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Development of a mathematical model for signal processing using laboratory data

In this paper, we consider a mathematical model for the interpretation of the radarograms which obtained by GPR systems. As noted in [1–3], in addition to testing the algorithms, it is necessary to compare the calculated data of the mathematical model with the real data obtained from the GPR. One of the reasons preventing the spread of GPR technologies is the complexity of data interpretation, which requires the involvement of highly qualified specialists. In connection with this research as a mathematical model and a comparison with the real data of the GPR in an ideal layered medium, will provide a method for interpreting radarograms. We have conducted a series of experimental studies using the Loza – A georadar at the newly created laboratory ground. A distinctive feature of these studies is the choice of several localized objects in the form of iron sheets placed in an ideal layered medium, namely in clean dry sand. The choice of such an environment is necessary for testing the algorithms, the mathematical models developed by us for determining the depth of localized several objects. A series of experimental studies were conducted using georadar and a number of radarograms were obtained to study the depth of objects. A cycle of calculations was carried out to verify the conformity of the results of mathematical modeling with real georadar data. Key words: electrodynamics equation, magnetic permeability, dobeshi wavelets, medium conductivity, dielectric permeability, Maxwell equation.

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1 Problem statement. Mathematical model

One of the main reasons preventing the wide spread of georadar technologies is the complexity of data interpretation, which at the present stage requires the involvement of highly qualified specialists. The way out of this situation is to create a mathematical apparatus for solving the inverse problem of radar sensing, which will minimize the operator's participation in obtaining the final result, as well as extract the maximum amount of information from georadar data. In connection with this research as a mathematical model and comparison with real data of ground-penetrating radar in the ideal layered medium, will provide the methodology of the interpretation of the GPR.

This kind of problems are related to the inverse and incorrect problems, the foundations of which were laid in the theory of the work Tikhonova, M. M. Lavrenteva, V. K. Ivanova, V. G. Romanova.

One of the main obstacles in the localization of underground facilities is the upper part of the soil lying above the desired objects. Passing through this area, the electromagnetic waves reflected from various objects interact with each other, can be amplified or, conversely, mutually reduced. One way to overcome this problem is to continue to solve the system of Maxwell's equations from the earth's surface in the direction of the location of the desired objects. The problem of continuation is one of the most difficult and incorrect problems of mathematical physics, complicated in this case by the presence of attenuation of the electromagnetic field in conducting media. Problems of continuation of solutions of equations of mathematical physics from the part of the boundary in many cases are strongly ill-posed problems in the classes of finite smoothness functions and are the first step in solving the coefficient inverse problems [4–7]. The approach of regularization of the field problem was proposed in the paper by V. Kozlov, V. G. Mazya, and V. Fomin in 1991 [8].

Consider the system of Maxwell's equations [9]:

$$\begin{cases} \varepsilon\frac{\partial E}{\partial t} - rot\, H + \sigma E + j^{cm} = 0, \\ \mu\frac{\partial H}{\partial t} + rot\, E = 0, \end{cases} \qquad (x,y,z) \in R^3, \quad x \neq 0, \quad t > 0. \tag{1}$$

There are positive functions $\varepsilon(x,y,z)$, $\sigma(x,y,z)$, $\mu(x,y,z)$ and the permittivity, conductivity and magnetic permeability of the medium, respectively.

$$R^3_- = \left\{ x,y,z \in R^3, \quad x < 0 \right\} - \text{air}, \; R^3_+ = \left\{ x,y,z \in R^3, \quad x > 0 \right\} - \text{earth}.$$

We consider that electromagnetic oscillations up to the moment of time $t = 0$ are absent:

$$(E,H) \mid_{t<0} \equiv 0 j^{cm} \mid_{t<0} \equiv 0$$

and then induced by a side current $j^{cm}(x,y,z,t)$.

Let us consider one of the simplest variants of the problem, when ε, σ and μ depend only on the depth x and one horizontal variable y, and the source of the external current is a sufficiently long (infinite) cable located in the center and stretched along the z axis:

$$j^{cm}(x,y,z,t) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} g(x,y)\; V(t). \tag{2}$$

Here, the function $g(x,y)$ describes the transverse dimensions of the source.

In this case, ignoring the influence of the cable ends, we conclude that only three components E_z,H_x, H_y. remain nonzero in the system of Maxwell equations.

After exclusion of the first equation of partial derivatives of the H_x component and H_y obtain regarding $E_z$the second order equation:

$$\mu\varepsilon\frac{\partial^2 E_z}{\partial t^2} + \mu\sigma\frac{\partial E_z}{\partial t} = \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} - g(x,y)V'(t), \tag{3}$$

to which we add the initial condition

$$E_z|_{t=0} = 0, \quad \left.\frac{\partial E_z}{\partial t}\right|_{t=0} = 0;$$

boundary conditions:

$$\left.\frac{\partial E_z}{\partial x}\right|_{x=x_1} = 0, \quad E_z|_{x=x_2} = 0, \; E_z\big|_{y=-y_1} = 0, \quad E_z\Big|_{y=+y_1} = 0$$

and conditions at the interface of media:

$$[E_z]_{x=x_1} = 0, \quad \left[\frac{\partial E_z}{\partial x}\right]_{x=x_2} = 0, [E_z]_{x=x_3} = 0, \quad \left[\frac{\partial E_z}{\partial x}\right]_{x=x_3} = 0,$$

where $\varepsilon = \varepsilon_0\varepsilon_{rel}$; $\varepsilon_0 = 8.854 \cdot 10^{-12} F/m$ dielectric constant; $\varepsilon_{rel}=$ relative dielectric constant

$$\mu_0 = 4\pi \cdot 10^{-7} g/m\sigma \;- sm/m, V(t) = \exp\left\{ -\frac{(t-t_0)^2}{t_1^2} \right\}, g(x,y) = \theta(a-x)\theta(a-y),$$

$a = 0.025\ m$ — the size of the source. $V'(t)$ — function describing the source of electromagnetic oscillations emitted by the transmitting antenna.

2 Numerical calculation

Consider a homogeneous medium at a site of 10 by 12 meters with an inclusion size of 0.6 by 0.4 m. The capacity of the pit at a depth of 0.6 m.

Parameters of homogeneous medium

Dry sand	$\varepsilon_1 = 6$	$\sigma_1 = 0.62$	$h_1 = 0.6$ m
Rectagular iron sheet thickness	$\varepsilon_2 = 1$	$\sigma_2 = 0.769 \cdot 10^7$	$h_2 = 0.005$ m
Wet sand	$\varepsilon_3 = 40$	$\sigma_3 = 0.005$	$h_3 = 0.5$ m or more

Calculation parameters: step x is equal to 0.01; step y equal to 0.01. The time step is considered from the Courant condition. The calculation time is from 0 to 60 ns. The problem is solved by a finite-difference method using an explicit scheme.

First, we consider a homogeneous medium without inclusions. Calculate field $E_z^{(1)}(x = 0, y, t)$.
Then consider the environment with inclusion and calculate the total field $E_z^{(2)}(x = 0, y, t)$. The anomalous field allows you to see the reflections from the localized object in the form of a hodograph (Fig. 1). In the case of two localized objects (Fig. 2).

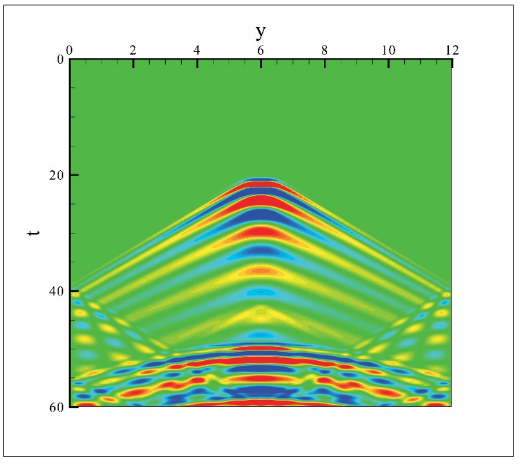


Figure 1. Anomalous field of a localized single object in a homogeneous medium

The calculated results are in good agreement with the measured GPR a GPR.

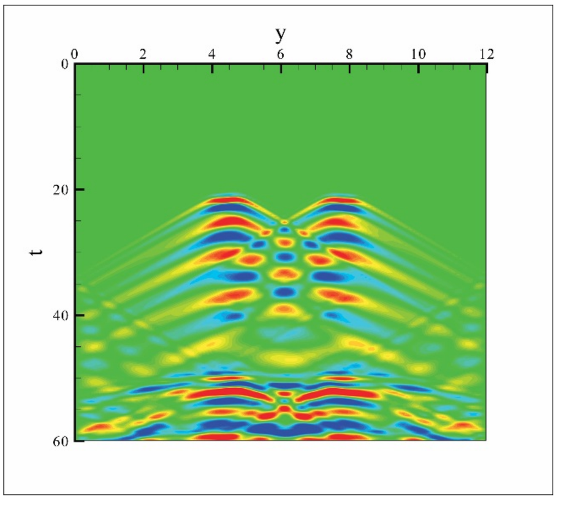


Figure 2. Anomalous field of two localized objects in a homogeneous medium

The results of the calculations are qualitatively the same as the radarogram measured by the georadar.

3 Processing of georadar signals. Cleaning the route from noise using wavelet transform

To improve the noise immunity of the georadar method, as a rule, pre-processing of experimental measurements is performed in order to isolate informative signals. The essence of the processing of georadar data is, first of all, in the allocation of a useful signal on the background of noise and noise. To distinguish useful signals, the difference between their characteristics and the corresponding characteristics of noise and interference waves is used [10, 11].

One of the ways of the primary processing of the radargram is the wavelet transform. With the help of wavelet digital signal conversion in the radar can reduce the influence of high-frequency components in the spectrum of the signal.

One of the ways of processing of the radargram is the wavelet transform. With the help of the digital signal wavelet transform in the radargram, it is possible to remove high-frequency components from the signal spectrum.

The wavelet transform of a one-dimensional signal is its representation as a generalized series or Fourier integral over a system of basis functions [12]:

$$\varphi_{ab}\left(t\right)=\left|a\right|^{-1/2}\varphi\left(\frac{t-b}{a}\right)$$

constructed from the parent (generating) wavelet $\psi(t)$, due to time - shift operations – b and time-scale changes – a. In the study, the signal is represented as a set of successive approximations of coarse(approximating) $Aj(t)$and refined(detailing) components:

$$f\left(t\right)=A_j\left(t\right)+\sum_{i=1}^{j}D_i(t)$$

with subsequent refinement by iterative method. Each step of refinement corresponds to a certain scale, that is, the level $j$of analysis (decomposition) and synthesis (reconstruction) of the signal. This representation of each component of the signal by wavelets can be considered in both the time and frequency domains. In a multiscale analysis, the signal $f(t)$ decomposes into two components:

$$f\left(t\right)=\sum_k a_k\ \varphi_k\left(t\right)+\sum_k d_k\ \psi_k\left(t\right).$$

The basis functions $\varphi(t)$ and $\psi(t)$are uniquely determined by the coefficients h_l:

$$\varphi\left(t\right)=2\sum_l h_l\varphi(2t-l);$$

$$\psi\left(t\right)=2\sum_l g_l\psi(2t-l).$$

In the transition from the current scale j to the next $j+1$, the number of wavelet coefficients is halved, and they are determined by the recurrence relations:

$$a_{j+1,k}=\sum_l h_{l-2,k}a_{j,k};$$

$$d_{j+1,k}=\sum_l g_{l-2,k}a_{j,k},$$

where

$$g_l=\left(-1\right)^l h_{2n-l-1}.$$

When restoring (reconstructing) a signal by its wavelet coefficients, the process proceeds from large to small scales and is described by the expression [12]:

$$a_{j-1,k}=\sum_l\left(h_{k-2l}a_{j,l}+g_{k-2l}a_{j,l}\right).$$

Daubechies wavelet DB4 were used to clear the signal from noise [13]. The Daubechies wavelets do not have analytical expressions and are determined only by the filters. In practical applications, approximating h_k and detailing g_k wavelet coefficients are used, without calculating the specific shape of the wavelets [14]. For Daubechies wavelet db4 the factors are the following: Decomposition into components of discrete Daubechies wavelets is carried out according to the formulas

$$a_i = h_0 s_{2i-1} + h_1 s_{2i} + h_2 s_{2i+1} + h_3 s_{2i+2};$$

$$d_i = g_0 s_{2i-1} + g_1 s_{2i} + g_2 s_{2i+1} + g_3 s_{2i+2} \, i=1,\ 2,\ldots,\ n/2\text{-}1;$$

$$a_{n/2} = h_0 s_{n-2} + h_1 s_{n-1} + h_2 s_0 + h_3 s_1;$$

$$d_{n/2} = g_0 s_{n-2} + g_1 s_{n-1} + g_2 s_0 + g_3 s_1.$$

Formula (1) is a pyramidal algorithm for calculating the wavelet coefficients of Mall [14]. These formulas digital filter h_n from the s_k signal allocates low frequencies, and the filter g_n allocates the upper frequencies. Wavelet transform the track radargram has been used on a software-controlled threshold processing of detail coefficients (thresholding). The algorithms thresholding the adaptive threshold limits established for each factor on the criterion of Stein's unbiased risk estimation (Stein's unbiased risk estimation) [15].

When restoring (reconstructing) a signal by its wavelet coefficients, the process proceeds from large to small scales and is described by formulas at each step [16]:

$$a_1 = h_2 s_{\frac{n}{2}} + h_1 s_n + h_0 s_1 + h_3 s_{\frac{n}{2}-1};$$

$$a_2 = g_0 s_{\frac{n}{2}} + g_1 s_{\frac{n}{2}-1} + g_2 s_1 + g_3 s_n;$$

$$a_i = h_2 s_{\frac{i-1}{2}} + h_1 s_{\frac{i-1}{2}+\frac{n}{2}} + h_0 s_{\frac{i-1}{2}+1} + h_3 s_{\frac{i-1}{2}+\frac{n}{2}-1} i = 3, 5, \ldots n/2 - -1 (i = 4, 6, ... n/2 \ \ (odd);$$

$$a_i = g_0 s_{\frac{i-1}{2}} + g_1 s_{\frac{i-1}{2}+\frac{n}{2}-1} + g_2 s_{\frac{i-1}{2}+1} + g_3 s_{\frac{i-1}{2}+\frac{n}{2}} i = 4, 6, \ldots n/2 \ \ (even).$$

provided that the detailing coefficients of the previous levels are recorded in place of the signal values.

The results of the transformation track of Daubechies wavelets is presented in Figure 3.

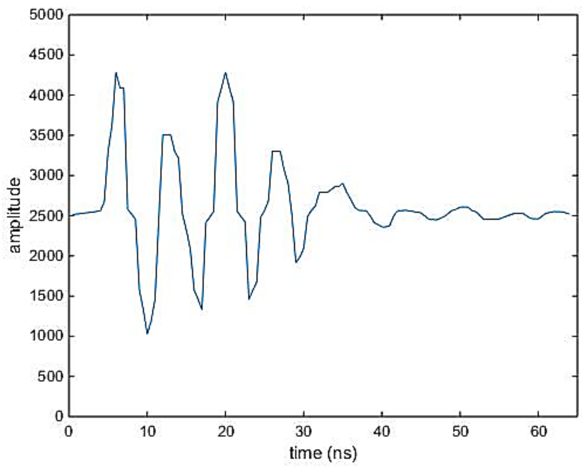


Figure 3. Chart of the radargram route after the wavelet transform

4 Testing of the model based on the device Loza - B. Experimental studies

To test the algorithm of the mathematical model to detect the depth of the objects and study its physical properties, a laboratory polygon was created, located 80 kilometers from Astana along the Kurgaldzhinskaya highway. A sand pit was chosen for the landfill, which corresponds to the model of the environment: air; clean dry sand; targets; and the underlying layer (wet sand), see figure 4 left fragment). The size of the pit: length 0.6 m.; width 0.5 m.; depth 0.65 m. the dimensions of the target - iron sheet rectangular: width 0.3 m; length 0.4 m.; thickness 0.005 m. Cm. Figure 4 (left fragment). The target is placed at a depth of 0.6 m. for georadar studies, it is necessary to mark the site where the object is located, see Figure 4 (right fragment).

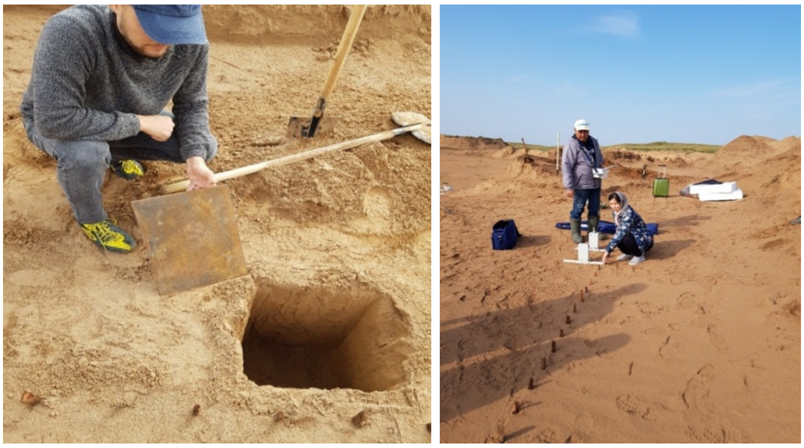


Figure 4. Metal disk

In order to test the algorithm, which will be given in the next section below, and to verify the results of numerical calculations for the detection of localized objects, a test experiment was conducted to detect a localized object using a georadar.

Georadar Loza - V was measured with the diversity of antennas. The measurements consist of 20 points, starts with 0 point and ends with 20 point. The transmitting antenna (source) is at point 0 and the receiving antenna measures at all other points. Then the transmitting antenna (source) is moved to the next point, and the receiving antenna goes through all the other points. And so continue until 20 point. See Figure 4 (right fragment), Figure 5. All data were digitized and summed. Then a comparative analysis of the measured data with the results of solving the inverse problem of detecting localized objects was carried out.

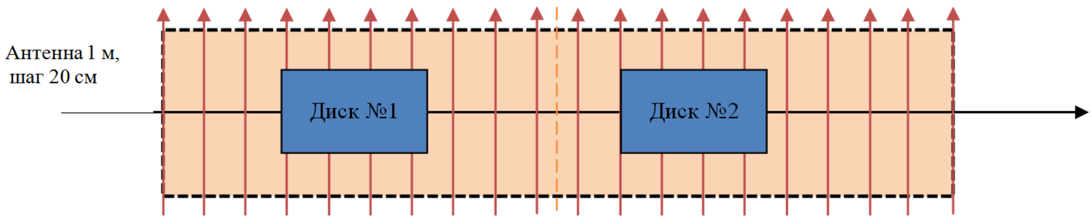


Figure 5. Scheme of the experiment with antenna diversity

Figure 6 shows the results of measurements carried out by the device in a homogeneous medium, i.e. without a target. This is necessary for us to analyze the radargrams obtained directly from the media in which the targets are placed.

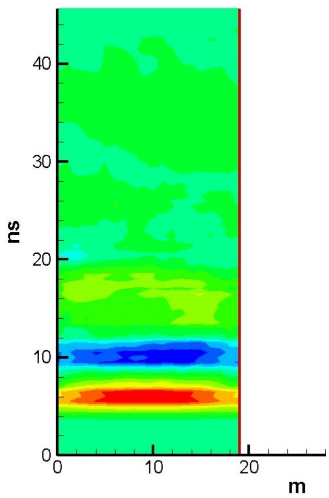
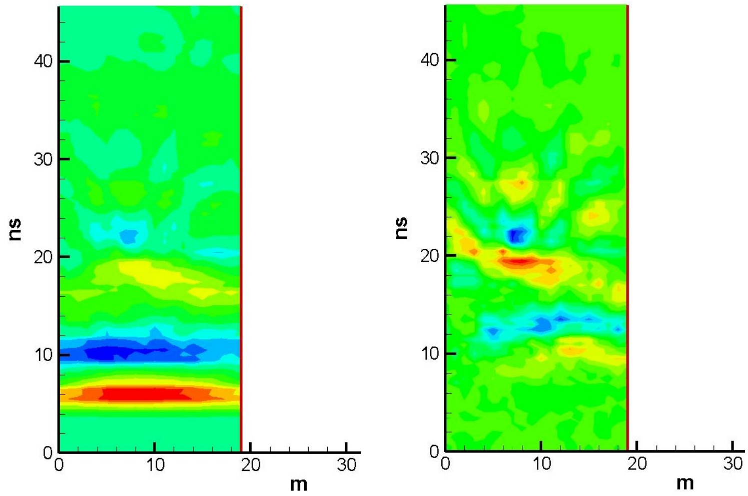


Figure 6. The radargram of a homogeneous medium

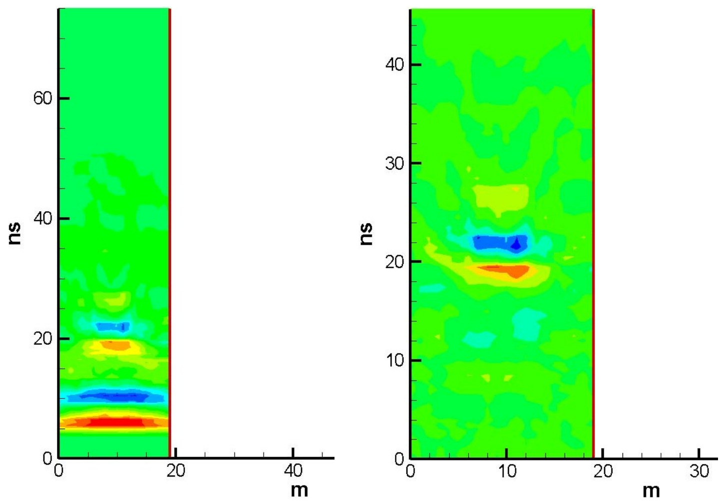
Figure 7 shows the results of measurements carried out by GPR according to the above scheme. Researched the area in which is hidden a single object – an iron sheet of a rectangular shape.



a b

Figure 7. a) Radargram of the area with one hidden object-iron sheet; b) The difference of the radargrams received from the environment with the target and without it on the same site

Similar experimental studies were carried out on a site in which two identical objects are hidden – iron sheets at a depth of 60 cm and at a distance from each other relative to the day surface by 20 cm.the results of the studies in the form of radargrams are presented in Figure 8 (a) with targets and in Figure 8 (b), the result is the same as in the past case of the difference.



a b

Figure 8. a) radargram of the area with hidden two identical objects-iron sheet; b) the difference of radargrams obtained from the medium with and without targets on the same site

By the type of the hodograph it is possible to assume, what layers with what parameters are in the studied environment. You can also use a hodograph to determine that a localized object is present in your environment, and you can specify its location.

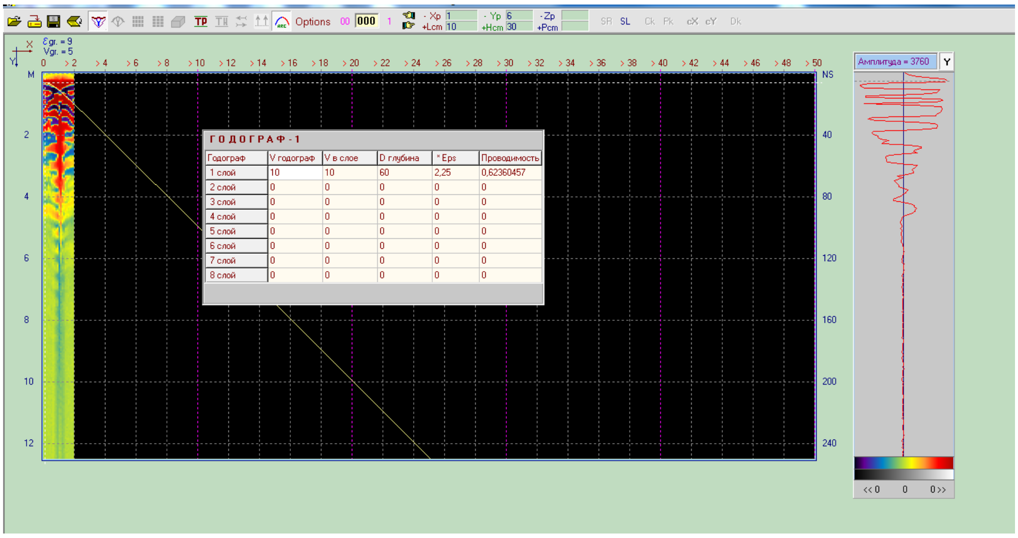


Figure 9. Radargram of the experiment

According to the hodograph, the depth of the localized object is determined, which is equal to 60 cm. (Fig. 9). In this paper, a mathematical model to determine the depth of localized objects is constructed.

A series of experimental studies, with the use of georadar Loza-B. a cycle of calculations to verify the compliance of the results of mathematical modeling of real georadar data.

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С.И. Кабанихин, Қ.Т. Искаков, Б.Б. Шолпанбаев, М.А. Шишленин, Д.Қ. Тоқсейіт

Зертханалық деректерді пайдалана отырып дабылды өңдеу үшін математикалық модельді әзірлеу

Мақалада георадиолокациялау жүйесімен алынған радар сигналдарын түсіндіру үшін математикалық модель қарастырылған. [1–3] жұмыстарда көрсетілгендей, алгоритмдерді тестілеуге қосымша математикалық модельдің есептік деректерін георадардан алынған нақты деректермен салыстыру қажет. Георадар технологияларының таралуына кедергі келтіретін себептердің бірі — жоғары білікті мамандарды тартуды талап ететін деректерді түсіндірудің күрделілігі. Осыған байланысты математикалық модельді және георадардың нақты деректерімен салыстыра отырып, идеалды деңгейдегі ортаны зерттеу радарограммаларды түсіндіру әдісін ұсынады.

Кілт сөздер: электрдинамика теңдеуі, магнит өткізгіштігі, Добеши вейвлеттері, ортаның өткізгіштігі, диэлектрлік өткізгіштігі, Максвелл теңдеуі.

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Разработка математической модели по обработке сигнала с использованием данных лабораторных исследований

В статье рассмотрена математическая модель для интерпретации радарограмм, полученных георадиолокационной системой. Как отмечено в работах [1–3], помимо апробации алгоритмов, необходимо сопоставить расчетные данные математической модели с реальными данными полученных от георадара. Одной из причин, препятствующих распространению георадарных технологий, является сложность интерпретации данных, требующая привлечения высококвалифицированных специалистов. В связи с этим исследования математической модели, а также сопоставление с реальными данными георадара в идеальной слоистой среде позволит получить методику интерпретации радарограмм.

Ключевые слова: уравнение электродинамики, магнитная проницаемость, вейвлеты Добеши, проводимость среды, диэлектрическая проницаемость, уравнение Максвелла.

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