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Calculation of the nuclear properties of Erbium $^{166}_{68}Er_{98}$ nucleus by using IBM-1 and VMI model

Iman T. Al-Alawy Ali K. Aobaid Waleed B. Salih

University of Anbar. College of Education for pure science.

A R T I C L E I N F O

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A B S T R A C T

The aim of the present work is to study some nuclear features of Erbium ($^{166}_{68}Er_{98}$) such as energy levels E(J), energy transition Eγ, band crossing, and back bending phenomena in the mass region(A=166,and total number of bosons N=15)of the dynamical symmetry SU(3)-O(6) using interacting boson model version-1(IBM-1) and a variable moment of inertia model(VMI)

In this study, we determined the most appropriate Hamiltonian that is needed for the present calculations of deformed understudy nucleus; these calculations have been estimated by best-fitting to the measured energies level.

The IBM-1 results have been compared with the previous experimental and theoretical (VMI-model) data and it was observed that they are agreed in the most of the states. The predictions from the VMI model are more accurate than those of the IBM-1.

Introduction

The interacting boson model-1(IBM-1) is an important subjects that is used to study some nuclear properties of all even-mass or odd-mass nuclei. This model has been proposed by Mariscotti . et al .(1969) [1]and Mariscotti (1970)[2]in order to study the energies of ground state rotational band of spherical nucleus.

This model is based on the well-known shell model and on geometrical collective models of the atomic nucleus [3, 4].

The interacting boson model-1 is suitable for describing the collective structure of nuclei with even number of protons and even number of neutrons which have positive parity (π+), and it builds on the interaction valence boson particles outside a nuclear closed shell or boson holes inside a closed shell.

* Corresponding author at: University of Anbar. College of Education for pure science, Iraq.

The total number of bosons (N) depends on the number of active nucleon (or hole) pairs outside a closed shell and it can be calculated by adding the number of neutrons pairs and protons pairs of (s and d) bosons which can be written as [4]:

N=ns+nd

Where: ns= number of s-bosons

nd= number of d-bosons.

The Interacting Boson Model-1 is very successful in studying the properties of many nuclei especially, When the total number of bosons N>>0, but it fails whenever N reaches zero, it completely fails in studying closed shells at 28, 50, 82, and 126 where N=0 because, there is no interacting between proton and neutron bosons (i.e there is no degree of freedom). Therefore, search began to find a new model to study these nuclei and other nuclei. One of these models is a Variable Moment of Inertia (VMI) Model", which proposes that moment of inertia is a variant.

Goldhaber and Scharff1978[5]studied the electric and dynamic quadrupole of even-even

Er(A=158-170)for the ground-state bands using only the energies of 2+ and 4+ states

After that Lin.,and Chern.(1979) [6]proposed another model called(A Microscopic-Macroscopic Model). This model is suggested for the study of high-spin of rotational SU (3) nuclear states. The model also combines the VMI model and the Microscopic treatment with exact angular momentum and particle number.

Raj et al(1988)[7]studied the energy levels for the ground –state bands of 158Er using the variable moment of inertia(VMI)model expressed in terms of the various orders of nuclear softnees(NS)

Otsuka and Honma.(1991)[8] proposed an algebraic super deformed interacting boson model .They found the inert core is smaller as compared with the normal interacting boson model and the number of valence bosons is increased by a factor of 2to3.

Myers and Swiatecki(1998)[9]studied the nuclear rotation of 160Er and some other nuclei using a statistical model of nuclear properties that combines the Thomas-Fermi assumption of two fermions per h3 of phase with an effective interaction between the nucleus.

Chiang. et al(1999)[10]calculated the s d super deformed energy bands of even-even Pb and Hg isotopes in the A=190 region using super deformed interacting boson model(IBM-1)and dynamic moment of inertia .They found that the calculated energy levels in general agree with the experimental data very well, and they also found that a slight renormalization of angular momentum-angular momentum interaction is needed to imitate the variation of the moment of inertia in a given energy band.

Garawi[2001[11]studied the nuclear structure of 160Dy using Variable Moment of Inertia Model and Interaction Boson Model of ground state bands.

Alenicheva .et al(2004)[12]studied the rotational bands of odd-mass deformed nuclei with mass number (43≤A ≤253)using generalization of the variable moment of inertia model for strong deformed nuclei. They found a good description of rotational energies.

Mohamed(2005)[13]studied the effect of back bending of the ground state rotational band of 162Er and 166Er deformed nuclei using variable moment of inertia with softness model(VMIS). The model shows good results in the rare earth and actinide regions.

Nurettin. and Ismail .(2007)[14] determined the most appropriate Hamiltonian that is needed for calculations of energy levels and electric transition probability B(E2) values of even-even Cerium(Ce128-138) nuclei which have a mass around A≈130 using IBM-1 and comparing them with some previous experimental and theoretical results.

The main purpose of the present work is to study the energy levels, energy bands, dynamical symmetry behavior, energy transition, and the electric transition probability, by using Interacting Boson Model-1 and Variable Moment of Inertia Model.

II-Theoretical Formalism:

II-1:The Solvable IBM-1 Hamiltonian:

The Hamiltonian form ,which connects the energy states as a function of angular momentum E(J) is written in the language of second quantization formalism and, as such, can involve allowed combinations of one-body and two-body interactions operators s(s†) and d(d†).

Where: s(s†)) is the annihilation (creation) operator for s-bosons d(d†) is the annihilation (creation)operator for d-bosons.

Arima and Iachello(1981)[15] proposed a new Hamiltonian form for IBM-1, and can be written as follows:

$H=\varepsilon_s n_s+\varepsilon_d n_d +½ \sum (2L+1)^{½} C_L\{ [d^\dagger\times d^\dagger]^{(L)} \times [\tilde{d} \times \tilde{d}]^{(L)}\}^{(0)}+(\acute{\upsilon}_{2/\sqrt{2}}\{[d^\dagger\times d^\dagger]^{(2)} \times[\tilde{d} \times \tilde{s}]^{(2)} +[d^\dagger\times s^\dagger]^{(2)} \times[\tilde{d} \times \tilde{d}]^{(2)}\}^{(0)} +½ \acute{\upsilon}_0\{[d^\dagger\times d^\dagger]^{(0)} \times[\tilde{s} \times \tilde{s}]^{(0)} +[s^\dagger\times s^\dagger]^{(0)} \times [\tilde{d} \times \tilde{d}]^{(0)}\}^{(0)} +u_0\{[d^\dagger\times s^\dagger]^{(2)}\times[\tilde{d} \times \tilde{s}]^{(2)}\}^{(0)}+½ u_0\{[s^\dagger\times s^\dagger]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)}\}^{(0)}$ ....(1)

Where: n_s and n_d as defined earlier, ε_s and ε_d are single boson energies for s and d-boson respectively. The C_L, $\acute{\upsilon}_2$, $\acute{\upsilon}_o$,u_2 ,u_o are corresponding interaction parameters.

The Hamiltonian in a bove equation has been reduced by Arima and Iachello to the form [15]:

$$H= \varepsilon + a_0(\hat{P}.\hat{P})+a_1(\hat{L}.\hat{L})+a_2(\hat{Q}.\hat{Q})+a_3(\hat{T}_3.\hat{T}_3)+a_4(\hat{T}_4.\hat{T}_4) ..(2)$$

Where:

$\varepsilon= n_d (\varepsilon_d - \varepsilon_s)-[6a_1+(11a_2/4) +(7a_3/5)+(9a_4/5)] \varepsilon_d+(N-5a_2) \varepsilon_s$ ....(3)

The form of Hamiltonian in equation(2) has been written in Fortran 77 lanquage using software Compaq Visual Fortran Version 6.6(CVF6.6) for compiling) linking and executing in order to calculate the energy levels E(J) and their transition energy E_γ.

II.2:Variable Moment of Inertia (VMI)Model

Mariscotti et al(1969)[1]showed that the rotational energy levels of the ground state bands in even-even deformed nuclei could be interpreted on the basis of the semiclassical model. In this model, the energy contains (in addition to the usual rotational term) a potential energy term which depends on the difference of the moment of inertia ℓ(J)(for the state of angular momentum J)from that of the ground state ℓ_0 . This model is called the "*Variable Moment of Inertia(VMI) Model*". In this model they assumed that there exists a variational expression for the energy levels E(J) in the form:

$$E(J)= \hbar^2/ 2\ell(J)[J(J+1)]+\frac{C}{2}(\ell(J)-\ell_0)^2 \text{ .....(4)}$$

Where:

ℓ(J) :is a moment of inertia as a function of angular momentum J

ℓ_0 : is a parameter defined as the ground state moment of inertia for J=0

C: is the restoring force constant

Equation (4) is valid for the ground state only. This equation have been developed by Mariscotti et al [1,2] to involve all bands, and it can be written as:

$$E(J)= E_k +\hbar^2/ 2\ell(J)[J(J+1)] + \frac{C}{2}(\ell(J)-\ell_0)^2 \text{ ....(5)}$$

Where:

E_k: energy of the band head.

In the present work, we assumed that E_k takes the following values:

E_k=0 for ground state

E_k=value of head band for β-band and a variable fitted parameter for γ-band

Deleplanque et al(2002)[16] determined an average moment of inertia along the yrast line at high spins. They characterize the yrast line by one moment of inertia $\ell_{exp.}$ Which is deduced from the slope of an energy level with J(J+1)plot, $\ell_{exp.}$is given by:

$E(J)=E_0 +J(J+1) \hbar^2/ 2\ell_{exp}$ ....(6)

The only dynamic parameter of the model is the effective moment of inertia ℓ(J) which can be determined for each level from the equilibrium condition[16]:

$$\frac{\partial E(J)}{\partial \ell(J)} = 0 \text{........(7)}$$

In the present work, we used equation (5) to calculate the moment of inertia as a function of angular momentum for all ${}^{166}_{68}Er_{98}$ state bands.

II.2:Backbending Phenomena In Even - Even ${}^{166}_{68}Er_{98}$ Nucleus:

In the present work an attempt is made to study the behavior of the back bending phenomena in the all state bands for ${}^{166}_{68}Er_{98}$ even-even nucleus using the VMI model. The interacting boson model predictions are excluded because of their complexity (seven fitted parameters in some cases). Furthermore, the well known energy expressions of the dynamical symmetry SU(5), SU(3) and O(6) are far from being suitable to describe the back bending phenomena in $(2\ell/\hbar^2)$ - $(\hbar\omega)^2$ plots. From the excitation energies E(J) of all state bands we deduce the nuclear moment of inertia$(2\ell/\hbar^2)$ and the squared rotational frequency $(\hbar\omega)^2$ by using the relations[6]:

$$\frac{2\ell}{\hbar^2} = \frac{4J-2}{E(J+2)-E(J)} \quad ....(8)$$

and

$$(\hbar\omega)^2 = (J^2 - J + 1)\left[\frac{E(J+2)-E(J)}{2J-1}\right]^2 \quad ....(9)$$ Resp

ectively

Equation(5) is used to study the energy band crossing phenomena from the plot between energy level E(J) and angular momentum (J) ,while the back bending phenomena from the plot between moment of inertia $(2\ell/\hbar^2)$ equation(8), and squared rotational energy$(\hbar\omega)^2$ equation(9).

III- Results and Discussions:

III-1: *Energy Levels Calculations*

To carry out a calculation of ${}^{166}_{68}Er_{98}$ using interacting boson model-1(IBM-1) which does not distinguish between the neutron- and proton- boson, we must evaluate the total number of bosons (N) and the dynamical symmetry. The total number of boson N=15(7-are protons-bosons and 8-neutrons-bosons). The theoretical and experimental calculations indicated that the ${}^{166}_{68}Er_{98}$ nucleus is belonging to the transitional regions SU(3)-O(6)and having (g, γ_1,β_1,β_2, γ_2,β_3) bands arrangement according to their appearance. The O(6) limit occurs because, the γ_1-band became before β_1.

The arrangement of these bands is very important because, it is used to calculate the behavior of selected nucleus and moment of inertia values for all bands.

The energy levels of the IBM-1 were calculated from the best fitted interaction parameters of equation (2) with the measured energies levels. These parameters are given in table (1).

While the calculations of Variable Moment of Inertia (VMI) model for any band have been estimated from the parameters (E_K,C, ℓ_0) of equation (5),

These parameters were chosen from smaller chi-square (χ^2) value as in the following equation[17]:

$$\chi^2 = \frac{(E_{cal} - E_{exp})^2}{E^2_{cal}} \quad ....(10)$$

The corresponding parameters with chi-square (χ^2) value are given in table(2) for all ${}^{166}_{68}Er_{98}$ bands.

Table (2) shows that the fitted parameter E_k equal zero for ground band, it takes the value of head (β_1,β_2, β_3) bands i.e the value of $0^+_1, 0^+_2 . 0^+_3, 0^+_4$, and it is a variable fitted for(γ_1, γ_2)bands, also this table shows that the smaller chi-square (χ^2) value was in β_3-band, This means that the calculated results are in quite agreement with experimental results[18,19].

The experimental and(IBM-1, VMI Model) calculations of the energy levels E(J) as a function of angular momentum(J) are shown in figures 1-6 for all bands apperance.

Figure(1)shows that the two sets of calculated results of g-band are very similar and agree with the experimental data reasonably well at low angular momentum(J<8),but at (J>8)the VMI calculation gives large better overall agreement with experimental than IBM-1.

Figure (2) shows that the same two sets of the calculated results but for γ_1-band .In this figure we noticed that the IBM-1 calculations gives slightly larger values than the VMI and experimental results, while the VMI calculations are very quite agreed with the experimental data.

Figure (3) shows that the IBM-1for the $\beta_1$calculations are slightly smaller than the VMI and experimental data, but the (VMI) calculations are very quite agreed with the experimental data.

Figure (4) shows that all calculations of β_2-band are agreed at (J<8), but at (J>8) the IBM-1 calculations are larger than the other calculations.

Figure (5) shows that the IBM-1for the γ_2-band calculations are slightly smaller than the VMI and experimental data at ($3\leq J\geq 13$), but it is in agreement with the others.

Figure (6) shows a comparison for the calculated energy levels of IBM-1, VMI as a function of angular momentum(J) for β_3-band.The comparison shows that the three sets are agreed until J=6, after that the IBM-1 results become higher than other calculations. The difference is very clear at high spin because, the fitted parameters of equation (2) are fitted for experimental data with low-lying spin.

Figure (7) shows the energy transition between two energy levels as a function of angular momentum (J) for g-band as an example for the experimental data, IBM-1, and VMI calculations. In this figure we noticed that all calculations are agreed with experimental data at low spin, but at high spin the variable moment of inertia calculations is in well agreement with experimental results better than IBM1.

III-2: The Energy Band Crossing

The band crossing phenomenon as illustrated in figure (8) is an important subject because, it is used to explain the effect of back bending phenomena. The bands (g, β3), (g,β2), (γ1,β3), (γ1,β2), (β1 ,β3), (β1,β2), (β1,γ1), (β2, β3), (γ2,β3) are crossing at angular momentum (Jc=16 18 14 15 13 17 15 12 8) respectively. The breakup of a pair of nucleus leads to band crossing and makes up a back bending phenomena. The band crossing also occurs due to reduction of the moment of inertia at large angular momentum. Also the Coriols force effect will reduce the energy of two nucleons [6, 11], and the unstability of these two excited states leads to crossing of certain band with another.

III-3: The Back bending Effect

The back bending effect will be used to study the effect of moment of inertia on nuclear structure.

Figure (9) shows the relation between moment of inertia (2ℓ/ħ2) and squared rotational energy (ħω)2 for g-band as an example. In this figure noticed that the VMI-calculations are in good agreement with the experimental data, but the IBM-1-calculations are very difference. Also there is no effect to the moment of inertia on the nuclear structure in this band. This is because the back bending is not present. In the other band in figure (10) the back bending effect occurs in γ1-band at angular momentum (J=11).

This phenomenon occurs due to the rapid increase of the moment of inertia with rotational frequency towards the rigid value. In other word, when the rotational energy exceeds the energy needed to break a pair of nucleon, the unpaired nucleon goes into different orbits, which results in a change of the moment of inertia. An explanation of this effect is that the pairing correlation due to coriols forces was disappearance.

The Conclusions

1-All the VMI model, and IBM-1, and the experimental calculations show that the nucleus understudy belonging to the dynamical symmetry SU(3)-O(6).

2-The variable moment of inertia model (VMI)and (IBM-1) succeeded in studying the nuclear structure of ${}^{166}_{68}Er_{98}$ nucleus, and in the dynamical symmetry SU(3)-O(6). The VMI model gives better agreement with experimental data than the IBM-1 calculations.

3-The results of these models used here have shown that the back bending phenomena does appear in the rotational bands of this nucleus, and the moment of inertia will effect the nuclear structure, but if the back bending phenomena does not appear the moment of inertia will not effect the nuclear structure,

4. The back bending phenomena is clear in the γ1-band with the dynamical symmetry either SU(3) or O(6), thus the nuclear structure of them have been affected.

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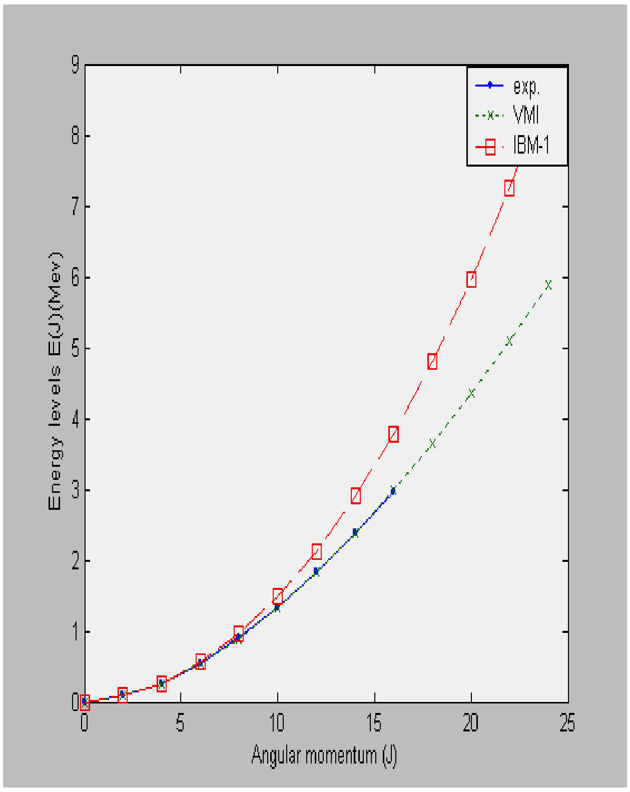
Table (1): The best fitted interaction parameters of equation (2) for the energies in(MeV)units

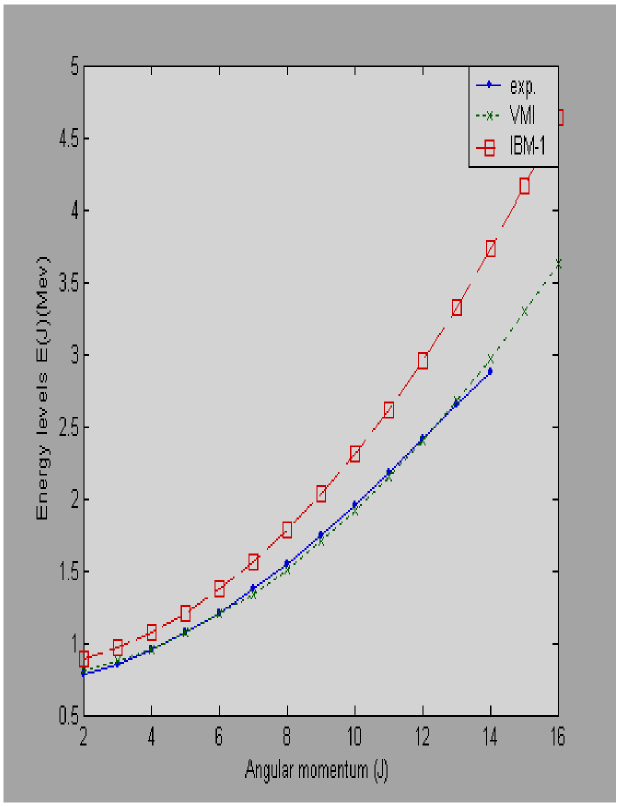
ε	$\hat{P}^\dagger.\hat{P}$	$\hat{L}.\hat{L}$	$\hat{Q}.\hat{Q}$	$\hat{T}_3.\hat{T}_3$	$\hat{T}_4.\hat{T}_4$	CHI
0.0000	**0.0240**	**0.0118**	**-0.011**	**-0.0096**	**0.0219**	**1.270**

Table(2):The best fitted parameters of equation(5)with ($\chi 2$) value.

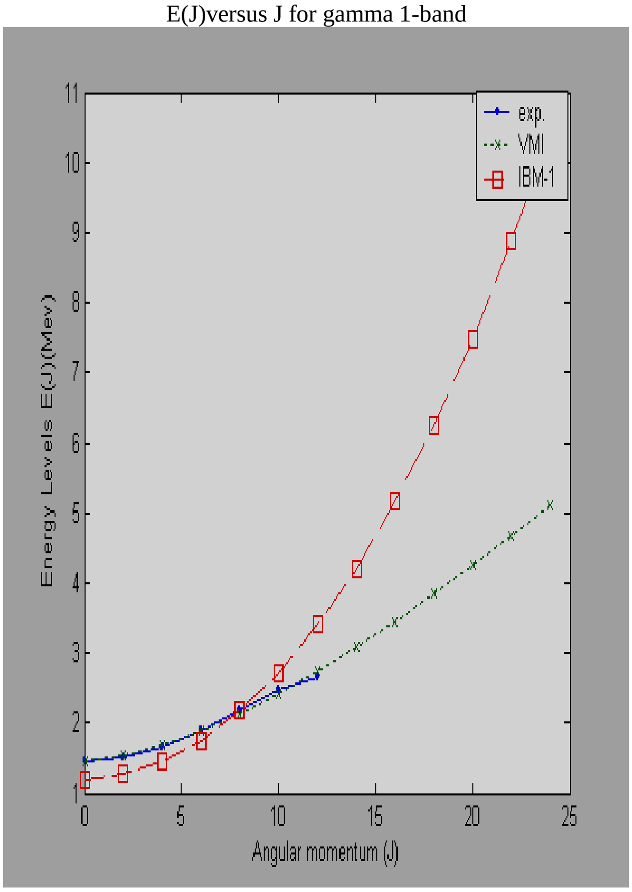
band	ℓ_0(MeV)$^{-1}$	C(MeV)^3	E_k(MeV)	χ^2
g	**36.900**	**0.0034**	**0.0000**	**0.0008**
β_1	**38.000**	**0.0003**	**1.4599**	**0.0052**
β_2	**28.000**	**0.0001**	**1.7031**	**0.0009**
β3	**47.000**	**0.00003**	**2.1870**	**0.0002**
γ_1	**47.000**	**0.0880**	**0.7542**	**0.0097**
γ_2	**38.000**	**0.6000**	**1.8400**	**0.0029**



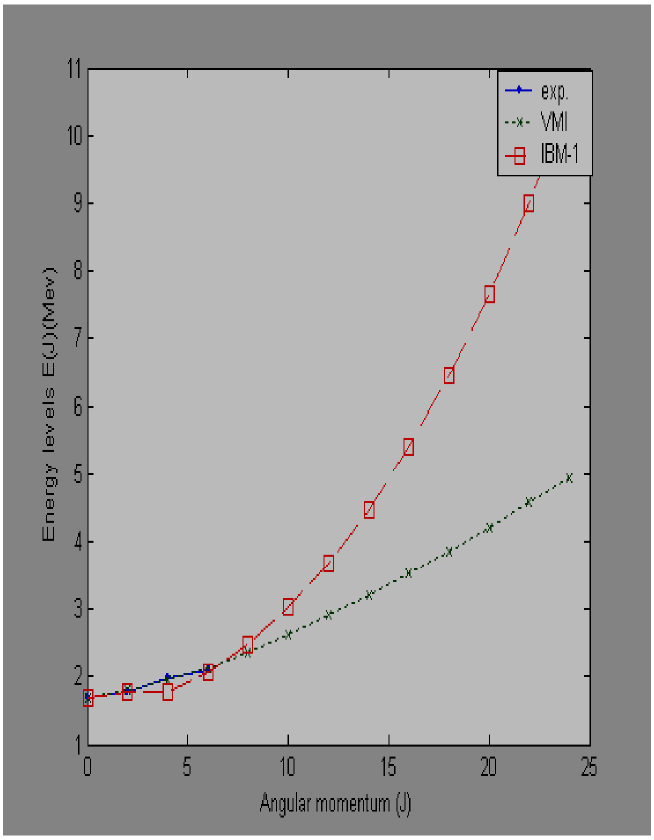
Figure(1):Three calculated IBM-1,VMI and experimental E(J)versus J for g-band



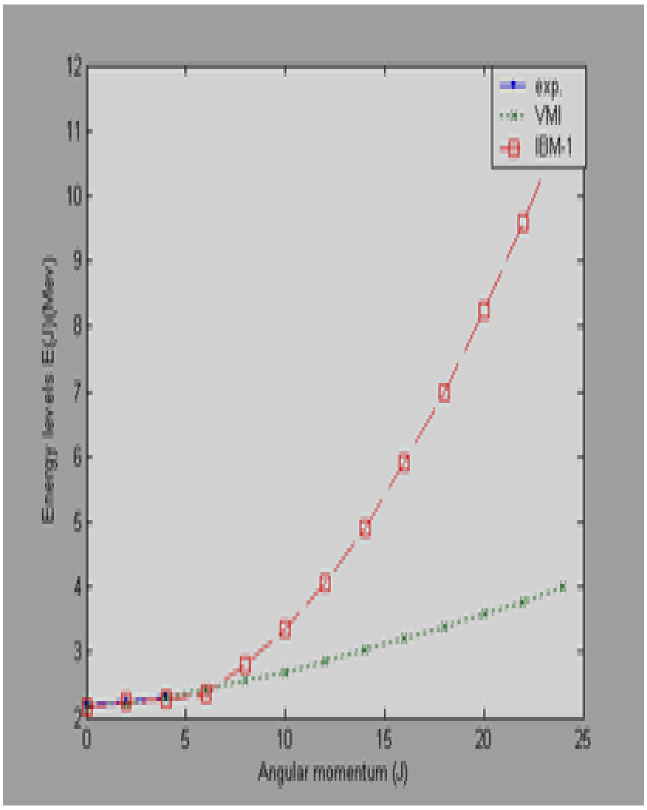
Figure(2):Three calculated IBM-1,VMI and experimental E(J)versus J for gamma 1-band



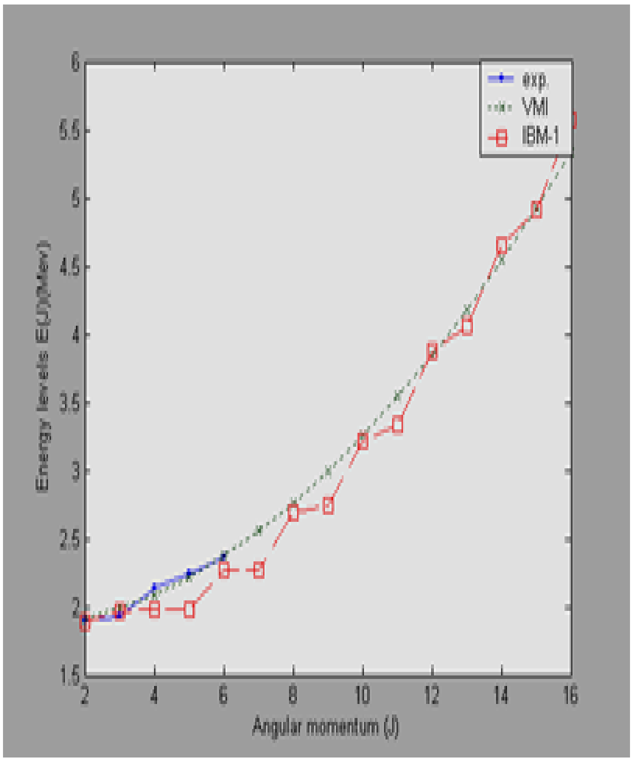
Figure(3):Three calculated IBM-1,VMI and experimental E(J)versus J for B_1-band



Figure(4):Three calculated IBM-1,VMI and experimental E(J)versus J for B_2-band



Figure(6):Three calculated IBM-1,VMI and experimental E(J)versus J for B_3-band



Figure(5):Three calculated IBM-1,VMI and experimental E(J)versus J for gamma 2-band

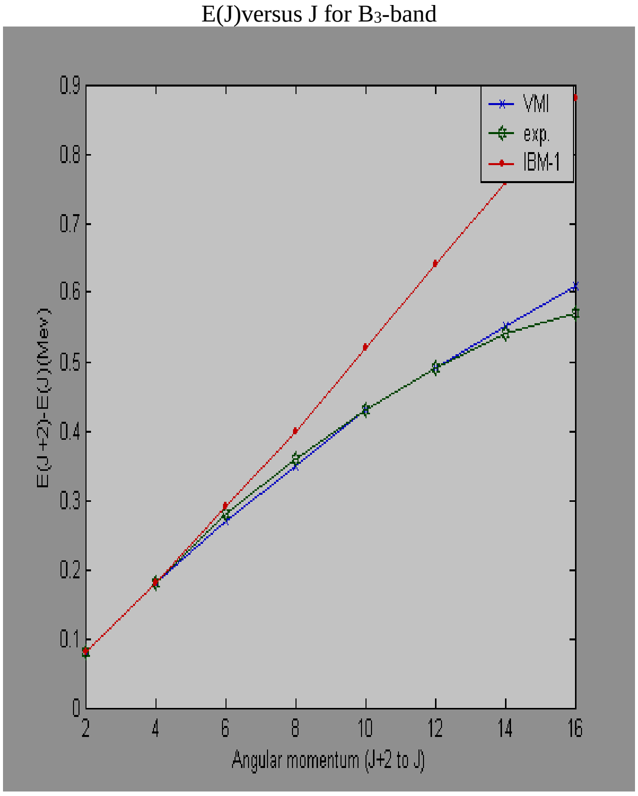
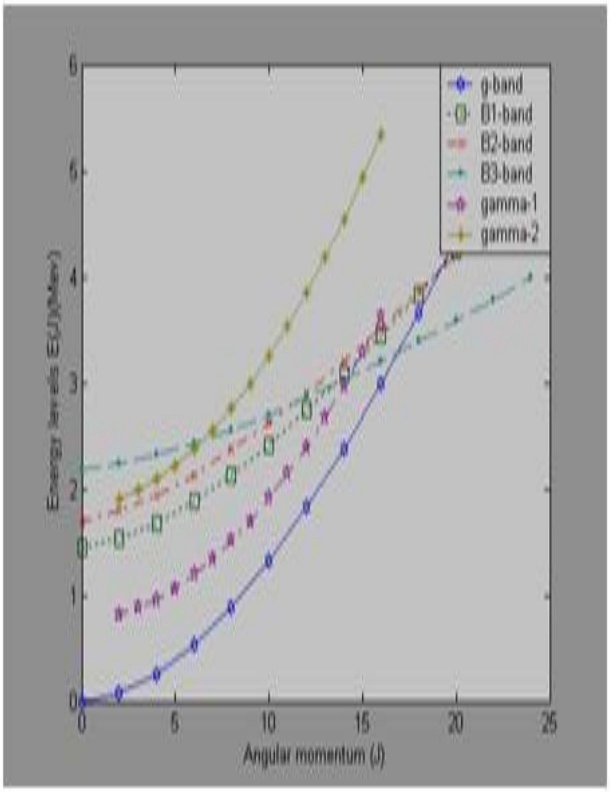


Figure (7)IBM-1 and VMI calculation s of energy transition compared with their experimental data for g=band.



Figure(8): The energy band crossing [E(J) as a function of angular momentum(j)]using VMI model. The parameters used for calculations are in table (2)

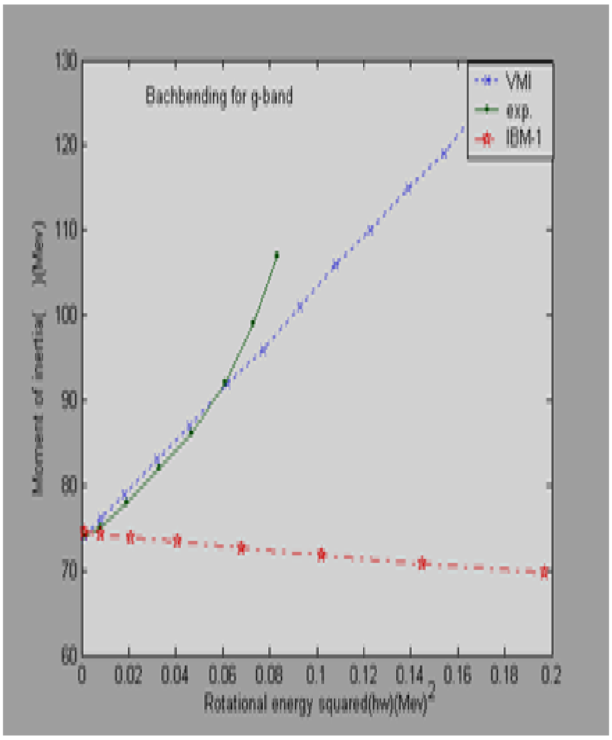


Figure (9)The calculated VMI,IBM-1, and experimental moment of inertia versus the rotational energy squared$(\hbar\omega)^2$for g-band

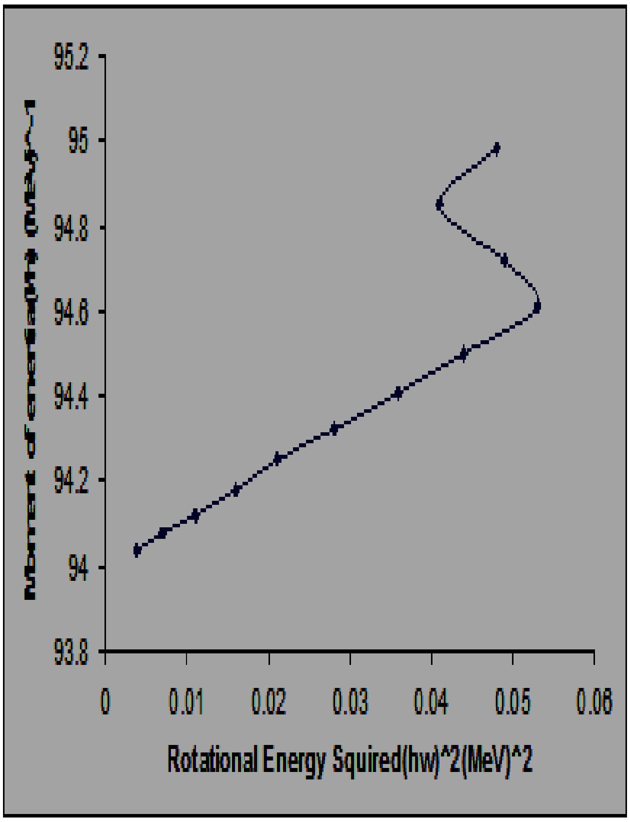


Figure (10): The back bending $(2\ell/\hbar^2)$ as a function of rotational energy squared $(\hbar\omega)^2$for γ_1-band using VMI model.

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حساب الخواص النووية لنواة الاريبيوم ($^{166}_{68}Er_{98}$) باستعمال نموذج البوزونات المتفاعلة-الأول ونموذج عزم القصور الذاتي المتغير(VMI)

إيمان طارق العلوي علي خلف عبيد وليد بديوي صالح

الخلاصة

فـــي البحـــث الحـــالي تـــم اســـتخدام نمـــوذج البوزونـــات المتفاعلـــة الأول (IBM-1) ونمـــوذج عـــزم القصـــور الـــذاتي المتغيـــر (VMI) لدراسـة بعض الخـواص النوويـة مثـل مستويات الطاقـة(E(J))،الطاقـة الانتقاليـة(Eγ)، تقـاطع الحـزم وظـاهرة الانحنـاء الخلفـي لنـواة الاريبيـوم ($^{166}_{68}Er_{98}$) التي تنتمي إلى التناظر الديناميكي SU(3)-O(6).

وقد تم مقارنة النتائج المحسوبة باستعمال نموذج البوزونات المتفاعلة الأول ونموذج تغير عزم القصور الذاتي مع النتائج العملية المتوفرة وكانت النتائج متطابقة بشكل جيد في كثير من المستويات كما إن النتائج المحسوبة باستعمال نموذج تغير عزم القصور الذاتي متطابقة بشكل أفضل من النتائج المحسوبة باستعمال نموذج البوزونات المتفاعلة- الأول.