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On generalization closed set and generalized continuity On Intuitionistic Topological spaces

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Abstract: In this paper, we continue the study of generalized closed sets in ITS, and related to each other's and with known's closed sets. We introduce also, different kinds of g-continuity and related to each other's and with known continuity in ITS.

Keywords: generalization , closed set , generalized continuity , Intuitionistic Topological spaces

Preliminaries

Since we shall use the following definitions and some properties, we recall them in this section.

Definition 1.1 [5] Let X be a non-empty set, an intuitionistic set (IS, for short) A is an object having the form $A = \langle x, A_1, A_2 \rangle$ where $A_1 \ and \ A_2$ are disjoint sub set of X. the set A_1 is called the set of members of A , while A_2 is called the set of non member of A.

Definition 1.2 [5] Let X be a non-empty set, and let A and B are IS, having the form $A = \langle x, A_1, A_2 \rangle$; $B = \langle x, B_1, B_2 \rangle$ respectively. Furthermore, let $\{A_i : i \in I\}$ be an arbitrary family of IS in X, where $A_i = \langle x, A_i^{(1)}, A_i^{(2)} \rangle$, then,

$$1 - \tilde{\emptyset} = \langle x, \emptyset, X \rangle; \ \tilde{X} = \langle x, X, \emptyset \rangle$$

$$2 - A \subseteq B, iff \ A_1 \subseteq B_1 \ and \ A_2 \supseteq B_2$$

$3 -$ the complement of A is denoted by $\bar{A}$ and defined by $\bar{A} = \langle x, A_2, A_1 \rangle$.

$$4 - \cup A_i = \langle x, \cup A_i^{(1)}, \cap A_i^{(2)} \rangle; \ \cap A_i = \langle x, \cap A_i^{(1)}, \cup A_i^{(2)} \rangle$$

Definition 1.3 [5] Let X, Y are non-empty sets and let $f : X \rightarrow Y$ be a function.

1. If $B = \langle x, B_1, B_2 \rangle$ is IS in Y, then the preimage of B under f is denoted by $f^{-1}(B)$ is IS in X defined by $f^{-1}(B) = \langle x, f^{-1}(B_1), f^{-1}(B_2) \rangle$.
2. If $A = \langle x, A_1, A_2 \rangle$ is an IS in X, the image of A under f is defined $f(A) = \langle y, f(A_1), \underline{f}(A_2) \rangle, where \ \underline{f}(A_2) = (f(A_2^c))^c$.

Definition 1.4 [14] An intuitionistic topology (IT, for short), on a non-empty set X, is a family T of an IS in X containing $\tilde{\emptyset}, \tilde{X}$ and closed under arbitrary unions and finitely intersections. The pair (X,T) is called an Intuitionistic topological space (ITS, for short). Any IS in T is known as an intuitionistic open set (IOS, for short) in X, the complement of IOS is called intuitionistic closed set (ICS, for short).

Definition 1.5 [5] Let (X,T) be an ITS and let $A = \langle x, A_1, A_2 \rangle$ be an IS in X, then the interior and closure of A are defined by;

$$int(A) = \cup \{G_i : G_i \in T, G_i \subseteq A\}$$

$$cl(A) = \cap \{K_i : K_i \text{ is ICS in } X \text{ and } A \subseteq K_i\}$$
.

Definition 1.6 [1] ,[11]An IS A in X in an ITS (X,T) is called:
An intuitionistic semi-open set (ISOS, for short) if $A \subseteq cl(int(A))$.
An intuitionistic pre-open set (IPOS, for short) if $A \subseteq int(cl(A))$.
An intuitionistic semipre-open set (ISPOS, for short) if $A \subseteq cl(int(cl(A)))$.
An intuitionistic presemi-open set (IPSOS, for short) if $A \subseteq int(cl(int(A)))$.
An IS A is called intuitionistic semi-closed (resp. intuitionistic pre-closed, intuitionistic semipre-closed and intuitionistic presemi-closed ISCS, IPCS, ISPCS and IPSCS, for short) if its complement is an ISOS (resp. IPOS, ISPOS, and IPSOS).
The semi-closure (resp. pre-closure, semipre-closure and presemi-closure) of A is the smallest ISCS (resp. IPCS, ISPCS and IPSCS) that containing A, [1, 11].
The semi – interior (resp. pre-interior, semipre-interior and presemi-interior) of an IS A is the largest ISOS (resp. IPOS, ISPOS, and IPSOS) that contained in A.

Definition 1.7 [3] ,[11] Let $f:(X,T) \to (Y,\Psi)$ be a mapping from ITS (X,T) to ITS (Y,Ψ), then f is called:

1. An intuitionistic open (S-open, PS-open, P-open, and Sp-open) mapping if $f(A)$ is an IOS (ISOS, IPSOS, IPOS, and ISPOS) in Y for every IOS A in X.
2. An intuitionistic closed(S-closed, P-closed, semipre-closed, and PS-closed) mapping if $f(A)$ is an ICS (ISCS, IPCS,ISPCS, and IPSCS) in Y for every ICS A in X, [3],[11].
3. Continuous (S-continuous, P-continuous, SP-continuous, and PS-continuous) if the inverse image of every intuitionistic closed in Y is closed (SCS, PCS, SPCS and PSCS) in X ,[13],[16],[12]
4. Homomorphism, if f is bijective and continuous and f^{-1} is continuous.

Definition 1.8 [14] Let $(X,T) \text{ and } (Y,\Psi)$ be two ITS and let $f:(X,T) \to (Y,\Psi)$ be a mapping, then the following statement are equivalent.

1. f is continuous
2. The inverse image of f is closed in X, for every closed set in Y.
3. $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for every subset B in Y.

Definition 1.9 [14] Let $(X,T) \text{ and } (Y,\Psi)$ be two ITS and let $f:(X,T) \to (Y,\Psi)$ be a mapping, then the following statement are equivalent.

1. f is homeomorphism
2. f is bijective, continuous and open.
3. f is bijective, continuous and closed.
4. $f(cl(A)) = cl(f(A))$ for every subset A of X.

Generalized closed sets in ITS's

We define in this section generalized-closed (g-closed for short), semi generalized-closed, pre generalized-closed, generalized semi-closed, generalized pre-closed, generalized semipre-closed, generalized presemi-closed and presemi generalized-closed sets $(resp. sg-closed,\ pg-closed, gs-closed,\ gp-closed,\ gsp-closed,\ gps-closed, \text{ and } psg-closed \text{ for short})$ .and illustrate the relation among other kinds of closed sets, some of its properties are given. As well as we give counter example for not true implications.

In [2, 4, 10, and 14] the notion submaximality, Extreme disconnectedness and g-closed set in general topological spaces are studied. Also the notion semi-precontinuous functions and its properties are studied in general topology. Weak and strong form $\beta-$irresoluteness are defined and studied in general topology. Our goal in this section is to generalized and study these notion in to ITS.

We start this section by the following definitions.

Definition 2.1

Let (X,T) be ITS, and let $A = \langle x, A_1, A_2 \rangle$ be IS, A is said to be

1- A g-closed $resp. gs-closed,\ psg-closed, gp-closed,\ gsp-closed,$ if

$cl(A) \subseteq U (resp. scl(A) \subseteq U, pscl(A) \subseteq U, pcl(A) \subseteq U$ $and\ spcl(A) \subseteq U)$ whenever $A \subseteq U$ and U is open set in X.

2- A $sg-closed(resp. gps-closed\ and\ pg-closed)$ if $scl(A) \subseteq U\ (resp.\ pscl(A) \subseteq U\ and\ pcl(A) \subseteq U)$ whenever U is semi-open(resp. presemi-open, pre-open) set in X.

3- The complement g-closed $resp. gs-closed,\ psg-closed, gp-closed,\ gsp-closed, sg-closed,\ gps-closed\ and\ pg-closed)$ is called g-open $resp. gs-open,\ psg-open, gp-open,\ gsp-open, sg-open,\ gps-open\ and\ pg-open)$ set

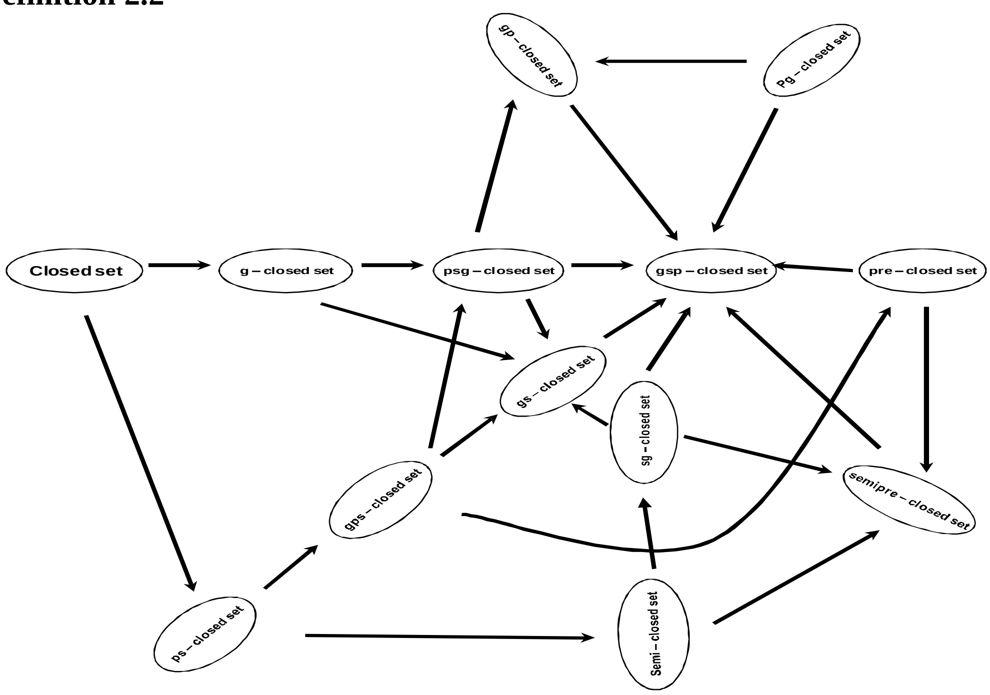
Definition 2.2

The intersection of all g-closed $resp. gs-closed,\ psg-closed, gp-closed,\ gsp-closed, sg-closed,\ gps-closed\ and\ pg-closed)$ containing a set A is called g-closure $resp. gs-closure,\ psg-closure, gp-closure,\ gsp-closure, sg-closure,\ gps-closure\ and\ pg-closure)$ of A and denoted by $gcl(A) (resp. gscl(A), psgcl(A), gpcl(A), gspcl(A), sgcl(A), gpscl(A) and\ pgcl(A))$

The following proposition give a relations among those notions appears in Definitions2.1 and 2.2.

Proposition 2.3

The following implications are true and not reversible.



We start with example shows g-closed is not imply closed set.

Example 2.4

Let $X = \{a, b, c\}$ and $T = \{\tilde{0}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{b\}, \{a, c\}\rangle, B = \langle x, \{a\}, \{b\}\rangle, C = \langle x, \{a, b\}, \emptyset\rangle$. Let $G = \langle x, \{a, c\}, \emptyset\rangle$, G is g-closed set in X, since the only open set containing G is X and $clG \subseteq X$. But it is clear that G is not closed in X.

The next example shows that the class of closed set is contained in class of presemi-closed sets.

Example 2.5

Let $X = \{a, b, c\}$ and $T = \{\tilde{0}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \{c\}\rangle,\ B = \langle x, \{a, b\}, \emptyset\rangle$

$PSOX = \{\tilde{0}, \tilde{X}, A, B, C, F, H\}$. Where $, C = \langle x, \{a\}, \emptyset\rangle, F = \langle x, \{a, b\}, \{c\}\rangle\ and\ H = \langle x, \{a, c\}, \emptyset\rangle$ so $\overline{H} = \langle x, \emptyset, \{a, c\}\rangle$ is PS-closed set in X, but it's not closed in X.

The following example shows that;

$1 - s - closed \nrightarrow ps - closed$

$2 - gs - closed \nrightarrow gps - closed$

$3 - gs - closed \nrightarrow sg - closed$

$4 - psg - closed \nrightarrow gps - closed$

$5 - gps - closed \nrightarrow ps - closed$

$6 - g - closed \nrightarrow gps - closed$

$7 - gsp - closed \nrightarrow gp - closed$

Example 2.6

Let $X = \{a,b,c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x,\{c\},\{a,b\}\rangle, B = \langle x,\{a\},\{b,c\}\rangle, C = \langle x,\{a,c\},\{b\}\rangle$.

SOX= $\{\tilde{\emptyset}, \tilde{X}, A, B, C, E, G, I, L, N\}$ where $E = \langle x,\{c\},\{a\}\rangle$; $G = \langle x,\{a,c\},\emptyset\rangle$, $I = \langle x,\{b,c\},\{a\}\rangle$; $L = \langle x,\{a\},\{c\}\rangle$ and $N = \langle x,\{a,b\},\{c\}\rangle$.

PSOX= $\{\tilde{\emptyset}, \tilde{X}, A, B, C, G\}$.

1- is s-closed set in X. since $intclE = int\bar{B} = A \subseteq E.$ but E is not psa-closed set in X, since $clintclE = clint\bar{B} = clA = \bar{B} \nsubseteq E$.

2- Since E is gs-closed (resp, $psg - closed, g - closed$) set in X. so the only open set containing E is X so $sclE \subseteq X.$ But E is not $gps - closed$ set in X, since the only PSOS in X containing E is G, but $clE = X \nsubseteq G$. And it is easy to verify that E is $g - closed \text{ but not } gps - closed$ and E is $psg - closed$ but not $gps - closed$.

3- Let $D = \langle x,\{c\},\emptyset\rangle$, D is gs-closed in X, since for all open set $D \subseteq U,\ sclD \subseteq X$. But D is not sg-closed, since the only IOS in X is G, but $sclD = X \nsubseteq G.$

4- Let $H = \langle x,\{b,c\},\emptyset\rangle$ so it very easy to verify gps-closed set in X. But H is not $ps - closed$.

5- Since A is IOS then it is easy to see that A is $gsp - closed$ in X, but A is not $gp - closed$ set in X.

The following example shows that:

$1 - sp - closed \nrightarrow s - closed$

$2 - sp - closed \nrightarrow sg - closed$

$3 - g - closed \nrightarrow sg - closed$

$4 - gp - closed \nrightarrow psg - closed$

$5 - gp - closed \nrightarrow pg - closed$

$6 - gsp - closed \nrightarrow pg - closed$

$7 - gsp - closed \nrightarrow p - closed$

$8 - gsp - closed \nrightarrow sp - closed$

$9 - gsp - closed \nrightarrow sg - closed$

$10 - gsp - closed \nrightarrow gs - closed$

$11 - gsp - closed \nrightarrow psg - closed$

$12 - p - closed \nrightarrow gps - closed$

Example 2.7

Let $X = \{a,b,c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x,\{a\},\{b\}\rangle$ $and\ B = \langle x,\{a,b\},\emptyset\rangle$.

$SPOX = \{\tilde{\emptyset}, \tilde{X}, A, B, C, E, F, G, H, I, J, K, L, M, O, S,$

. Where $C = \langle x,\{a\},\emptyset\rangle, D = \langle x,\{a\},\{c\}\rangle$, $E = \langle x,\{a,b\},\{c\}\rangle$, $F = \langle x,\{a,c\},\emptyset\rangle$, $G = \langle x,\{a,c\},\{b\}\rangle$, $H = \langle x,\{a\},\{b,c\}\rangle$, $I = \langle x,\{b,c\},\emptyset\rangle$, $J = \langle x,\{b,c\},\{a\}\rangle$, $K = \langle x,\{c\},\emptyset\rangle$, $L = \langle x,\{c\},\{a\}\rangle$, $S = \langle x,\emptyset,\{b\}\rangle$, $W = \langle x,\emptyset,\{c\}\rangle$, $N = \langle x,\{c\},\{a,b\}\rangle$, $P = \langle x,\{b\},\emptyset\rangle$ and $Y = \langle x,\{b\},\{c\}\rangle$ .M= $\langle x,\{c\},\{b\}\rangle$, $O = \langle x,\emptyset,\{c,b\}\rangle$ SOX=PSOX= $\{\tilde{\emptyset}, \tilde{X}, A, B, C, F, G\}$.

1- Since D is p-closed set so it is SP-closed set in X, since $clintD = \emptyset \subseteq D$. But D is not S-closed and not sg-closed set in X, since the only ISOS in X that contained D is B and F $intclD = X$ is not subset of B or F. To see that D is $gp-closed\ (resp.gsp-closed\ and\ p-closed$). We have B the only IOS in X that contain D, and $inclint\ D = D \subseteq B$. But D is not $psg-closed(resp.gs-closed\ and$ [illegible] set in X, since the only open set and $ps-open$ set in X that contain D are B and F, but $clD = X$ is not contained in B or F.

2- Since the only open set in X that contain F is X, so F $is\ g-closed\ (resp.gp-closed\ and\ gsp-closed)$. But F is not $sg-closed(resp.pg-closed\ and\ p-closed)$, because the only ISOs IPOS in X that contain F is X , but $clF = x\ and\ intF = X$.

3- We are going to show in the following example that,

1 $-sg-closed \nrightarrow g-closed$

$2-gs-closed \nrightarrow g-closed$

$3-gs-closed \nrightarrow psg-closed$

Example 2.8

Let $X = \{a,b,c\}$ and $T = \{\tilde{\emptyset},\tilde{X},A,B,C\}$ where $A = \langle x,\{c\},\{a,b\}\rangle, B = \langle x,\{b\},\{c\}\rangle, C = \langle x,\{b,c\},\emptyset\rangle$.

SOX= $\{\tilde{\emptyset},\tilde{X},A,B,C,F,M\}$ where $F = \langle x,\{c\},\{b\}\rangle$; $M = \langle x,\{a,b\},\{c\}\rangle$. We have B is $sg-colsed\ and\ gs-closed$ set in X, since the only IOS and ISOS in X that contain B are B , C and M, but $clB = \bar{A}$ is not subset of B or C.

The following example shows that

1 $-gps-closed \nrightarrow g-closed$

$2-psg-closed \nrightarrow g-closed$

Example 2.9

Let $X = \{a,b,c\}$ and $T = \{\tilde{\emptyset},\tilde{X},A,B\}$, where $A = \langle x,\{c\},\{a,b\}\rangle, B = \langle x,\{b,c\},\{a\}\rangle,$.

PSOX= $\{\tilde{\emptyset},\tilde{X},A,B,C,D,E,F,G,H\}$, where $C = \langle x,\{c\},\{a\}\rangle$; $G = \langle x,\{a,c\},\{b\}\rangle$, $D = \langle x,\{c\},\emptyset\rangle$; $E = \langle x,\{c\},\{b\}\rangle$ and $F = \langle x,\{a,c\},\emptyset\rangle$, $H = \langle x,\{b,c\},\emptyset\rangle$. Let $M = \langle x,\{b\},\{a,c\}\rangle$; M is $gps-closed\ and\ psg-closed$ set in X, since the only IOS and IPSOS in X that contain M are H and B, and $clintclM = M$ which is sub set of B only. But M is not –g-closed, since the only IOS in X that contain M is B only, but $clM = \bar{A}$ is not subset of B.

In the last example we show that $sg-closed\ set \nrightarrow s-closed\ set$.

Example 2.10

Let $X = \{a,b,c\}$ and $T = \{\tilde{\emptyset},\tilde{X},A,B,C\}$ where $A = \langle x,\{c\},\{a,b\}\rangle, B = \langle x,\{a\},\{c\}\rangle, C = \langle x,\{a,c$.

SOX= $\{\tilde{\emptyset},\tilde{X},A,B,C,E,M\}$ where $E = \langle x,\{c\},\{a\}\rangle, M = \langle x,\{a,b\},\{c\}\rangle$. Let H $= \langle x,\{b,c\},\emptyset\rangle$. It is clear that H $sg-closed$, but not $s-closed$ set.

We end this chapter by the following remarks

Remark 2.11

The notion g-closed and sg-closed sets are independent notions.

Example 2.7 (3) and Example 2.8(1) show the case.

Remark 2.12

The notion g-closed and gps-closed sets are independent notions.

Example2.6 (6) and Example 2.9(1) show the case.

Generalized continuous function on ITS's

This section is devoted to introduce the definitions of a $g-continuous, sg-continuous, gs-continuous, gps-continuous, psg-continuous, gp-continuous, pg-continuous \text{ and } gsp-continuous.$ And investigate the relations among them by giving a diagram illustrate these relations. As well as we gives some counter example to show that the converse of true implications are not true in general.

In [10, 9] preregular- closed and θ-generalized closed are introduced and a g-continuous function in general topological space are given in this section we generalized these notions on ITS.

Definition 3.1

Let $(X,T),(Y,\Psi)$ be two ITS, and let $f:X\to Y$ be a function. Then f is said to be $g-continuous(resp.\, gs-continuous, sg-continuous, gps-continuous, psg-continuous, gsp-continuous, pg-continuous \text{ and } gp-continuous)$ function if $f^{-1}(V)$ is $g-closed\ (resp.\, gs-closed, sg-closed, gps-closed, psg-closed, gsp-closed, pg-closed \text{ and } gp-closed)$ set in X for every closed set V in Y.

The notions g-continuity and sg-continuity are independent.

Example 3.2

Let $X=\{a,b,c\}$ and $T=\{\tilde{\emptyset},\tilde{X},A,B\}$ where $A=\langle x,\{b\},\{c\}\rangle$, $B=\langle x,\{a,b\},\emptyset\rangle$. And Let $Y=\{1,2,3\}$ and $T=\{\tilde{\emptyset},\tilde{Y},C,D\}$ where $C=\langle x,\{1\},\{2\}\rangle,\ D=\langle x,\{1,3\},\emptyset\rangle$. Define a function $f:X\to Y, by\ f(a)=1, f(c)=2 \text{ and } f(b)=3$. f is g-continuous but f is not sg-continuous.

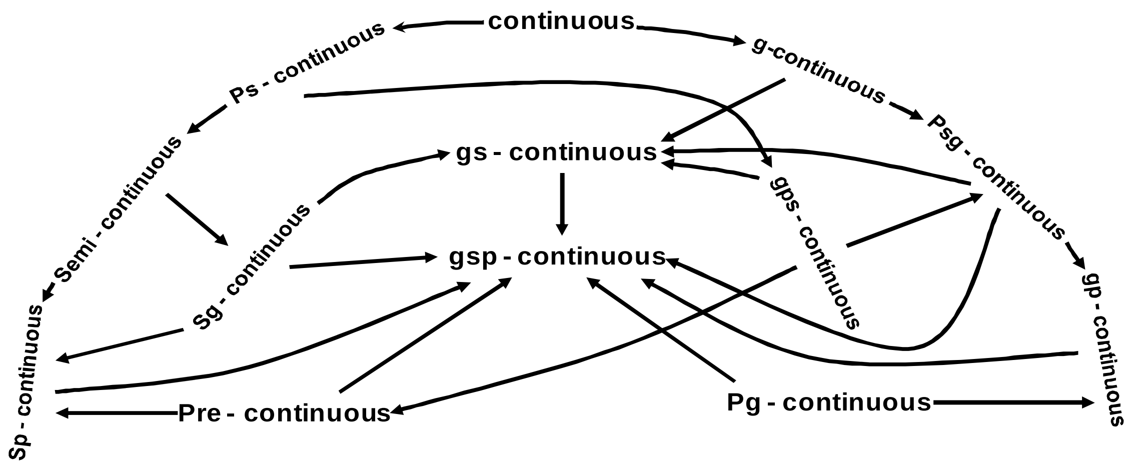
Example 3.3

Let $X=\{a,b,c\}$ and $T=\{\tilde{\emptyset},\tilde{X},A,B,C\}$ where $A=\langle x,\{c\},\{a,b\}\rangle$, $B=\langle x,\{b\},\{c\}\rangle \text{ and } C=\langle x,\{b,c\},\emptyset\rangle$. And Let $Y=\{1,2,3\}$ and $T=\{\tilde{\emptyset},\tilde{Y},E,D\}$ where $D=\langle y,\{2,3\},\emptyset\rangle,\ E=\langle y,\{2\},\{3\}\rangle$. Define a function $f:X\to Y, by\ f(a)=1, f(b)=3 \text{ and } f(c)=2$. is f sg-continuous, but f is not g-continuous.

The following proposition gives a relations among these concepts appears in above definition.

Proposition 3.4

The following implications are valid and not reversed



Proof: is direct consequences by using Prop. 2.3 and corresponding definition of continuity.

We start with example showing f is g-continuous, but f is not continuous.

Example.3.5

Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \{b, c\}\rangle$, $B = \langle x, \{b, c\}, \{a\}\rangle$. And Let $Y = \{1,2,3\}$ and $T = \{\tilde{\emptyset}, \tilde{Y}, C, D\}$ where $C = \langle x, \{1\}, \{2,3\}\rangle$, $D = \langle x, \{1,3\}, \{2\}\rangle$. Define a function $f: X \to Y, by\ f(a) = f(c) = 1\ and\ f(b) = 2$. f is g-continuous since $F = f^{-1}(\bar{C}) = f^{-1}(\bar{D}) = \langle x, \{b\}, \{a, c\}\rangle$ is g-closed in X, since the only IOS in X contain F is B, and $clF = \bar{A} \subseteq B$. But it is clear that F is not closed set in X.

The following example shows that:

1- $sp - cont. \nrightarrow s - cont.$ 2 $- sp - cont. \nrightarrow sg - cont.$

Example 3.6

Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{a\}, \{b, c\}\rangle$. And Let $Y = \{1,2,3\}$ and $T = \{\tilde{\emptyset}, \tilde{Y}, B\}$ where $B = \langle x, \{2\}, \{1\}\rangle$. Define a function $f: X \to Y, by\ f(a) = 3, f(b) = 1\ and\ f(c) = 2$.

$SPOX = \{\tilde{\emptyset}, \tilde{X}, A, C, D, E, F, G, H, I, J, L, M, O, P, Q, R\}$ where $C = \langle x, \{a\}, \emptyset\rangle$, $D = \langle x, \{a\}, \{b\}\rangle$, $E = \langle x, \{a\}, \{c\}\rangle$, $F = \langle x, \{a, b\}, \emptyset\rangle$, $G = \langle x, \{a, b\}, \{c\}\rangle$, $H = \langle x, \{a, c\}, \emptyset\rangle$, $I = \langle x, \{a, c\}, \{b\}\rangle$, $J = \langle x, \{b\}, \emptyset\rangle$, $L = \langle x, \emptyset, \{b, c\}\rangle$, $M = \langle x, \emptyset, \{b\}\rangle$, $O = \langle x, \{b\}, \{c\}\rangle$, $P = \langle x, \emptyset, \{c\}\rangle, Q = \langle x, \{c\}, \emptyset\rangle, R = \langle x, \{c\}, \{b\}\rangle$, $N = \langle x, \{b, c\}, \emptyset\rangle$.

$SOX = \{\tilde{\emptyset}, \tilde{X}, A, C, D, E, F, G, H, I\}$. f is sp-cont., since $f^{-1}(\bar{B}) = O \in SPCSX$. But f is not s-cont., since $intcl(f^{-1}(\bar{B})) = intclO = int\ X = X \not\subseteq O = f^{-1}(\bar{B})$. Also f is not sg-cont, since the the only ISOS in X that contains $f^{-1}(\bar{B})$ is G and F, but $intcl(f^{-1}(\bar{B})) = intclO = int\ X = X \not\subseteq G\ or\ F.$

Example 3.7

Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \{b\}\rangle, B = \langle x, \{a, c\}, \emptyset\rangle$. And Let $Y = \{1,2,3\}$ and $\Psi = \{\tilde{\emptyset}, \tilde{Y}, C, D\}$ where $C = \langle y, \{2\}, \{3\}\rangle, D = \langle x, \{2,3\}, \emptyset\rangle$. Define a function $f: X \to Y, by\ f(a) = f(b) = 3\ and\ f(c) = 2$. $PSOX = \{\tilde{\emptyset}, \tilde{X}, A, B, E, H, J\}$ where $E = \langle x, \{a\}, \emptyset\rangle$, $J = \langle x, \{a, c\}, \{b\}\rangle$, $H = \langle x, \{a, b\}, \emptyset\rangle$, f is gs-cont., since $f^{-1}(\bar{C}) = I = \langle x, \{a, b\}, \{c\}\rangle$. Is gs-closed in X, since the only open set V in X such that $I \subseteq V, such\ that\ sclI \subseteq X$ is only $f^{-1}(\bar{D}) = \langle x, \emptyset, X\rangle = \tilde{\emptyset}$. But f is not gp, gps also f is not sg-cont. s-cont., since the only IPSOS and ISOS in X that contain I is H, but $clI = X$ is not subset of H.

In the next example we show that:

$gps - cont. \nrightarrow ps - cont.$ 2 $- sg - cont. \nrightarrow s - cont.$

Example 3.8

Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \{b, c\}\rangle, B = \langle x, \{b, c\}, \{a\}\rangle$. And Let $Y = \{1,2,3\}$ and $\Psi = \{\tilde{\emptyset}, \tilde{Y}, C, D, E\}$ where $C = \langle y, \{1,2\}, \{3\}\rangle, D = \langle y, \{2,3\}, \{1\}\rangle, E = \langle y, \{2\}, \{1,3\}\rangle$. Define a function $f: X \to Y, by\ f(a) = f(b) = 3\ and\ f(c) = 1$. $PSOX = \{\tilde{\emptyset}, \tilde{X}, A, B\} = SOX = T$.

So 1- f is gps-continuous, But its clear that f is not ps-continuous,2-f is sg-continuous and f is not semi-continuous .

In the next example we show that:

1- $gs - cont. \nrightarrow g - cont.$

2- $gs - cont. \nrightarrow psg - cont.$

3- $gsp - cont. \nrightarrow psg - cont.$

Example 3.9

Let $X = \{1,2,3\}$ and $T = \{\tilde{0}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{2\}, \{1,3\} \rangle$, $B = \langle x, \{1\}, \{2\} \rangle, C = \langle x, \{1,2\}, \emptyset \rangle$.

And Let $Y = \{a,b,c\}$ and $\Psi = \{\tilde{0}, \tilde{Y}, D, E, F\}$ where $D = \langle y, \{a\}, \{c\} \rangle, E = \langle x, \{c\}, \{a,b\} \rangle$ and $F = \langle y, \{a,c\}, \emptyset \rangle$ .Define a function $f: X \to Y, by\ f(2) = a, f(3) = b\ and\ f(1) = c$ .It is easily to satisfy that f is gs-cont and gsp-cont. but f is not g-continuous and not psg-continuous, since the only IOS in X, that contain $I = f^{-1}(\bar{E}) = \langle x, \{2,3\}, \{1\} \rangle$ is is a subset of X only, but $cl\ I = \bar{A}$ is not contained in B or C.

In The following example we show that; 1- $psg - cont. \nrightarrow gps - cont.$ 2- $gsp - cont. \nrightarrow sg - cont.$

Example 3.10

Let $X = \{a,b,c\}$ and $T = \{\tilde{0}, \tilde{X}, A, B\}$ where $A = \langle x, \{c\}, \{b\} \rangle, B = \langle x, \{a,c\}, \emptyset \rangle$.

And Let $Y = \{1,2,3\}$ and $\Psi = \{\tilde{0}, \tilde{Y}, C, D\}$ where $C = \langle y, \{1,2\}, \emptyset \rangle, D = \langle y, \{1\}, \{3\} \rangle$, . Define a function $f: X \to Y, by\ f(a) = 1, f(b) = 3\ and\ f(c) = 2$ $PSOX = \{\tilde{0}, \tilde{X}, A, B, E, H, I\} = SOX$. ,where $E = \langle x, \{c\}, \emptyset \rangle$, $H = \langle x, \{a,c\}, \{b\} \rangle$, $I = \langle x, \{b,c\}, \emptyset \rangle$. We can see that f is psg-cont. gsp-cont. since $K = f^{-1}(\bar{D}) = \langle x, \{b\}, \{a\} \rangle$ is psg-closed and gsp-closed set in X, because the only open set V in X such that K$\subseteq V$ is X , the $pscl(K) \subseteq X, spclK \subseteq X$, $\bar{B} = f^{-1}(\bar{C}) = \langle x, \emptyset, \{a,c\} \rangle$ is psg-closed and gsp-closed set in X. But f is not gps-cont. and not sg-cont. since the only PSOS and SOS in X that contain K is I , but $clK = X \not\subseteq I$.

Next we show that:

1- $gsp - cont. \nrightarrow pg - cont.$

2- $gp - cont. \nrightarrow pg - cont.$

3- $gsp - cont. \nrightarrow p - cont$.

4- $gp - cont. \nrightarrow psg - cont.$

5- $sp - cont. \nrightarrow p - cont.$

Example 3.11 Let $X = \{a,b,c\}$ and $T = \{\tilde{0}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{b\}, \{a,c\} \rangle, B = \langle x, \{a\}, \{b\} \rangle\ and\ C = \langle x, \{a,b\}, \emptyset \rangle$. And Let $Y = \{1,2,3\}$ and $\Psi = \{\tilde{0}, \tilde{Y}, D, E, F\}$ where $D = \langle y, \{2\}, \{3\} \rangle, E = \langle y, \{3\}, \{1,2\} \rangle, F = \langle y, \{2,3\}, \emptyset \rangle$. Define a function $f: X \to Y, by\ f(a) = 1, f(b) = 3\ and\ f(c) = 2$. $SPOX = \{\tilde{0}, \tilde{X}, A, B, C\} \cup_{i=1}^{21} G_i$ where It is easy to see that f is gsp-cont. (resp. gp-cont. and sp-cont.) but f is not pg-cont. and p-cont. since the only POS in X that contain $I = f^{-1}(\bar{D}) = G_3$ is C, $G_1, G_3\ and\ G_5$, but $cl\ int\ G_3 = G_1 \not\subseteq G_3\ or G_4$. Since $cl\ int\ G_3 = clA = \bar{B} \not\subseteq G_3$ so it is not p-closed in X. also, since G_{20} is gp-closed in X , but it is not psg-closed.

$gsp - cont \nrightarrow gp - cont$ is shown in the following example.

Example 3.12

Let $X = \{1,2,3,4\}$ and $T = \{\tilde{0}, \tilde{X}, A, B, C, D\}$

where $A = \langle x,\{3\},\{2,4\}\rangle, B = \langle x,\{1,2\},\{3,4\}\rangle, C = \langle x,\emptyset,\{2,3,4\}\rangle\ D = \langle x,\{1,2,3\},\{4\}\rangle$. And Let $Y = \{a,b,c,d\}$ and $\Psi = \{\tilde{\emptyset},\tilde{Y},E,F,G\}$

where $E = \langle y,\{a\},\{b,c\}\rangle, F = \langle y,\{b,c\},\{a,d\}\rangle\ and\ G = \langle y,\{a,b,c\},\emptyset\rangle$

Define a function $f: X \to Y, by\ f(1) = f(3) = d, f(4) = a\ and\ f(2) = c$. We can see that f is gsp-cont. But f is not gp-cont., since $W = f^{-1}(\tilde{E}) \subseteq D$, but $cl\ int\ W = \langle x,\{2,4\},\emptyset\rangle \nsubseteq D$.

Example 3.13

We show in this example that there is a function f such that f is gsp-cont., but not sp-cont.

Let $X = \{a,b,c\}$ and $T = \{\tilde{\emptyset},\tilde{X},A,B\}$

where $A = \langle x,\{a,c\},\emptyset\rangle$, $B = \langle x,\{c\},\{b\}\rangle$. And Let $Y = \{1,2,3\}$ and $\Psi = \{\tilde{\emptyset},\tilde{Y},C,D\}$

where $C = \langle y,\{1\},\{2\}\rangle, D = \langle y,\{1,3\},\emptyset\rangle$. Define a function $f: X \to Y, by\ f(c) = f(b) = 2\ and\ f(a) = 3$.

$SPOX = \{\tilde{\emptyset},\tilde{X},A,B\} \cup_{i=1}^{17} G_i$, where

$G_1 = \langle x,\{c\},\emptyset\rangle, G_2 = \langle x,\{c\},\{a\}\rangle, G_3 = \langle x,\{a,c\},\{b\}\rangle, G_4 = \langle x,\{c\},\{a,b\}\rangle, G_5 = \langle x,\{b,c\},\emptyset\rangle, G_6 = \langle x,\{b,c\},\{a\}\rangle, G_7 = \langle x,\{a\},\emptyset\rangle, G_8 = \langle x,\{a\},\{b\}\rangle, G_9 = \langle x,\{a\},\{c\}\rangle, G_{10} = \langle x,\{a$ [illegible] $\langle x,\{a,b\},\{c\}\rangle,\ G_{13} = \langle x,\{b\},\emptyset\rangle, G_{14} = \langle x,$ [illegible] $G_{15} = \langle x,\emptyset,\{a\}\rangle, G_{16} = \langle x,\emptyset,\{b\}\rangle, G_{17} = \langle x,\emptyset,\{a,b\}\rangle$

It is very easy to see that f gsp-cont., but f is not sp-cont.

Example 3.14

Let $X = \{a,b,c\}$ and $T = \{\tilde{\emptyset},\tilde{X},A,B\}$

where $A = \langle x,\{c\},\{a,b\}\rangle$, $and\ B = \langle x,\{b,c\},\{a\}\rangle$. And Let $Y = \{1,2,3\}$ and $\Psi = \{\tilde{\emptyset},\tilde{Y},C,D,E\}$

where $C = \langle y,\{1,3\},\{2\}\rangle\ D = \langle y,\{2\},\{3\}\rangle, E = \langle y,\emptyset,\{2$

. Define a function $f: X \to Y, by\ f(a) = 1, f(b) = 2\ and\ f(c) = 3$. We can show in this example that f psg-cont.. but f is not g-cont.

Example 3.15

We show in this example that there is a function f such that f is gsp-cont., but f is not gs-cont. Let $X = \{a,b,c\}$ and $T = \{\tilde{\emptyset},\tilde{X},A,B,C\}$

where $A = \langle x,\{b\},\{a,c\}\rangle, B = \langle x,\{a\},\{b\}\rangle\ and\ C = \langle x,\{a,b\},\emptyset\rangle$.

And Let $Y = \{1,2,3\}$ and $\Psi = \{\tilde{\emptyset},\tilde{Y},D,E\}$

where $D = \langle y,\{1,2\},\emptyset\rangle, E = \langle y,\{2\},\{1\}\rangle$.

Define a function $f: X \to Y, by\ f(a) = 3, f(b) = 1\ and\ f(c) = 2$.

It is easy to verify that f is gsp-cont., but f is not gs-cont.

At last example, we show that there is a function f which is p-cont., but not gps-cont.

Example 3.16

Let $X = \{a,b,c\}$ and $T = \{\tilde{\emptyset},\tilde{X},A,B\}$

where $A = \langle x,\{b\},\{a,c\}\rangle, and\ B = \langle x,\{a,b\},\emptyset\rangle$

. And let $Y = \{1,2,3\}$ and $\Psi = \{\tilde{\emptyset},\tilde{Y},E,F\}$

where $E = \langle y,\{1\},\{2\}\rangle, F = \langle y,\{1,3\},\emptyset\rangle$.

Define a function $f: X \to Y, by\ f(a) = 1, f(b) = 3\ and\ f(c) = 2$

$PSOX = \{\tilde{\emptyset},\tilde{X},A,B\} \cup_{i=1}^{6} G_i$, where

$G_1 = \langle x,\{b\},\emptyset\rangle, G_2 = \langle x,\{b\},\{c\}\rangle, G_3 = \langle x,\{b,c\},\emptyset\rangle, G_4 = \langle x,\{b,c\},\{a\}\rangle, G_5 = \langle x,\{b\},\{a\}, G_6\langle x,\{a,b\},\{c\}\rangle, POX = PSOX \cup \{G_i\}_{i=7}^{14}$

. where

$G_7 = \langle x,\emptyset,\{a\}\rangle, G_8 = \langle x,\emptyset,\{c\}\rangle, G_9\langle x,\{a,c\},\emptyset\rangle, G_{10} = \langle x,\{a\},\emptyset\rangle, G_{11} = \langle x,\{c\},\{a\}\rangle, G_{12} = \langle x,\emptyset,\{a,c\}\rangle,\ G_{13} = \langle x,\{a\},\{c\}\rangle, and\ G_{14} = \langle x,\{c\},\emptyset\rangle$

. We can verify that f is p-cont. But not gps-cont.

Next we give the following proposition.

Proposition 3.17

Let $(X,T),(Y,\Psi)$ be two ITS and let $f:X\to Y$ be a function, then the following statements are equivalent.

1. f is g-continuous (resp. $sg-cont., gs-cont., gps-cont., psg-cont., gsp-cont., gp-cont. and\ pg-cont$

The inverse image of each open set in Y is $g-open$, (resp. $sg-open, gs-open, gps-open, psg-open, gsp-open, gp-open\ and\ pg-open$

Proof: $\mathbf{1 \Rightarrow 2}$

Let V be any open set in Y, then $\bar{V}$ is a closed set in Y. since f is $g-cont$.(resp. $sg-cont., gs-cont., gps-cont., psg-cont., gsp-cont., gp-cont. and\ pg-cont$.),then $f^{-1}(\bar{V})$ is $g-closed$, (resp. $sg-closed, gs-closed, gps-closed, psg-closed, gsp-closed, gp-closed\ and\ pg-closed$) in X. But since $f^{-1}(\bar{V})=\overline{f^{-1}(V)}$, then $f^{-1}(V)$ is $g-open$, (resp. $sg-open, gs-open, gps-open, psg-open, gsp-open, gp-open\ and\ pg-open$)sets in X.

$\mathbf{2 \Rightarrow 1}$ Let V be any closed set in Y , then $\bar{V}$ is an open set in Y, so $f^{-1}(\bar{V})$ is $g-open$, (resp. $sg-open, gs-open, gps-open, psg-open, gsp-open, gp-open\ and\ pg-open$) in X by hypothesis. But $f^{-1}(\bar{V})=\overline{f^{-1}(V)}$, then $f^{-1}(V)$ is $g-closed$, (resp. $sg-closed, gs-closed, gps-closed, psg-closed, gsp-closed, gp-closed\ and\ pg-closed$) in X. Therefore f is $g-cont$. (resp. $sg-cont., gs-cont., gps-cont., psg-cont., gsp-cont., gp-cont. and\ pg-cont$.). ■

The following propositions was proved in [6] in general topological space we generalize them in to ITS.

Proposition 3.18

Let $(X,T),(Y,\Psi)$ be two ITS and let $f:X\to Y$ be a function. If f is sg-continuous, then $f(sgcl(A))\subseteq cl(f(A)), \forall A\subseteq X$.

Proof

Let A be any subset of X, so $cl(A)$ is closed set in X. since $A\subseteq cl(A)$, then $f(A)\subseteq cl(f(A))$, so $f^{-1}(f(A))\subseteq f^{-1}(cl(f(A)))$. We have $A\subseteq f^{-1}\left(cl\left(f(A)\right)\right)$. Now $cl(f(A))$ is closed set in Y, $f^{-1}(cl(f(A)))$ is a sg-closed set containing A. Consequently $sgcl(A)\subseteq f^{-1}\left(cl(f(A))\right)$. So $f(sg(cl(A)))\subseteq f(f^{-1}(cl(f(A))))\subseteq cl(f(A))$. ■

Proposition 3.19

Let $(X,T),(Y,\Psi)$ be two ITS and let $f:X\to Y$ be a function, then the following statement are equivalent.

1. For any subset A of X, $f(sgcl(A))\subseteq cl(f(A))$.
2. For each subset B of Y, $sgcl(f^{-1}(B))\subseteq f^{-1}(cl(B))$

Proof $\mathbf{1 \Rightarrow 2}$

Suppose that A be any sub set of X and B be any subset of Y, clB is a closed set in Y. since $B\subseteq clB$, so $f^{-1}(B))\subseteq f^{-1}(clB)$, we get from hypothesis that $f(sgcl(A))\subseteq cl(f(A))$, then

$f(sgcl(f^{-1}(B)))\subseteq cl(f(f^{-1}(B)))\subseteq clB$

[by replacing A by $f^{-1}(B)$].

Hence
$sgcl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$.

2 ⇒ 1 *Suppose that B any subset of Y, let* $B = f(A)$ *where A is a subset of X. Since from hypothesis*
$sgcl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$, *then*
$sgcl(f^{-1}(f(A))) \subseteq sgcl(f^{-1}(B)) \subseteq f^{-1}(cl(f(A)))$
. So
$sgcl(A) \subseteq A \subseteq f^{-1}(cl(f(A)))$.
Therefore,
$f(sgcl(A)) \subseteq cl(f(A))$.■

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حول المجموعات المغلقة المعممة و الاستمرارية المعممة في الفضاءات التبولوجية الحدسية

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الخلاصة

يهدف هذا البحث الىالاستمرار بدراسة المجموعات المغلقة المعممة في الفضاءات التبولوجية الحدسية وعلاقتها ببعضها البعض ومع المجموعات المغلقة المعروفة. قدمنا كذلك انواع مختلفة من الاستمرارية المعممة في الفضاءات التبولوجية الحدســية وعلاقتها ببعضها والانواع المعروفة منها.