Risk and Risk Aversion Effects in Contests with Contingent Payments

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Abstract: Contests by their very nature involve risk, winning and losing are both possible, and the gain from winning can itself be uncertain. The participants in a contest use resources to increase their chance of winning. The main focus of this analysis is on the effects of risk aversion and risk in contests where only winners pay for resources used to compete. When payment is contingent on winning, the effect of risk aversion is in the opposite direction of what occurs when costs are paid by both winners and losers. A number of contests observed in the marketplace that exhibit this contingent payment property are discussed.

Key Words: risk aversion; contests; contingent payments; self-protection
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1. Introduction

The participants in a contest know they can win or lose, and they use resources to increase their chance of winning. For some contests the cost of these resources is paid by the contestant at the time the resources are used. For others, however, the contestant only bears the cost after the contest is resolved. This allows the payment to be contingent on the outcome of the contest. Much of the existing research concerning contests assumes that the costs are paid up front regardless of the outcome of the contest. This is an important case, and we modestly extend the known results for contests with upfront payment of costs. The majority of our analysis, however, is for contests with contingent payment of costs where only the winner of the contest pays. A good example of such a contest occurs in professional golf, where new golfers are often sponsored by an investor who agrees to pay all expenses associated with competing, and in return the investor acquires a share of the winnings when the sponsored golfer wins a tournament. From the contestant/golfer's perspective, payment for resources used to compete is only made after the contest is resolved and only if the contest is won. It is after winning that the golfer pays the sponsor.

Contests by their very nature involve risk, winning and losing are both possible, and the gain from winning can itself be uncertain. The main focus of this paper is on the effects of risk aversion and risk in contests with contingent payment of costs. These results are then contrasted with those determined for contests with upfront payment of costs.

The literature on contests began many years ago and is very extensive.[1] Many different models of contests exist. In these models the contest success function, which gives the probability of winning, can be assumed to take on a general form, or alternatively, in other instances this functional form is quite specific. For our purposes, a general formulation is best suited for the analysis of risk and risk aversion impacts, so no specific form for the probability function is assumed. Only modest restrictions on how the probability of winning is affected by the efforts of the contestants are imposed. Models of contests also assume different goals for contestants. Here contestants are assumed to maximize expected utility, and the utility functions are assumed to be increasing and concave. Again, for the analysis of risk aversion and risk effects, specifying a particular form for the utility function, or imposing a particular risk aversion property such as constant absolute risk aversion, is not desirable because varying the risk taking properties of the contestants is an important part of the analysis. Finally, many models of contests focus on the symmetric Nash equilibrium resulting from the competition among contestants, and that practice is followed here.

As others have noted, participants in contests, when viewed as individual decision makers, face a decision much like the decision to self-protect (Ehrlich and Becker 1972). The self-protection decision has been extensively analyzed, including analysis for situations where the payment for self-protection is contingent on the outcome (Liu et al. 2009). That work proves useful in the derivations presented here. Because contests involve more than one participant, however, the equilibrium condition used to conduct comparative static analysis is a Nash equilibrium condition in a non-cooperative game

[1]See, for example, Tullock (1980), Nitzan (1994), Baye et al. (1996) and Skaperdas (1996). Contest models have been used to analyze a wide range of strategic interactions in litigation, political campaigning or lobbying, R&D contests, government procurement and sports competition.

rather than a first-order condition for expected utility maximization. This implies that only some of the results from the analysis of the self-protection decision with a contingent payment carry forward to this analysis.

The contingent payments considered here assume that payment for the cost of participation in a contest is made only in the winning state by the winning contestant. Not paying when losing a contest reduces or eliminates many of the anomalies that arise in the basic self-protection decision model. The lowest possible outcome does not get smaller. Paying only when winning also significantly alters the effects of risk and risk aversion on the outcome of the contest.[2] As a general summary, relative to contests where costs are paid up front, the symmetric Nash equilibrium is unique, the effects of increased risk aversion are reversed, and the effects of increased risk are maintained when costs are paid only by the winner. The details of these claims are presented in the body of the paper.

The paper is organized as follows. First, in the next section, the contest model used throughout the paper is presented, and the known results when costs are paid up front, regardless of whether the contest is won or lost, are reviewed. The model used is very similar to one formulated by Konrad and Schlesinger (1997) and used recently by Treich (2010). In this model, Treich characterizes the symmetric Nash equilibria, and shows that risk-averse and prudent contestants choose to devote fewer resources to winning than do risk-neutral contestants. Treich also shows that risk-averse and prudent

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[2] Structural aspects of the contest – including the prize distribution, the number of contestants and the contest success function – are assumed to be exogenous in order to focus on the effects of risk and risk aversion. For more complicated contest models with endogenous contest structures of various degrees (usually with the assumption of risk-neutral contestants), see, for example, Gradstein and Konrad (1999), Moldovanu and Sela (2001) and Fu and Lu (2010).

contestants devote fewer resources to winning the contest when the gain from winning is random rather than certain. The analysis here adds to these findings by comparing the equilibria reached by two groups of risk-averse and prudent (i.e., downside risk averse) contestants, with one group being both Ross more risk averse and Ross more downside risk averse than the other. It is not surprising that the finding we obtain is in the same direction as that determined by Treich.

Section 3 contains the main contribution of the paper. In this section, the contest model is modified so that the payment for the resources used to improve the chance of winning is contingent upon winning. That is, the contestant chooses the quantity of resources to employ to enhance the chance of winning, but pays for these resources after the contest is resolved, and only pays when a winner.[3] The golf pro example mentioned earlier is one example of such a contest, but there are many others. The analysis in Section 3 is purposely kept very general, working with a general contingent cost function, and deferring the discussion of several concrete specifications of the contingent cost function and their examples to the next section. Compared with the results presented in Section 2 under upfront payment, the results obtained in this section under contingent payment reverse the effects of risk aversion, but uphold the effects of making the prize random.

Section 4 focuses on several specific contingent cost functions that are observed in the contest world. These cost functions specify exactly the algorithm used to determine the size of the payment made by the winner of the contest. These algorithms are discussed so that the general results can be interpreted, and also because each

[3] Since there are only two outcomes, winning and losing, the contingency here is to pay in one state, winning, and pay zero when losing.

algorithm represents a contingent payment contract that is observed in the marketplace. Included in the discussion are contingent payment contracts that require the winner to pay a fixed share of the prize, that require the winner to pay the costs incurred by all participants, and that require the winner to pay only her own expenses and losers to pay nothing.[4] Section 5 concludes and summarizes the results presented here.

2. The Contest Model with Upfront Payment of Costs

The contest model posed by Konrad and Schlesinger (1997) and modified and used by Treich (2010) employs a general contest success function and a general utility function. The number of contestants is arbitrary and denoted *n*. The contestants are identical in that they maximize expected utility for the same utility function, $u(\cdot)$, begin with same wealth, *w*, and earn the same nonrandom prize from winning, which has a monetary value $b > 0$.[5] The utility function $u(\cdot)$ is assumed to satisfy $u'(\cdot) > 0$ and $u''(\cdot) < 0$. The investment of resources used by contestant *i* to increase the probability of winning is denoted x_i. This is sometimes referred to as the effort put forth by the contestant.

The probability of individual *i* winning the contest depends on the resources he uses, as well as the resources expended by all other contestants. This probability is represented by an *n*-variable function $p_i \equiv p_i(x_1, \cdots, x_n)$. While there is a contest success function for each contestant, hence the subscript *i*, these functions are assumed to be the same for all contestants. These functions are also assumed to be twice differentiable and

[4] The third specification of the contingent cost function is discussed by Skaperdas and Gan (1995) and Yates (2011).
[5] The model can be generalized to have multiple prizes (and hence multiple winners) of the same size without altering the findings in the paper.

nonnegative. Of course, since they represent probabilities, these functions satisfy

$$\sum_{i=1}^{n} p_i = 1 .$$

To provide some structure to the contest success function we follow Konrad and Schlesinger (1997) and Treich (2010) and assume throughout the paper that for all *i* and all $j \neq i$:

A1. $\frac{\partial p_i}{\partial x_i}(x_1,\cdots,x_n) \geq 0$ and $\frac{\partial p_i}{\partial x_j}(x_1,\cdots,x_n) \leq 0$, for all x_i ($1 \leq i \leq n$).

A2. $\frac{\partial^2 p_i}{\partial x_i^2}(x,\cdots,x) < 0$, for all x.

A3. $\frac{\partial^2 p_i}{\partial x_j \partial x_i}(x,\cdots,x) \leq 0$, for all x.

A4. $p_i(x,\cdots,x) = 1/n$, for all x.

These assumptions on contest success functions have become standard. Assumption A1 requires that the probability of winning be nondecreasing in one's own investment, and nonincreasing in any other contestant's investment. Assumptions A2 and A3 reflect the notion of diminishing marginal returns to investment for all contestants. Finally, the last assumption, A4, indicates that if all players expend the same amount of resources when participating in the contest, then each is equally likely to win. This assumption is consistent with the other symmetry assumptions made in the contest model. The main two implications of these assumptions for the analysis of the symmetric Nash equilibrium are $p_x \equiv \frac{\partial p_i}{\partial x_i}(x,\cdots,x) \geq 0$ and $\frac{dp_x}{dx} < 0$.[6]

[6] These assumptions are natural extensions (with symmetry) of the similar assumptions in self-protection to the strategic, multiple-player environment. Note also that these assumptions are satisfied by the logistic contest success functions that have solid axiomatic foundations and are dominant in the literature on contests (e.g., Tullock 1980, Nitzan 1994 and Skaperdas 1996).

Konrad and Schlesinger and also Treich assume that the cost of resources used in the contest is paid by the contestant up front. That is, each player incurs a sunk cost determined before the contest is resolved, and this cost is paid whether they win or lose the contest. We shall refer to this as upfront payment, and use UP as superscripts in various expressions. Player *i*'s expected utility under upfront payment is:

(1) $$p_i u(w+b-x_i)+(1-p_i)u(w-x_i).$$

Konrad and Schlesinger (1997) and Treich (2010) limit the comparative static analysis to symmetric interior Nash equilibria, and we do also.[7] Imposing symmetry on the first order condition derived from (1), Treich identifies the following equation as one that characterizes the symmetric interior Nash equilibria under upfront payment of costs:[8]

(2) $$F^{UP}(x) \equiv p_x\left[u(w+b-x)-u(w-x)\right]-\left[\tfrac{1}{n}u'(w+b-x)+(1-\tfrac{1}{n})u'(w-x)\right]=0.$$

Treich discusses conditions ensuring a unique symmetric Nash equilibrium, and also considers the multiple equilibrium case in his analysis since multiple symmetric equilibria are possible with upfront payment of costs.

Treich provides two main findings concerning the effects of risk aversion and risk in this contest model. He first examines the effect of changing the risk preferences of the contestants from risk neutral to risk averse. As in self-protection decision models, the assumption of prudence, $u'''(\cdot)>0$, proves to be a useful one.

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[7] The existence of symmetric and asymmetric equilibria in contests with risk aversion are studied in Skaperdas and Gan (1995), Treich (2010) and Cornes and Hartley (2012) under various assumptions on the utility function and the contest success functions.

[8] As discussed in Treich (2010), the symmetric equilibria characterized by (2) may be multiple.

Proposition 1: (Treich) Risk aversion decreases the equilibrium investment of resources to win the contest under prudence.

This result indicates that the equilibrium value for x in the symmetric Nash equilibrium for this contest is smaller when the contestants are risk averse than when they are risk neutral.

Treich's second main result examines the effect of making the gain from winning a contest random rather than certain. The following proposition states that replacing the certain b with a random $\tilde{b}$ with $E\tilde{b} = b$ decreases the equilibrium effort for risk-averse and prudent contestants.

Proposition 2: (Treich) Suppose that the symmetric equilibrium characterized by (2) is unique. Then risk-averse and prudent contestants invest less when the prize is risky.

To summarize, assuming upfront payment of contest participation costs, Treich shows that risk-averse and prudent (or downside risk-averse) contestants invest less in the contest than those who are risk neutral, and also invest less when the prize is risky rather than certain. [9] A natural follow-up question is to ask whether the effects of being more risk averse and more downside risk averse can be determined. That is, can a comparison be made when the reference contestant is not risk neutral? As is often the case when determining the effects of an increase in, rather than the introduction of, risk aversion, additional assumptions on the risk preferences of the contestants are required to provide an answer. The following proposition uses the Ross definition of more risk averse and similar definition of more downside risk averse to provide an answer.

[9] Both prudence and downside risk aversion are characterized by $u''' > 0$.

Proposition 3: Suppose that $F^{UP}(x)$ defined in (2) is strictly decreasing in x.[10] Ross more risk-averse and Ross more downside risk-averse contestants invest (weakly) less resources.

To demonstrate this proposition, several definitions and lemmas are used.

Definition 1: (Ross) u(x) is strongly/Ross more risk averse than v(x) on [A, B] if there exists a constant k > 0 such that $\frac{u''(x)}{v''(x)} \geq k \geq \frac{u'(y)}{v'(y)}$ for all x and y in [A, B].

Definition 2: (Modica and Scarsini) Suppose that both u(x) and v(x) are prudent (i.e., downside risk averse). u(x) is strongly/Ross more downside risk averse than v(x) on [A, B] if there exists a constant k > 0 such that $\frac{u'''(x)}{v'''(x)} \geq k \geq \frac{u'(y)}{v'(y)}$ for all x and y in [A, B].

Lemma 1: u(x) is both Ross more risk averse and Ross more downside risk averse than v(x) on [A, B] if and only if there exists a constant k > 0 and $\phi(x)$ such that $u(x) \equiv kv(x) + \phi(x)$, where $\phi'(x) \leq 0$, $\phi''(x) \leq 0$ and $\phi'''(x) \geq 0$ on [A, B].

Lemma 2: (Eeckhoudt and Gollier) For $d \geq c$, define $f(c,d) = 0.5[u'(c) + u'(d)] - [u(d) - u(c)]/(d-c)$. Function f is positive (negative) if and only if u' is convex (concave).

Definition 1 is provided by Ross (1981), and Definition 2 is given by Modica and Scarsini (2005).[11] For these two definitions, it is assumed that u(x) and v(x) have

[10] Note that this is a sufficient condition for the unique symmetric equilibrium condition in Proposition 2.

positive first derivatives, negative second derivatives (in the case of Definition 1) and positive third derivatives (in the case of Definition 2) on [A, B]. Lemma 1 is related to several findings in the literature, but requires a brief proof and this is given in the appendix.[12] Lemma 2 is stated and demonstrated in Eeckhoudt and Gollier (2005). With these definitions and lemmas, Proposition 3 can be demonstrated.

<u>Proof of Proposition 3</u>: Suppose u is both Ross more risk averse and Ross more downside risk averse than v. Then, according to Lemma 1, there exists a constant k > 0 and $\phi(x)$ such that $u(x) \equiv kv(x) + \phi(x)$, where $\phi'(x) \le 0$, $\phi''(x) \le 0$ and $\phi'''(x) \ge 0$. Denote x_u and x_v the equilibrium contest investment for u and v, respectively. Evaluating $F^{UP}(x)$ given in (2) at x_v, we have

$$
\begin{aligned}
F^{UP}(x_v) &= p_x(x_v)\left[u(w+b-x_v)-u(w-x_v)\right]-\left[\tfrac{1}{n}u'(w+b-x_v)+(1-\tfrac{1}{n})u'(w-x_v)\right] \\
&= p_x(x_v)\left[\phi(w+b-x_v)-\phi(w-x_v)\right]-\left[\tfrac{1}{n}\phi'(w+b-x_v)+(1-\tfrac{1}{n})\phi'(w-x_v)\right] \\
&\le \tfrac{1}{b}\left[\phi(w+b-x_v)-\phi(w-x_v)\right]-\left[\tfrac{1}{n}\phi'(w+b-x_v)+(1-\tfrac{1}{n})\phi'(w-x_v)\right] \\
&\le \tfrac{1}{b}\left[\phi(w+b-x_v)-\phi(w-x_v)\right]-\left[\tfrac{1}{2}\phi'(w+b-x_v)+\tfrac{1}{2}\phi'(w-x_v)\right] \\
&\le 0.
\end{aligned} \tag{3}
$$

The reason for the first inequality in (3) is the following. Treich shows that x_v is smaller than the value for *x* that satisfies $p_x = 1/b$ which is the equilibrium condition for the risk-neutral case (Proposition 1). Since $dp_x/dx < 0$, this implies that $p_x(x_v) > 1/b$. Also note that $\phi(w+b-x_v)-\phi(w-x_v) \le 0$ since $\phi'(x) \le 0$ is assumed. The second inequality in (3)

[11] Ross more risk averse implies, but is not implied by, Arrow-Pratt more risk averse. Extensions of the Ross notion of comparative risk aversion to the general *n*th-degree are studied in Jindapon and Neilson (2007), Li (2009), Denuit and Eeckhoudt (2010) and Liu and Meyer (2013).

[12] It immediately follows from Lemma 1 that the linearly-restricted increase and the quadratically-restricted increase in risk aversion defined in Eeckhoudt et al. (2017) each imply both Ross more risk averse and Ross more downside risk averse.

results from $\phi''(x) \le 0$, and the third inequality uses both $\phi'''(x) \ge 0$ and Lemma 2. Because $F^{UP}(x)$ is strictly decreasing in *x*, $F^{UP}(x_u) = 0$ implies $x_u \le x_v$. Q.E.D.

In a two-player contest model, Skaperdas and Gan (1995) find that, under some restrictive assumptions on the contest success function and the utility function (including the assumption that the utility function satisfies constant absolute risk aversion), the more risk-averse player makes a smaller investment than the less risk-averse player in equilibrium. This result is consistent with the spirit of Proposition 3.

3. The Contest Model with Contingent Payment of Costs

The contest model of Konrad and Schlesinger (1997) and Treich (2010) which was discussed and extended in Section 2 assumes upfront payment of costs and these costs are paid by both contest winners and losers. In this section that assumption is changed in a significant way. The assumption made in this section, and in the remainder of the paper, is that only the winner pays. That is, the payment for resources used to compete is contingent on winning. The other assumptions in Treich's contest model are maintained. The term contingent cost function is used to describe the amount paid by the winning contestant. The contingent cost function is denoted $c_i \equiv c_i(x_1, \cdots, x_n)$ and specifies the amount that contestant *i* is required to pay if he wins the contest. Player *i*'s payment is zero if he does not win. This type of payment of costs is referred to as contingent payment, and CP is used where convenient.

Much like for the contest success functions, these cost functions c_i are assumed to be the same for each contestant (i.e., symmetric), twice differentiable and positive over a bounded domain for x_i ($1 \le i \le n$). In addition, it is assumed throughout the paper that for all *i* and all $j \ne i$:

C1. $\frac{\partial c_i}{\partial x_i}(x_1, \cdots, x_n) > 0$ and $\frac{\partial c_i}{\partial x_j}(x_1, \cdots, x_n) \ge 0$, for all x_i ($1 \le i \le n$).

C2. $\frac{\partial^2 c_i}{\partial x_i^2}(x, \cdots, x) \ge 0$, for all x.

C3. $\frac{\partial^2 c_i}{\partial x_j \partial x_i}(x, \cdots, x) \ge 0$, for all x.

Assumption C1 indicates that the payment made by the winner is increasing in his own expenditure of resources, and nondecreasing in the investments made by others. Assumptions C2 and C3 represent the notion of nondecreasing marginal cost. These are very modest assumptions.

Similar to what is done by Treich for the contest success function, one can use this contingent cost function of *n* variables to define the following one-variable functions $\bar{c}(x) \equiv c_i(x, \cdots, x)$ and $c_x(x) \equiv \frac{\partial c_i}{\partial x_i}(x, \cdots, x)$.

The contingent cost function $c_i(x_1, \cdots, x_n)$, with the modest assumptions imposed here, is chosen because it is general enough to contain several important special cases, yet restricted enough to allow several important comparative static propositions to be demonstrated. The special cases specify more exactly the amount to be paid when the contest is won. For example, the cost function can be $c_i(x_1, \cdots, x_n) = \alpha(x_i)b$, where $0 < \alpha(x_i) < 1$, $\alpha'(x_i) > 0$ and $\alpha''(x_i) \ge 0$,[13] which indicates that the winning contestant pays a share of the prize to a third party who incurs the cost of x_i, and the share can

[13] Because x_i belongs to a bounded domain, $0 < \alpha(x_i) < 1$ can be consistent with $\alpha'(x_i) > 0$ and $\alpha''(x_i) \ge 0$.

depend on the resources expended. Another special case occurs when the winner pays everyone's cost of participating; that is, $c_i(x_1,\cdots,x_n)=\sum_{i=1}^{n} x_i$. Yet another special case occurs when the winner only pays his or her own cost and losers pay nothing, $c_i(x_1,\cdots,x_n)=x_i$. These specific cases and their examples are discussed in detail in Section 4.

For the general contingent cost function, Player *i*'s expected utility is:

$$p_i u(w+b-c_i)+(1-p_i)u(w)\,. \tag{4}$$

Imposing symmetry on the first order condition derived from (4), the following equation characterizes the symmetric interior Nash equilibria under contingent payment of costs:

$$F^{CP}(x)\equiv p_x\left[u(w+b-\overline{c}(x))-u(w)\right]-\tfrac{1}{n}u'(w+b-\overline{c}(x))c_x=0\,. \tag{5}$$

A sufficient condition for a unique symmetric interior Nash equilibrium is that $F^{CP}(x)$ is strictly decreasing in *x*, or equivalently,

$$\frac{dp_x}{dx}\left[u(w+b-c(\overline{x}))-u(w)\right]-u'(w+b-\overline{c}(x))\left[p_x\overline{c}'(x)+\frac{1}{n}\frac{dc_x}{dx}\right]+\frac{1}{n}u''(w+b-\overline{c}(x))\overline{c}'(x)c_x<0\,. \tag{6}$$

In this expression, each of the three terms is negative under the stated assumptions on the utility, contest success and contingent cost functions. Thus, very different from the upfront payment case, issues of multiplicity of equilibria do not arise, and carrying out comparative static analysis is less cumbersome because there are not multiple equilibria issues to consider.

As indicated in Section 2, Treich (2010) establishes that under upfront payment of costs, risk aversion combined with prudence is sufficient to show that the equilibrium amount expended to win the contest is less than what would occur under risk neutrality.[14] Proposition 3, also in Section 2, extends this finding to comparing two groups of contestants with one group being Ross more risk averse and Ross more downside risk averse than the other. The question addressed next asks whether similar effects of risk aversion and increased risk aversion occur when the payment of participation costs is contingent rather than up front. The answer is composed of two parts, the first comparing risk aversion with risk neutrality, and then examining the case where one group is Arrow-Pratt more risk averse than another.

To compare risk aversion with risk neutrality, the unique symmetric equilibrium investment level for any concave utility function $u(\cdot)$ given by (5) is compared with the unique symmetric equilibrium investment level under risk neutrality which is given below as (7) and is obtained from (5) by replacing $u(\cdot)$ with a linear function.

(7) $$f^{CP}(x) \equiv p_x[b - \bar{c}(x)] - \tfrac{1}{n} c_x = 0 .$$

The proposition below states that compared with risk neutrality, risk aversion increases equilibrium expenditure of resources when payment is contingent upon winning.

[14] Jindapon and Whaley (2015) complements Treich (2010) by showing that under upfront payment, risk loving and imprudent contestants increase contest investment above the risk-neutral outcome. Both Treich (2010) and Jindapon and Whaley (2015) are consistent with the analyses emphasizing the role played by prudence (i.e., downside risk aversion) in self-protection under upfront payment (Chiu 2005, Eeckhoudt and Gollier 2005, Menegatti 2009, Dionne and Li 2011, Ebert 2015, Denuit et al. 2016 and Peter 2017). In particular, Denuit et al. (2016) explain that the composite change in the final wealth distribution caused by an increase in self-protection effort includes a component of downside risk increase in the sense of Menezes et al. (1980) that is disliked by downside risk averse decision makers.

Proposition 4: The equilibrium contest investment is higher under risk aversion than under risk neutrality.

Proof: Note that $f^{CP}(x)$ is strictly decreasing in *x*. Therefore, to prove that the solution to $F^{CP}(x)=0$ is larger than the solution to $f^{CP}(x)=0$, it is sufficient to show that $f^{CP}(x)$ is negative at the solution to $F^{CP}(x)=0$.

Indeed, at the solution to $F^{CP}(x)=0$ (equation 5),

$$p_x = \frac{\frac{1}{n}u'(w+b-\bar{c}(x))c_x}{u(w+b-\bar{c}(x))-u(w)}.$$

So

$$\begin{aligned} f^{CP}(x) &= \tfrac{1}{n}c_x \frac{u'(w+b-\bar{c}(x))(b-\bar{c}(x))-[u(w+b-\bar{c}(x))-u(w)]}{u(w+b-\bar{c}(x))-u(w)} \\ &= \tfrac{1}{n}c_x \frac{[u'(w+b-\bar{c}(x))-u'(\theta)](b-\bar{c}(x))}{u(w+b-\bar{c}(x))-u(w)} \\ &< 0, \end{aligned}$$

where $\theta \in (w, w+b-\bar{c}(x))$ is a value that exists according to the intermediate value theorem. Note that the inequality above requires that $u(\cdot)$ be concave. Q.E.D.

The finding in Proposition 4 is the direct opposite of what occurs when payment of costs is upfront. It is also the case that when the payment for resources is contingent on winning, prudence need not be assumed; that is, both prudent and imprudent risk-averse contestants use more resources to win a contest that risk-neutral contestants.

When comparing one risk aversion level with another, Konrad and Schlesinger (1997), assuming upfront payment of costs, find that contest effort can increase or decrease with an increase in risk aversion. This result is consistent with and similar to the findings in the self-protection decision model with upfront payment.[15] In addition, Proposition 3 in Section 2 indicates that to make an unambiguous prediction concerning the effects of an increase in risk aversion under upfront payment, additional assumptions such as Ross more risk averse and Ross more downside risk averse are needed. On the other hand, Liu et al. (2009) show that an increase in risk aversion according to the less demanding Arrow-Pratt notion always induces a larger amount of effort to self-protect under contingent payment. Then a question arises: which utility function, $u(\cdot)$ or $v(\cdot)$, where $u(\cdot)$ is Arrow-Pratt more risk averse than $v(\cdot)$, would give rise to a larger equilibrium contest investment, when contest investments are paid contingent on winning?

To resolve this question, the symmetric equilibrium contest investment for $u(\cdot)$, denoted x_u, which is the solution to $F^{CP}(x) = 0$ and given as (5), must be compared with the symmetric equilibrium contest investment for $v(\cdot)$, denoted x_v, which is the solution to (5') below that is obtained by replacing $u(\cdot)$ with $v(\cdot)$.

(5') $$p_x\left[v(w+b-\bar{c}(x))-v(w)\right]-\tfrac{1}{n}v'(w+b-\bar{c}(x))c_x=0.$$

The proposition below states that for the general contingent cost function assumed here, a higher degree of risk aversion in the sense of Arrow-Pratt implies a larger equilibrium contest investment.

<u>Proposition 5</u>: $x_u \ge x_v$ if $u(\cdot)$ is Arrow-Pratt more risk averse than $v(\cdot)$.

[15] For the lack of clear-cut general results about the relationship between (the degree of) risk aversion and self-protection investment under upfront payment, see Dionne and Eeckhoudt (1985) and Briys and Schlesinger (1990).

Proof: First, $F^{CP}(x)$ is strictly decreasing in *x*. Therefore, to prove that x_u (the solution to $F^{CP}(x)=0$) is weakly larger than x_v (the solution to equation (5')), it is sufficient to show that $F^{CP}(x_v)\geq 0$.

Because $u(\cdot)$ is Arrow-Pratt more risk averse than $v(\cdot)$, there exists a strictly increasing and weakly concave function $\varphi(v)$ such that $u(\cdot)\equiv\varphi(v(\cdot))$ (Pratt 1964). Therefore,

$$\begin{aligned}
&F^{CP}(x_v)=p_x(x_v)\left[u(w+b-\bar{c}(x_v))-u(w)\right]-\tfrac{1}{n}u'(w+b-\bar{c}(x_v))c_x(x_v)\\
&=p_x(x_v)\left[\varphi(v(w+b-\bar{c}(x_v)))-\varphi(v(w))\right]-\tfrac{1}{n}\varphi'(v(w+b-\bar{c}(x_v)))v'(w+b-\bar{c}(x_v))c_x(x_v)\\
&=p_x(x_v)\varphi'(\theta)\left[v(w+b-\bar{c}(x_v))-v(w)\right]-\tfrac{1}{n}\varphi'(v(w+b-\bar{c}(x_v)))v'(w+b-\bar{c}(x_v))c_x(x_v)\\
&\geq\varphi'(v(w+b-\bar{c}(x_v)))\left\{p_x(x_v)\left[v(w+b-\bar{c}(x_v))-v(w)\right]-\tfrac{1}{n}v'(w+b-\bar{c}(x_v))c_x(x_v)\right\}\\
&=0,
\end{aligned}$$

where $\theta\in(v(w),v(w+b-\bar{c}(x_v)))$ is a value that exists according to the intermediate value theorem. Note that the inequality above is due to $\varphi(v)$ being weakly concave. This concludes the proof. Q.E.D.

The finding in Proposition 5 is quite different from what occurs when upfront payment is assumed. This proposition indicates that increased risk aversion leads to increased expenditure of resources to win the contest, the opposite of the finding in Proposition 3, and also indicates that there is no indeterminacy like that pointed out by Konrad and Schlesinger (1997). Changing to contingent payment of costs significantly alters the effect of risk aversion on the outcome of a contest. Skaperdas and Gan (1995) also find that, for a two-player setting and the special type of contingent payment where

$c_i(x_1,\cdots,x_n)=x_i$, increases in risk aversion increase, rather than decrease, the equilibrium contest investment. Proposition 5 generalizes this result.

The final proposition that is demonstrated using the general contingent cost function yields a finding similar to that of Treich (2010). Recall that with upfront payments of costs, Treich shows that replacing a nonrandom prize *b* with a random prize $\tilde{b}$ whose expected value is *b* causes a risk-averse and prudent contestant to expend fewer resources when competing in a contest. Proposition 6 below states that a similar result occurs when contingent payment of costs is assumed instead.[16]

With contingent payment of costs, the equilibrium contest investment under a nonrandom prize *b* is determined by (5), which takes the following more general form when *b* is replaced with a random $\tilde{b}$ with $E\tilde{b}=b$:

(8) $$p_x\left[Eu(w+\tilde{b}-\bar{c}(x))-u(w)\right]-\tfrac{1}{n}Eu'(w+\tilde{b}-\bar{c}(x))c_x=0\,.$$

Proposition 6: (i) The equilibrium contest investment increases as *b* increases; (ii) the equilibrium contest investment decreases as *b* is replaced with a random $\tilde{b}$ with $E\tilde{b}=b$ if contestants are also prudent (in addition to risk averse).

Proof: (i) In equation (5) that characterizes the equilibrium investment with contingent payment of costs, the partial derivative of $F^{CP}(x)$ with respect to *b* is always positive. That is, for any *x*, $F^{CP}(x)$ is larger when *b* is larger. This, together with $F^{CP}(x)$ being strictly decreasing in *x*, implies that the equilibrium contest investment increases as *b* increases.

[16] It is straightforward to extend this result for a first-degree stochastically dominant change in $\tilde{b}$ or a mean-preserving spread over $\tilde{b}$ in the sense of Rothschild and Stiglitz (1970).

(ii) In equation (5), the second-order partial derivative of $F^{CP}(x)$ with respect to *b* is always negative under the condition of prudence. That is, for any *x*, the left side of (8) is smaller than $F^{CP}(x)$. This, together with $F^{CP}(x)$ being strictly decreasing in *x*, implies that the equilibrium contest investment decreases as *b* is replaced with a random $\tilde{b}$ with $E\tilde{b} = b$ if contestants are prudent. Q.E.D.

Therefore, changing to contingent payment of costs does not alter the effect of increased risk on the outcome of a contest.

4. Specific Algorithms for the Contingent Payment of Costs

The analysis in Section 3 uses a general contingent cost function and a number of unambiguous comparative static propositions are presented that provide information concerning the effects of risk aversion and risk in contests. This section indicates that the general contingent cost function includes several payment plans observed in many real-world contests. Three specific algorithms that determine the exact amount paid by the winner of the contest are discussed. In each, the losers pay zero.

Three algorithms are discussed in this section. 1) $c_i(x_1, \cdots, x_n) = \alpha(x_i)b$, where $0 < \alpha(x_i) < 1$, $\alpha'(x_i) > 0$ and $\alpha''(x_i) \geq 0$. For this algorithm, the winner pays a share of the prize, denoted $\alpha(x_i)$, and the share depends on the resources expended x_i. 2) $c_i(x_1, \cdots, x_n) = \sum_{i=1}^{n} x_i$. Here the winner pays for all the resources expended by all the contestants. 3) $c_i(x_1, \cdots, x_n) = x_i$. In this case, the winner pays only his own costs. This was discussed in Skaperdas and Gan (1995) and Yates (2011).

It is obvious that all three contingent cost functions specified above satisfy assumptions C1 – C3. Therefore, for each of these contingent cost functions, $F^{CP}(x)$ is strictly decreasing in *x*, the symmetric interior Nash equilibrium is unique, and the propositions demonstrated in Section 3 apply. Now that it has been verified that results from Section 3 apply to each of these three specific algorithms for determining the payment made by the winner, each is discussed in more detail.

The professional golf example of the first algorithm, where $c_i(x_1,\cdots,x_n)=\alpha(x_i)b$, has already been mentioned. In this example, the contestant contracts with an investor. The investor pays for all resources used to compete, and in return receives a share of the prize. The fraction received by the investor can depend on the resources expended. With such a contingent payment arrangement, a risk-neutral or risk-averse investor would require that the expected amount received from a winner to be equal to or greater than the cost of the resources expended by the contestant. That is, $p_i\alpha(x_i)b \geq x_i$ would be required, and a strict inequality would hold when the investor is risk averse. The investor is accepting risk and would require a positive expected value in order to do so.

A risk-averse contestant would choose to participate in such a contract in part because the risk to him is reduced. To see this, observe that with upfront payment the contestant earns ($w + b – x_i$) and ($w – x_i$) with probabilities p_i and ($1 – p_i$), respectively. With this particular contingent payment algorithm, the contestant earns ($w + (1 - \alpha)\cdot b$) or w, again with probabilities p_i and ($1 – p_i$). The wealth distribution under this contingent payment plan has a lower mean when $p_i\alpha(x_i)b \geq x_i$, but is less risky than the wealth distribution that occurs under the upfront payment. Thus, a risk-averse contestant may choose such a contingent payment plan.

This description can also intuitively explain the finding that increased risk aversion leads to an increase in x. With increased risk aversion, the contestant is even more willing to accept a reduction in the mean value in order to also reduce risk (Liu and Meyer 2017), and the way to accomplish this is to expend resources to win the contest. Likely, one of the more important reasons for observing such contest payment structures in the marketplace is to transfer risk from the contestant to the investor.

In the symmetric Nash equilibrium with this contingent cost function, it must be that the prize exceeds the total amount of resources spent by the contestants. That is, there must be a risk-averse investor who sponsors each contestant in equilibrium, and as a result, the prize must be large enough so that $\alpha \cdot b/n$, the expected payment to the investor sponsoring any one contestant, is larger than x. This in turn implies that b must be larger than $n \cdot x$ which in words indicates that the prize must exceed the resources expended to win the prize. It is also the case that if one investor chose to sponsor all contestants, that investor would bear no risk since he is guaranteed to have sponsored the winner. Even with only this one investor who sponsors all, the risk borne by the contestants is still reduced. This investor/sponsor could be viewed a facilitator who allows all contestants to reduce their risk without requiring that the investor bear any risk. This is analogous to perfect mutual insurance.

A second algorithm used to collect payment from the winner of a contest is to ask the winner to pay all contest expenses, that is, $c_i(x_1, \cdots, x_n) = \sum_{i=1}^{n} x_i$. While this is a contest rule that can be directly implemented, it sometimes occurs as a result of a procedure that is less direct. For instance a venture capital firm or a government agency can have a fixed budget that is partially used to encourage contestants to expend resources to develop a product, and to also then pay for the production of the product by the contest winner. In such situations, the firm or agency offers seed money to contestants who expend resources to design and develop a prototype of the desired product. After having examined the prototypes, the firm or agency declares a winner and awards a production contract to the winner of the contest. The winner accepts the residual of the fixed budget to pay for production costs. Although not directly the case, the outcome of such a contingent cost procedure is that the winner in fact pays the expenses of all participants.

With this method of paying for resources used when competing in a contest, the general result from Section 3 indicates that more risk-averse contestants use more resources to compete. There does not appear to be a risk sharing or risk transfer argument that provides the intuition for this result. Instead, using more resources when competing increases the probability of winning, but also decreases the size of net prize. It must be that the expected net prize is reduced because if not, all risk-averse persons would increase resource use. Thus, any contestant who is choosing the use of resources optimally is exhibiting a risk-return tradeoff and increased risk aversion induces even larger use of resources.

The final algorithm for paying for costs of competing is quite different from the other two because, the cost of resources used when competing is not a sunk cost, but a promise to expend resources if and when the contest is won. When this is the case, contestants propose spending resources (x_1, x_2, …,x_n), and based on this proposal, the winner is selected and the winner pays what she has proposed. This can be represented by the contingent cost function $c_i(x_1,\cdots,x_n)=x_i$. This particularly cost function was discussed recently by Yates (2011). An example of this is when several firms bid for a local government's construction project, and each firm specifies the amenities it will provide as a winning contractor.

For this contingent cost function, Yates focuses on things other than the effects of risk or risk aversion. Thus, this special case of our general contingent cost function is included mainly to indicate that our findings have another application. It is interesting to note however, that increased risk aversion leads to more expenditure of resources to compete rather than less even when those resources are not a sunk cost, but only a promise to pay if the contest is won. Again spending additional resources alter the risk return tradeoff and more risk-averse contestants are willing to give up more in terms of mean value in order to reduce risk by increasing the probability of winning.

5. Conclusion

In the existing research on contests, it is typically assumed that contest investments are paid up front by each contestant, regardless of whether the contestant wins the contest. In this context, it has been shown that risk aversion combined with prudence (i.e., downside risk aversion) implies a smaller equilibrium contest investment than risk neutrality, although an increase in risk aversion, in the sense of Arrow-Pratt, does not necessarily imply a decrease in the equilibrium contest investment. The present paper adds to the findings in contests with upfront payment of costs by showing that Ross more risk aversion combined with Ross more downside risk aversion implies a smaller equilibrium contest investment.

As it is argued in the paper, however, many real-world contests have an arrangement of contingent payment where only the winner of the contest pays. The main focus of the paper is on the effects of risk aversion and risk in contests with contingent payment of costs. The most striking findings resulting from this change in how costs are allocated are that risk aversion implies a larger equilibrium contest investment than risk neutrality, and that more risk aversion in the sense of Arrow-Pratt implies a larger equilibrium contest investment, reversing the corresponding findings obtained under upfront payment of costs. In contrast, introducing additional risk by making the prize random has the same effect under contingent payment as under upfront payment, which is to decrease the equilibrium contest investment when the contestants are both risk averse and prudent.

Given that alternative payment methods can make a big difference in contests, a natural question arises as to which payment method of the contest costs – out of at least the four discussed in this paper (upfront payment and the three specific contingent payment methods) – leads to a larger/smaller equilibrium contest investment, or is more efficient under various goals of contest design. Another interesting question to ask is what would happen if the payment method is changed to loser-pay, an arrangement that is relevant in litigation and international military conflicts. These questions are left to future research.

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Appendix

Proof of Lemma 1:

"only if" – Suppose that u(x) is both Ross more risk averse and Ross more downside risk averse than v(x) on [A, B]. Then, according to Definitions 1 and 2, there exists a constant $k_1 > 0$ such that $\frac{u''(x)}{v''(x)} \geq k_1 \geq \frac{u'(y)}{v'(y)}$ for all x and y in [A, B], and there is also another constant $k_2 > 0$ such that $\frac{u'''(x)}{v'''(x)} \geq k_2 \geq \frac{u'(y)}{v'(y)}$ for all x and y in [A, B]. Let k = min (k_1, k_2) and define $\phi(x) \equiv u(x) - kv(x)$. It is straightforward to check that $\phi'(x) = u'(x) - kv'(x) \leq 0$, $\phi''(x) = u''(x) - kv''(x) \leq 0$, and $\phi'''(x) = u'''(x) - kv'''(x) \geq 0$ on [A, B].

"if" – Suppose that there exists a constant k > 0 and $\phi(x)$ such that $u(x) \equiv kv(x) + \phi(x)$, where $\phi'(x) \leq 0$, $\phi''(x) \leq 0$ and $\phi'''(x) \geq 0$ on [A, B]. Then we have $u'(x) = kv'(x) + \phi'(x) \leq kv'(x)$, $u''(x) = kv''(x) + \phi''(x) \leq kv''(x)$, and $u'''(x) = kv'''(x) + \phi'''(x) \geq kv'''(x)$. Therefore, $\frac{u''(x)}{v''(x)} \geq k \geq \frac{u'(y)}{v'(y)}$ for all x and y in [A, B], and $\frac{u'''(x)}{v'''(x)} \geq k \geq \frac{u'(y)}{v'(y)}$ for all x and y in [A, B]. Q.E.D.