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Sliding Mode Contouring Control for Biaxial Feed Drive Systems with a Nonlinear Sliding Surface

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Abstract

Control input variance is one of the important criteria in machining because it affects the surface roughness, machining precisions and consumed energy. This paper presents a nonlinear controller design for biaxial feed drive systems for reducing the control input variance while maintaining the motion accuracy. The contour error, which is defined as the error component orthogonal to the desired contour curve, is considered to design the controller because it directly affects the precision of machined work-piece profile. The proposed nonlinear controller allows to adjust a controller gain from its low value to high value as the contour error changes from low value to high value and vice versa, and hence a closed-loop system simultaneously achieves low overshoot and settling time, resulting in a smaller error. In order to design the variable controller gain, a sliding mode control based on a nonlinear sliding surface is employed. Experimental results demonstrate a significant performance improvement in terms of control input variance while maintaining the motion accuracy

Keywords: Feed drive system; Contouring control; Nonlinear control; Sliding mode control; Nonlinear sliding surface

1. Introduction

High-precision machining requires multi-axis feed drive systems to accurately follow specified contours. Due to disturbances such as friction, cutting force and modeling error, tracking errors usually appear in industrial applications such as X-Y tables, computer numerical control (CNC) machines and industrial manipulators. For machining, error components orthogonal to the desired contour curve, called contour errors, represent good indicators of the machining precision [1]. Koren used the tracking error in each drive axis to estimate the contour error in biaxial contour-following tasks and proposed the cross-coupled controller (CCC) [2]. However, a disadvantage of the CCC is that both tracking and contour errors are used to calculate control inputs, and a degradation in the contour tracking performance occurs. To address this problem, Lo and Chung proposed a contouring control method for biaxial feed drive systems based on a coordinate transformation [3], in which tracking errors are transformed into errors with components that are orthogonal and tangential to the desired contour curves. By decomposing the contour error into normal tracking error and advancing tangential error, Ho et al. applied a dynamic decoupling procedure to the system dynamics [4]. Chiu and Tomizuka proposed a task coordinate frame approach for contouring control of feed drive systems [5]. Cheng and Lee proposed a real-time contour error estimation algorithm and employed an integrated motion control scheme to improve the machining accuracy for a contour following task [6].

Not only motion precision but also control input variance is an important criterion in machining because it directly affects the surface roughness and consumed energy. This study presents a controller design for reducing the control input variance while maintaining the contouring performance. Sliding mode control (SMC) is a viable and effective method with a strong robustness property and fast error convergence characteristics for nonlinear systems subjected to external

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disturbances and parameter variations by emulating a prescribed reduced-order system [7][8]. The conventional sliding mode controllers utilize a linear sliding surface resulting in a constant damping ratio by which quick response produces high overshoot increasing the tracking errors and consumed energy, otherwise, the low overshoot results in a slow response leading to tracking errors. To address this particular problem, a nonlinear sliding surface is introduced to achieve a variable gain for improving the response of the closed loop system as the tracking error increases [9].

This study applies the sliding mode control to a contouring control problem of biaxial feed drive systems based on a coordinate transformation approach. The main advantage of the proposed approach is that it achieves a quick response and a small overshoot, and thereby providing smaller control input variance while maintaining the contouring performance. Experiments were conducted to demonstrate the effectiveness of the proposed design.

2. Feed drive dynamics

2.1. Definition of contour error

Contour error is defined as the shortest distance between an actual position of a feed drive system and a desired contour. The relationships between the contour and tracking errors in each feed drive axis are shown in Fig. 1. The coordinate frame $\sum w$ is a fixed frame and its axes x and y correspond to feed drive axes. The desired position of the point of the machined part at time t_r, defined in $\sum w$, is $r = \left[r_x, r_y\right]^T$. The actual position of the feed drive system is represented by $q = \left[q_x, q_y\right]^T$, which is also defined in the fixed frame. The tracking error in each feed drive axis is defined as

$$e_w = \left[e_x, e_y\right]^T = q - r \tag{1}$$

Because calculating the actual contour error e_c in Fig. 1 in real time for complex contour is an intensive computational task, we define several local coordinate frames as shown in Fig.1 and transform the error e_w into these frames. The coordinate frame $\sum L$ is attached at r and its axes are t and n. The axis t is in the tangential directional vector of the desired trajectory at r, and the directional vector n is perpendicular to t. The tracking error vector can be expressed with respect to $\sum L$ as

$$e_L = [e_t, e_n]^T = L^T e_W,\ L = [\mathrm{t} \quad \mathrm{n}] \tag{2}$$

We assume that the distance between the desired position r and the point s on the desired trajectory is approximately equal to the tangential error e_t. In addition, the desired velocity along this segment is nearly constant and equal to the desired velocity at r. The required time to traverse this segment t_d is estimated as follows:

$$t_d = \frac{-e_t}{\|\dot{r}\|}, \dot{r} \neq 0 \tag{3}$$

A new coordinate frame $\sum \tilde{L}$ corresponding to the

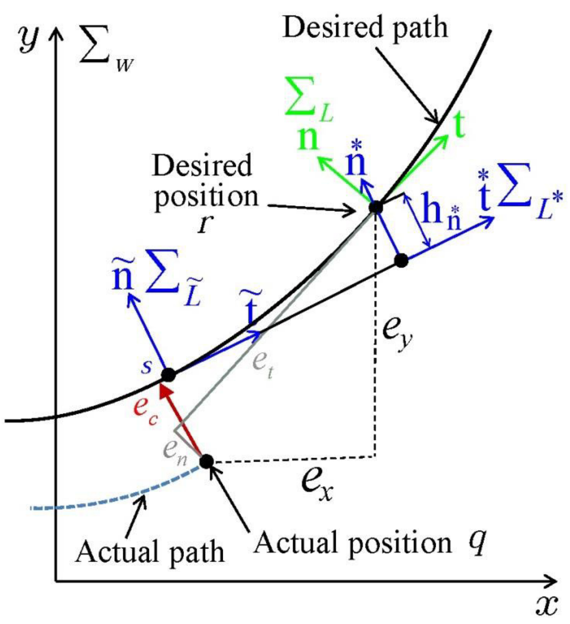

Fig. 1 Contour error definition and its estimate

instantaneous time $t_s = t_r - t_d$ is defined by two unit vectors as follows:

$$\tilde{\mathrm{t}} = \mathrm{t}(t_s),\ \tilde{\mathrm{n}} = \mathrm{n}(t_s) \tag{4}$$

The corresponding error vector in Fig. 1 is represented as follows:

$$e_{\tilde{L}} = [e_{\tilde{t}}, e_{\tilde{n}}]^T = \tilde{L}^T e_W, \tilde{L} = L(t_s) \tag{5}$$

The normal component is adjusted according to a new coordinate frame $\sum L^*$ shown in Fig. 1. The adjusted error can be expressed as follows:

$$\begin{aligned} e_{L^*} &= [e_{t^*}, e_{n^*}]^T = \tilde{L}^T e_W + h^* \\ h^* &= [0 \quad h_n^*]^T = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tilde{L}^T \{r(t_s) - r(t_r)\} \end{aligned} \tag{6}$$

We regard the error component e_{n^*} as an estimate of the contouring error e_c. This approach provides a good approximation of e_c while maintaining the tangential error (i.e., $e_{t^*} \cong e_t$) even when the error e_t is significant as shown in Fig. 1.

2.2. Feed drive dynamics

This study considers a typical biaxial feed drive system represented by the following decoupled second order system:

$$\begin{aligned} &M\ddot{q} + C\dot{q} = u - d \\ &M = \mathrm{diag}\{m_i\}, C = \mathrm{diag}\{c_i\}, i = x, y \\ &u = \left[u_x, u_y\right]^T, d = \left[d_x, d_y\right]^T \end{aligned} \tag{7}$$

where $m_i (> 0), c_i (> 0), d_i$ and u_i are the mass coefficient, viscous friction coefficient, external disturbance and control input voltage for the drive axis i, respectively. All terms in Eq. (7) has a dimension of voltage. The notation $\mathrm{diag}(a_i)$ denotes a diagonal matrix with the element a_i at the i th diagonal position.

3. Sliding Mode Contouring Control

3.1. Controller Design

In this section, the design of the contouring controller with a nonlinear sliding surface for biaxial feed drive systems will be considered. The proposed controller achieves a quick response and a small overshoot, thereby providing smaller control input variance while maintaining the contouring performance based on sliding model control using a nonlinear sliding surface. We consider the following nonlinear sliding surface [10][11]:

$$S=[F-\varphi P \quad I][e_{L^*} \quad \dot{e}_{L^*}]^T,\ S\in R^2 \tag{8}$$

where $F\in R^{2\times 2}$ is the linear gain matrix of the sliding surface, which is chosen such that the dominant poles have small damping ratios to achieve a fast response. $P\in R^{2\times 2}$ is a positive definite matrix to adjust the final gain. $I\in R^{2\times 2}$ is the identity matrix. φ is a 2×2 diagonal matrix with non-positive nonlinear entries depending on the transformed errors and is used to change the gain of the system. The choice of φ is not unique, and we employ the following nonlinear function [12]:

$$\varphi=\mathrm{diag}\left\{-\beta_i\frac{\exp(-k_i\tilde{e}_i)+\exp(k_i\tilde{e}_i)}{2}\right\} \tag{9}$$

$$\tilde{e}_i=\begin{cases} e_i & if\ |e_i|\le e_{imax} \\ e_{imax}\mathrm{sgn}(e_i) & if |e_i|>e_{imax}\end{cases},\ i=t^*,n^*$$

where e_{imax}, β_i and k_i are positive tunning parameters used to adjust the maximum bound, minimum bound and variation rate of the nonlinear function magnitude φ, respectively. $\mathrm{sgn}(e_i)$ represents the sign function of the error signal e_i. From Eqs. (1) and (7), the tracking error dynamics of the feed drive system in the fixed coordinate frame $\sum w$ is expressed as:

$$\ddot{e}_w=M^{-1}(-C\dot{q}-d+u)-\ddot{r} \tag{10}$$

The transformed error dynamics can be estimated by differentiating Eq. (6) twice with respect to time as follows:

$$\ddot{e}_{L^*}=\tilde{L}^T\ddot{e}_w+2\dot{\tilde{L}}^T\dot{e}_w+\ddot{\tilde{L}}^T e_w+\ddot{h}^* \tag{11}$$

Based on the proposed nonlinear sliding surface (Eq. (8)), assuming that the reference velocity and acceleration are given, substituting Eq. (10) into (11), and considering the feed drive dynamics, we design the following controller:

$$u=M\left\{\ddot{r}-\tilde{L}\left[(F-\varphi P)\dot{e}_{L^*}+\ddot{\tilde{L}}^T e_w+2\dot{\tilde{L}}^T\dot{e}_w+\ddot{h}^*-\dot{\varphi}Pe_{L^*}+K_cS+Q\mathrm{sgn}(S)\right]\right\}+C\dot{q} \tag{12}$$

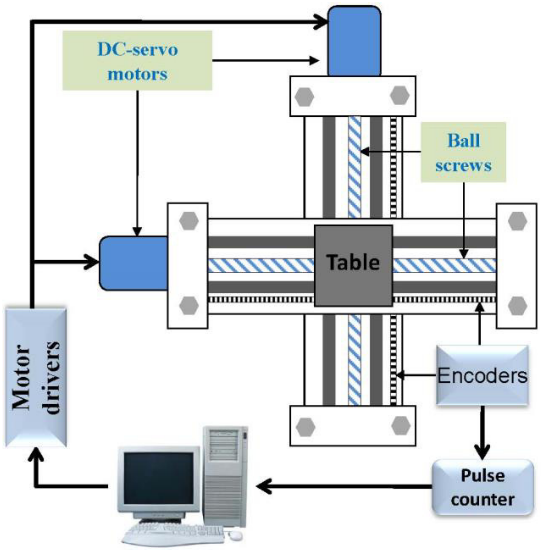

Fig. 2 Experimental system

where $K_c\in R^{2\times 2}$ is a diagonal gain matrix, $\mathrm{sgn}(S)$ consists of the signs of the sliding surface vector S and Q=diag$\{Q_i\}$ is a diagonal matrix with diagonal elements chosen from the maximum bound of the uncertainty as follows:

$$Q_i\ \ge\ \max(|\tilde{d}_i|) \tag{13}$$

where $\tilde{d}_i$ is an element of $\tilde{L}^TM^{-1}d$.

3.2. Stability Analysis

In order to insure that the controller forces the transformed errors onto the desired nonlinear sliding surface as the time goes infinity, we consider the following Lyapunov function candidate:

$$V\ =\frac{1}{2}S^TS \tag{14}$$

The time derivative of the Lyapunov function is

$$\dot{V}\ =\ S^T\{\ddot{e}_{L^*}+(F-\varphi P)\dot{e}_{L^*}-\dot{\varphi}Pe_{L^*}\} \tag{15}$$

Substituting Eqs. (10) - (12) into (15) leads to

$$\dot{V}\ =\ S^T\{-\tilde{L}^TM^{-1}d-K_cS-Q\mathrm{sgn}(S)\} \tag{16}$$

Thus, with Eq. (13), we have

$$\dot{V}<\ 0 \qquad (17)]$$

4. Experiment

To verify the effectiveness of the proposed controller, the control law in Eq. (12) was implemented with C++ language

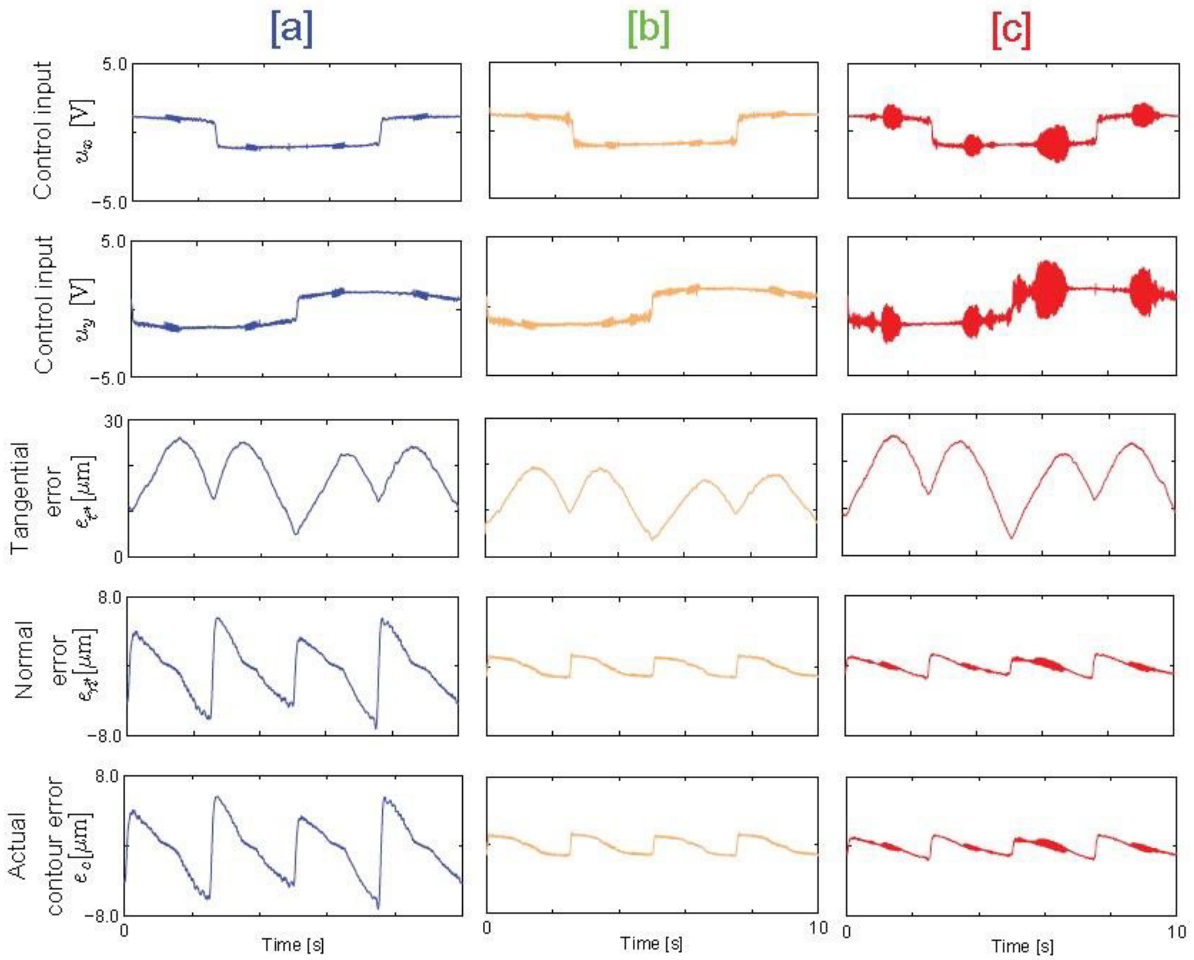

Fig. 3 Experimental results: (a) Linear sliding surface $F = \text{diag}\{30,\ \ 50\}$, (b) Linear sliding surface $F = \text{diag}\{30,\ \ 50\}$ (proposed), (c) Linear sliding surface $F = \text{diag}\{30,\ \ 210\}$

on a personal computer with a sampling time of 5 ms. The control input signal has been applied to a biaxial feed drive system that is driven by two DC servo motors via a DA board as shown in Fig. 2. The motors are coupled to, and drive, two ball screws through high-reduction gears to provide required torque. In addition, a rotary encoder with 0.314 μ rad resolution (corresponding to 0.025 μm in the drive axis) is attached to each feed drive axis to measure the position of the biaxial feed drive system. A pulse counter board is used to count the encoder pulses. The nominal parameter values of the biaxial feed drive system in Eq. (7) are $m_1 = 0.061\text{Vs}^2\text{mm}^{-1}$, $m_2 = 0.045\text{Vs}^2\text{mm}^{-1}$, $c_1 = 0.21\text{Vsmm}^{-1}$ and $c_2 = 0.24\text{Vsmm}^{-1}$. A circular reference trajectory (radius 5mm, frequency 0.1 Hz) is used in the experiment.

First, we compared the performance of the proposed controller with a nonlinear sliding surface in Eq. (8) and the conventional sliding mode controller with a linear sliding surface ($\varphi = 0$ in Eq. (8)). In this comparison, the controller gain K_c, the positive definite matrix P and the linear term of the sliding surface F are set to the same values in both controllers for fare comparison. We set the controller gain K_c, positive definite matrix P and the linear term of the sliding surface F to diag{20, 50}s^{-1}, diag{1.5, 1.7} s^{-1} and diag{30, 50} s^{-1}, respectively, while the nonlinear tunning parameters $\beta_{t^*}, \beta_{n^*}, k_{t^*}$ and k_{n^*} in Eq. (9) are set to 10 s^{-1}, 10 s^{-1}, 5 s^{-1} and 3000 mm^{-1}, respectively (some explanations will be given later on the selection of these parameters).

Figure 3(a) shows the experimental results for a controller with a linear sliding surface where the control input of the feed drive axis, transformed error components and actual contour error are plotted. The same quantities for the proposed nonlinear sliding surface are shown in Fig. 3(b). It can be confirmed that the proposed sliding surface achieves better performance in terms of the contour error. A similar contour error profile to that shown in Fig. 3(b) can be obtained by increasing the elements of linear gain F in the controller with a linear sliding surface (i.e., the controller used in experiment shown in Fig. 3(a)). The results of this case are shown in Fig. 3(c) where F is adjusted to be diag{30, 210} s^{-1} (from the view point of machining, the normal error components is more important than the tangential one and this motivated us to increase only the normal component of F). Achieving a similar contouring performance with the linear sliding surface increased the control input variance as shown

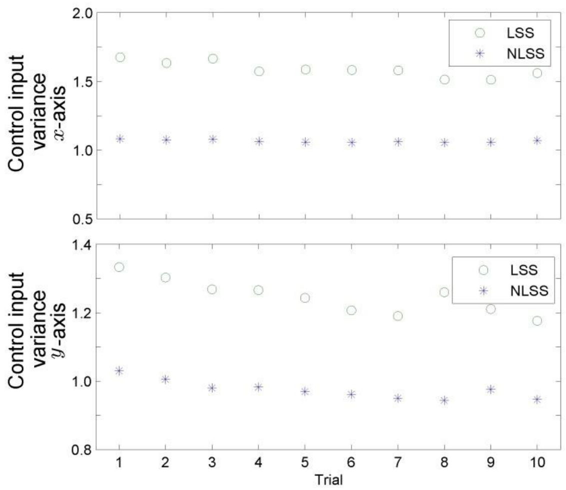

Fig. 4 Control input variance of linear and nonlinear sliding surfaces

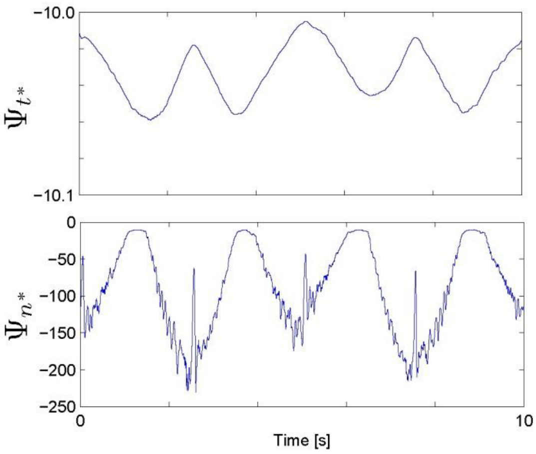

Fig. 5 Diagonal elements of the matrix $F - \varphi P$

in Fig. 3(c).

The control input variance for the controller used in Figs. 3(b) and (c) is calculated as follows:

$$\sigma_j^2 = \frac{\sum_{i=1}^{I}\left(u_{ji} - \mu_j\right)^2}{I}, j = x, y \quad (18)$$

where u_{ji} denotes the control input value at the i th sampling instant of the j th axis, I is the total number of sampling instants (i=1,...,I), and μ_j is the mean of all of the control input values of the j th axis. As shown in Fig. 4, the proposed approach provided the smaller control input variance by about 32.8 % and 21.8 % for x and y-axis, respectively.

Another advantage of the proposed approach is that it is easy to tune the nonlinear term because only the function φ needs to be tuned. The choice of tuning parameters β_{t^*} and β_{n^*} to be small (both are set to $10\ \mathrm{s}^{-1}$) is to ensure small initial magnitude of the function φ and to prevent the overshoot. Because allowable normal error magnitude is very small compared to the tangential one, the tuning parameters k_{t^*} and k_{n^*} are selected to be $5\ \mathrm{mm}^{-1}$ and $3000\ \mathrm{mm}^{-1}$, respectively. The above two points allow the nonlinear function to have a small initial magnitude and to decrease when the error values increase so that the total gain of the sliding surface is increased as shown in Fig. 5.

5. Conclusions

This paper presents a sliding mode contouring controller with a nonlinear sliding surface for biaxial feed drive systems based on a coordinate transformation. The advantage of the proposed approach is that the sliding surface gain varies according to the contour error so that the system simultaneously achieves low overshoot and a small settling time, resulting in a smaller error. To verify the effectiveness of the proposed control approach, we conducted experiment with circular reference trajectories. The results indicated that the proposed controller can significantly reduce the control input variance while maintaining the contouring performance by adjusting tuning parameters of the nonlinear function. Verification of consumed energy reduction and application to three and five-axis machines are left for future work.

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