

# Calculation the Cross Sections for $^{64}\text{Cu}(n,p)^{64}\text{Ni}$ Reaction

## By Reciprocity Theory



Sameera A. Ebrahem\* Shaimaa A. Abbas\* Adil M. Ibraheim\*\* Israa A. Abbas\*\*\*

\* University of Baghdad - College of Education

\*\* Ministry of Education - Baghdad

\*\*\* Al-Mustansrya University – College Of Education.

### ARTICLE INFO

Received: 22 / 11 /2012

Accepted: 20 / 11 /2012

Available online: 16/2/2014

DOI: 10.37652/juaps.2013.85007

Keywords:

### ABSTRACT

In this study intermediate elements  $^{64}\text{Ni}$ ,  $^{64}\text{Cu}$  for  $^{64}\text{Ni}$  ( $p,n$ )  $^{64}\text{Cu}$  reaction with proton energy from (1.0) MeV to (132) MeV with threshold energy (2.496) MeV are used according to the available data of reaction cross sections. We calculated the cross sections for  $^{64}\text{Cu}(n,p)^{64}\text{Ni}$  reaction by application in nuclear technology (reciprocity theory). In reciprocity theory we derive the mathematical formula for  $^{64}\text{Cu}(n,p)^{64}\text{Ni}$  and we deduced high probability to produced  $^{64}\text{Ni}$  because it is very important such as it used in technology field .The evaluated cross sections as a functions of neutron energy between ( $E_n = 0.504\text{MeV}$ ) to ( $E_n=129.506\text{MeV}$ ) of (0.0106barn) (0.254barn) respectively and statistical factor ( $g_{p,n}=1$  and  $g_{n,p}=1/3$ ).

### Introduction

When two charged nuclei, overcoming their Coulomb repulsion, a rearrangement of the constituents of the nucleus may occur. Similar to the rearrangement of atoms in reacting molecules during a chemical reaction this may result as a nuclear reaction. Nuclear reactions are usually produced by bombarding a target nucleus with a nuclear projectile, in most cases a nucleon (neutron or proton) or a light nucleus such as a deuteron or an  $\alpha$ -particle [1].

At low excitation energies (< 10 MeV), the majority of nuclear reactions involve the formation of two nuclei, one nearly equal in charge and mass number to the target nucleus. Such reactions are represented by an equation of the type:



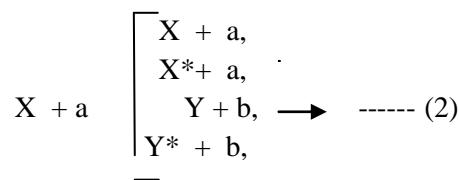
Or  $X(a,b)Y$

Where (a) is the light projectile nucleus (proton, neutron, deuteron,  $^3\text{H}$ ,  $^3\text{He}$ , or  $^4\text{He}$ ) and (X) is the target nucleus at rest in the laboratory system.

\* Corresponding author at: University of Baghdad - College of Education. E-mail address:

(Y) is the produced nucleus and (b) is a light nuclear particle which carries away the major share of the kinetic energy. If the product nucleus (Y) is left in an excited state after the emission of the light particle (b), it usually subsequently decays by radiating one or more gamma rays. Alternatively if (Y) is beta unstable, it decays at some later date by electron or positron emission followed by gamma emission [2].

Nuclear reactions of low excitation energies include the following types : (n, $\gamma$ ), (n,p), (n, $\alpha$ ), ( $\alpha$ ,n), (p, $\gamma$ ), (p,n), (d,n), (d,p), ....etc.



In the first two reactions of the set (2) the outgoing particle is of the same kind as the incident particle, and the process is called scattering. The first reaction represents elastic scattering and the second reaction represents inelastic scattering in which the target nucleus (X) is raised into an excited state ( $X^*$ ). The other reactions of the set represent different

possible nuclear transmutations in which the product nuclei may be found in their ground states or, more often, in excited states. The excited product nucleus usually decays very quickly to the ground state with the emission of  $\gamma$ -rays.

### Cross Sections Of Nuclear Reactions:

To characterize the probability that a certain nuclear reaction will take place, it is customary to define an effective area of the nucleus for that reaction, called a cross section [1]. The reaction cross section data provides information of fundamental importance in the study of nuclear systems. The cross section is defined by [3]:

$$\sigma = N_r / N_i \quad \text{----- (3)}$$

where  $(\sigma)$  is the cross section,

$(N_r)$  is the number of reactions per unit time per nucleus.

$(N_i)$  is the number of incident particles per unit time per unit area,=flux

The cross section has the units of area and is of the order of the square of nuclear radius and a commonly used unit is the barn:

$$1 \text{ barn} = 10^{-24} \text{ cm}^2$$

In general, a given bombarding particle and target can react in a variety of ways producing a variety of light reaction products per unit time. The total cross section is then defined as [4]:

$$\sigma_{tot} = \sum_i \sigma_i \quad \text{----- (4)}$$

Where  $\sigma_i$  is the partial cross section for the process[1].

### 3. Reverse Reaction:

If the cross-sections of the reaction  $A(p,n)B$  are measured as a functions of  $T_p$  ( $T_p$  = Kinetic energy of incident proton), the cross-sections of the inverse reaction  $B(n,p)A$  can be calculated as a function of  $T_n$  ( $T_n$  = Kinetic energy of neutron) using the reciprocity theorem [5] which states that :

$$\frac{\sigma_{(p,n)}}{\sigma_{(n,p)}} \lambda_p^2 = \frac{\sigma_{(n,p)}}{g_{(n,p)} \lambda_n^2} \quad \text{----- (5)}$$

Where  $\sigma_{(p,n)}$  and  $\sigma_{(n,p)}$  represent cross-sections of  $A(p,n)B$  and  $B(n,p)A$  reactions respectively,  $g_{(p,n)}$  and  $g_{(n,p)}$  represent a statistical factors of  $A(p,n)B$  and  $B(n,p)A$  reactions respectively  $\lambda$  is the de-Broglie wave length divided by  $2\pi$  and is given by [6]

$$\lambda = \frac{\hbar}{MV} \quad \text{----- (6)}$$

Where  $\hbar$  is Dirac constant ( $h/2\pi$ ),  $h$  is Plank constant,  $M$  and  $V$  are mass and velocity of  $p$  or  $n$ .

From eq.(6),we have

$$\lambda^2 = \frac{\hbar^2}{2MT} \quad \text{----- (7)}$$

The statistical g-factors are givens by [5]

$$g_{(p,n)} = \frac{2J_c + 1}{(2I_A + 1)(2I_p + 1)} \quad \text{----- (8)}$$

and

$$g_{(n,p)} = \frac{2J_c + 1}{(2I_B + 1)(2I_n + 1)} \quad \text{----- (9)}$$

The conservation low of the momentum and parity implique that :

$$I_A + I_p = J_c = I_B + I_n \quad \text{----- (10)}$$

and

$$\pi_A \cdot \pi_p (-1)^{\ell_p} = \pi_c = \pi_B \cdot \pi_n (-1)^{\ell_n} \quad \text{----- (11)}$$

$J_c$  and  $\pi_c$  are total angular momentum and parity of the compound nucleus .

$I_A$  and  $\pi_A$  are total angular momentum and parity of nucleus  $A$ .

$I_B$  and  $\pi_B$  are total angular momentum and parity of nucleus  $B$ .

$I_p$  and  $\pi_p$  are total angular momentum and parity of proton.

$I_n$  and  $\pi_n$  are total angular momentum and parity of neutron .

$$\pi_p = \pi_n = +1 \quad \text{----- (12)}$$

$$I_p = s_p + \ell_p \quad \text{----- (13)}$$

$$I_n = s_n + \ell_n \quad \text{----- (14)}$$

where

$s_p$  is spin of proton =  $1/2$

$\ell_p$  is the orbital angular momentum of proton

$s_n$  is spin of neutron =  $\frac{1}{2}$

$\ell_n$  is the orbital angular momentum of neutron

From eq.(10),we have :

$$|J_c - I_A| \leq I_p \leq J_c + I_A \quad \text{---- (15)}$$

and

$$|J_c - I_B| \leq I_n \leq J_c + I_B \quad \text{---- (16)}$$

The reactions  $A(p,n)B$  and  $B(n,p)A$  can be represented with the compound nucleus C as in the following schematic diagram. It is clear that there are some important and useful relations between the kinetic energies of the neutron and proton.

$$E = S_p + \frac{M_A}{M_A + M_p} T_p \quad \text{---- (17a)}$$

$$E = S_n + \frac{M_B}{M_B + M_n} T_n \quad \text{---- (17b)}$$

One can calculate the separation energies of proton ( $S_p$ ) and neutron ( $S_n$ ) using the following relations:

$$S_p = 931.5 [M_A + M_p - M_c] \quad \text{---- (18)}$$

$$S_n = 931.5 [M_B + M_n - M_c] \quad \text{---- (19)}$$

Combining (17a), (17b), (18), (19)

And the equation of Q- value of the reaction  $A(p,n)B$  which is given by :

$$Q = 931.5 [M_A + M_p - M_B - M_n] \quad \text{---- (20)}$$

We get that :

$$Q = \frac{M_B}{M_B + M_n} T_n - \frac{M_A}{M_A + M_p} T_p \quad \text{---- (21)}$$

Or :

$$T_n = \frac{M_B + M_n}{M_B} \left[ \frac{M_A}{M_A + M_p} T_p + Q \right] \quad \text{---- (22)}$$

Then the threshold energy  $E_{th}$  is :

$$E_{th} = \left| -Q \frac{M_A + M_p}{M_A} \right| \quad \text{---- (23a)}$$

Or

$$Q = -\frac{M_A}{M_A + M_p} E_{th} \quad \text{---- (23b)}$$

Then

$$T_n = \frac{M_B + M_n}{M_B} \times \frac{M_A}{M_A + M_p} (T_p - E_{th}) \quad \text{---- (24)}$$

Thus eq . (5) can be written as follows:

$$\sigma_{(n,p)} = \frac{g_{(n,p)} M_p T_p}{g_{(p,n)} M_n T_n} \sigma_{(p,n)} \quad \text{---- (25)}$$

It is clear from this equation that the cross sections of reverse reaction are related by a variable parameters which can be calculated if the nuclear characteristics of the reactions are known.

### Results and Discussion:

The evaluated cross sections as a function of proton energy are listed in table (1) and these data are plotted in fig (2)[7] .

By using the reciprocity theory we derive the mathematical formula for  $^{64}\text{Cu}$  ( $n,p$ )  $^{64}\text{Ni}$  reaction by ground state :

$$\sigma_{(n,p)} = \frac{g_{(n,p)} M_p T_p}{g_{(p,n)} M_n T_n} \sigma_{(p,n)}$$

Where  $g_{(p,n)}=1$      $g_{(n,p)}=1/3$      $M_p=\text{proton mass}$   
 $M_n=\text{neutron mass}$

The evaluated cross sections as a function of neutron energy from (0.504 MeV) to (129.506MeV) of present work are listed in table (2) .These data are plotted in fig.(3). From fig . (3) we observed the maximum cross section in neutron energy (27.5043 MeV) is (0.3692 barn) to produced  $^{64}\text{Ni}$  by bombard  $^{64}\text{Cu}$  by neutron .

The cross sections are increased when neutron energy between(0.504 – 27.5043 )MeV but these are decreased smoothly and we get mathematical formula representing the cross sections distribution in the indicate range of neutron energy as follow :

$$y = 3.7*10^{-13}x^6 + 1.3*10^{-6}x^5 - 9*10^{-8}x^4 + 1.6*10^{-5}x^3 - 0.0011x^2 + 0.035x + 0.00068$$

### Reference:

- [1] Meyerhof W.E., (1967) "Elements of Nuclear Physics", McGraw- Hill Book Co.

- [2]Smith C. M. H., (1964), "Nuclear Physics " .
- [3] Cottingham W.N. and Greenwood D.A., (2001)," An Introduction to nuclear physic", 2<sup>nd</sup> ed. S Cambridge univ . press.
- [4] Jean L.B., James R., and Michel S., (2005),"Fundamentals in Nuclear physics" springer, p.14- 6 .
- [5] Macklin R.L. and Gibbons J.H. ;phys. Rev. 165 ,1147(1968).
- [6] Ebrahem S. A. ; " Calculation the Cross Sections of (*n,a*) and (*n,p*) Reactions by Using the Reciprocity Theory for the First Exited State" P-6, (2011).
- [7] Chiba, Chadwick,ate.; ENDF/B-VII Library ; (1999).

**Table (1):The cross sections of  $^{64}\text{Ni}$  (*p,n*) $^{64}\text{Cu}$  reaction as a function of proton energy**

p - energy (MeV)	Cross section s (mbar)	p - energy (MeV)	Cross section s (mbar)	p - energy (MeV)	Cross section s (mbar)
1.000	0.000	25.472 4	1.1545	51.000	1.0102
1.367	0.000	26.000	1.1563	52.000	1.001
2.000	0.0008	26.847 8	1.1579	52.224 2	0.999
2.4956	0.0087	27.000	1.1581	52.702 4	0.9946
3.000	0.0332	27.023 7	1.1581	53.000	0.992
4.000	0.1407	27.252	1.1583	53.201 5	0.9902
5.000	0.2719	28.000	1.1581	54.000	0.9832
6.000	0.3096	29.000	1.1568	54.058 7	0.9827
7.000	0.2947	29.055 2	1.1567	55.000	0.9747
7.285	0.3108	30.000	1.1542	55.169 8	0.9733
7.5493	0.333	30.587 6	1.1521	56.000	0.9667
8.000	0.3822	30.973 7	1.1507	57.000	0.9589
8.1398	0.4001	31.000	1.1506	57.530 2	0.955
8.245	0.4146	31.090 5	1.1503	58.000	0.9515
8.7951	0.4921	31.517 1	1.1487	59.000	0.9444
9.000	0.5154	32.000	1.1468	59.969 9	0.9379

10.000	0.6088	32.908 8	1.1429	60.000	0.9377
10.536 6	0.6549	33.000	1.1425	61.000	0.9311
11.000	0.6941	33.239 2	1.1414	62.000	0.9249
12.000	0.7757	33.845 5	1.1385	62.283 4	0.9232
12.746	0.8319	34.000	1.1377	62.609 4	0.9212
13.000	0.8498	35.000	1.1321	63.000	0.9189
14.000	0.9089	35.731 8	1.1277	63.959 4	0.9135
14.940 4	0.95	35.749 9	1.1276	64.000	0.9132
15.000	0.9522	36.000	1.1261	65.000	0.9079
15.269 2	0.9619	37.000	1.1200	65.871 4	0.9035
15.478 1	0.9693	38.000	1.1139	66.000	0.9028
16.000	0.9866	38.416 1	1.1113	67.000	0.898
16.072 4	0.9889	39.000	1.1077	67.648 4	0.895
16.405 2	0.9991	40.000	1.1016	68.000	0.8934
16.920 6	1.0142	40.973 5	1.0957	69.000	0.8891
17.000	1.0164	41.000	1.0955	70.000	0.885
18.000	1.0425	42.000	1.0894	71.000	0.8812
18.904 3	1.0636	42.202 4	1.0881	72.000	0.8775
19.000	1.0657	42.481 8	1.0864	73.000	0.874
19.107 6	1.0681	42.742 7	1.0848	74.000	0.8708
20.000	1.0865	42.924 3	1.0837	75.000	0.8677
21.000	1.1048	43.000	1.0832	75.814 9	0.8654
21.561	1.114	44.000	1.077	76.000	0.8649
21.672 5	1.1157	44.614	1.073	77.000	0.8621
22.000	1.1208	45.000	1.0705	78.000	0.8595
23.000	1.1343	46.000	1.0625	79.000	0.857
23.036 4	1.1347	47.000	1.0523	80.000	0.8547
23.047 5	1.1349	47.287 6	1.0491	81.000	0.8525
23.457 7	1.1394	47.558 1	1.046	82.000	0.8504
24.000	1.1446	48.000	1.041	83.000	0.8484
24.090 5	1.1454	49.000	1.0297	84.000	0.8465
25.000	1.152	50.000	1.0195	85.000	0.8447
25.301 7	1.1537	50.473 3	1.0151	86.000	0.843

**Table (1):The cross sections of  $^{64}\text{Ni}$  (*p,n*) $^{64}\text{Cu}$  reaction as a function of proton energy**

p - energy (MeV)	Cross sections (mbar)	p - energy (MeV)	Cross sections (mbar)	p - energy (MeV)	Cross sections (mbar)
87.00	0.841 3	97.000	0.827 8	114.00	0.810 5
88.00	0.839 7	98.000	0.826 6	116.00	0.808 6
89.00	0.838 2	99.000	0.825 5	118.00	0.806 7
90.00	0.836 8	100.00	0.824 4	120.00	0.804 9
91.00	0.835	102.00	0.822	122.00	0.803

0	4	0	3	0	1
92.00	0.834	104.00	0.820	124.00	0.801
0	0	0	2	0	3
93.00	0.832	106.00	0.818	126.00	0.799
0	7	0	2	0	5
94.00	0.831	108.00	0.816	128.00	0.797
0	4	0	2	0	7
95.00	0.830	110.00	0.814	130.00	0.795
0	2	0	3	0	9
96.00	0.829	112.00	0.812	132.00	0.794
0	0	4	0	0	1

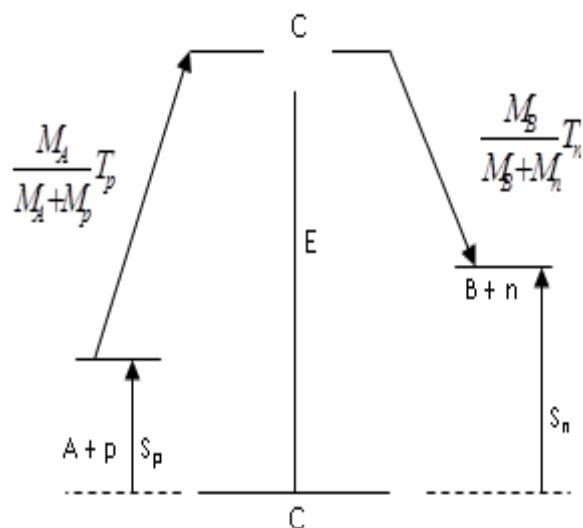
Table (2):The cross sections of  $^{64}\text{Cu}$  (n,p)  $^{64}\text{Ni}$  reaction as a function of neutron energy

n - energy (MeV)	Cross sections (barn)	n - energy (MeV)	Cross sections (barn)	n - energy (MeV)	Cross sections (barn)
-1.496	0.002	22.976	0.369	48.504	0.323
	8	6	3	6	1
-1.129	0.000	23.504	0.369	49.504	0.320
	3	3	9	6	2
-0.496	0	24.352	0.370	49.728	0.319
	1	1	4	8	5
-0.0004	0	24.504	0.370	50.207	0.318
	3	3	4	1	2
0.504	0.010	24.528	0.370	50.504	0.317
	6	4	6	6	3
1.504	0.045	24.756	0.370	50.706	0.316
	3	5	1	7	
2.504	0.087	25.504	0.370	51.504	0.314
	3	3	4	6	5
3.504	0.099	26.504	0.37	51.563	0.314
	3	3	37	3	3
4.5041	0.094	26.559	0.37	52.504	0.311
	3	5	7	8	
4.789	0.099	27.504	0.369	52.674	0.311
	4	3	2	4	3
5.0533	0.106	28.092	0.368	53.504	0.309
	5	5	7	2	
5.5041	0.122	28.478	0.368	54.504	0.306
	3	1	1	7	7
5.6439	0.128	28.504	0.368	55.034	0.305
	4	1	9	9	5
5.749	0.132	28.594	0.367	55.504	0.304
	6	9	9	7	4
6.2992	0.157	29.021	0.367	56.504	0.302
	4	4	4	7	1
6.5041	0.164	29.504	0.366	57.474	0.300
	9	4	8	6	0
7.5041	0.194	30.413	0.365	57.504	0.299
	7	1	6	7	9
8.0407	0.209	30.504	0.365	58.504	0.297
	5	4	5	7	8
8.5041	0.222	30.743	0.365	59.504	0.295
	6	1	7	8	
9.5041	0.248	31.349	0.364	59.788	0.295
	1	9	2	2	3
10.250	0.266	31.504	0.363	60.114	0.294
	1	4	9	2	7
10.504	0.271	32.504	0.362	60.504	0.293
	8	4	1	8	9
11.504	0.290	33.236	0.360	61.464	0.292
	1	7	2	7	2
12.444	0.303	33.254	0.360	61.504	0.292
	5	9	3	7	1
12.504	0.304	33.504	0.360	62.504	0.290
	2	6	4	2	8
12.773	0.307	34.504	0.358	63.376	0.289
	3	7	4	2	
12.982	0.310	35.504	0.356	63.504	0.288
	3	1	4	8	8
13.504	0.315	35.920	0.355	64.504	0.287
	2	6	6	5	2
13.576	0.316	36.504	0.354	65.153	0.286

5	3	5	3	2	3
13.909	0.319	37.504	0.352	65.504	0.285
4	6	5	4	8	8
14.424	0.324	38.478	0.350	66.504	0.284
8	4	5	5	8	4
14.504	0.325	38.504	0.350	67.504	0.283
2	1	5	4	8	1
15.504	0.333	39.504	0.348	68.504	0.281
2	5	5	5	8	9
16.408	0.340	39.706	0.348	69.504	0.280
5	2	9	5	9	7
16.504	0.340	39.986	0.347	70.504	0.279
2	9	3	5	9	6
16.611	0.341	40.247	0.347	71.504	0.278
8	6	2	7	9	5
17.504	0.347	40.428	0.346	72.504	0.277
2	5	8	6	9	6
18.504	0.353	40.504	0.346	73.319	0.276
2	4	5	5	8	8
19.065	0.356	41.504	0.344	73.504	0.276
2	3	5	5	9	6
19.176	0.356	42.118	0.343	74.504	0.275
8	9	5	2	9	8
19.504	0.358	42.504	0.342	75.504	0.274
2	5	5	4	9	9
20.504	0.362	43.504	0.339	76.504	0.274
3	8	5	9	9	1
20.540	0.363	44.504	0.336	77.505	0.273
7	6	6	6	7	4
20.551	0.363	44.792	0.335	78.505	0.272
8	2	6	7	7	7
20.961	0.364	45.062	0.334	79.505	0.272
9	5	7	6	7	
21.504	0.366	45.504	0.333	80.505	0.271
3	1	6	6	7	4
21.594	0.366	46.504	0.329	81.505	0.270
8	4	6	4	8	8
22.504	0.368	47.504	0.326	82.505	0.270
3	5	6	1	8	2
22.806	0.369	47.977	0.324	83.505	0.269
	9	7	7	8	7

Table (2):The cross sections of  $^{64}\text{Cu}(n,p)^{64}\text{Ni}$  reaction as a function of neutron energy

n-energy (MeV)	Cross section s (barn)	n-energy (MeV)	Cross section s (barn)	n-energy (MeV)	Cross section s (barn)		
84.505	0.2691	95.505	0.2644	115.50	0.2581		
85.505	1	0.2686	2	0.2641	6	0.2575	
86.505	1	0.2681	2	0.2637	6	0.2569	
87.505	1	0.2677	2	0.263	121.50	6	0.2563
88.505	1	0.2672	5	0.2624	6	0.2557	
89.505	1	0.2668	5	0.2617	6	0.2552	
90.505	1	0.2664	5	0.2611	6	0.2546	
91.505	1	0.2659	5	0.2605	6	0.254	
92.505	1	0.2656	5	0.2599	----	----	
93.505	2	0.2652	5	0.2592	----	----	
94.505	2	0.2648	5	0.2587	----	----	



Schematic diagram of the reactions

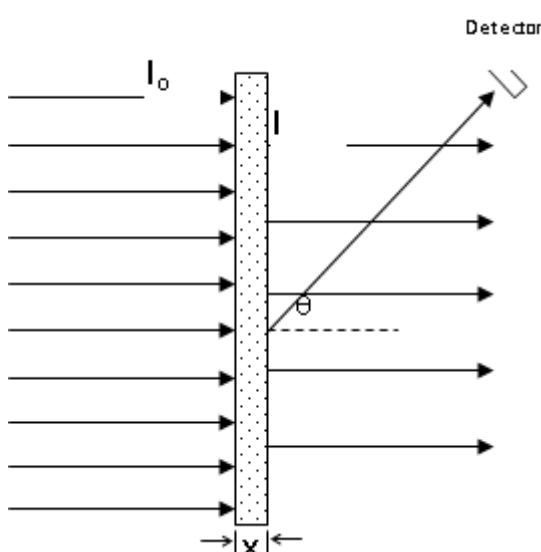
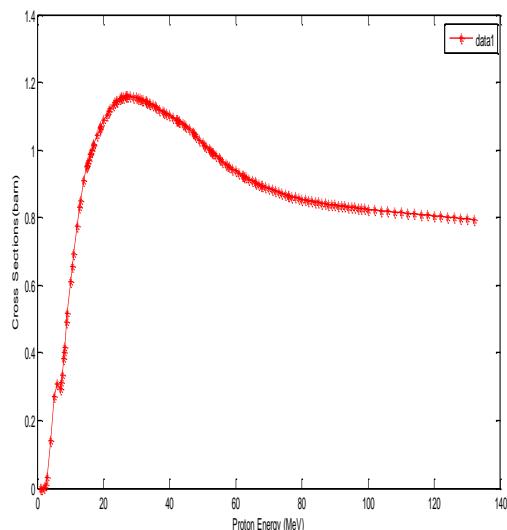
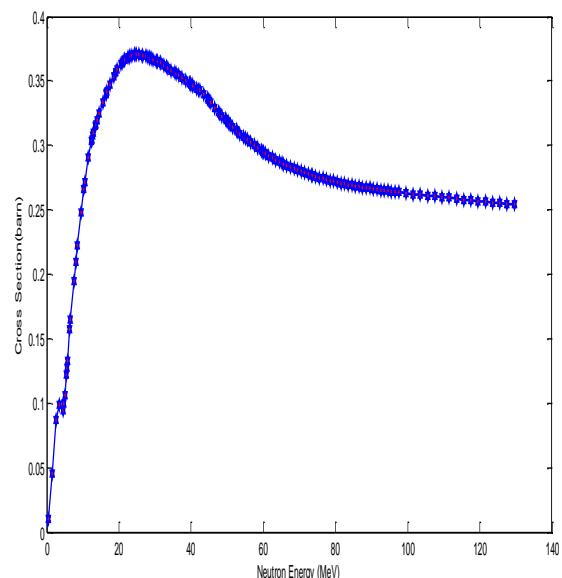


Figure (1): A schematic diagram illustrating the definition of total cross section in terms of the reduction of intensity



Figure(2): Cross section as a function of proton energy of  $^{64}\text{Ni}(p,n)^{64}\text{Cu}$  Reaction[7]



Figure(3): Cross section as a function of neutron energy of  $^{64}\text{Cu}(n,p)^{64}\text{Ni}$  Reaction by inverse reaction

## حساب المقاطع العرضية للتفاعل $^{64}\text{Cu}(\text{n},\text{p})^{64}\text{Ni}$ بواسطة نظرية المعاكس

سميرة أحمد أبراهيم      شيماء اكرم عباس      عادل محمود أبراهيم      أسراء اكرم عباس

E.mail:

الخلاصة

في هذه الدراسة تم حساب المقاطع العرضية للنوى المتوسطة  $^{64}\text{Ni}$  للبيانات المتوفرة في الادبيات العالمية وللمدى الطيفي من (1.0) MeV إلى (132.0) MeV وبطاقة عنبه مقدارها (2.496) MeV كدالة للمقاطع العرضية. تم حساب المقاطع العرضية للتفاعل  $^{64}\text{Cu}(\text{n},\text{p})^{64}\text{Ni}$  بواسطة تقنية التفاعل المعاكس وتم تحديد الاحتمالية الاكبر لأنتج النikel لأهميته الكبيرة في التقدم التكنولوجي. بواسطة التفاعل المعاكس تم استئناف المعادلة . قيم المقاطع العرضية كدالة لطاقة النيوترون تتراوح ما بين ( $E_n=0.504\text{MeV}$ ) إلى ( $E_n=129.506\text{MeV}$ ) . على التوالي اما العامل الأحصائي فهو ( $g_{n,p}=1$ ) و ( $g_{p,n}=1/3$ ) .