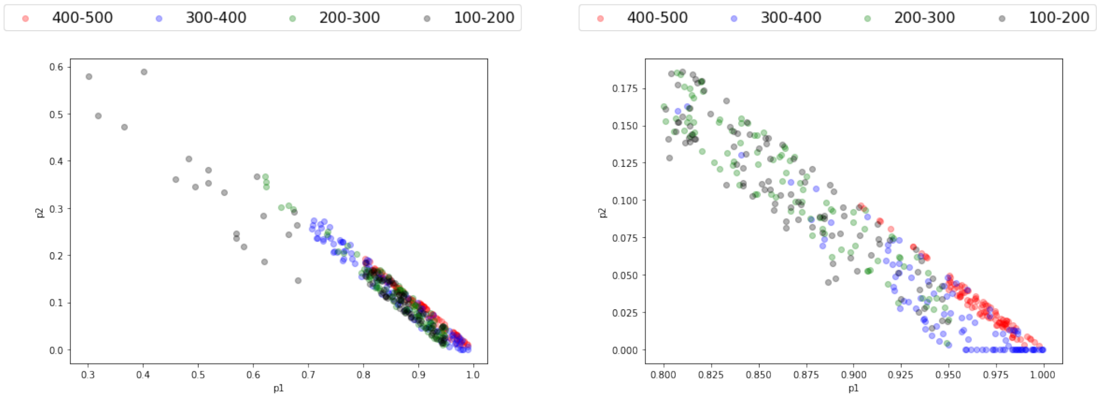
For CDA markets, the winning rates against the strategies S3 and S4 are almost 100%, which supports our findings in Section 4.4.1 that strategies S1 and S2 dominate S3 and S4. The winning rates against S1 and S2 are relatively low, which can be explained by the fact that playing pure strategy S1 has similar performance to playing pure strategy S2.

For frequent call markets, we notice that the calculated equilibrium strategy profiles are basically the pure strategy S1 (the probabilities of playing S1 are 0.999, 1.000, and 1.000, respectively). Thus, we only compare the equilibrium strategy profiles to pure strategies S2, S3, and S4. It turns out that the winning rates against other strategies are rather high.

Second, we look at the robustness of EGTA by examining the dispersion level of simulated results in terms of the length of the horizon and the number of agents. We use a vector $x_t = (p_1, p_2, p_3, p_4)_t$ to denote an equilibrium strategy profile, where p_i is the probability of the strategy S_i in the profile, $i = 1,2,3,4$. We fill the coordinate system with these vectors to get a geometric intuition. In our empirical study, we find that the p_4s are always 0 and the p_3s are 0 in most equilibrium strategy profiles, so we choose the two-dimensional projection to visualize the dispersion level, i.e., the scatter diagram drawn by values of (p_1, p_2).

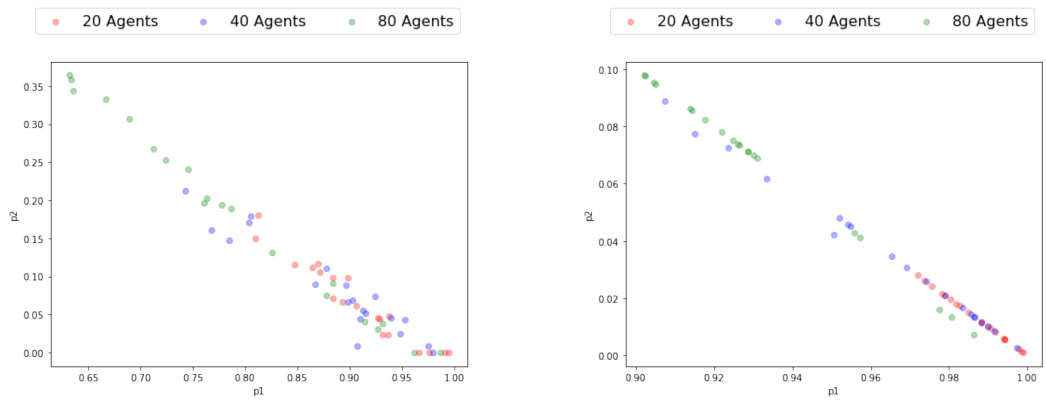
Figure 4.2 shows the scatter diagram of the equilibrium strategy profiles with different horizon lengths. Generally, scatters are located around the straight line $p_1 + p_2 = 1$. For FCMs, the results are more robust than for CDA markets, because there are few outliers. In terms of the horizon length, the results show that longer horizons make the simulated equilibrium strategy profiles more robust in both markets, as we can see the red points are quite aggregated. Outliers usually occur in the simulations without a long enough horizon.

Figure 4.3 shows the scatter diagram of the equilibrium strategy profiles with different numbers of agents. Similarly, the results are more robust in frequent call markets than in CDA markets. However, we find a trend in both market types; when the number of agents



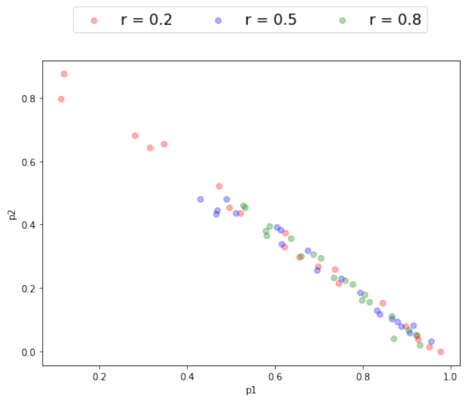
(a) CDA, $r = 0.8, N = 40$ (b) FCM, $r = 0.8, N = 40$

Fig. 4.2 Robustness of EGTA with respect to Horizon Length

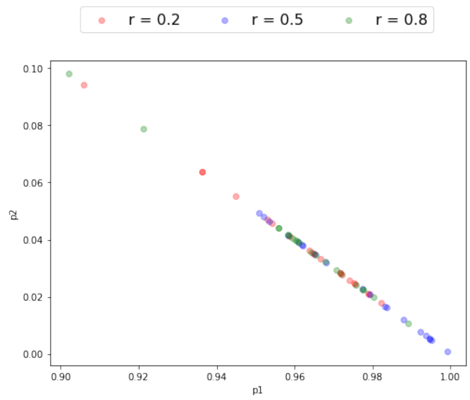


(a) CDA, $r = 0.8, T = 500$ (b) FCM, $r = 0.8, T = 500$

Fig. 4.3 Robustness of EGTA with respect to Number of Agents increases, the scatters of the equilibrium strategy profiles will move towards the upper left corner along the straight line $p_1 + p_2 = 1$. This might happen because of the possible loss from decreased transactions due to the required extra surplus (Strategy S2) being made up by the increased number of agents.



(a) CDA, $r = 0.8, T = 500$



(b) FCM, $r = 0.8, T = 500$

Fig. 4.4 Robustness of EGTA with respect to Reversion Rate

Figure 4.4 shows the scatter diagram of the equilibrium strategy profiles with different reversion rates. Here, we do not see much difference in the robustness of EGTA equilibria in markets with different reversion rates. We, therefore, tend to believe that the stability of the markets has little impact on the robustness of EGTA results.

To summarize, the EGTA method generates sound equilibrium strategy profiles. The level of robustness depends on the environment setup. In general, applying EGTA in frequent call markets with a long horizon and a large number of agents tends to generate more robust results.

4.4.4 Comparison of Homogeneous and Heterogeneous EGTA

In previous research applying EGTA to different market mechanisms, the agents play the same strategy throughout the whole game. We call this approach homogeneous EGTA. Here, we relax this assumption by allowing the agents to play different strategies at each round, based on the current state of the environment. Ideally, this novel approach, termed heterogeneous EGTA, would reflect more realistic agent behavior and lead to better conclusions about the agent interaction and the market overall. For heterogeneous EGTA, the large game is divided into many sub-games, and equilibrium strategy profiles of each sub-game are calculated. In more detail, in our experiments, the large game is turned into 500 sub-games according to the length of the initial game horizon (i.e., one sub-game for each step). For the first round, we calculate the equilibrium strategy profiles via EGTA and let the agents play the equilibrium strategies for one round. The environment, including the limit order book, will be updated via the simulation, and the updated environment will be the initial condition for the next round. At this point, new equilibrium strategy profiles are computed via EGTA based on the altered environment conditions. The process repeats till the last sub-game.

We measure the difference between homogeneous EGTA and heterogeneous EGTA in two ways. First, we look at the market performance with both methods to see whether the conclusions from homogeneous EGTA still hold when applying heterogeneous EGTA. Second, when the environmental conditions change, the corresponding equilibrium strategy profiles would alter accordingly. We then examine the variation level of the equilibrium strategy profiles series.

Table 4.8 Market performance comparison

Market	Reversion Rate	Efficiency	
		Homogeneous	Heterogeneous
CDA	0.8	0.824	0.850
CDA	0.5	0.844	0.848
CDA	0.2	0.835	0.855
FCM	0.8	0.950	0.946
FCM	0.5	0.948	0.953
FCM	0.2	0.947	0.940

From the efficiency comparison shown in Table 4.8, we can conclude that in general, the main findings on the relative efficiency of CDA markets and FCMs are still valid. Whereas in CDA markets, the efficiency of the simulated equilibrium via the heterogeneous EGTA increases slightly, for frequent call markets, there is almost no difference between the homogeneous and heterogeneous approaches. The reason for this could lie in the variation level of the equilibrium strategy profiles series of the two market types.

We notice that in frequent call markets, the elements of the equilibrium strategy profiles series are highly aggregated, close to the profiles in Table 4.3. However, the equilibrium strategy profiles of sub-games do vary in CDA markets. To measure the variation level, we introduce the generalized variance (GV) as the metric. Suppose we have t samples, then the GV is defined in Equation (4.6):

$$\mathrm{GV} = \det(\Sigma) \tag{4.6}$$

where Σ is the covariance matrix. A greater GV shows a higher dispersion degree. Due to the fact that the probability of strategies S3 and S4 are quite small in all equilibrium strategy profiles (which will cause GV to be around 0), we consider the probabilities of strategies S1 and S2 only.

Table 4.9 Generalized variance

Market	Reversion Rate	Generalized Variance
CDA	0.8	7.14e-20
CDA	0.5	1.38e-21
CDA	0.2	1.74e-05
FCM	0.8	≈0.0
FCM	0.5	≈0.0
FCM	0.2	≈0.0

From the comparison of GV shown in Table 4.9, we can see that in frequent call markets, the equilibrium strategy profiles computed via homogeneous EGTA constitute a good approximation of those found via heterogeneous EGTA. For CDA markets where local equilibrium strategy profiles show much variation, there is no impact on the major conclusion.

The benefit of heterogeneous EGTA is that it allows the achievement of more reasonable equilibria as it takes the changes in the environmental conditions into consideration. However, an obvious drawback is that the computation cost increases drastically, as there are T rounds of EGTA (which is computationally very heavy, T times the computation cost of the homogeneous) while there is only one in the homogeneous approach.

4.5 Conclusion

This chapter explores the equilibria of agent-based markets via EGTA. Specifically, we provide insights into the following questions:

- How does the equilibrium strategy profile look like and how is it generated?
- Is any pure strategy good enough to approximate the market equilibrium?
- What is the quality and robustness of the equilibrium strategy profiles computed by EGTA?
- Should EGTA be modified?

The first question is addressed in Section 4.4.1. We find that the strategies that lead to good performance in markets where only pure strategies are played account for a large chunk of the probability mass in mixed strategy equilibria. Specifically, agents tend to play

S1 and S2 more than S3 and S4. From the agent's perspective, the telling-truth strategy is better than greedy strategies, but the incentives could be small in thick markets for appropriate levels of greediness.

Section 4.4.1 and Section 4.4.2 answer the second question. Under certain conditions (for example, in frequent call markets or markets with simple agent designs), playing a pure telling-truth strategy could be an approximation of the equilibrium strategy profile so that the time-consuming EGTA could be avoided. However, in general markets, especially in CDA markets or when we take the agents' attitude to risk into consideration, no pure strategy could approximate the market equilibrium. We thus conclude that EGTA is needed when we evaluate such agent-based markets.

We address the next question in Sections 4.4.2 and 4.4.3, where we analyze the soundness and robustness of EGTA. In general, the EGTA methods generate sound and robust equilibrium strategy profiles in frequent call markets. For CDA markets, longer experiment horizons and more elaborate agent designs are helpful in increasing soundness and robustness. However, when risk neutrality cannot be guaranteed, the findings are not very robust, and new experiments are needed to account for this dimension.

The last question is explored in Section 4.4.4. We compare the performance of heterogeneous EGTA against the results of homogeneous EGTA. It turns out that both methods give similar performance, although the equilibrium strategy profiles can vary (as in the case of CDA markets). Considering the extra computation cost, we tend to believe that homogeneous EGTA has a good balance between the robustness of its conclusions and its speed.

Chapter 5

The Spoofing Resistance of Frequent Call Markets

We study the effects of spoofing attacks on frequent call markets (FCMs). Spoofing—a strategic manipulation to mislead market participants by creating spurious limit orders—could harm market efficiency and has been declared illegal in many countries. However, this practice is still very common, which inspired extensive research on measuring, detecting, and curbing such manipulation in the popular market model of continuous double auctions (CDAs). In this chapter, we extend this research to frequent call markets and use agent-based modeling to simulate spoofing in this context. Specifically, we apply empirical game-theoretic analysis (EGTA) to compute equilibria of agent-based markets and demonstrate that while spoofing may be profitable in both market models, it has less impact on FCMs as opposed to CDAs. Finally, we explore several FCM mechanism designs to help curb this type of market manipulation even further.

5.1 Introduction

In order-driven markets, the market data reveals the universal beliefs on the market and is a key information source for traders. If the market data, including the best ask and bid price and their sizes, are manipulated, other participants could be misled and suffer from potential loss. It is believed that market manipulation can harm market liquidity (Comerton-Forde and Putniņš, 2014; Cumming et al., 2020), reduce market efficiency (Pirrong, 1995), and raise spreads and volatility (Williams and Skrzypacz, 2020).

Spoofing is a simple and popular market manipulation method that can be defined as "strategically placing and canceling orders in order to move prices and trade later in the opposite direction" (Williams and Skrzypacz, 2020). Spoofing is illegal in many countries (Lee et al., 2013; Lin et al., 2019). For example, the Dodd-Frank Wall Street Reform and

Consumer Protection Act made spoofing illegal in 2010 in the U.S. However, spoofing is still common in financial markets; for example, JP Morgan Chase was fined by U.S. regulators in 2020 for price manipulation in precious metal and treasury bill markets (Williams and Skrzypacz, 2020). The impact of spoofing on participants and markets is then an important research question in the context of market mechanism design.

Continuous-time double auctions (CDAs) are a very popular trading mechanism applied in modern order-driven exchanges. In a CDA market, the order submissions and withdrawals are processed serially (Budish et al., 2015). However, CDA markets are believed to lead to the latency arms race problem because traders could have huge benefits if they have only tiny access-time advantages over others Budish et al. (2015); Sparrow (2012). The frequent call market, where orders arriving during the clearing interval are accumulated and processed in batches at the end of the interval, is taken as the alternative mechanism addressing the latency arms race problem (Budish et al., 2015; Wah and Wellman, 2013). In this chapter, we aim to investigate the impact of spoofing on frequent call markets (FCMs).

We adopt the agent-based method to model the market and simulate the interaction amongst strategic agents and between agents and the market. We consider three types of strategic agents: one group generates their bidding or asking price, i.e., the strategy, only based on their valuation; the second group also takes historical information into consideration, and the last group plays a spoofing strategy. We then apply empirical game-theoretic analysis (EGTA) (Wellman, 2006) to account for the strategic response of agents to market rules and each other's actions. EGTA helps to find equilibrium states of such agent-based markets and we compare a set of market metrics for agents and markets before and after introducing spoofing. We also compare the impact on FCMs to the impact on CDA markets to examine whether FCMs curb this kind of manipulation. Furthermore, we explore additional designs of FCMs, which might decrease the risk of spoofing. To the best of our knowledge, this is the first attempt to analyze the impact of spoofing on frequent call markets.

The chapter is organized as follows: Section 5.2 overviews the literature on this topic. Section 5.3 describes the research design and experimental setting. The main empirical results and their analysis are shown in Section 5.4. The last section summarises our findings.

5.2 Related Work

We have seen much research on spoofing and price manipulation in order-driven markets. Related research mainly lies in five fields: an empirical study of the behavior and performance of spoofing traders, the impact of spoofing on markets, the study of spoofing strategies, detecting or predicting spoofers, and the mechanism designed to mitigate spoofing in markets. Lee et al. (Lee et al., 2013) study behavior and performance of spoofing traders in Korea Exchange. Lin et al. (Lin et al., 2019) examine the market manipulation in the Singapore Exchange. Mendonca et al. (Mendonça and De Genaro, 2020) conduct an empirical study on spoofing in the Brazilian capital market. Williams et al. (Williams and Skrzypacz, 2020) claim that spoofing would raise the spread and market volatility. The harm to market liquidity caused by spoofing is also supported by the research of Comerton, Forde, and Cumming (Comerton-Forde and Putniņš, 2014; Cumming et al., 2020). Pirrong believes that spoofing also reduces market efficiency (Pirrong, 1995). Carter et al. (Cartea et al., 2020) put forward an optimal spoofing strategy based on an imbalance in volumes. Tao et al. (Tao et al., 2020) also study the optimal spoofing strategy in high-frequency trading and, in turn, detect any spoofing in the market. Martìnez-Miranda et al. study a model used to predict active spoofing (Martínez-Miranda et al., 2016). Wang and Wellman (Wang and Wellman, 2020) apply an adversarial learning framework to detect spoofing. Moreover, Wang et al. (Wang and Wellman, 2017) adopt agent-based modeling to demonstrate the effectiveness of spoofing in CDA markets. They follow the same framework to investigate the mechanism to mitigate spoofing (Wang et al., 2018) and trading strategies in the face of spoofing (Wang et al., 2019).

However, all related research papers focus on the modeling of the consequences caused by spoofing, failing to investigate the underlying reason why spoofing will work from the perspective of the behavior of strategic agents. Furthermore, possible changes to the market mechanism to reduce or even avoid the existence of spoofing is not investigated yet. To the best of our knowledge, we are the first to explain how spoofing works from the micro viewing, and based on that, we raise the mechanism design which is helpful in reducing the spoofing.

5.3 Experimental Setup

In this work, we employ a parameterized single-asset financial market model inspired by previous research (Brinkman and Wellman, 2017; Liu et al., 2021a; Wang and Wellman, 2017), which is believed to capture the qualitative phenomena found in real financial markets Brinkman and Wellman (2017). We adopt the agent-based modeling method to study the complex system and solve the equilibrium strategies using EGTA. The surplus of agents will be explored based on equilibrium strategies. We will compare the agents' performance and market measures with and without spoofing.

5.3.1 Market Model

We assume a single-asset market filled with N strategic agents who can submit limit orders, cancel existing orders, or take no action following their assigned strategies. We consider a finite and discrete trading horizon T. At each time step t, each agent will decide whether to enter the market—this is controlled by a Poisson process with arrival rate λ. Upon entering the market, an agent observes current and historical level 1 market data, i.e., the best bid and ask price. It plays the role of a buyer or a seller uniformly at random. Before placing new orders, agents will cancel all existing orders. The size of the limit order is set to be 1 unit.

We focus on the frequent call auction mechanism. Thus, the limit order will not be executed as soon as it arrives, even if it crosses the limit order book. In frequent call auctions, there is a clearing interval with length l, and all orders collected during the clearing interval will be aggregated and executed in a batch at the end of the clearing interval.

As for the asset, we use the same mean-reverting stochastic process which can be seen in (4.1) to model fundamental value dynamics.

5.3.2 Valuation Model

At each time t, the fundamental value alters, and then all agents will generate their own valuations loosely following the setup in Brinkman and Wellman (2017); Wah et al. (2016a). Specifically, each agent's valuation at time t is the sum of two components, common and private.

The common component is the individual estimation of the fundamental value with a *valuation bias*, which is independently generated from a normal distribution $N(0, \sigma^2_{\text{bias}})$. We use $\lambda_{i,t}$ to denote the valuation bias of agent i at time t.

The private component is a measurement of the personal valuation of different positions. We express this through a vector Θ_i that we call the private value vector of agent i. Assume agents can long or short the asset and the maximum size allowed to long or short is Q, then the collection of allowed positions is $\{-Q, -Q+1, \ldots, Q-1, Q\}$. The element of the private value vector is the marginal surplus of obtaining one more unit of the asset when the agent is in a certain position; thus the length of each Θ_i is $2Q$ and the specific form of Θ_i is $(\theta_{i,-Q}, \theta_{i,-(Q-1)}, \ldots, \theta_{i,0}, \ldots, \theta_{i,Q-1})$, where the element $\theta_{i,q}$ is the marginal surplus of obtaining one more unit of the asset when agent i is in position q. We generate the private value vectors in the following way: we first have $2Q$ independent samples from a normal distribution $N(0, \sigma^2_{pv})$, then we sort these $2Q$ values in descending order and fill the vector Θ. The private value vector for each agent is fixed throughout the full horizon T. The valuation is the sum of common and private components and we can define the valuation of agent i at time t in position q as

$$v_{i,t,q} = \begin{cases} \lambda_{i,t} + f_t + \theta_{i,q}, & \text{if buying} \\ \lambda_{i,t} + f_t - \theta_{i,q-1}, & \text{if selling} \end{cases} . \tag{5.1}$$

5.3.3 Trading Strategies

We consider three types of agents in our experiments. The first group places orders only based on their valuations; we call this the background trading strategy. The second group considers the market information; we call theirs the *heuristic belief learning* (HBL) strategy. Finally, the last group plays the spoofing strategy.

Background Trading Strategy

If an agent places a limit order with its real valuation, we call it the truth-telling bidding strategy. However, telling the truth might not be the dominant strategy; we follow the strategy space design of Brinkman et al. Brinkman and Wellman (2017) and consider the situation where agents require an extra bonus from a transaction. We set up a required surplus range $[\alpha_{\min}, \alpha_{\max}]$, where $\alpha_{\min}$ represents the minimum acceptable surplus and $\alpha_{\max}$ denotes the maximum expected surplus from trading. If an agent enters the market, its surplus demand is uniformly drawn from the surplus range and the limit order price is exactly the sum of its valuation and the surplus demand. Specifically, the background trading strategy can be described as

$$p_{i,t,q} \sim \begin{cases} U[v_{i,t,q} - \alpha_{\max}, v_{i,t,q} - \alpha_{\min}], & \text{if buying} \\ U[v_{i,t,q} + \alpha_{\min}, v_{i,t,q} + \alpha_{\max}], & \text{if selling} \end{cases} . \tag{5.2}$$

where $p_{i,t,q}$ is the price of the limit order to be submitted and drawn from the uniform distribution with the lower bound $v_{i,t,q} - \alpha_{\max}$ and the upper bound $v_{i,t,q} - \alpha_{\min}$ if it is a bid order and the uniform distribution with the lower bound $v_{i,t,q} + \alpha_{\min}$ and the upper bound $v_{i,t,q} + \alpha_{\max}$ if it is a ask order. To decrease the computational cost, we choose a limited strategy space denoted by B1 to B5 as shown in Table 5.1. Obviously, strategy B1 is exactly the truth-telling strategy and the latter strategies are more greedy than the former ones. We note that related work on EGTA discussed above uses a slightly larger strategy space, excluding B1 and including greedier strategies. However, these greedy strategies account for very small probabilities at equilibrium, while the truth-telling strategy is an important component of the equilibrium according to further analyses. Thus, we believe our limited strategy space is more solid.

Table 5.1 Background trading strategy space

Strategy	B1	B2	B3	B4	B5
$\alpha_{\min}$	0	0	0	20	50
$\alpha_{\max}$	0	50	100	100	100

HBL Trading Startegy

An agent following the HBL strategy (simply referred to as HBL agent below) observes the current level 1 market data, i.e., the best bid/ask price and the execution price, and stores the observation in its memory. When HBL agents enter the market, their strategy is generated from their belief function defined as follows: suppose L is the memory length of HBL agents, i.e., when they enter the market at time t, they can only take the market data between $t-L$ and t into consideration. The design of the HBL strategy follows the research on spoofing in CDA markets (Wang and Wellman, 2017). Some required variables are explained in Table 5.2. Only the market data within the memory length will be used to define these variables.

Table 5.2 Parameters of HBL strategy

Variable	Explanation
EB(p)	Volume of executed bid orders with price $\leq p$
SA(p)	Volume of ask orders with price $\leq p$
NB(p)	Volume of non-executed bid orders with price $\geq p$
EA(p)	Volume of executed ask orders with price $\geq p$
GB(p)	Volume of bid orders with price $\geq p$
NA(p)	Volume of non-executed ask with price $\leq p$

The belief function is defined by:

$$g_t(p) = \begin{cases} \frac{\mathrm{EB}_t(p)+\mathrm{SA}_t(p)}{\mathrm{EB}_t(p)+\mathrm{SA}_t(p)+\mathrm{NB}_t(p)}, & \text{if buying} \\ \\ \frac{\mathrm{EA}_t(p)+\mathrm{GB}_t(p)}{\mathrm{EA}_t(p)+\mathrm{GB}_t(p)+\mathrm{NA}_t(p)}, & \text{if selling} \end{cases}. \tag{5.3}$$

The belief function is an estimation of the probability that orders with different price levels will be matched and executed in the market. An HBL agent will choose the price that maximizes the expected surplus from the trade based on her valuation and belief function.

Specifically, the strategy, i.e., selected price of agent i with position q at time t is:

$$P^*_{i,t,q} = \begin{cases} \arg\max_p (v_{i,t,q} - p) g_t(p) \text{ if buying} \\ \arg\max_p (p - v_{i,t,q}) g_t(p) \text{ if selling} \end{cases} . \tag{5.4}$$

We consider two HBL strategies, denoted by HBL1 and HBL2, with memory lengths of 10 and 50, respectively.

Spoofing Strategy

We consider a spoofing agent aiming to sell assets at the end of the horizon. Thus, it will spoof bid orders. Her spoofing strategy is to place a large amount V of limit bid orders with a price of just 1 tick smaller than the best bid price. Suppose the temporary supply exceeds the temporary demand, i.e., more people are selling than those who are buying. If the spoofing agent observes any updates of the best bid price, she will cancel previous spoof orders and place new spoof bid orders. The execution price will not drop a lot within a short time because of the large amount bid orders submitted by the spoofer. It will provide a signal to the market that the asset's price is strong, and more people will hold a belief that there is a great chance that its price will go up in the future. As a result, more and more people hold such beliefs, and the demand will exceed the supply, resulting in an increase in the price. Finally, the spoofer can sell the inventory at a relatively high price. This strategy expects that the feigned interest to buy will mislead the market belief of HBL agents and the spoofer could benefit from the manipulation.

5.3.4 Metrics

We consider metrics for both agents and the market. The surplus of agent i, as defined in (5.5) where s_{it} is the surplus from the transaction at time t defined in (5.6) and W_i is the wealth at the end of the trading horizon, is the main metric to measure the agent's performance.

$$\textbf{Surplus}_i = \sum s_{i,t} + W_i \tag{5.5}$$

$$s_{i,t} = \begin{cases} v_{i,t,q} - P^*_{i,t,q} \text{ if buying} \\ P^*_{i,t,q} - v_{i,t,q}) \text{ if selling} \end{cases} . \tag{5.6}$$

The key feature of the market we are concerned with is market efficiency. It is measured by the ratio between the realized and the potential total surplus which is calculated by supposing all involved orders are matched at the base of maximum surplus rather than how they are processed in reality. We also look at the difference between the mid-price time series and the fundamental value time series to reveal the price discovery. To study this difference, we will consider the root-mean-squared deviation (RMSD) of the difference between these two time series. RMSD is defined by (5.7) where N is the number of trading rounds and the (m_t) is the mid-price time series. Therefore, lower RMSD indicates better price discovery.

$$\mathbf{RMSD} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(f_t - m_t)} \tag{5.7}$$

5.3.5 Environment Parameters

In our comparative analysis, the markets and agents being compared share the same parameters except the market mechanism and the strategies they employ. Some common parameters including $\bar{f}, \sigma_s^2, \sigma_{\text{bias}}^2, \lambda_i, \sigma_{\text{pv}}^2$ are fixed through all the experiments. The asset price is a discrete integer ranging from 1 to 1000 in all experiments. We consider the thickness of the market, assigning 100 agents to a thin market and 200 agents to a thick market. The length of the simulation horizon T is 2000 and the length l of the clearing interval for FCMs is 10. We also consider the stability of the fundamental value which is controlled by the reversion rate r. A higher reversion rate makes more stable fundamental dynamics. In our experiments, the reversion value r has two options: 0.8 and 0.2. We run 100 simulations for each game and take the average value as an approximation. Suppose there are m different strategies, so we need to simulate C_{N+m-1}^{m} cases to cover all possible profiles for each market setup when applying EGTA. The values of the most common parameters are listed in Table 5.3.

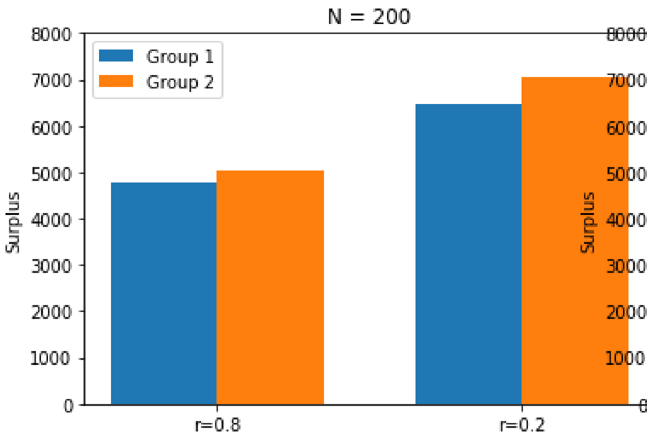
Table 5.3 Environment parameters

N	$\bar{f}$	r	σ_s^2	σ_{bias}^2	σ_{pv}^2	λ	V
100,200	500	0.8,0.2	100	50	25	0.01	200

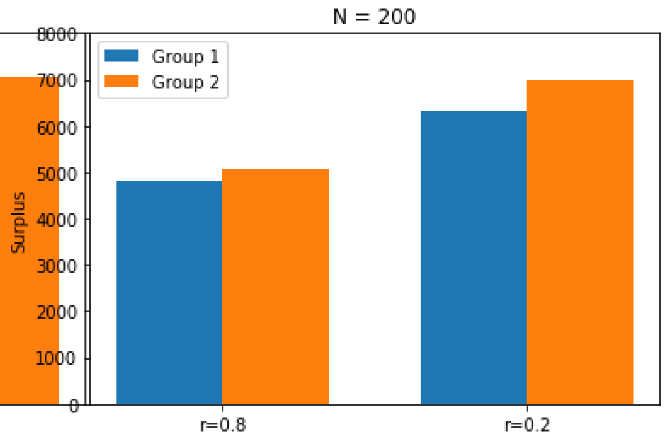
We also list all non-spoofing trading strategies in Table 5.4.

Table 5.4 Non-spoofing trading strategy space

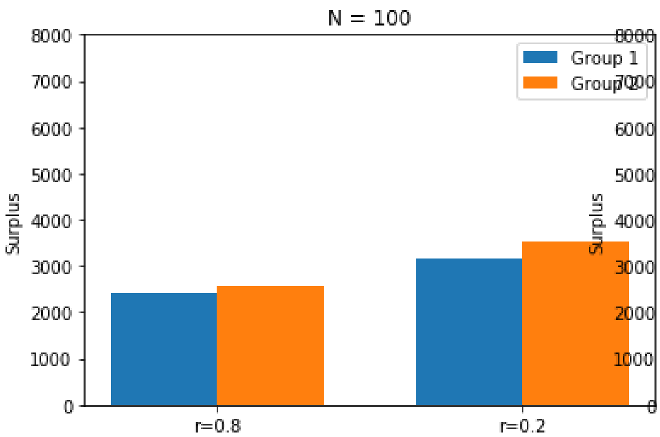
	B1	B2	B3	B4	B5	HBL1	HBL2
α_{min}	0	0	0	20	50	-	-
α_{max}	0	50	100	100	100	-	-
Memory length	-	-	-	-	-	10	50



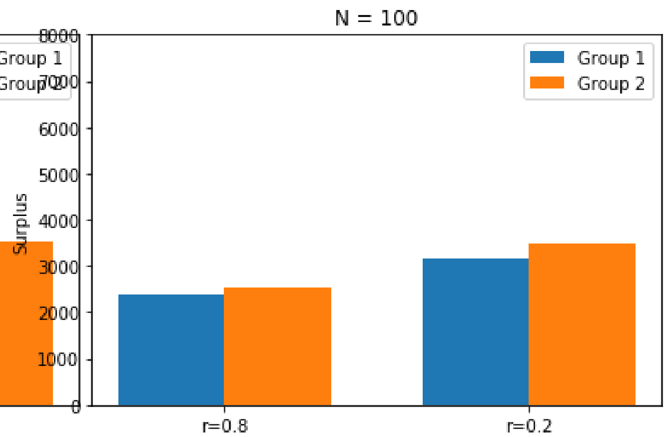
(a) Group 1 vs Group 2 (HBL1), N=200



(b) Group 1 vs Group 2 (HBL2), N=200



(c) Group 1 vs Group 2 (HBL1), N=100



(d) Group 1 vs Group 2 (HBL2), N=100

Fig. 5.1 Agent surplus: Background vs HBL

5.4 Experiments And Analysis

Our analysis contains several steps. We first verify that HBL strategies are profitable in the frequent call market without spoofing. Then we introduce the spoofing agent to examine whether spoofing does harm to the market and the other agents. In the next step, we compare the harm caused by spoofing in the frequent call market and the traditional continuous double auction market, leaving all the other parameters unchanged. Finally, we investigate additional features which help curbing spoofing. In what follows, we refer to the average market efficiency, average total surplus, and average trading volume simply as *Efficiency, Surplus* and *Volume*.

5.4.1 HBL Strategy Profitability

Our investigation starts with the verification of the profitability of HBL strategies in the frequent call market. We divide the agents into two equal groups. Group 1 plays background trading strategies shown in Table 5.1, while Group 2 plays HBL strategies. After solving the equilibrium for Group 1, the average surplus for each group in different market setups is listed in Figure 5.1 (in cases with multiple equilibria, we choose the equilibrium with the smallest regret value).

We can conclude that HBL strategies are profitable compared with background strategies, especially in less stable markets and thick markets. The parameter *memory length* in HBL strategies has little impact on the metrics at equilibrium.

5.4.2 Introducing Spoofing

In the following experiments, we let one agent play the spoofing strategy and the rest of the agents select their strategy from the extended non-spoofing strategy space listed in Table 5.4. To do the comparison between the markets with and without spoofing, we also run controlling experiments where all agents select their strategy from Table 5.4 using the same market setups.

Metrics for Agents

We focus on the changes in agents' surplus after introducing spoofing. The results in FCMs are listed in Table 5.5 while the results in CDAs are listed in Table 5.6.

Table 5.5 Agent surplus comparison, FCMs

Agents	r	No Spoofing	Spoofing	
			normal agents	spoofer
200	0.2	6417	6739	-20796
200	0.8	5345	5517	-23405
100	0.2	3146	3337	-5135
100	0.8	2509	2621	-3518

We find that in CDA markets, the introduction of the spoofing strategy does harm the non-spoofing agents, who suffer a sharp decrease in their surplus. On the other hand, the agent playing a spoofing strategy has more surplus at the end of the trading horizon than the others. Our experiments support the profitability of spoofing in CDA markets. However, when we introduce the spoofing strategy to frequent call markets, non-spoofing agents all have additional benefits (average surplus of normal agents shown in Table 5.5 is greater than that shown in Table 5.6) while the agent playing the spoofing strategy bears a

Table 5.6 Agent surplus comparison, CDAs

Agents	r	No Spoofing	Spoofing	
			normal agents	spoofer
200	0.2	6420	6049	6664
200	0.8	4948	4709	4749
100	0.2	3211	2929	3391
100	0.8	2438	2326	2357

huge loss (average surplus of the spoofer shown in Table 5.5 is smaller than that shown in Table 5.6).

Finally, we adjust the strategy space to contain background trading strategy, HBL strategy and spoofing strategy and calculate the equilibria in all market setups. The equilibrium profiles are listed in Table 5.7, where these numbers are probabilities of playing the corresponding strategies in the equilibrium. We see that agents will not select a spoofing strategy in equilibrium in FCMs, which reveals that the frequent call mechanism is better at curbing spoofing than traditional CDA markets.

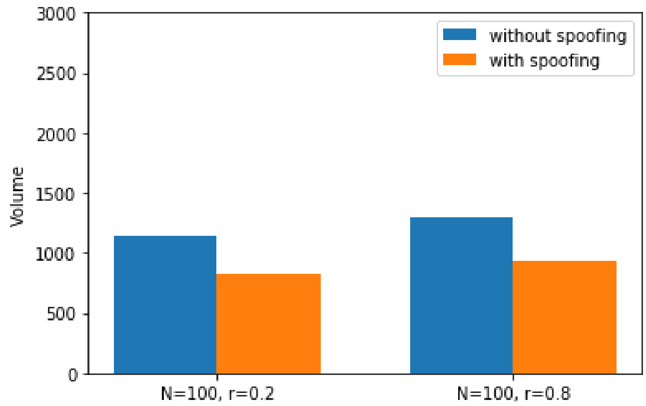
Table 5.7 Equilibrium profiles

Market, N, r	Background	HBL	Spoofing
CDA, 200, 0.2	0.21	0.50	0.29
CDA, 200, 0.8	0.30	0.52	0.18
FCM, 200, 0.2	0.56	0.44	0.00
FCM, 200, 0.8	0.64	0.36	0.00

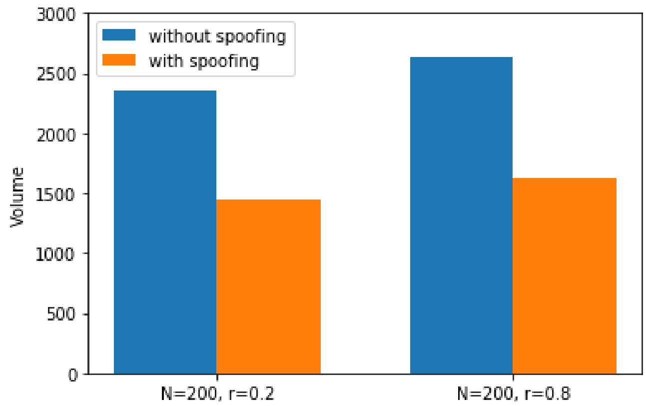
Metrics for Markets

We next look at the market performance. The market efficiency is summarized in Table 5.8. The results reveal that spoofing has negative effects on market efficiency. We also compare the trading volume of markets with and without spoofing in Figure 5.2, which shows that spoofing decreases the trading volume of the market, showing that it might decrease market liquidity.

Finally, we look at the price discovery with and without spoofing in different markets. The RMSDs of the difference between mid-price time series and fundamental value time series are listed in Table 5.9. We can conclude that the introduction of spoofing has negative effects on price discovery. However, the frequent call markets have better price discovery than CDA markets, with or without spoofing.



(a) Order volume comparison, FCM, N=100



(b) Order volume comparison, FCM, N=200

Fig. 5.2 Volume of traded orders

Table 5.8 Market efficiency, with & without spoofing

Market Type	Agents	r	efficiency (No Spoofing)	efficiency (Spoofing)
CDA	200	0.2	0.68	0.50
CDA	200	0.8	0.62	0.52
CDA	100	0.2	0.64	0.55
CDA	100	0.8	0.67	0.51
FCM	200	0.2	0.77	0.60
FCM	200	0.8	0.75	0.62
FCM	100	0.2	0.80	0.63
FCM	100	0.8	0.75	0.63

Table 5.9 Price discovery, with & without spoofing

Market Type	Agents	r	RMSD (No Spoofing)	RMSD (Spoofing)
FCM	100	0.2	120.49	125.56
CDA	100	0.2	136.45	140.89
FCM	200	0.2	120.33	134.37
CDA	200	0.2	143.78	140.49
FCM	100	0.8	162.23	162.11
CDA	100	0.8	177.51	179.13
FCM	200	0.8	170.30	179.18
CDA	200	0.8	179.35	179.28

5.4.3 Slow Spoofer

We investigate the underlying reason why spoofing causes losses in FCMs. We notice that a large amount of feigning orders will be placed in spoofing. If the spoofer fails to update its feigning orders, these orders could become stale, which means they contain no recent information about the market and could become the source of a huge loss if they are traded. We count the volume of traded feigning orders in previous experiments and list the figures in Table 5.10. It is clear that many more spoofing orders are traded during the trading horizon in FCMs. We believe that in CDA markets, the update of the best bid price is a clear signal for the spoofer to update its feigning orders. However, the FCMs will not update during the clearing interval and the spoofer has no up-to-date information to make any decision except leaving its orders to become stale, which might explain why the spoofer will fail in FCMs.

To verify our argument, we test several CDA markets sharing the same environment parameters but different spoofers. The only difference in spoofers is their response time to stale orders. We plot the surplus trends for both non-spoofing agents and the spoofer in Figure 5.3. The left end of each plot is the surplus in the market with fast spoofer while

Table 5.10 Volume of traded feigning orders

Market Type	Agents	r	Volume of Traded Feigning Orders
CDA	200	0.2	3.2
CDA	200	0.8	3.0
CDA	100	0.2	1.5
CDA	100	0.8	1.8
FCM	200	0.2	1457.3
FCM	200	0.8	1605.4
FCM	100	0.2	485.0
FCM	100	0.8	570.7

the right end shows the surplus in the market with slow spoofer. The results show that the response time is the key to taking advantage of the other non-spoofing agents. Slow spoofers could suffer from huge losses because of failure to cancel stale orders.

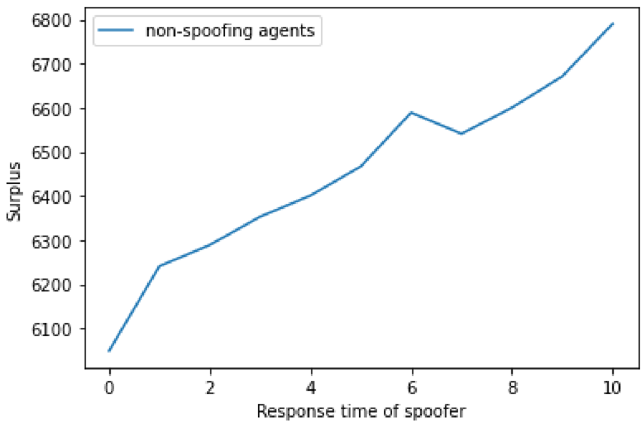
5.4.4 Random Clearing Interval Length

In continuous double auction markets, it is difficult to regulate the response time of agents. However, in frequent call markets, adopting random clearing interval length could lead to further curb spoofing. The underlying principle could be explained as follows: it is risky to leave feigning orders in the limit order book while the random completion of the auction gives no clear signals for agents to update their stale orders. In other words, when an agent plays the spoofing strategy and places a large amount of feigning orders, there exists a great probability that she cannot update her orders in time, and failure to do so will cause losses, finally decreasing the incentive to play the spoofing strategy.

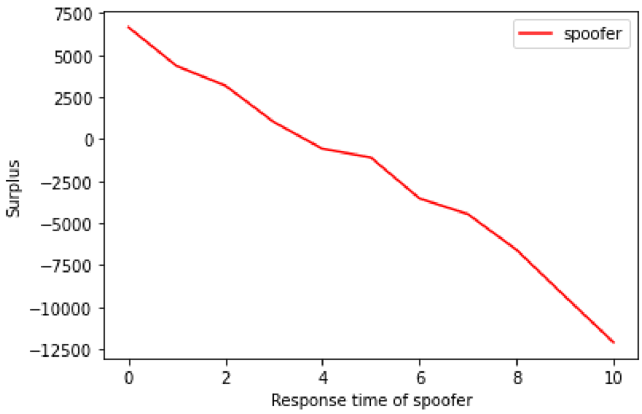
In the previous experiments, the length of the clearing interval is set to be 10. After adopting a random ending rule for the auction, the average clearing interval is also 10, but the length of each clearing interval is generated from a uniform distribution $U[0,20]$. We compare the impact caused by spoofing on FCMs with fixed endings with the impact on random endings, denoted RFCMs. The experiment results are summarized in Table 5.11 and Table 5.12, showing the metrics for agents and markets, respectively.

Table 5.11 Metrics for agents: FCM vs RFCM

Market, N, r	Normal Agents	Spoofer	Feigning Orders
FCM,200,0.2	6739	-20796	1457.3
RFCM,200,0.2	6874	-22267	1667.2
FCM,200,0.8	5517	-23405	1605.4
RFCM,200,0.8	5643	-24508	1789.1



(a) Surplus trend of non-spoofing agents



(b) Surplus trend of spoofing agents

Fig. 5.3 Surplus trends, CDA, N=200, $r = 0.2$

Table 5.12 Metrics for markets: FCM vs RFCM

Market, N, r	Efficiency	Price Discovery
FCM,200,0.2	0.60	134.37
RFCM,200,0.2	0.63	133.56
FCM,200,0.8	0.62	179.18
RFCM,200,0.8	0.62	180.89

We can conclude that a random ending of the auction has enhanced effects on curbing spoofing compared with FCMs with fixed clearing intervals, with no sacrifice in terms of market efficiency or the level of price discovery.

5.5 Conclusion

This chapter explores the effects on traders' surplus and market performance of introducing a spoofing strategy to frequent call markets using agent-based modeling and the EGTA method.

We conclude from our experimental results that spoofing will decrease market efficiency and order volume of the market, having further negative effects on market liquidity. An interesting finding is that spoofing is not as profitable in the frequent call markets as in traditional continuous double auction markets. We compare the agent surplus between CDA markets and FCMs sharing the same parameters and conclude that spoofing is profitable in CDAs, whereas it will benefit others and cause huge losses to the spoofer in FCMs.

Investigating the reasons underpinning these differences, we find that the effect of spoofing is related to the volume of traded feigning orders, which is also equivalent to the speed of accessing updated level 1 market data. If the spoofing agent is able to update its feigning orders immediately after the best bids and asks to alter, it can avoid loss from unexpected trades. However, the call auction mechanism delays its access to the latest market data and increases the risk of executing spoofing orders. A mechanism that forces the agents to have a longer response to the latest market information should then give less incentive to spoof. Following this idea, we test a so-called slow spoofer in CDA markets and the results support our argument.

Finally, we follow this design idea in the frequent call market and set up a random auction ending. This trading mechanism increases the risk for spoofing agents to stay in the market because their stale spoofing orders could be traded at any time before they make the decision from the observation of the market. Our experimental results reveal that this design is beneficial in that it curbs spoofing.

Chapter 6

On the Impact of Iceberg Orders in Financial Markets

6.1 Introduction

Continuous-time double auctions (CDAs) are a very popular trading mechanism applied in modern order-driven exchanges. In a CDA market, the order submissions and withdrawals are processed serially (Budish et al., 2015). In these order-driven financial markets, some market information, usually the price and accumulated volumes of the best limit orders, is open to the market participants so that traders have information to reach trading decisions and submit their orders. These orders provide liquidity to the market. However, transparency is not always desirable for the market or its participants. When a trader seeks to execute an order for a large amount of the asset, opportunistic traders could be attracted. They would attempt to hurt the effective execution of the trade (Frey and Sandås, 2017) and profit (activity known as front running). On the other hand, such large orders could leak information about the true value of the asset to the market, and other participants would adjust their strategies and stop the execution of the large trade at the pre-specified price (Esser and Mönch, 2007). To address these concerns, some exchanges adopt a particular order type, called *iceberg order*.

An iceberg order is very similar to a normal limit order, except for the revealed volume. A limit order is comprised of a price and a volume, with the meaning that the originator would like to buy a certain quantity (specified by the volume) up to a certain limit (defined by the price). When a trader submits an iceberg order, the price, the volume, and the peak size will be specified, with the peak size always smaller than the volume. After the iceberg order arrives in the limit order book, the peak size instead of the total volume is displayed whilst the remaining volume is hidden. If the order is executed for the peak size, another peak size is automatically displayed in the order book with a new time stamp, and the hidden part is reduced by the corresponding number of shares (Esser and Mönch, 2007). Some also argue that a trader using iceberg orders faces a loss-in-priority when she hides her intentions, as most electronic limit order books penalize the usage of hidden liquidity.

Against this background, in this chapter, we aim to investigate the impact of iceberg orders on the market performance and the choice of trading strategies of the traders. We adopt an agent-based method to model the market and simulate the interactions among strategic agents and between the agents and the market. We consider two types of strategic agents: one group, comprised of what we call the *normal agents*, submit limit orders; the second group contains just one agent, the *iceberg trader*, seeking to execute a large volume trade within a limited time interval via an iceberg order. We consider three trading strategies seen in previous research (Brinkman and Wellman, 2017; Gode and Sunder, 1993; Wang and Wellman, 2017) for normal agents. We then apply empirical game-theoretic analysis (EGTA) (Wellman, 2006) to account for the strategic response of agents to market rules and each other's actions. EGTA helps to find equilibrium states of such agent-based markets, and we compare a set of market metrics for agents and markets before and after introducing iceberg orders.

We extend our analysis to frequent call markets (FCMs), which are another important class of trading mechanisms as they address the latency arms race problem caused by the CDA mechanism (Budish et al., 2015).

At a very high level, we find that in CDAs, placing a large volume order will harm the market efficiency and make it difficult to be fully executed, because many speculators will be attracted, whereas placing an iceberg order to hide some trading interests will instead help maintain a higher market efficiency and improve one's own profit. On the contrary, our results show little difference after placing large volume orders in FCMs, thus suggesting that this trading mechanism is more robust in this context.

The chapter is organized as follows: Section 6.2 overviews the literature on this topic. Section 6.3 describes the research design and experimental setting. The main empirical results and their analysis are shown in Section 6.4. The last section summarizes our findings.

6.2 Related Work

We have seen much research on hidden liquidity in order-driven markets. Related research mainly lies in three fields: an empirical study of detecting and predicting hidden liquidity including iceberg orders, analysis of behavior and performance of iceberg orders, and the optimal strategy of submitting iceberg orders. Many papers (De Winne and D'hondt, 2007; D'Hondt et al., 2004; Pascual and Veredas, 2009; Tuttle, 2003) report the widespread use of hidden orders. Zotikov et al. (Zotikov and Antonov, 2021) propose a method for detecting and predicting hidden liquidity on the Chicago Mercantile Exchange (CME). Esser et al. (Esser and Mönch, 2007) analyze the rationale for the use of iceberg orders in continuous trading by assessing the costs and benefits. Frey et al. (Frey and Sandas, 2008; Frey and Sandås, 2017) use samples from Deutsche electronic trading platform Xetra to perform an empirical study of the impact of iceberg orders on the price and order flow dynamics in limit order books and also report evidence that iceberg orders can be detected using public information and that market participants follow the state-dependent order submission strategies. De Winne et al. (De Winne et al., 2009) explain the importance of hidden orders and the resulting hidden liquidity in their research. Bessembinder et al. (Bessembinder et al., 2009) assess the costs and benefits of order exposure and study the extent to which non-displayed size is truly hidden. Cebiroglu et al. (Cebiroglu and Horst, 2011) develop a sequential trade model of iceberg order execution in a limit order book.

All recent research on iceberg orders is more about the empirical study, illustrating the stylized facts of the statistical properties caused by the iceberg orders. However, there is no micro-perspective research on the impact of iceberg orders on the financial market and seeking the optimal iceberg order strategy. To the best of our knowledge, we are the first to fill the research gaps by employing ABM and EGTA methods.

6.3 Experimental Setup

We employ the same parameterized single risky asset financial market model which can be seen in 5.3.1 and 5.3.2 in this work. We consider four types of agents in our experiments: the background agents whose trading strategy can be seen in 5.3.3, the HBL agents (see 5.3.3), the speculating agent, and the *iceberg agent* who seeks to sell a large number of assets in a short time, and will consider submitting iceberg orders to achieve her goal. We assume that there is only one iceberg trader in each market. The rest are normal agents whose order size is fixed to be 1. This choice guarantees that normal agents will not cause a shock to the market with their orders. To account for different possible trading strategies for these agents, we consider the following three trading strategy families, inspired by previous research (Liu et al., 2021b; Wang and Wellman, 2017): the *background trading strategy*, the *heuristic belief learning (HBL) strategy*, and the *speculating strategy*.

We find from previous chapters that the greedy background strategy like B5 in Section 5.3.3 is seldom adopted by agents but HBL strategies are dominated. To decrease the computational cost, we choose a limited background strategy space denoted by B1 to B4 but expand the HBL strategy pool (see Table 6.3).

Table 6.1 Speculating strategy

Action	Condition
Place market ask order	if $v \leq$ best bid price
Place market bid order	if $v \geq$ best ask price
Wait	otherwise

Speculating Strategy

We particularly consider the speculating strategy in this work because, in our experiments, the iceberg agent will submit a limit order with a huge volume, and such a scenario attracts speculators aiming to have an immediate trade if agents observe the mispricing opportunity. As from Table 6.1, an agent playing the speculating strategy will keep her eye on the market, and if the best bid price is greater than or equal to her valuation, she will submit a market ask order, while if the best ask price is lower than or equal to her valuation, she will submit a market bid order.

Iceberg Strategy

We use two parameters to describe the action of the iceberg trader – i.e., the only agent seeking to sell a large amount of assets. The first parameter is the *Total Size*, which is the number of assets to be executed overall, and the second parameter is the *Peak Size Ratio*, which is the ratio between the peak size and the Total Size. For example, if the Peak Size Ratio is set to be 0.1, then only 10% of the full amount will be displayed in the limit order book. In our experiments, we use a Total Size of 5, 10, 15, and 20, following the previous research (Frey and Sandas, 2008) that shows that the ratio between the average total size of an iceberg order and the average size of a limit order lies in this range. These two parameters will be included in the combination of the environment parameters and pre-set before each game.

6.3.1 Metrics

We evaluate the performance of both the agents and the market. The key feature of the market we are concerned with is its efficiency, measured as the ratio between the realized and the potential total surplus. The main metric to measure the agents' performance is surplus, which is defined as the sum of differences between the realized values of the portfolio and its valuation, and the net cash at the end of the trading horizon. We particularly care about the goal of the iceberg trader – i.e., to what extent her order is fulfilled. We use the non-executed volume ratio to measure this dimension.

6.3.2 Environment Parameters

In our comparative analysis, the markets and the agents being compared share the same parameters, except the market setup and the strategies they employ. Some common parameters include $\bar{f}, \sigma_s^2, \sigma_{\text{bias}}^2, \lambda_i, \sigma_{\text{pv}}^2$; these are fixed throughout all the experiments. We introduce the same limit order book at the beginning of each game. For FCMs in our experiments, the time during which the market only collects orders with no execution (a.k.a., clearing interval), L_{clearing}, is fixed to be 10. The length of the simulation horizon, T, is 400, and we let the iceberg agent enter the market at $t = 200$. We run 100 simulations for each game and take the average value (surplus, bid-ask spread, and volume) as an approximation. The values of the most common parameters are listed in Table 6.2.

Table 6.2 Environment parameters

N	$\bar{f}$	r	σ_s^2	σ_{bias}^2	σ_{pv}^2	λ	T	L_{clearing}
181	500	0.2	100	50	25	0.05	400	10

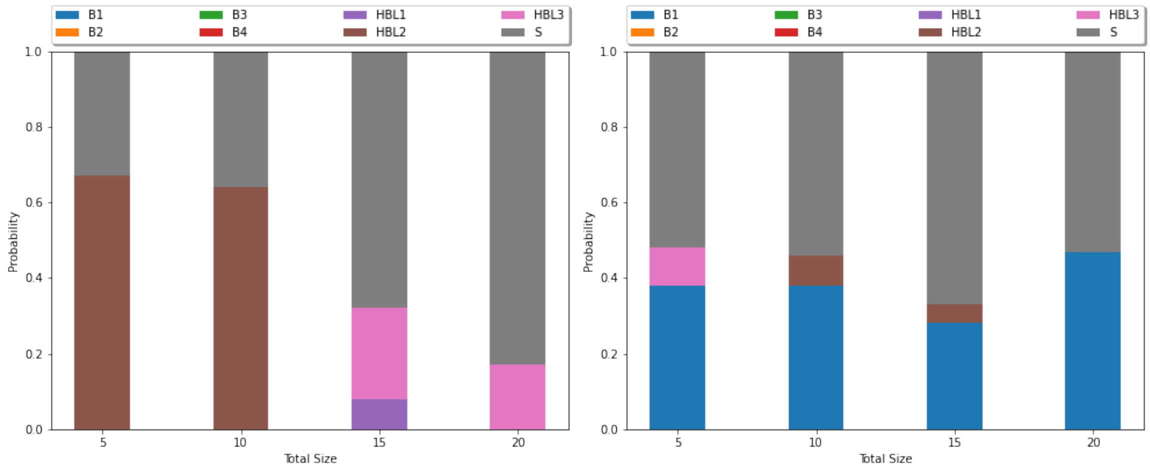
We also list all trading strategies for normal agents in Table 6.3, where $B1$ to $B4$ are background trading strategies, $HBL1$ to $HBL3$ are HBL trading strategies, and S is the speculating strategy.

Table 6.3 Trading strategy space for normal agents

	B1	B2	B3	B4	HBL1	HBL2	HBL3	S
α_{min}	0	0	0	20	-	-	-	-
α_{max}	0	20	50	50	-	-	-	-
Memory	-	-	-	-	10	20	50	-

6.4 Experiments And Analysis

Our analysis contains several steps. We first study the impact of large volume orders, including the changes in agents' responses and the market performance. We then introduce the iceberg strategies and compare the differences in the market and agent metrics when the iceberg trader chooses to submit an iceberg order instead of a simple large-volume order. We are also interested in the impact of such behavior on the iceberg trader herself. Finally, we draw some insights about the optimal strategy for the iceberg trader.



(a) Mixed strategy equilibrium in CDAs

(b) Mixed strategy equilibrium in FCMs

Fig. 6.1 Mixed strategy equilibrium when large volume orders enter the market

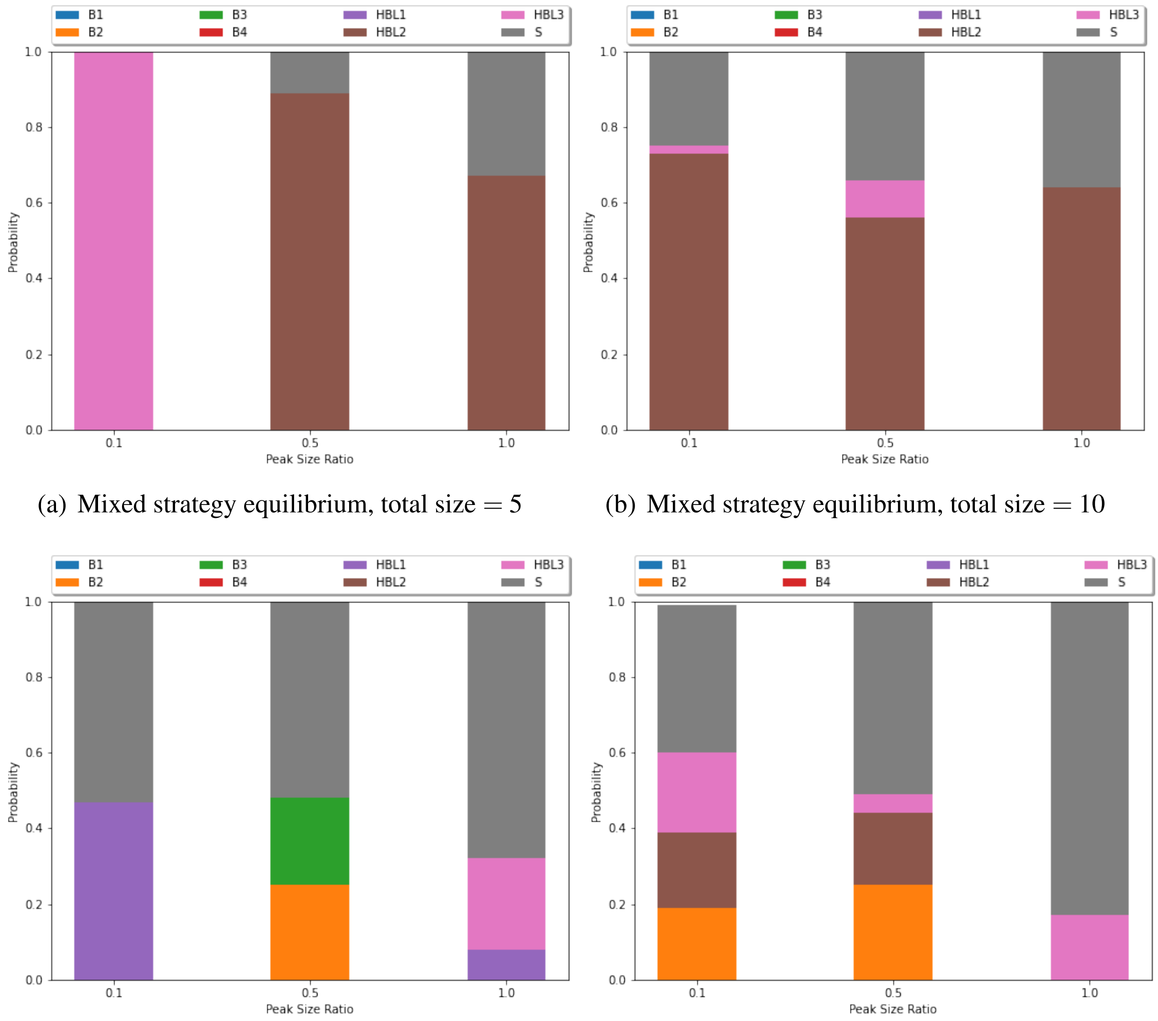
6.4.1 Impact of Large Volume Orders on Agents' Behavior

Our investigation starts with the impact of large-volume orders. In this experiment, all the agents are normal and one of them will submit a sell order of large size. We consider four possible environments determined by the Total Size. In each sub-game, normal agents choose their actions from the strategy space shown in Table 6.3. After solving for the equilibrium (in cases with multiple equilibria, we choose the one with the smallest regret value), we compare the performance metrics for the market and its participants.

We first look at the equilibrium strategy profiles shown in Figure 6.1. In CDAs, the speculating strategy accounts for a higher proportion in the equilibrium strategy as the Total Size increases; we can then conclude that the large volume order has a significant impact on the trading strategy of the agents involved in CDAs, while the historical market information becomes less reliable when the agents reach their trading decisions. On the contrary, in FCMs, there is no obvious trend for the probability of the speculating strategy with the increasing Total Size. It is interesting to find that the FCM mechanism encourages the truth-telling strategy even when a large volume order enters the market.

We introduce different Peak Size Ratios as an environment parameter for extended experiments to assess the impact of different iceberg strategies on the agents' behavior. The results are visualized in Figures 6.2–6.3. Note that when the Peak Size Ratio is 1.0, the iceberg strategy is equivalent to submitting the limit order of full size.

From these results, we can conclude that in CDAs, when the iceberg trader reveals more information about her hidden liquidity, the other (normal) agents tend to play the speculating strategy in response. Moreover, in this case, market information and historical information are helpful for normal agents as the HBL strategy family accounts for the



(a) Mixed strategy equilibrium, total size = 5

(b) Mixed strategy equilibrium, total size = 10

(c) Mixed strategy equilibrium, total size = 15

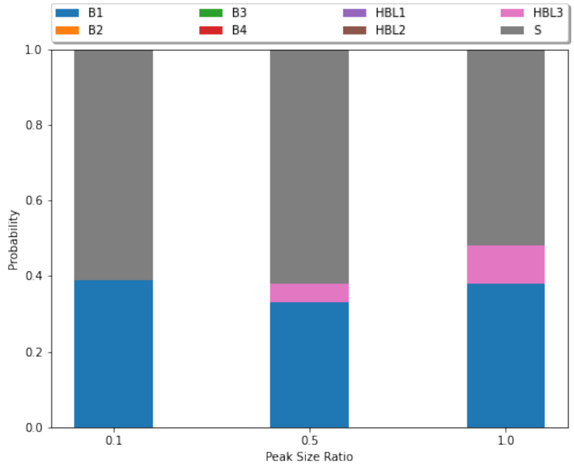
(d) Mixed strategy equilibrium, total size = 20

Fig. 6.2 Mixed strategy equilibrium, CDA

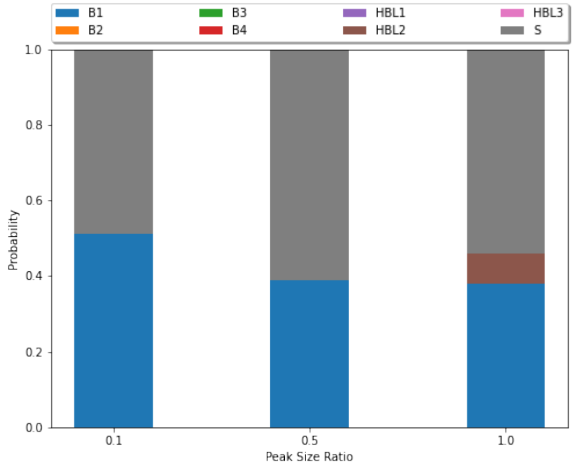
second highest probability in almost every equilibrium. In FCMs, instead, the truth-telling strategy and the speculating strategy still account for the highest probability in all equilibria, and there is no obvious correlation between the Total Size parameter and the Peak Size Ratio parameter.

6.4.2 Impact of Iceberg Orders on the Market

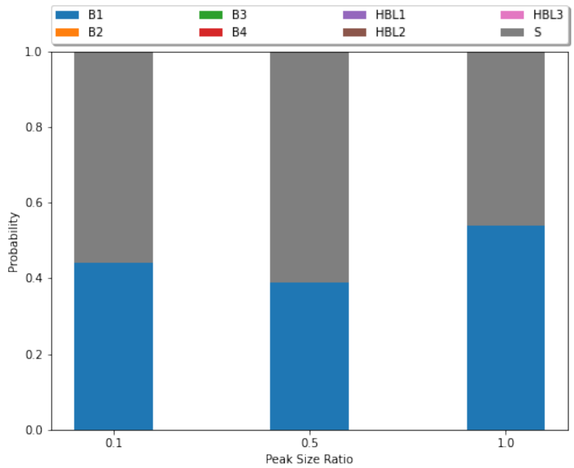
In the following experiments, we let one agent play the role of the iceberg trader, and the rest of the agents select their strategies from the strategy space for normal agents listed in Table 6.3. We keep different Peak Size Ratios as an environment parameter to see the impact on the market performance under the application of different iceberg strategies. The results are shown in Table 6.4.



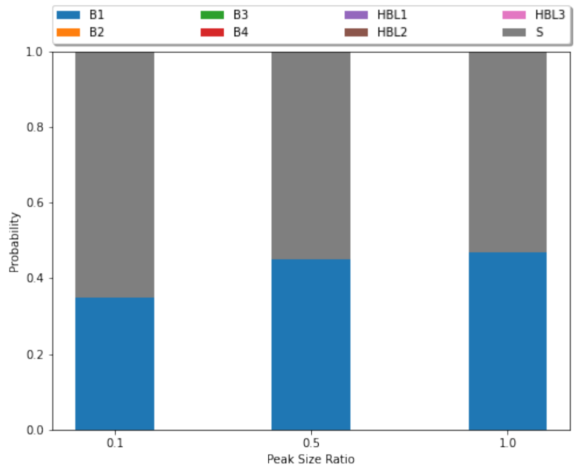
(a) Mixed strategy equilibrium, total size = 5



(b) Mixed strategy equilibrium, total size = 10



(c) Mixed strategy equilibrium, total size = 15



(d) Mixed strategy equilibrium, total size = 20

Fig. 6.3 Mixed strategy equilibrium, FCM

In terms of market efficiency, it is clear that iceberg orders are helpful in increasing market efficiency, and the smaller the Peak Size Ratio is, the more significant the effects we see in CDAs. This could be explained by our previous conclusion that fewer speculators are attracted to the market when the iceberg trader submits iceberg orders. In FCMs, the market efficiency is stable under the variations of the Peak Size Ratio; this could also be explained by the stable equilibrium profiles shown in Figure 6.3.

6.4.3 Impact of Iceberg Orders on Originator

In the following comparison, we look at the performance of the iceberg trader herself and consider the non-executed volume ratio when the trading ends. Obviously, a smaller non-executed volume ratio is better for the iceberg trader. We obtain the results after

Table 6.4 Market efficiency

Total Size	Peak Size Ratio	Market Type	
		CDA	FCM
5	0.1	0.881	0.934
	0.5	0.841	0.927
	1.0	0.838	0.930
10	0.1	0.880	0.919
	0.5	0.815	0.924
	1.0	0.791	0.921
15	0.1	0.817	0.926
	0.5	0.865	0.915
	1.0	0.801	0.924
20	0.1	0.830	0.918
	0.5	0.821	0.910
	1.0	0.776	0.915

computing the equilibrium; see Table 6.5. Here, CDA and FCM denote the experiments where the large-volume order is placed directly in the CDA and FCM markets, whereas CDA+I and FCM+I denote the experiments where the large-volume order is submitted as the iceberg order in the CDA and FCM, respectively.

We find that in CDAs, when the iceberg trader seeks to trade 5, 10, and 15 units, respectively, playing iceberg strategies helps to decrease the non-executed volume ratio sharply and hence increase her surplus during trading. However, in the last market, where the iceberg trader seeks to trade 20 times the best bid size, the non-executed volume ratio goes up when submitting iceberg orders. We believe that this is because the order size is too large to be filled in the pre-defined time when the trader hides the trading signal. If she submits a normal limit order with the full amount, speculators will be attracted and help fill these orders. This highlights a trade-off between the loss from the speculators and that from non-execution. Thus, it would be interesting to find out whether an optimal iceberg strategy could exist. In FCMs, on the other hand, placing iceberg orders has little effect on reducing the non-executed volume ratio; however, we note that the non-executed volume ratio is relatively smaller than that in CDAs with the same environment setting, both with and without the iceberg strategy.

6.4.4 Optimal iceberg strategy

We finally investigate the optimal iceberg strategy for traders seeking a large trade. In our experiments, we have a particular private value vector for the iceberg trader. Specifically,

Table 6.5 Non-executed volume ratio

Total Size	Peak Size Ratio	Market Type			
		CDA	CDA+I	FCM	FCM+I
5	0.1	0	0	0	0
	0.5	-	0	-	0
	1.0	-	0	-	0
10	0.1	0.27	0	0	0
	0.5	-	0	-	0
	1.0	-	0.27	-	0
15	0.1	0.48	0.39	0.09	0.11
	0.5	-	0.41	-	0.13
	1.0	-	0.48	-	0.09
20	0.1	0.53	0.60	0.16	0.13
	0.5	-	0.48	-	0.12
	1.0	-	0.53	-	0.16

we assume that the iceberg trader plays the seller role, aiming to sell a certain (large) amount of assets. Thus, she will have a negative payoff when she gets one more unit of the asset, meaning that the marginal element in a positive position, i.e., θ with positive index q, is negative. We also consider the effect of the trade volume and conduct four comparative experiments with different trade volumes.

There are four optional strategies in the iceberg strategy space, including three iceberg orders with different peak size ratios and one non-iceberg order, i.e., a full-size limit order. The strategy space is listed in Table 6.6, where I1, I2, and I3 are iceberg orders, and the *Peak Size Ratio* sets up the proportion of displayed amount. I4 is the non-iceberg strategy, which also can be seen as a special iceberg order with a 100% peak size ratio. Other participants strategically choose actions from the space in Table 6.3. The equilibria are summarized in Tables 6.7 and 6.8.

Table 6.6 Iceberg trading strategy space

	I1	I2	I3	I4
Peak Size Ratio	0.1	0.2	0.5	1.0

Regarding the optimal iceberg strategy in CDAs, surprisingly, there are three pure strategy equilibria, in each of which the iceberg trader plays the pure strategy I1 and the normal agents play the pure strategy HBL3. The results thus indicate that in these three environments, the optimal strategy is to display 10% of the total amount. At the same time, normal agents play the HBL strategy; that is, more information is helpful.

Table 6.7 Equilibrium profiles, CDA

Trader Type	Strategy	Peak Size Ratio			
		5	10	15	20
Iceberg Trader	I1	1.00	1.00	1.00	-
	I2	-	-	-	-
	I3	-	-	-	-
	I4	-	-	-	1.00
Normal Agents	B1	-	-	-	-
	B2	-	-	-	-
	B3	-	-	-	-
	B4	-	-	-	-
	HBL1	-	-	-	0.23
	HBL2	-	-	-	-
	HBL3	1.00	1.00	1.00	-
	S	-	-	-	0.76

Although the strategy space in this investigation is somewhat limited, we conjecture that the iceberg optimal strategy here is to display as little amount as possible. However, in the last experiment, the iceberg trader plays the pure strategy I4 in equilibrium – i.e., she reveals the full amount instead of submitting any hidden liquidity; as a result, the normal agents are attracted to play the speculating strategy. We believe that this is because when the iceberg trader submits iceberg orders, it takes much longer to execute the large trade than when revealing the full amount to attract the speculators, and the loss from slow execution is greater than the loss from speculators. This suggests that there is a threshold below which the iceberg strategy is optimal and above which the non-iceberg strategy performs better; the evaluation of this threshold is an interesting direction for future work.

In contrast, in FCMs, the iceberg trader always tends to play the pure strategy I4, i.e., to place the full-size limit order in most markets, and the normal agents play a mixed strategy combining the speculating strategy and the truth-telling strategy, which is consistent with what we found in previous experiments.

6.5 Conclusions

This chapter employs agent-based modeling and empirical game-theoretic analysis to explore the impact of iceberg orders on the traders' and market performances in continuous double auctions and frequent call markets.

From our experimental results, we conclude that in CDAs, the order volume has significant effects on the agents' strategy choice and market performance. In general, when

Table 6.8 Equilibrium profiles, FCM

Trader Type	Strategy	Peak Size Ratio			
		5	10	15	20
Iceberg Trader	I1	-	-	1.00	-
	I2	-	-	-	-
	I3	-	-	-	-
	I4	1.00	1.00	-	1.00
Normal Agents	B1	0.48	0.45	0.43	0.54
	B2	-	-	-	-
	B3	-	-	-	-
	B4	-	-	-	-
	HBL1	-	-	-	-
	HBL2	-	-	-	-
	HBL3	-	-	-	-
	S	0.52	0.55	0.57	0.46

agents observe a large volume order, they tend to speculate instead of providing liquidity. In terms of the market performance, the market efficiency will be harmed. In many cases, submitting a large volume order is not ideal for the originator herself, as this leads to a high non-executed volume ratio, meaning that her trading goal is not achieved.

Investigating the changes in the agent and market metrics resulting from submitting iceberg orders, we find that iceberg orders help address the problem caused by high volume orders: Specifically, fewer speculators are attracted, while market efficiency increases. It is also good for the iceberg trader herself, who obtains more surplus and has a smaller volume that is not executed. However, these benefits can not always be achieved, and we conjecture that, if the target volume is large enough, submitting the large volume order to attract the speculators could be better than taking the risk of having fewer trades executed when submitting iceberg orders. This is supported by our investigation into the optimal iceberg strategy for traders seeking a large trade.

For FCMs, we found that placing the large volume order has little impact on the behavior of the other agents while placing the iceberg order has little effect on the market efficiency. We thus believe that the FCM mechanism provides hidden liquidity, as the orders placed during the clearing interval are invisible to the public and, as a result, placing iceberg orders has little further impact, especially when compared to the issues we observed for CDAs. Overall, our results prove that FCMs are much more robust to orders of large size and suffer almost no impact from iceberg orders.

Chapter 7

Call Markets with Adaptive Clearing Intervals

Trading mechanisms play a fundamental role in the health of financial markets. For example, it is believed that continuous double auctions — wherein the market continuously clears matching offers in order of their arrival — constitute fertile soil for predatory behavior and toxic order flows. To this end, frequent call markets — where the matching is done at regular time intervals — have been proposed as an alternative design choice to address the latency arbitrage opportunities built in those markets.

This chapter studies the extent to which adaptive rules to define the length of the clearing intervals — that might move in sync with the market fundamentals — affect the performance of frequent call markets. We adopt an empirical game-theoretic analysis to show that matching orders in accordance with these rules can increase efficiency and selfish traders' surplus in a variety of market conditions. In so doing, our work paves the way for a deeper understanding of the flexibility granted by adaptive call markets.

7.1 Introduction

Continuous-time double auctions (CDAs) are a very popular trading mechanism applied in modern financial exchanges to the point that they are considered almost a synonym of order-driven markets. The key feature of a CDA market is that order submissions and withdrawals are processed serially, that is, in order of submission to the market (Budish et al., 2015). However, CDA markets are believed to lead to the latency arms race problem because traders can enjoy huge benefits from having just a tiny access-time advantage over the others (Sparrow, 2012). In other words, CDAs create the perfect environment for the proliferation of high-frequency traders, seen by many as an unhealthy feature of modern financial markets, that leads to flash crashes and, more generally, highly volatile markets (Kirilenko et al., 2017).

A way to address this problem is by switching from continuous-time clearing mechanisms to a discrete-time clearing model called batch auction or call market. Specifically, in a call market, there is a clearing interval: orders received during this time interval are accumulated and processed in batch at the end of the interval only, rather than serially in the order of arrival. In modern financial markets, there are thousands of trades within a tiny time period. Thus, the research on call markets nowadays lies mainly in the high clearing frequency scenario, a.k.a. frequent call markets. Frequent call markets have been shown to successfully address arbitrage problems caused by latency arms race (Budish et al., 2015; Wah and Wellman, 2013), which encouraged their substantial use of high-speed connection and processing devices, especially in the determination of the open price of some exchanges (such as London Stock Exchange). Naturally, call markets motivate a further design problem concerning the length of the clearing interval:

What is the "best" way to determine how often orders should be matched?

Against this background, this work aims to investigate the effects of clearing interval design on market performance. Specifically, we consider adaptive call markets (ACMs) where the clearing interval is not constant but moves in rhythm with the market: for example, the matching of orders could be activated by properties of the market microstructure. A special subclass of adaptive call markets is a simple random call market (RCM) where the outcome of a random process determines the length of the clearing interval. Alternatively, much of the previous research on batch auctions focused on what we call deterministic call markets (DCM), in which the physical clearing interval length is fixed. We explore the extent to which adaptive call markets can "improve" the market over deterministic batch auctions. Our comparison focuses on average market efficiency defined by the ratio between the realised and the potential total surplus, average size of spread, average trader surplus, and average traded volumes as measurements of the market performance. We employ agent-based modeling to simulate market interactions. To obtain solid conclusions from the simulations, we apply an *empirical game-theoretic analysis* (EGTA) to compute the empirical equilibria that take into account the strategic responses of traders, guided by their incentives, to the trading mechanism designs we propose.

Our first observation is that call markets outperform CDAs in many aspects, including market efficiency, total surplus, and the size of the spread. We also conclude that the uncertainty on the duration of the clearing interval is, by itself, not a strong enough feature to change the market qualitatively. In fact, comparing RCMs with DCMs, we note that market performance is only linked to the clearing frequency. However, we show that, similarly to DCMs, adaptivity combined with market-based clearing rules can also lead to performing markets. Furthermore, we explore other market-based clearing rules and draw a picture that supports the exploration of different combinations of termination rules (depending on market conditions) that could be used to make the market perform as intended.

The chapter unfolds as follows: Section 7.2 presents related literature on this topic. The research design and experimental settings are introduced in Section 7.3. The main results and their analysis are shown in Section 7.4. The last section concludes the chapter.

7.2 Related Work

Budish et al. (Budish et al., 2015) were the first to compare continuous and frequent call markets. They conducted empirical experiments to show that the arbitrage opportunities are built on the continuous trading mechanism and how a frequent call auction mechanism can address this. They also put forward the implementation details for frequent call markets in (Budish et al., 2014). The effectiveness of batch auctions is supported by Wah et al. (Wah et al., 2016a) and Haas et al. (Haas and Zoican, 2016). Aldrich and Vargas (Aldrich and Vargas, 2019) implemented a financial market in the laboratory with low communication latency. They found that frequent call markets exhibit less predatory trading behavior, lower investments, lower transaction costs, and lower volatility in market spreads and liquidity. Chang et al. (Chang et al., 2020) implemented call markets to the closing trading mechanisms to form the closing price. At the same time, Brunner (Brünner, 2019) studied the price formation when insider information exists in the frequent call market. Brinkman et al. (Brinkman and Wellman, 2017) conducted an empirical study on the optimization of (deterministic) clearing intervals. Some researchers also focused on randomized frequent call markets. Farmer and Skouras (Farmer and Skouras, 2012) theoretically discussed the costs, risks, and benefits of replacing the continuous double auction with frequent pro rata sealed bid randomized auctions.

However, previous research has been undertaken in the limited context of frequent call markets with fixed clearing interval lengths. In other words, the investigation of frequent call markets with adaptive clearing intervals, for example, a random interval length, has not been conducted. In this work, we not only study and compare the frequent call markets with deterministic and random clearing intervals but also explore the possibility of introducing other termination rules not just determined by time. We raise the stability-based and volume-based termination rules to the frequent call markets and use ABM and EGTA methods to measure their performances.

7.3 Experimental Setup

To compare markets with different trading mechanisms, we employ a parameterized single-asset financial market model inspired by Wah et al. (Wah et al., 2016a), since, arguably, it captures the qualitative phenomena found in real financial markets (Brinkman and Wellman, 2017). The trading mechanisms compared will be CDAs and call auctions, DCMs, and ACMs – our focus will be on the extent to which changing the matching rules of call markets qualitatively changes the market.

7.3.1 Market Model

In our model, there are N agents trading a single security in the market. Agents are allowed to submit limit orders (that is, an order specifying a direction – buy or sell – and a price) when they wish to trade. The limitation of the size of each order is set to 1 unit at the beginning of our experiments and will be relaxed when we test the performance with "extreme" orders (see Section 7.4.5 for the definition). As in previous works, only the price quotes reflecting the best outstanding bid and ask price are revealed to the public, while other details of the shape of the order book are invisible. In a CDA market, the orders will be matched continuously following the price-time priority rule (i.e., orders at either side of the market are in a FIFO queue). In contrast, in frequent call markets, the order book will only record new orders until it receives the matching signal generated by the matching rule. To simplify, we adopt fine-grained but discrete prices and times to simulate both the CDA and the frequent call markets. We also put a price range of the security to avoid unnecessary complexity – the scaling of prices does not affect our conclusions. The trading occurs over a finite horizon, T, which is long enough to generate reliable estimators. The price range and trading horizon will be instantiated below.

We use the same fundamental model and valuation model which can be seen in 5.3.1 and 5.3.2. We aim to explore the general scenario when the market mechanisms are changed, so we consider the background agents described in Section 5.3.3 only but expand the size of the background trading strategies (see Table 7.2).

7.3.2 Environment Parameters

In our comparative analysis, the markets and agents being compared share the same parameters except the market mechanism and the strategies they employ. Some common parameters including $\bar{f}, \sigma_s^2, \sigma_{\text{bias}}^2, \lambda_i, \sigma_{\text{pv}}^2$ are fixed through all the experiments. The asset price is a discrete integer ranging from 1 to 1000 in all experiments. We consider the thickness of the market, assigning 25 agents to a thin market and 91 agents to a thick market considering the computation cost and the requirement of the EGTA method. We also consider the stability of the fundamental value which is controlled by the reversion rate r. A higher reversion rate makes more stable fundamental dynamics. In our experiments, the reversion value r has two options: 0.8 and 0.2. We run 100 simulations for each game and take the average value of target market measurements (see Section 7.3.3) as the approximation. The values of the most common parameters are listed in Table 7.1. We also list some typical trading strategies that describe different levels of greed in Table 7.2 as the action space in our games.

Table 7.1 Environment parameters

N	$\bar{f}$	r	σ_s^2	σ_{bias}^2	σ_{pv}^2	λ	V	r_{entry}
25,91	500	0.8,0.2	100	50	25	0.01	200	0.1

7.3.3 Market Measures

The key feature of the market we are concerned with here is market efficiency. It is measured by the ratio between the realized and the potential total surplus. Liquidity is another vital measurement. A market with high liquidity ensures that “there is enough interest in an asset to maintain a reasonable amount of trading without steep price changes” (Das, 2008). It means that there is a high probability for a limit order to be executed in a reasonable amount of time. The depth of the limit order book and the spread of bid and ask are good measures of liquidity. In our experiments, we adopt the average size of spread as the measure, because it is quite straightforward to implement for the exchange (and also for the traders without access to level II data). Finally, we choose the average trading volume to see the activeness of the market. In what follows, we regard the average market efficiency, average total surplus, average spread, and average trading volumes simply as *Efficiency, Surplus, Spread* and *Volume*.

Table 7.2 Strategy space for our EGTA analysis

Index	1	2	3	4	5	6	7
α_{min}	0	0	0	0	20	20	50
α_{max}	0	20	50	100	50	100	100

7.4 Experiments And Analysis

Our aim is to identify a new market design that overcomes the limitations of existing mechanisms. We start with verifying the Latency Arm Race problem raised by Budish (Budish et al., 2015) using ABM and EGTA.

7.4.1 Latency Arms Race

Asymmetric information is a hot topic in the study of market mechanism design and results in the issue of adverse selection. Outstanding orders in the book reflect outdated information and become stale when the situation changes. Those with better information will update their orders in time and trigger transactions with stale orders. The advantages of better and faster information create arbitrage rents (Budish et al., 2015). The pursuit of the advantage position contributes to an arms race for speed, which is believed to be harmful to social welfare (Budish et al., 2015; Sparrow, 2012; Wah and Wellman, 2013). Frequent call auctions reduce the advantage of tiny speed, as earlier arrival orders will not always be executed before late orders.

To experimentally examine arbitrage rents and their elimination, we select one agent as the fast agent while the rest are called the informed agents. The only difference between the fast and informed agents is the speed of accessing the exchange.

To model the speed arms race, we simply put the submissions of fast agents before other agents' submissions, while the order of submissions among informed agents is randomized. This allows fast agents to submit orders earlier than the others given the same information. According to price-time priority, the earliest orders will be matched first with outstanding orders in the order book, and the order price is not considered in generating the queue of incoming orders. It is possible that the order with the same price but submitted later fails to execute.

To see why frequent-call markets help alleviate the latency arms race, we compare CDAs and DCMs. In this experiment, the market is thick (91 agents) and the clearing interval length is set to be 4. The measurement of the effects of the speed advantage is the average surplus obtained per agent, and the result is shown in the first two columns of Figure 7.1. The fast agent gains more surplus than informed agents. Our results confirm, however, that in the frequent-call market, the difference between allocated surplus sharply decreases, thus showing that frequent-call auctions help eliminate the advantages of speed.

7.4.2 Deterministic Call Markets

We first compare the efficiency of DCMs with different clearing interval lengths and a typical CDA market in Figure 7.2. We can conclude that if the clearing interval length is in

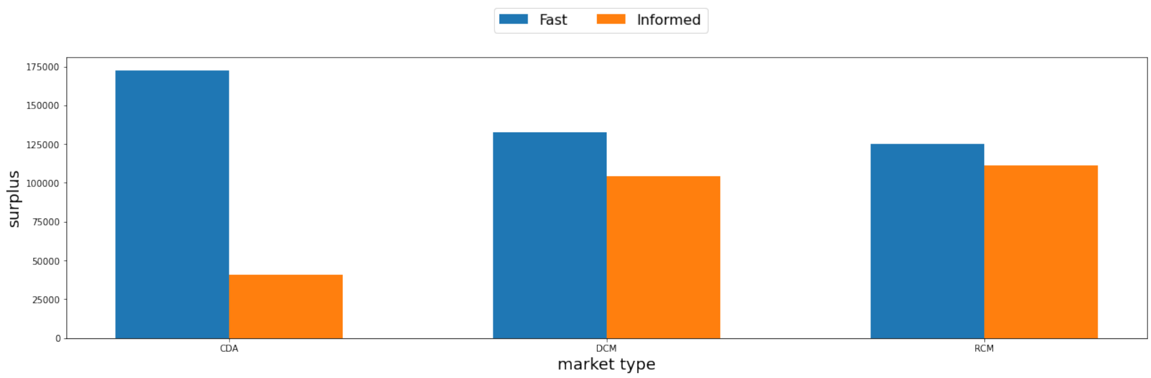


Fig. 7.1 Latency arm race

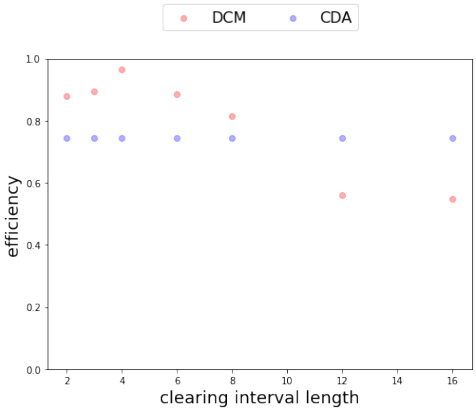
a reasonable range, the DCM shows a higher market efficiency than a CDA market in all market settings. However, if the clearing lasts too long, its efficiency will decrease sharply and even be poorer than that of the CDA market. We also notice that when the length of the clearing interval is close to the average number of trades (that is, 2.75, 9.16, 2.81, and 8.43 in our four experiments), the efficiency is always high (albeit not maximal). We have the hypothesis that there might exist an optimal length of clearing that moves in sync with the market fundamentals, but it should be examined by large amount of simulations with different setups, and we leave it as future work. Our first conclusion is that a reasonably high frequency of clearing will keep the efficiency at a high level.

It is also notable that in real markets, transactions will not happen uniformly(Petrov et al., 2019), which is opposite to our experiment environment. In other words, the optimal deterministic interval length should be the intrinsic time rather than the physical time, which makes estimating the fixed physical length of the optimal clearing interval more difficult.

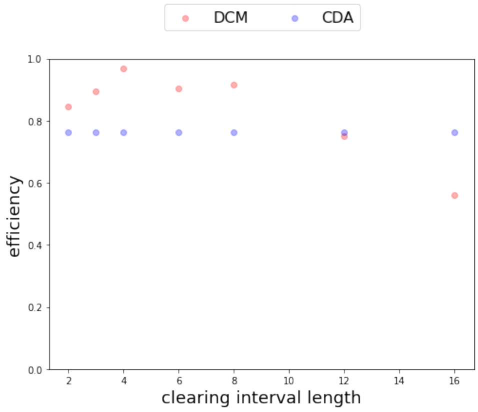
From this experiment, we learned that we need only focus on high clearing frequencies in our research. Firstly, since the fundamental value stochastic process uses intrinsic time, we cannot always use the optimal length in DCMs, and, looking at the results, it is better to err by overestimating Δt. Secondly, even in ACMs, we cannot exactly follow the clock of Δt to clear since it is not obvious or easy to know exactly how the dynamics (5.1) evolves. In fact, ACMs could be seen as an attempt to mimic this through clearing rules defined upon the market behavior.

7.4.3 Random Call Markets

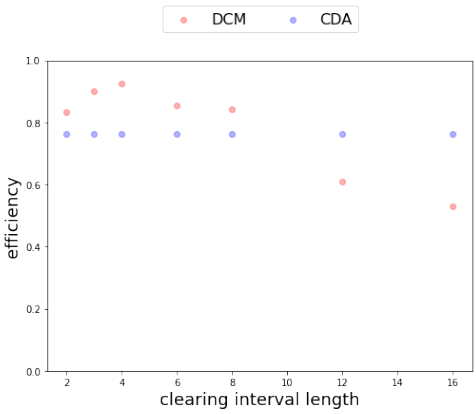
To begin our exploration in adaptive call markets, we experiment with a large subclass – RCMs – where the length of clearing interval is not fixed and is generated from a distribution parameterized by a fixed length of time. Suppose l is the pre-set average



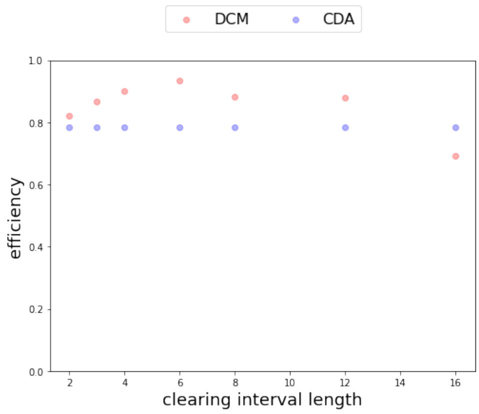
(a) Thin market, $r = 0.2$



(b) Thick market, $r = 0.2$



(c) Thin market, $r = 0.8$



(d) Thick market, $r = 0.8$

Fig. 7.2 Market efficiency: CDA vs DCM

clearing interval length, then each clearing interval length of the RCM is generated independently and randomly from the set $\{1, 2, \ldots, 2l-1\}$. We want to understand if the superiority of DCMs over CDAs is only due to slowing down the clearing or rather to a regular pattern of fixed clearing intervals. In other words, we wonder if irregular clearing intervals would somehow counteract, or even accentuate, the advantages of call markets over CDAs.

We start from the study of the speed arms race in RCMs, and the result is shown in the last column of Figure 7.1. The smaller difference between the surplus of fast and informed agents vis-à-vis ordinary FCMs indicates that the random clearing scheme further eliminates the advantages of speed. We believe the underlying reason is that the opportunity to cancel orders is not always realized for fast agents. To examine whether the fixed-clearing-length call market and the random-time-length call market differ in market efficiency given the same average clearing length, we make the simulations in four sets of markets. We also put the performance of a CDA market into the comparison. The results are shown in Figure 7.3, 7.4 and 7.5.

We find that the values of the three measurements are very close in the same market set but differ among different sets. With the increasing clearing frequency, which is equivalent to the decreasing of clearing interval lengths, the efficiency, and the surplus go down while the spread increases. We can also see a trend; when the clearing frequency goes up, the market performance is closer to the performance of the CDA market. This is because when the clearing frequency is high enough, or, in other words, the clearing interval is short enough, RCMs become equivalent to CDA markets.

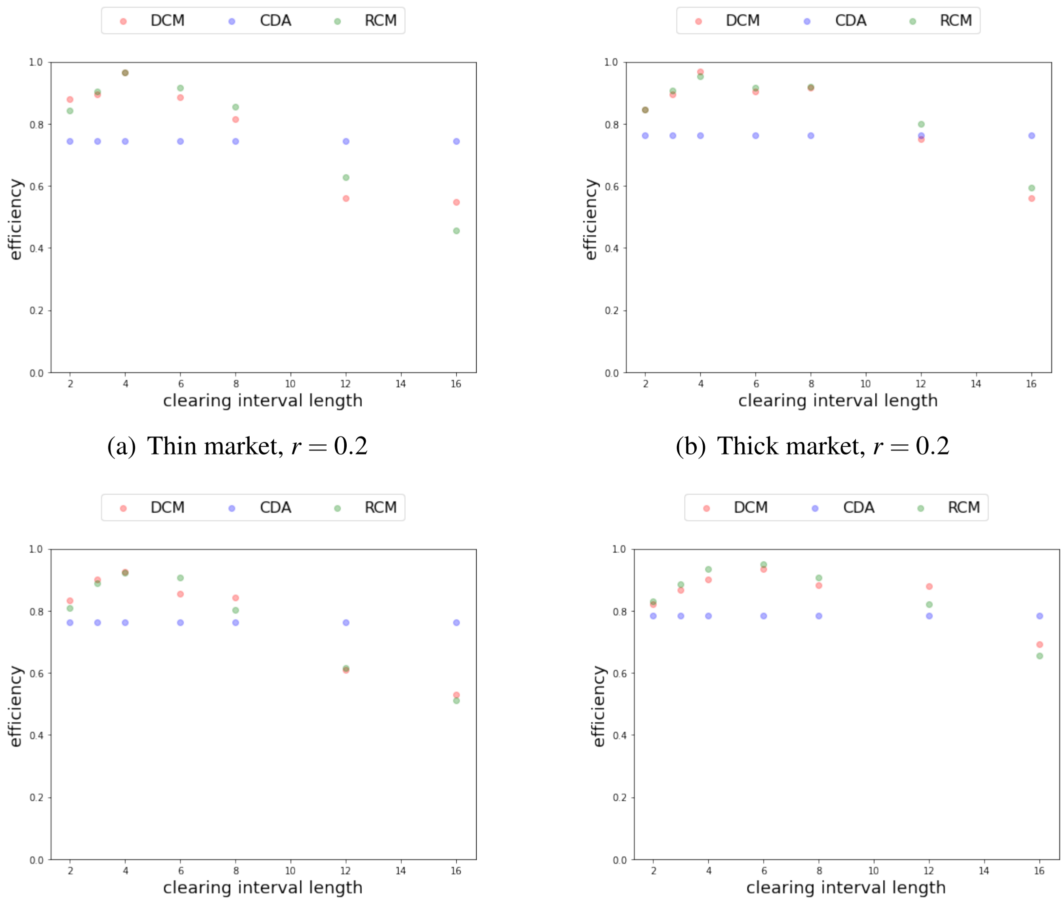
Another conclusion is that the market performance is only dependent on the clearing frequency, the irregularity of clearing intervals being irrelevant. In summary, deterministic and random call markets have very similar performances as long as the frequency is equal.

7.4.4 Stability-driven Adaptive Call Markets

Adaptive call markets are very flexible because the clearing can be determined by different rules. These rules might take internal information, which is invisible to the public, as input and send clearing signals to the order book when the conditions are met. Crucially, and differently from random clearing times discussed above, we can also design clearing rules that aim to make the market behave as desired.

In this section, we aim to have a stable market and analyze Stability-driven Adaptive Call Markets (SACMs). A stable market is believed to produce an efficient economic outcome (Issing, 2002).

Definition 1 *If the mid-price increases or decreases by over 10% in the next transaction, we call it a sharp movement.*



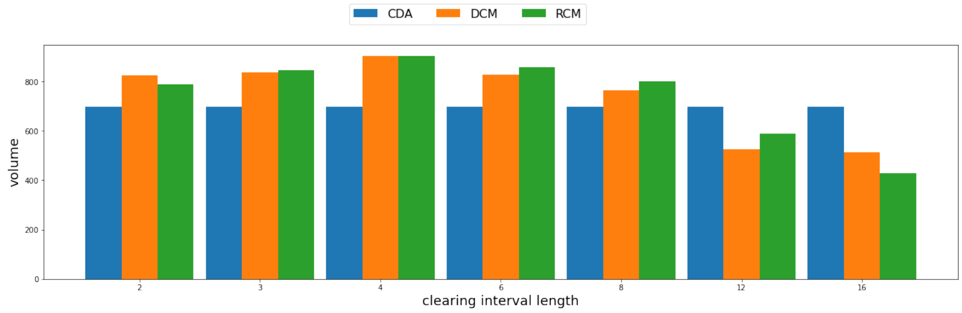
(a) Thin market, $r = 0.2$

(b) Thick market, $r = 0.2$

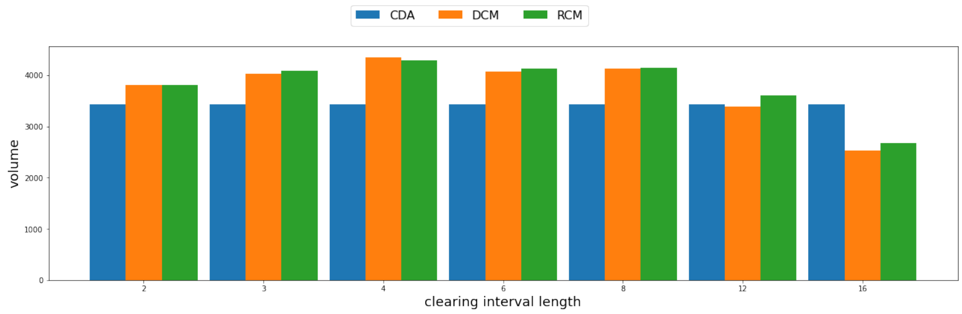
(c) Thin market, $r = 0.8$

(d) Thick market, $r = 0.8$

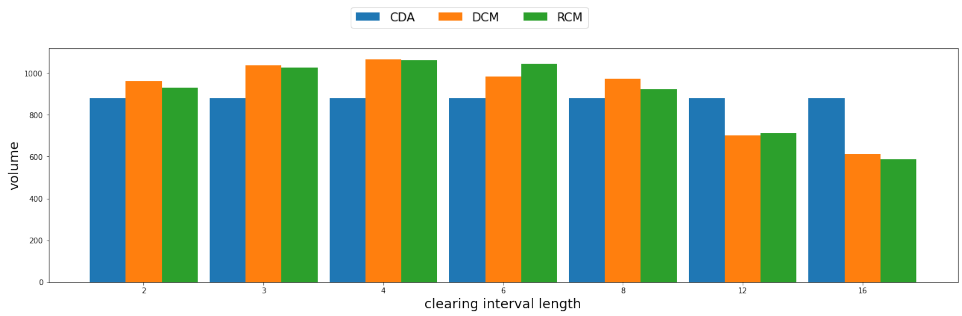
Fig. 7.3 Market efficiency: CDA vs DCM vs RCM



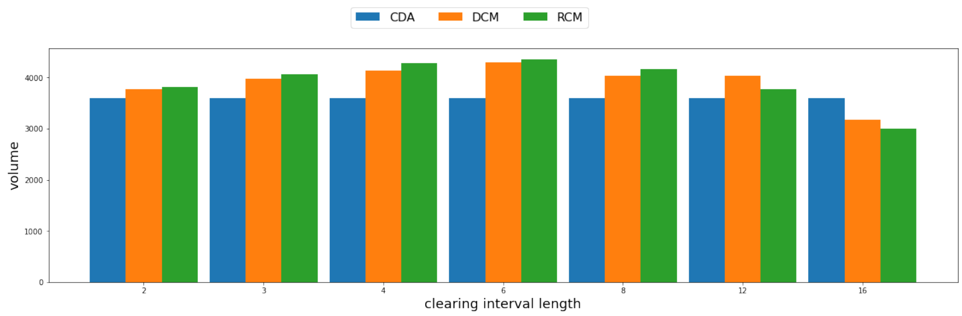
(a) Thin market, $r = 0.2$



(b) Thick market, $r = 0.2$

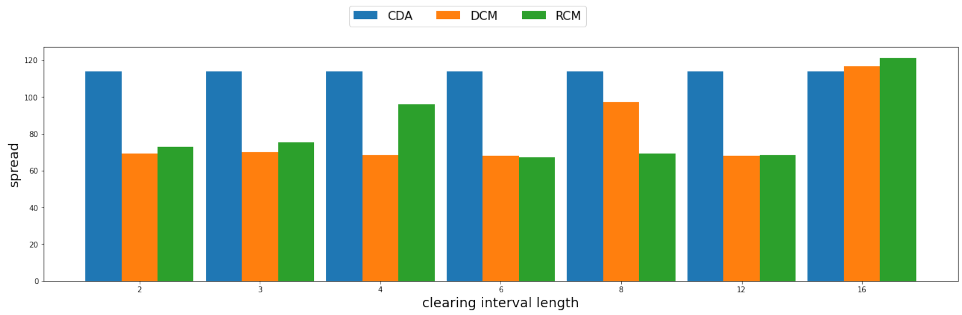


(c) Thin market, $r = 0.8$

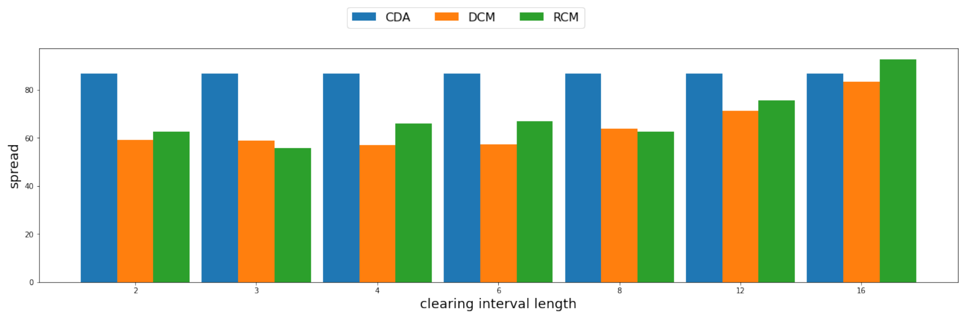


(d) Thick market, $r = 0.8$

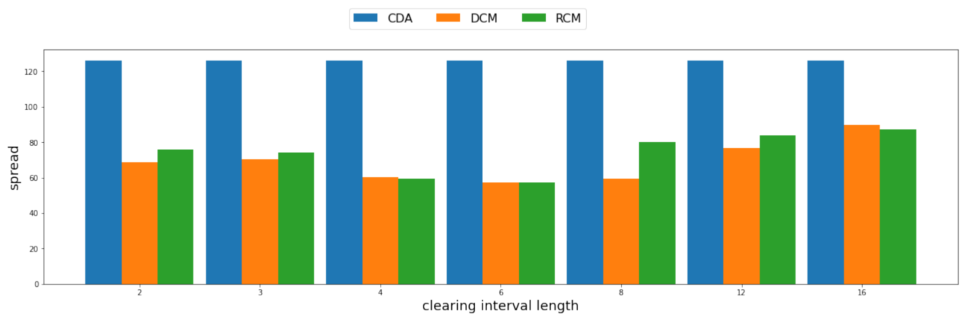
Fig. 7.4 Market volume: CDA vs DCM vs RCM



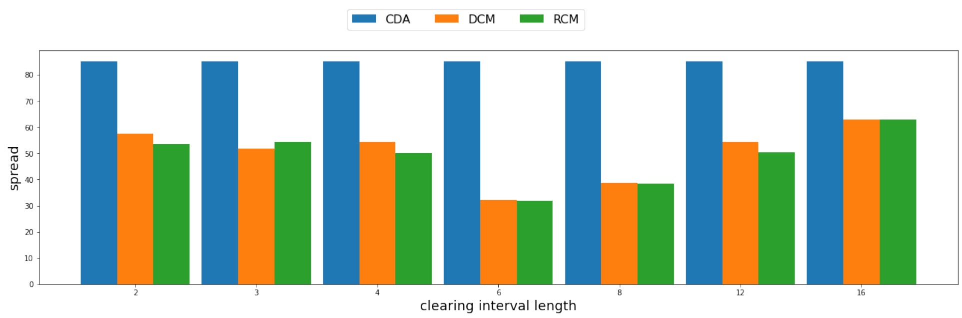
(a) Thin market, $r = 0.2$



(b) Thick market, $r = 0.2$



(c) Thin market, $r = 0.8$



(d) Thick market, $r = 0.8$

Fig. 7.5 Market spread: CDA vs DCM vs RCM

We use the sharp movement ratio, i.e., the ratio between the number of sharp movements and the length of the mid-price series, to measure the stability level. Figure 7.6 shows the ratio of sharp movement in the CDA and DCM. Although DCM achieves a higher efficiency, it also shows less stability. If the price of the asset is overly unstable in the market, trading could be risky and less attractive to risk-averse investors. The idea behind the SACM is to clear when the market dynamics lie in the preset "safe" range and make the market hold when it is more unstable than expected. Like DCMs, this gives the chance to "slow" traders to cancel the orders in periods of high volatility and avoid sudden changes in execution prices. Differently from the DCMs, however, SACMs keep the market moving at an adaptive pace, in order to get over the bumps more quickly. In a SACM, the market will keep collecting orders and calculate the mid-price as if the limit order book were cleared continuously. These "mid-prices" are tracked, and if the movement of this virtual mid-price hits the threshold, the book starts to clear and then goes to the next clearing round. Specifically, let M denote the mid-price right after the previous clearing and let M' be the virtual mid-price, updated as new orders are collected like the CDA although the updating process is virtual and no transaction will happen. A SACM with threshold d clears if

$$\frac{|M-M'|}{M} \leq d.$$

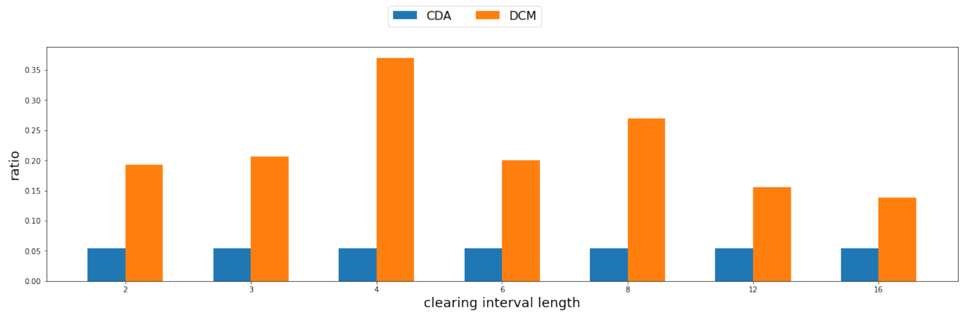
In other words, the market clears when the percentage change in mid-price is at most $100d$. So d denotes the percentage change in mid-price that we allow before we suspend clearing. If the mid-price moves within the allowed threshold, the clearing is triggered following RCM scheme; the clearing resumes when the mid-price returns to the safe range.

We run the simulations under different thresholds and compare the results with a CDA market and DCMs. Since the price grid is sparse (integers in $[0,1000]$) we need to set up seemingly large thresholds d to capture market movements; we test when d is 0.1. The results are shown in Figure 7.7.

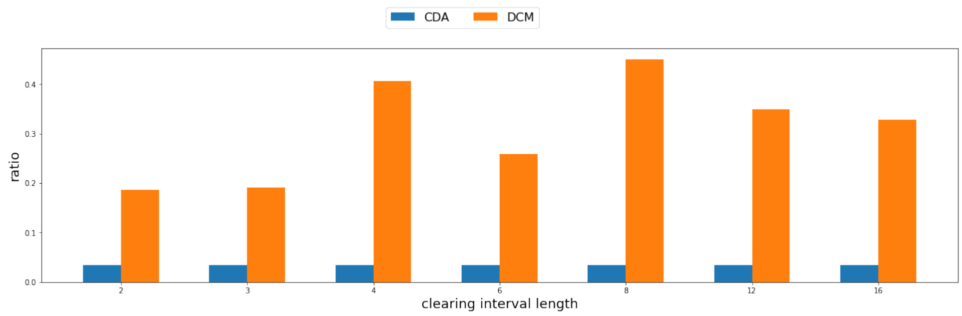
We can see from the plots that the market is able to maintain a high efficiency by adding the stability level d.

7.4.5 Volume-driven Adaptive Call Markets

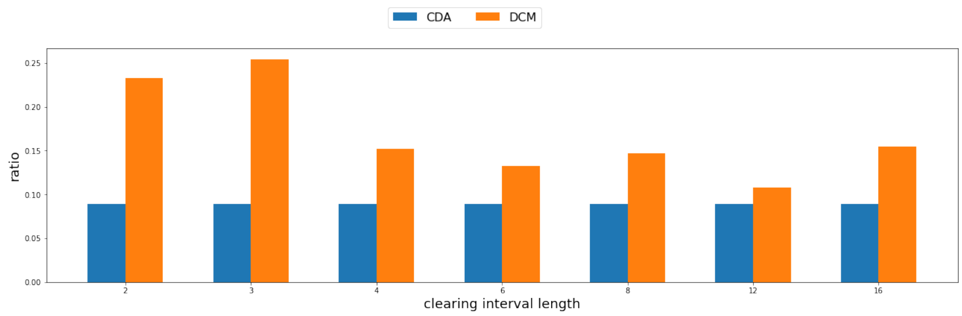
Another intuitive idea on the design of clearing termination rules is to record the aggregate volume during the clearing interval and when it reaches a threshold, the market will stop collecting orders and start to clear. More specifically, let v denote the volume threshold – the market will not clear until the cumulative volume of received orders since the last clearing reaches v. We call this market cumulative-volume adaptive call market (CVACM). However, when the aggregate volume time series is highly correlated with the length of



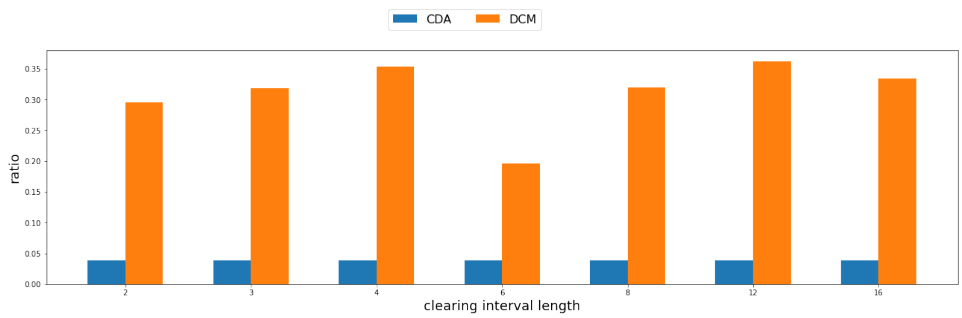
(a) Thin market, $r = 0.2$



(b) Thick market, $r = 0.2$

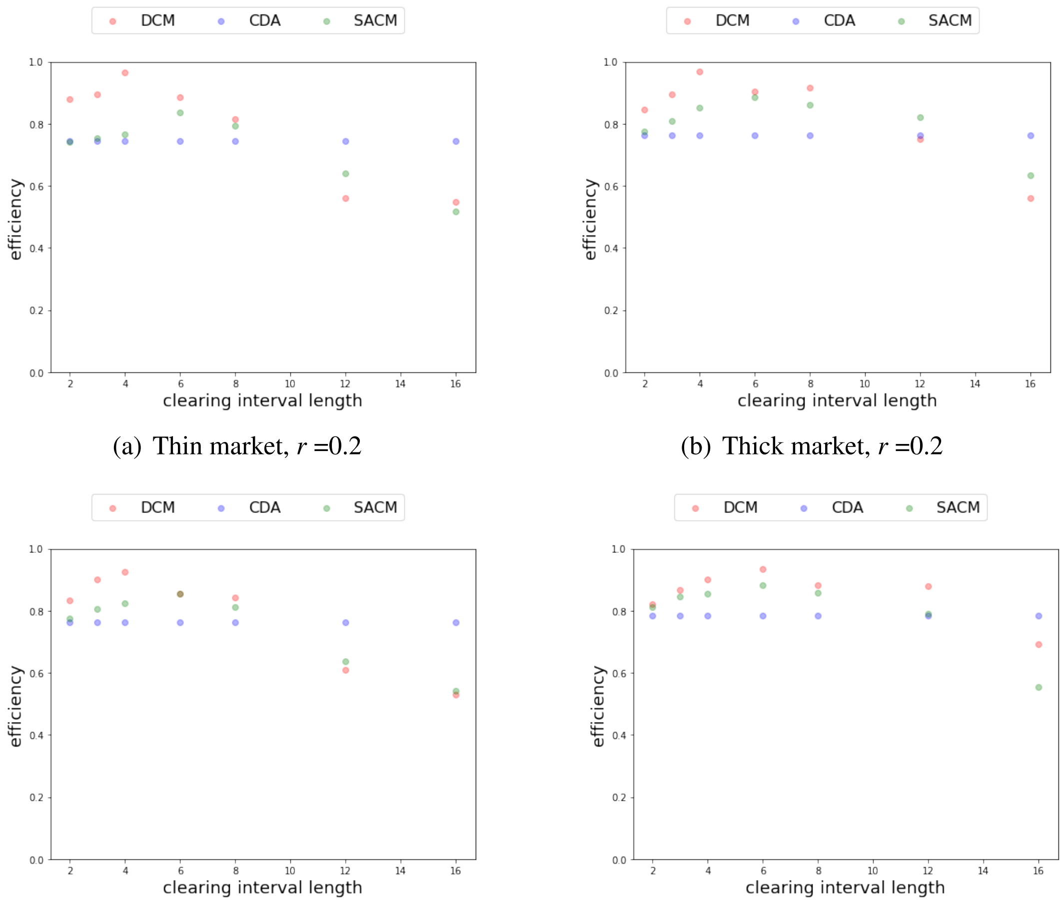


(c) Thin market, $r = 0.8$



(d) Thick market, $r = 0.8$

Fig. 7.6 Sharp movement ratio: CDA vs DCM



(a) Thin market, r =0.2

(b) Thick market, r =0.2

(c) Thin market, r =0.8

(d) Thick market, r =0.8

Fig. 7.7 Market efficiency: SACM

submission interval, then this market will have a performance similar to fixed interval call markets and, therefore, will not be discussed in detail here.

However, this approach inspires a new method to address the problem of large one-side orders caused by serial order processing of CDA markets. The book keeps collecting orders during clearing intervals and updates the bid-ask curve. New orders that will be executed are labeled as effective orders.[1] Let R be the ratio between the cumulative volume of effective ask orders and the cumulative volume of effective bid orders. Moreover, let L denote an extreme-volume threshold. When R lies in the range $[\frac{1}{L}, L]$, we say that the volume ratio is in normal status, while if it is outside the range, we say it is in an extreme status. The clearing rule is activated when the volume ratio changes status, from normal to extreme or from extreme to normal. This is a safety termination condition to avoid the squeeze of oversized orders and is used as an addition to other sets of termination rules. We focus, in particular, on a market design called Extreme-Volume ACM (EVACM, for short), which combines the rule above with a DCM. That is, an EVACM will clear when the deterministic clearing time occurs (from the DCM) or the extreme-volume threshold is hit (as from above).

Let us give a simple example of this approach. Consider the order book, with the following outstanding orders after the last clearing

Bids	**Asks**
95	102
94	101
93	100
92	99

each for one unit. Now assume that the following orders arrive at the market in this order: ask at 90, ask at 89, ask at 88, ask at 87, bid at 99, bid at 100, and bid at 101. In a CDA, the execution prices would be continuously decreasing (i.e., 90, 89, 88, 87) first and then sharply increasing (i.e., 99,100,101). But if we use a DCM and all the orders above arrive within the fixed clearing intervals, we would not be clearing and the imbalance might be reoccurring. But if we use an EVACM with an extreme-volume threshold of 2 for example, the market will not clear when the four asks and the first bid are received. In fact, in all these cases, R is an extreme status. However, when the second bid arrives, then $R = 2$ and the volume ratio moves to normal status, and a clearing point will be triggered. Similarly, if several effective ask orders suddenly arrive before the next DCM clearing point, then R will go into an extreme status, and the market will be cleared again, although it is not the clearing time according to DCM rules.

[1]Note that orders remain effective in our simulation environment during a bidding interval.

We run experiments to test an extreme scenario where the cumulative bid order size is 100 times the cumulative ask order size. We set the extreme-volume threshold to 20 and add this clearing rule to a DCM in an EVACM. We also include CVACM in the comparison shown in Table 7.3. These experiments are run in the thick market, and we set the number of calls of both the DCM and RCM to be 20, about one-eighth of the number of agents. The results show that the extreme-volume termination rule is helpful in stopping "flash crashes" and vertical increases and, in turn, contributes to the stability of the market.

Table 7.3 Changes in price in extreme scenarios

Type	DCM	RCM	CVACM	EVACM
Proportion of change	32%	35%	41%	21%

7.4.6 Exploration of Combined Termination Rules

EVACM helps to avoid extreme loss, which indicates that it can be an additional termination rule to others. We have a first exploration of combined termination rules and examine the performance of RCMs with stability termination rule and Extreme-Volume termination rule (CACM for short). We have a quick experiment in a thin market with parameters $r = 0.2, l = 4, d = 0.1, R = 5$. We introduce a new setting to these experiments: an external market bid order with size 50 will enter the market at the same time. The experiment results are shown in Table 7.4.

Table 7.4 Market performance: CDA vs DCM vs CACM

Markets	Efficiency	Volume	Spread	Sharp Movement Ratio	Maximum Price Change
CDA	0.636	763	135	4.3%	62%
DCM	0.811	927	96	28.7%	35%
CACM	0.762	826	67	0	10%

We can conclude that the CACM performs a trade-off between the high efficiency and the stability compared with CDA and DCM. We also find that both the stability termination rule and the Extreme-Volume termination rule contribute to the constant decrease in the aspect of market spread, sharp movement ratio, and maximum price change from DCM to CACM by delaying the trigger of the clearing.

7.5 Conclusions

In this chapter, we investigate the design of call markets with variable clearing intervals. We measure the market performance by its efficiency, size of spread, surplus, and trading volumes. We highlight that when the clearing interval matches the update clock used by the intrinsic time underlying the stochastic process, the market efficiency is optimized. Similarly, the clearing frequency determines the market performance: a higher frequency results in a slight decrease in efficiency and average individual surplus, and a wider bid-ask spread. Call markets outperform traditional CDA markets as long as the clearing frequency is lower than the serial order processing of the CDA market. Whether the length of the clearing interval is fixed or not has little implication on the market performance: a deterministic market and a random-clearing-time call market (called simply random call markets above) have a similar market performance if they have the same clearing frequency. This suggests that in the design of real frequent call markets, a key step is to estimate the average updating time of the asset fundamental value and make the length of the clearing interval stick to it. If we are not able to have a precise estimation of updating time, we need to make sure that the clearing interval is shorter than the updating interval to avoid loss of market efficiency. We also examine adaptive call markets with different termination rules. We show that the liquidity-driven, mid-price-tracking call market is able to provide a balance between good market performance and acceptable price stability. Extreme-volume adaptive call markets are helpful in extreme scenarios where the size of orders belonging to one side is much greater than the size of orders on the other side and reduce the risk of sharp price movements caused by such events.

As highlighted by the approach we used in Section 7.4.5, adaptive call markets are flexible with the termination rules. If we believe that there exists a good clearing condition, we can add it to the set of termination rules. For example, if we successfully constructed an indicator of the updating process of the fundamental value, we could combine it with the mid-price tracking rule and extreme-volume rule so that the market is healthy and efficient as we hope. This flexibility is the key advantage of ACMs over traditional CDA markets and deterministic call markets. Leveraging this flexibility and studying these combinations of clearing rules deserves further research. We also believe that ACMs would benefit from being studied in a variety of market conditions: central-concentrated and diffused fundamental value processes.

Chapter 8

Conclusion

In this thesis, we use agent-based models to study how the market structure affects the strategy profiles of involved agents and the resulting market performance. We apply the EGTA method to solve the equilibrium and achieve my conclusions. My contributions can be summarized as follows:

- In Chapter 4, we investigate the equilibrium strategy profiles, including their induced market performance, and their robustness to different simulation parameters. For two mainstream trading mechanisms, continuous double auctions (CDAs) and frequent call markets (FCMs), we find that EGTA is needed for solving the game as pure strategies are not a good approximation of the equilibrium. Moreover, EGTA gives generally sound and robust solutions regarding different market and model setups, with the notable exception of agents' risk attitudes. We also consider heterogeneous EGTA, a more realistic generalisation of EGTA whereby traders can modify their strategies during the simulation, and show that fixed strategies lead to sufficiently good analyses, especially taking the computation cost into the consideration.

- In Chapter 5, we study the effects of spoofing attacks on CDA and FCM markets. We let one spoofer (agent playing the spoofing strategy) play with other strategic agents and demonstrate that while spoofing may be profitable in both market models, it has less impact on FCMs than on CDAs. We also explore several FCM mechanism designs to help curb this type of market manipulation even further.

- In Chapter 6, we study the impact of iceberg orders on the price and order flow dynamics in financial markets. We find that the volume of submitted orders significantly affects the strategy choice of the other agents and the market performance. In general, when agents observe a large volume order, they tend to speculate instead of providing liquidity. In terms of market performance, both efficiency and liquidity will be harmed. We show that while playing iceberg-order strategy can alleviate the problem caused by the high volume orders, submitting a large enough order and attracting speculators is better than taking the risk of having fewer trades executed with iceberg orders.

- In Chapter 7. We verify that CDAs constitute fertile soil for predatory behaviour and toxic order flows and that FCMs address the latency arbitrage opportunities built in those markets. This chapter studies the extent to which adaptive rules to define the length of the clearing intervals — that might move in sync with the market fundamentals — affect the performance of frequent call markets. We show that matching orders in accordance with these rules can increase efficiency and selfish traders' surplus in a variety of market conditions. In so doing, our work paves the way for a deeper understanding of the flexibility granted by adaptive call markets.

There are also many limitations existing in this thesis. First, the experiment project of each main chapter was finished separately; it lacks consistency throughout all these four main projects. For example, the background strategy pools are different in these experiments. The detailed designs of the fundamental process and agents are also various, especially the valuation model for each agent. I will carefully design the environment and agents in future Agent-Based Modeling research to ensure that similar experiments share a common design of the environment and the agents so that the outcomes of experiments are comparable and easy to replicate.

The second main limitation lies in the presentation of simulation results. I follow the framework of the previous ABM research and generate the point estimations as the result to be presented and further investigated. For example, the average market efficiency or the average surplus are presented in this thesis. However, the point estimations make less statistical sense compared with interval estimations. I will choose to generate the confidence intervals rather than the estimated values to illustrate the statistical meanings.

The third main limitation is that these four projects all employ synthetic data generated by a pre-set mean-reverting stochastic process. A gap exists between my research and the application to the real world. I will emphasize the connections to real-world problems in future ABM research and attempt to use real data to construct the environment and agents to ensure the agent-based models are more realistic.

Apart from these limitations discussed above, I also see some open questions for future work.

- Although Gode and Sunder claimed in their work that ZI agents perform as well as humans, we critically argue that further investigations are needed because they only consider 12 agents and humans, and involved human traders are not as skilled as those in financial markets. we also introduce other strategic agents in this thesis, including the spoofing agent, the speculating agent, and the iceberg agent. It is worth investigating whether the variety of involved strategic agents will affect the findings of the market structure. For example, we showed that FCMs achieve higher market efficiency than CDAs when filled with background agents. Will it still hold if agents replace the background agents with other strategic behavior?

- Due to the computation time restriction, only one spoofer is considered in the mechanism design to reduce the spoofing. Only one iceberg agent is employed to study the impact of iceberg orders on the market. If more spoofers or more iceberg agents are introduced, will our findings still hold?

- We have seen the development of deep learning models and their more and more successful applications in financial markets. Will the agents equipped with machine learning tools alter the market microstructure? Another similar open question lies in the modeling of the microstructure of newborn markets. For example, the cryptocurrency market shows different statistical properties from those of the traditional financial market because of many reasons. It is worth investigating whether ABM is helpful in re-producing similar stylized facts with the real cryptocurrency markets. With the development of green finance, the carbon emission market will play a more and more important role in our daily lives. Can we forecast the microstructure of the carbon emission market and exploit the forecasted results to improve the effectiveness of the carbon emission market?

References

Aldrich, E. M. and Vargas, K. L. (2019). Experiments in high-frequency trading: comparing two market institutions. *Experimental Economics*, pages 1–31.

Arrow, K. J. (1965). *Aspects of the theory of risk-bearing*. Helsinki: Yrjö Jahnsonian Sätiö.

Arthur, W. B., Holland, J. H., LeBaron, B., Palmer, R., and Tayler, P. (1996). Asset pricing under endogenous expectations in an artificial stock market. *The economy as an evolving complex system II*, 27.

Bacry, E., Jaisson, T., and Muzy, J.-F. (2016). Estimation of slowly decreasing hawkes kernels: application to high-frequency order book dynamics. *Quantitative Finance*, 16(8):1179–1201.

Baur, D. G. and Schulze, N. (2009). Financial market stability—a test. *Journal of international financial markets, institutions and money*, 19(3):506–519.

Bessembinder, H., Panayides, M., and Venkataraman, K. (2009). Hidden liquidity: an analysis of order exposure strategies in electronic stock markets. *Journal of Financial Economics*, 94(3):361–383.

Brinkman, E. (2018). *Understanding financial market behavior through empirical game-theoretic analysis*. PhD thesis.

Brinkman, E. and Wellman, M. P. (2017). Empirical mechanism design for optimizing clearing interval in frequent call markets. In *Proceedings of the 2017 ACM Conference on Economics and Computation*, pages 205–221.

Brünner, T. (2019). Price formation in call auctions with insider information. *Studies in Economics and Finance*.

Budish, E., Cramton, P., and Shim, J. (2014). Implementation details for frequent batch auctions: Slowing down markets to the blink of an eye. *American Economic Review*, 104(5):418–24.

Budish, E., Cramton, P., and Shim, J. (2015). The high-frequency trading arms race: Frequent batch auctions as a market design response. *The Quarterly Journal of Economics*, 130(4):1547–1621.

Cartea, Á., Jaimungal, S., and Wang, Y. (2020). Spoofing and price manipulation in order-driven markets. *Applied Mathematical Finance*, 27(1-2):67–98.

Cason, T. N. and Friedman, D. (1996). Price formation in double auction markets. *Journal of Economic Dynamics and Control*, 20(8):1307–1337.

Cebiroglu, G. and Horst, U. (2011). Optimal display of iceberg orders. Technical report.

Chang, Y.-K., Chou, R. K., and Yang, J. J. (2020). A rare move: The effects of switching from a closing call auction to a continuous trading. *Journal of Futures Markets*, 40(3):308–328.

Cliff, D. and Bruten, J. (1998). Zero is not enough: On the lower limit of agent intelligence for continuous double auction markets.

Comerton-Forde, C. and Putniņš, T. J. (2014). Stock price manipulation: Prevalence and determinants. *Review of Finance*, 18(1):23–66.

Consiglio*, A., Lacagnina, V., and Russino, A. (2005). A simulation analysis of the microstructure of an order driven financial market with multiple securities and portfolio choices. *Quantitative Finance*, 5(1):71–87.

Cont, R. (2007). Volatility clustering in financial markets: empirical facts and agent-based models. In *Long memory in economics*, pages 289–309. Springer.

Cont, R. and De Larrard, A. (2012). Order book dynamics in liquid markets: limit theorems and diffusion approximations. *Available at SSRN 1757861*.

Cont, R. and De Larrard, A. (2013). Price dynamics in a markovian limit order market. *SIAM Journal on Financial Mathematics*, 4(1):1–25.

Cont, R., Kukanov, A., and Stoikov, S. (2014). The price impact of order book events. *Journal of financial econometrics*, 12(1):47–88.

Cont, R., Stoikov, S., and Talreja, R. (2010). A stochastic model for order book dynamics. *Operations research*, 58(3):549–563.

Cumming, D., Ji, S., Peter, R., and Tarsalewska, M. (2020). Market manipulation and innovation. *Journal of Banking & Finance*, 120:105957.

Das, S. (2008). The effects of market-making on price dynamics. In *Proceedings of the 7th international joint conference on Autonomous agents and multiagent systems-Volume 2*, pages 887–894. International Foundation for Autonomous Agents and Multiagent Systems.

Daskalakis, C., Goldberg, P. W., and Papadimitriou, C. H. (2009). The complexity of computing a nash equilibrium. *SIAM Journal on Computing*, 39(1):195–259.

De Winne, R. and D'hondt, C. (2007). Hide-and-seek in the market: placing and detecting hidden orders. *Review of Finance*, 11(4):663–692.

De Winne, R., D'Hondt, C., et al. (2009). Hidden liquidity in a pure order-driven market. *Finance Letters, forthcoming*, 14:22.

D'Hondt, C., De Winne, R., and Francois-Heude, A. (2004). Hidden orders on euronext: Nothing is quite as it seems. *Available at SSRN 379362*.

Ehrentreich, N. (2003). A corrected version of the santa fe institute artificial stock market model. *Complexity*, page 23.

Esser, A. and Mönch, B. (2007). The navigation of an iceberg: The optimal use of hidden orders. *Finance Research Letters*, 4(2):68–81.

Farmer, D. and Skouras, S. (2012). Review of the benefits of a continuous market vs. randomised stop auctions and of alternative priority rules (policy options 7 and 12). *Manuscript, Foresight. Government Office for Science*.

Frey, S. and Sandas, P. (2008). Iceberg orders and the compensation for liquidity provision.

Frey, S. and Sandås, P. (2017). The impact of iceberg orders in limit order books. *Quarterly Journal of Finance*, 7(03):1750007.

Gil-Bazo, J., Moreno, D., and Tapia, M. (2007). Price dynamics, informational efficiency, and wealth distribution in continuous double-auction markets. *Computational Intelligence*, 23(2):176–196.

Gode, D. K. and Sunder, S. (1993). Allocative efficiency of markets with zero-intelligence traders: Market as a partial substitute for individual rationality. *Journal of political economy*, 101(1):119–137.

Haas, M. and Zoican, M. A. (2016). Discrete or continuous trading? hft competition and liquidity on batch auction markets. *Université Paris-Dauphine*.

Hasbrouck, J. (2007). *Empirical market microstructure: The institutions, economics, and econometrics of securities trading*. Oxford University Press.

Hendershott, T., Jones, C. M., and Menkveld, A. J. (2011). Does algorithmic trading improve liquidity? *The Journal of finance*, 66(1):1–33.

Issing, O. (2002). Why stable prices and stable markets are important and how they fit together. In *Speech at the First Conference of the Monetary Stability Foundation*, volume 5.

Jordan, P. R., Schvartzman, L. J., and Wellman, M. P. (2010). Strategy exploration in empirical games. In *Proceedings of the 9th International Conference on Autonomous Agents and Multiagent Systems: volume 1-Volume 1*, pages 1131–1138.

Kirilenko, A., Kyle, A. S., Samadi, M., and Tuzun, T. (2017). The flash crash: High-frequency trading in an electronic market. *The Journal of Finance*, 72(3):967–998.

Kominers, S. D., Teytelboym, A., and Crawford, V. P. (2017). An invitation to market design. *Oxford Review of Economic Policy*, 33(4):541–571.

LeBaron, B. (2002). Building the santa fe artificial stock market. *Physica A*, pages 1–20.

LeBaron, B. (2006). Agent-based computational finance. *Handbook of computational economics*, 2:1187–1233.

Lee, E. J., Eom, K. S., and Park, K. S. (2013). Microstructure-based manipulation: Strategic behavior and performance of spoofing traders. *Journal of Financial Markets*, 16(2):227–252.

Lettau, M. (1997). Explaining the facts with adaptive agents: The case of mutual fund flows. *Journal of Economic Dynamics and Control*, 21(7):1117–1147.

Levy, H. (1994). Absolute and relative risk aversion: An experimental study. *Journal of Risk and uncertainty*, 8(3):289–307.

Lin, Y., Michayluk, D., and Zou, M. (2019). Can random auction ending curb market manipulation? *Available at SSRN 3404507*.

Liu, B., Polukarov, M., Ventre, C., Li, L., and Kanthan, L. (2021a). Call markets with adaptive clearing intervals. In *Proceedings of the 20th International Conference on Autonomous Agents and Multi-Agent Systems*, pages 1587–1589.

Liu, B., Polukarov, M., Ventre, C., Li, L., Kanthan, L., Wu, F., and Basios, M. (2021b). The spoofing resistance of frequent call markets. In *Proceedings of the 21st International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*.

Madhavan, A. (2000). Market microstructure: A survey. *Journal of financial markets*, 3(3):205–258.

Manski, C. F. (2006). Interpreting the predictions of prediction markets. *economics letters*, 91(3):425–429.

Martínez-Miranda, E., McBurney, P., and Howard, M. J. (2016). Learning unfair trading: A market manipulation analysis from the reinforcement learning perspective. In *2016 IEEE Conference on Evolving and Adaptive Intelligent Systems (EAIS)*, pages 103–109. IEEE.

Maslov, S. (2000). Simple model of a limit order-driven market. *Physica A: Statistical Mechanics and its Applications*, 278(3-4):571–578.

Mendonça, L. and De Genaro, A. (2020). Detection and analysis of occurrences of spoofing in the brazilian capital market. *Journal of Financial Regulation and Compliance*.

Muth, J. F. (1961). Rational expectations and the theory of price movements. *Econometrica: Journal of the Econometric Society*, pages 315–335.

O'hara, M. (1997). *Market microstructure theory*. Wiley.

Othman, A. (2008). Zero-intelligence agents in prediction markets. In *Proceedings of the 7th international joint conference on Autonomous agents and multiagent systems-Volume 2*, pages 879–886. International Foundation for Autonomous Agents and Multiagent Systems.

Palmer, R., Arthur, W. B., Holland, J. H., and LeBaron, B. (1999). An artificial stock market. *Artificial Life and Robotics*, 3(1):27–31.

Palmer, R. G., Arthur, W. B., Holland, J. H., LeBaron, B., and Tayler, P. (1994). Artificial economic life: a simple model of a stockmarket. *Physica D: Nonlinear Phenomena*, 75(1-3):264–274.

Parlour, C. A. (1998). Price dynamics in limit order markets. *The Review of Financial Studies*, 11(4):789–816.

Pascual, R. and Veredas, D. (2009). What pieces of limit order book information matter in explaining order choice by patient and impatient traders? *Quantitative Finance*, 9(5):527–545.

Petrov, V., Golub, A., and Olsen, R. B. (2019). Intrinsic time directional-change methodology in higher dimensions. *SSRN*.

Pirrong, S. C. (1995). The self-regulation of commodity exchanges: the case of market manipulation. *The Journal of Law and Economics*, 38(1):141–206.

Pratt, J. W. (1978). Risk aversion in the small and in the large. In *Uncertainty in economics*, pages 59–79. Elsevier.

Raberto, M., Cincotti, S., Focardi, S. M., and Marchesi, M. (2001). Agent-based simulation of a financial market. *Physica A: Statistical Mechanics and its Applications*, 299(1-2):319–327.

Roşu, I. (2009). A dynamic model of the limit order book. *The Review of Financial Studies*, 22(11):4601–4641.

Rust, J., Miller, J. H., and Palmer, R. (2018). Behavior of trading automata in a computerized double auction market. In *The Double Auction Market Institutions, Theories, and Evidence*, pages 155–198. Routledge.

Schvartzman, L. J. and Wellman, M. P. (2009). Stronger cda strategies through empirical game-theoretic analysis and reinforcement learning. In *Proceedings of The 8th International Conference on Autonomous Agents and Multiagent Systems-Volume 1*, pages 249–256.

Sparrow, C. (2012). The failure of continuous markets. *The Journal of Trading*, 7(2):44–47.

Tao, X., Day, A., Ling, L., and Drapeau, S. (2020). On detecting spoofing strategies in high frequency trading. *Available at SSRN 3746263*.

Tuttle, L. A. (2003). Hidden orders, trading costs and information. *Trading Costs and Information (November 29, 2003)*.

Vorobeychik, Y., Wellman, M. P., et al. (2008). Stochastic search methods for nash equilibrium approximation in simulation-based games. In *AAMAS (2)*, pages 1055–1062.

Wah, E., Hurd, D., and Wellman, M. (2016a). Strategic market choice: Frequent call markets vs. continuous double auctions for fast and slow traders. *EAI Endorsed Transactions on Serious Games*, 3(10).

Wah, E., Lahaie, S., and Pennock, D. M. (2016b). An empirical game-theoretic analysis of price discovery in prediction markets. In *IJCAI*, pages 510–516.

Wah, E. and Wellman, M. P. (2013). Latency arbitrage, market fragmentation, and efficiency: a two-market model. In *Proceedings of the fourteenth ACM conference on Electronic commerce*, pages 855–872.

Wah, E., Wright, M., and Wellman, M. P. (2017). Welfare effects of market making in continuous double auctions. *Journal of Artificial Intelligence Research*, 59:613–650.

Wang, X., Hoang, C., and Wellman, M. P. (2019). Learning-based trading strategies in the face of market manipulation. In *ICML-19 Workshop on AI in Finance*.

Wang, X., Vorobeychik, Y., and Wellman, M. P. (2018). A cloaking mechanism to mitigate market manipulation. In *Twenty-Seventh International Joint Conference on Artificial Intelligence*.

Wang, X. and Wellman, M. P. (2017). Spoofing the limit order book: An agent-based model. In *Workshops at the Thirty-First AAAI Conference on Artificial Intelligence*.

Wang, X. and Wellman, M. P. (2020). Market manipulation: An adversarial learning framework for detection and evasion. In *IJCAI*, pages 4626–4632.

Wellman, M. P. (2006). Methods for empirical game-theoretic analysis. In *AAAI*, pages 1552–1556.

Wellman, M. P., Kim, T. H., and Duong, Q. (2013). Analyzing incentives for protocol compliance in complex domains: A case study of introduction-based routing. *arXiv preprint arXiv:1306.0388*.

Wiedenbeck, B. and Wellman, M. P. (2012). Scaling simulation-based game analysis through deviation-preserving reduction. In *AAMAS*, pages 931–938.

Williams, B. and Skrzypacz, A. (2020). Spoofing in equilibrium. *Available at SSRN*.

Yang, T.-W. and Zhu, L. (2016). A reduced-form model for level-1 limit order books. *Market Microstructure and Liquidity*, 2(02):1650008.

Zotikov, D. and Antonov, A. (2021). Cme iceberg order detection and prediction. *Quantitative Finance*, 21(11):1977–1992.