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A.R.Yeshkeyev

Ye.A.Buketov Karaganda State University;
Institute of Applied Mathematics, Karaganda
(E-mail: Modth1705@mail.ru)

Properties of a stability for positive Jonsson theories

Actually, we study the connections of the Δ-*PM*-theories with their centers in the enrich signature. The properties of various companions of some Δ-*PM*-theories and their connection with this theory are considered on the language of the central types of positive Jonsson theory.

Key words: Jonsson theory, existentially closed model, forking, central type, the syntactic and semantic similarity of jonsson theories.

The main result of this article is the following theorem.

Theorem 1.2. Let $T-\Delta-PM$-theory, α-Jonsson, perfect, complete for $\Sigma_{\alpha+1}$ sentences. Then the following conditions are equivalent:

1) the ratio *PJNF* satisfies axioms 1–7 relatively theory *T*;

2) T^* stable and for any $p \in P$, $A \in A$ $((p, A) \in P\backslash JNF \Leftrightarrow p$ does not forks over *A* (in the sense of Shelah).

Under the above notions, for example we obtained the following.

Theorem 2.1. Let $T\Sigma_{\alpha+1}$ — complete, perfect $\Delta-PM$ theory. Thenthe following conditions are equivalent:

1) theory $T^c - P$-λ-stable in the sense of [1];

2) theory $T^* - P$-λ-stable.

In studying the properties of forking for positive Jonsson theories weconsidered axiomatic approach. Such analogs was considered in [2], respectively, for Jonsson theory.

Introduction

It is well known that using the concept of the forking outstanding specialist in the Model Theory S.Shelah was resolved the problem of classification of complete theories regarding the spectrum. Thus the concept of forking is a very important concept in the Model Theory. But, at the same time, it should be noted that the above concept of forking was determined to complete theories.

This article describes an attempt to transfer the concept a forking to the certain class of theories, which generally are not complete, but at the same time, this class is wide enough and natural.

Jonsson's conditions are the natural algebraic requirements that arise in studying a wide class of algebras. To Jonsson's properties satisfied such theories as group theory, the theory of Abelian groups, the theory of fields of fixed characteristic, the theory of Boolean algebras, the theory of ordered sets, thetheory of polygons (*S*-Acts, where *S* is a monoid), and many others. Let us to recall the definition of Jonsson theory.

Definition 1. The theory *T* is Jonsson's theory if it satisfies to followingconditions:

1) *T* has a infinite model;

2) *T* it is inductive;

3) *T* admits a joint embedding property (*JEP*);

4) *T* admits a property of an amalgamation (*AP*).

As is evident from the of the list, that the obtaining technic of such results for Jonsson theories applying can be quite broad. In this paper, the object of our research will focus on a class of theories related to the notion jonssoness and positivity. A subject of there search such theories related to the so-called «Eastern» model theory. This conventional definition and separation of the general model theory into two main areas: «Western» and «East» well-known expert in model theory Keisler H.J., identified in his survey article [3]. However, he notes that the western model theory studies the complete theories, and the eastern model theory correspondingly Jonsson theories. This work is a review of results concerning researches of notion of some kind of positive Jonsson theories and its a class of models. All necessary information about Jonsson theories can be found in [2, 4–7].

In the work [8] of I.Ben-Yaacov, was introduced a positive model theory, and within it were considered so-called *CAT*s. One can find as that syntactic feature of this work is the elimination of symbols of an universal quantifier and a negation in the basic formulas. Semantic feature is to consider as morphisms of continuations and immersions. It is easy to note that the problematic of positive Jonsson's theories and *CAT*s are very dense connected.

We recall the following definition concerning some particular type of positive Jonsson theory.

Let L be a first-order language.At is the set of atomic formulas of the language. B^+ *(At)* — with respect to a closed set of positive Boolean combinations (conjunction and disjunction) of all atomic formulas and their subformulae change of variables. $L^+= Q\ (B^+(At))$ is a set of formulas in normal prenex form obtained by applying quantifiers (∀ and ∃) to B^+ (*At*). We mean a formula positive if it belongs to L^+. Axiomatizable theory is called positive if its axioms are positive. $B\ (L^+)$ is an arbitrary Boolean combination of formulas L^+. When $\Delta=B$ *(At)* we get the usual Jonsson theory with the only difference that it has only positive axioms.

Let $0 \leq n \leq \omega$. Let Π_n^+ — the set of all formulas of a language L^+withthe form ∀∃...φ (i.e. the formulae from with a change of quantifiers beginning from ∀).

Let $\Delta \subseteq \Pi_n^+ \subseteq L^+$.

Recall the following definition of some kind of positive theory from [4].

Definition 2. Theory T is called Δ-positive mustafinien-(Δ-*PM*)-theory, if

1) theory$T$ has infinite model;

2) theory$T$ is Π_{n+2}^+ axiomatizable;

3) theory$T$ admits $\Delta - JEP$;

4) theory$T$ admits $\Delta - AP$.

Definition 3. The T theory is called Δ − mustafinien (Δ − *M*)-theory, if in the definition 2 we considered as morphisms only immersions following [8].

Remark: If the length of prefix of considered axioms exactly equal to two, then the above definitions 2 and 3 give to us, respectively, definitions of Δ-positive Jonsson (Δ − *PJ*) theory and Δ-Jonsson (Δ − *J*) theory.

We should say that presenting results about models of Δ − *PJ*-theories [4] which are positive generalization of Jonsson's theories, if they are, in general,such, because the are exist the samples of non-Jonsson, but positive Jonssonin any type above mentioned meaning. But we will not go beyond the first-order. Even in the case where Δ − *PJ*-theory is not Jonsson, uses the idea ofthe generalization of a semantic method [4] for Jonsson theories. The essenceof this generalization is that properties of Δ − *PJ*-central completion will betranslated on Δ − *PJ*-preimage.

If Δ − *PJ*-theory is Jonssonien, we will to work with the *ModT* like withthe class of models of a Jonsson theory. If Δ − *PJ*-theory is not Jonssonien, then as with the *ModT*, we consider E_T^+ — a positive class of existentially closedmodels of this theory. This approach for the class E_T — class of existentiallyclosed models of any universal theory T has been studied in [9]. Since relatively Jonsson Theories there are two possibilities: the perfect and imperfect cases ofones, we will adhere to the following. It is known in the [4] that if the Jonssontheory T is perfect, the class of its existentially closed models E_T elementaryand coincides with the *ModT**, where *T**— its center. In the opposite case, i.e, if the theory T is does not perfect, we do as in the [9], i.e instead of *ModT* weare working with the class E_T^+. When we consider an arbitrary Δ − *PJ*-theory *T*, the class E_T^+ considered as extension of the class E_T (both classes are alwaysavailable), and depending on the perfectness and incompleteness of the model-theoretic properties of a class E_T^+ represent interest. In this article usuallywe considered that Δ − *PJ*-theory are Δ − *PJ*-perfect, and it is a natural generalization of perfectness in Jonsson's case. It is clear that all results for Δ−*PM*-theories one can trivial transfer to other types of positive Jonssontheories (Δ − *PJ*, Δ − *M*, Δ − *J*), so we will prove just for Δ − *PM*-case ofpositive Jonsson theory.

The greatest progress has been made in the description of perfect Jonsson theories [4]. It is turn out that when theory is perfect then its center became a model companion of this theory. The idea of central-type dates back to the various enrichments of signatureand types of expressions through their forgetting in the old signature. And in the first and second cases, these ideas allow you to transfer the basic model-theoretic concepts defined for complete theories, theories on Jonsson and positive generalizations that generally incomplete.

The idea of the central type appears when considering enriched signature.

Let T an arbitrary $\Delta - PM$-theory in the language of the signature σ. Let C-semantic model theory of T. $A \subseteq C$. Let $\sigma_\Gamma(A) = \sigma \cup \{c_a | a \in A\} \cup \Gamma$ where $\Gamma = \{P\} \cup \{c\}$. Consider the following theory $T_\Gamma^{PM}(A) = Th_{\Pi_{\alpha+2}^+}(C,a)_{a \in A} \cup \{P(c)\} \cup \{"P \subseteq "\}$, where $\{"P \subseteq "\}$ there are infinite number of sentences, which says that the interpretation of characters P has positively existentially closed sub model in the signature σ. This theory is not necessarily complete.

Through S_Γ^{PM} denote the set of all $\sum_{\alpha+1}^+$ – completions. Theory T is $P - \lambda$-stable if

$\left|S_\Gamma^{PM}\right| \le \lambda$ for any A, such that $|A| \le \lambda$.

Let us consider all completions of the center T^* of the theory T in the new

Signature σ_Γ where $\Gamma = \{c\}$. By virtue $\Delta - PM$-ness of the theory T^*, there is its center, and we denote it as T^c. When restricted T^c to the signature σ, the theory T^c becomes a complete type. This type we call the central type of theory T.

§1 About forking in the class of the $\Delta - PM$ theories

Our aim is to define the concept of forking by axiomatic way for the $\Delta - PM$ theory when it perfect α–Jonsson theory. We go by generalizing the results of [10, 2]. Give the following definitions.

Definition 1.1. Let $M - \sum_{\alpha+1}^+ -$ saturated Δ-positive $\alpha+1-$existentiallyclosed model of cardinality k (k large enough cardinal) $\Delta - PM$ theory ofT ($\sum_{\alpha+1}^+$ saturation is saturation-type relatively $\sum_{\alpha+1}^+$ to its capacity). Recall that the model M of the theory T is called Δ-positive existentiallyclosed if for each Δ-homomorphism and every $a \in M$, and $\varphi(\bar{x},\bar{y}) \in \Delta : N \models \exists \bar{y}\, \varphi(f(\bar{a}),\bar{y}) \Rightarrow M \models \exists \bar{y}\, \varphi(\bar{a},\bar{y})$. Let $T\Delta - PM$ theory, $S^{PM}(X)$ the set of all positive $\sum_{\alpha+1}^+$ complete n-types, over X joint with T for each finite n.

Let A — class of all subsets of the M, P class of all $\sum_{\alpha+1}^+$ types (not necessarilycomplete), let $PJNF \subseteq P \times A$ — a binary relation. We impose on the $PJNF$ (positive Jonsson nonforking) the following axioms:

Axiom 1. If $(p, A) \in PJNF$, $f \in Aut(M)$, $f(A) = B$, that $(f(p), B) \in PJNF$.

Axiom 2. If $(p, A) \in PJNF$, $q \subseteq p$, then $(q, A) \in PJNF$.

Axiom 3. If $A \subseteq B \subseteq C$, $p \in S^{PM}(C)$, then $(p, A) \in PJNF \Leftrightarrow (p, B) \in PJNF$ and $(p|B, A) \in PJNF$.

Axiom 4. If $A \subseteq B, dom(p) \subseteq B, (p, A) \in PJNF$, then $\exists q \in S^{PM}(B)$ ($p \in q$ and $(q, A) \in PJNF$).

Axiom 5. There is a cardinal μ such that if $A \subseteq B \subseteq C$, $p \in S^{PM}(B), (p, A) \in PJNF$, then $\left|\{q \in S^{PM}(C) : p \subseteq q \text{ and } (q, A) \in PJNF\}\right| < \mu$.

Axiom 6. There is a cardinal ρ such that $\forall p \in P, \forall A \in A$ if $(p, A) \in PJNF$, then $\exists A_1 \subseteq A, (|A_1| < \rho \text{ and} (p, A_1) \in PJNF)$.

Axiom 7. If $p \in S^{PM}(A)$, then $(p, A) \in PJNF$.

The classical notion of forking belongs to Shelah.

Definition 1.2. Set of formulas $\{\varphi(\bar{x},\bar{a}_i) : i < k\} = p$ called k–in consistent for some positive integer k, if every finite subset p of cardinality k is inconsistent, $ie \models \neg\bar{x}\left(\varphi\left(\bar{x},\overline{a_{i_1}}\right) \wedge \ldots \wedge \varphi\left(\bar{x},\overline{a_{i_k}}\right)\right)$ for each $i_1 < \ldots < i_k < k$.

Partial type on a variety of relatively $k \in \omega$ divisible if there is a formula $\varphi(\bar{x},\bar{a})$ and a sequence $\langle a_i : i \in \omega \rangle$ such that:

1) $p \vdash \varphi(\bar{x},\bar{a})$;

2) $tp(\bar{a} / A) = tp(\bar{a}_i / A)$ for all i;

3) $\{\varphi(\bar{x},\bar{a}_i): i\in\omega\}$ k — is incompatible.

Also *p* divided over *A* the relatively certain *k*. In addition *p* is forked over *A* in *T* if there are exist formulas $\varphi_0(\bar{x},\bar{a}_0),...,\varphi_n(\bar{x},\bar{a}_n)$ such that:

(i) $p|=\bigcup_{0\le i\le n}\varphi_i(\bar{x},\bar{a}_n)$;

(ii) $\varphi_i(\bar{x},\bar{a}_i)$ is divided over *A* for each *i*.

Following agreement is important. In fact, we will talk about the semantic aspect of the $\Delta-PM$-theory. If the $\Delta-PM$-theory of *T* is α–Jonsson, then with $ModT$ we work as with the class some models Jonsson theory. If the $\Delta-PM$-theory of *T* is not α–Jonsson, then as a class $ModT$ we will consider it positively existentially closed models $\sum_{\alpha+1}^{+}T$. This approach for class $\sum_{\alpha+1}^{+}T$. of existentially closed models of arbitrary universal theory *T* was considered in [9]. Since relatively Jonsson theories are two possible cases: perfect and imperfect, we will adhere to the following. It is well known [4] that if the theory *T* perfect Jonsson, the class of its existentially closed models of elementary and coincides with $ModT^*$, where T^* is its center. Otherwise, i.e. if the theory *T* is not perfect, we proceed in a similar [9], but instead $ModT$ working with the class $\sum_{\alpha+1}^{+}T$. This class is considered as an extension E^T class of existentially closedmodels (both classes always exist), and depending on the theory of T perfectand imperfect model-theoretic properties of a class of $\sum_{\alpha+1}^{+}T$ special interest.In this article, when considered Δ considered $\Delta-PM$-theory $\Delta-PM$ – are perfect, which is a natural generalization of perfect sense in Jonsson.

Definition 1.3. Following [3], we say that the model $A\in K$ is simple in theclass *K*, if for any $B\in K$ such that there exists a homomorphism $h: A\to B$, that is an h embedding. We say that the theory *T* satisfies the condition (*S*), ifeach model $A\in K$ is simple in the class *K*. In [3] observed that (*S*) is equivalentto the syntactic properties: (S') is «Each existential formula *L* is equivalent *T* to some positive existential formula».

Easy to see that not Jonsson $\Delta-PM$-theory *T* into force of the agreement $ModT$ satisfies the property (S').

We will use in the proof of Theorem 1.2. The following results:

Theorem 1.1. (Ramsey F.P.). Let *I* be an infinite set, $n<\omega$, $|I|^n$ the family of all subsets of the set I, which consists precisely of the n elements. If $|I|^n=A_0\cup...\cup A_{k-1}$, $k<\omega$, $A_i\cap A_j=\varnothing$ with $i<j<k$ there exists an infinite $J\subset I$ such that $|J|^n\subset A_i$ for some $i<k$.

Lemma 1.1. [10, Lemma 14.9]. Let *T* stable theory, *M* saturated modelof the power μ^+ types $p_1,p_2\in S(M)$ each does not forks over *A*. Then if $p_1\restriction A=p_2\restriction A$ there exists an *A* elementary identity monomorphism *f* suchthat $f(d_1)\sim d_2$, where d_1,d_2 the schema defining p_1,p_2 respectively.

The class of all Δ positive α + 1-existentially closed models of the theory *T* is denoted by $\sum_{\alpha+1}^{+}T$.

Definition 1.4. We say that the $\Delta-PM$-theory T $PM-\lambda$ is stable if for any model $A\in\sum_{\alpha+1}^{+}T$, any subset X of set $A|X|\le\lambda\Rightarrow|S^{PM}(X)|\le\lambda$. $\Delta-PM$-theory *T PM*-stable if it is $PM-\lambda$-stable, for some λ.

Theorem 1.2. Let $T-\Delta-PM$ theory, α Jonsson, perfect, complete for $\sum_{\alpha+1}$ sentences. Then the following conditions are equivalent:

1) the ratio *PJNF* satisfies axioms 1–7 relatively theory *T*;

2) T^* stable and not for any $p\in P, A\in A((p,A)\in PJNF\Leftrightarrow p$ does not forks over *A* (in the sense of Shelah).

Proof.

1 ⇒ 2. Let $\lambda=2^{\rho\cdot|T|\cdot\mu}$, where λ, ρ, μ are the cardinals, which corresponded to axioms 1–7. We now show that T $P-\lambda$ — is stable. Then, by [11], we have that $T^*\lambda$ is stable. Obviously, that $\lambda^\rho=\lambda$. Let $|A|=\lambda$. If $p\in S^{PM}(A)$, then, by Axiom 7, $(p,A)\in PJNF$ and by Axiom 6 there is $A_p\subseteq A$ such that

$\left|A_p\right| < \rho$ and $(p, A_p) \in PJNF$. Then by Axiom 3 $(p \upharpoonright A_p, A) \in PJNF$. We denote $p \upharpoonright A_p$. Although $g(p)$. By axiom $5 \left|\left\{q \in S^{PM}(A) : g(q) = g(p)\right\}\right| < \mu$. Consequently $\left|S^{PM}(A)\right| \le \left|\left\{g(p) : p \in SPM(A)\right\}\right| \cdot \mu \le \left|A^{\rho}\right| \cdot 2^{\rho \cdot |T|} \cdot$ $\cdot \mu \le \lambda^{p} \cdot \lambda \cdot \lambda = \lambda^{\rho} = \lambda.$ Consequently, T $PM - \lambda$ stable. And we conclude that $T^{*}\lambda$-stable by [11]. Suppose now $(p, A) \in PJNF$. We show that ρ it does not a forks overA. Let $B = dom(p)$. Then by Axiom 4, there exists $q \in SPM(B)$ such that and $(q, A) \in PJNF$. We prove that q does not forks over A (then p doesnot forks over A by Axiom 2). Assume the contrary. Then by the perfectness of theory T and definitions 1.2., and also (S') there is a finite set of positive existential formulas Σ_0^{+} such that $q \left|- \bigcup \left\{\varphi : \varphi \in \Sigma_0^{+}\right\}\right.$ and each formula $\varphi \in \Sigma_0^{+}$ is divided over A. Let $C = B \cup D$, D — the set of constants appearing at least one of the formulas of Σ_0^{+}. By Axiom 4, there exists $q_0 \in S^{PM}(C)$ such that $q \in q_0$ and $(q_0, A) \in PJNF$. Clearly, therefore $q_0 \left|- \bigcup \left\{\varphi : \varphi \in \Sigma_0^{+}\right\}\right.$, there $\varphi(\overline{x}, \overline{a}) \in q_0 \cap \Sigma_0^{+}$. Using Theorem 1.1., Compactness theorem and divisibility $\varphi(\overline{x}, \overline{a})$ over A, we can show the existence of a sequence $\left\langle \overline{a_\alpha} : \alpha < \mu^{+} \right\rangle$ and elementary monomorphisms $f_\alpha, \alpha < \mu^{+}$ identical to $\overline{a_0} = \overline{a}, \overline{a_\alpha} = f_\alpha(\overline{a}), \alpha < \mu^{+}$ — and $\left\{\varphi(\overline{x}, \overline{a_\alpha}) : \alpha < \mu^{+}\right\}$ such that k is inconsistent for some $< \omega$.

Let $E = C \cup \left\{\overline{a_\alpha} : \alpha < \mu^{+}\right\}$, $q_\alpha = f_\alpha(q_0)$. $0 < \alpha < \mu^{+}$. By Axiom 1, $(q_0, A) \in PJNF$, $\alpha < \mu^{+}$, by Axiom 4, there exist $q_\alpha^{'} \in S^{PM}(E)$ such that $q_\alpha \subseteq q_\alpha^{'}$ and $\left(q_\alpha^{'}, A\right) \in PJNF$. Clearly, that $\varphi(\overline{x}, \overline{a_\alpha}) \in q_\alpha^{'}, q_\alpha \subseteq q_\alpha^{'}, \alpha < \mu^{+}$. We have $\left|\left\{q_\alpha^{'} : \alpha < \mu^{+}\right\}\right| = \mu^{+}$ as $\left\{\varphi\left(\overline{x}, \overline{a_\alpha}\right) : \alpha < \mu^{+}\right\}$ k-incompatible. This contradicts the axiom 5. Consequently, q does not forks over A. Thus, we have that if $(p, A) \in PJNF$ than p does not forks over A.

Let us prove the opposite direction. Let P not forking over A. Since the theory T is perfect that T^{*}, it is model-complete [8], and enough for us to workonly with existential types, furthermore into force (S') with positive existential types, and consider $\Sigma_{\alpha+1}^{+}$-saturated positive — α + 1 existentially closed models of the theory T. We need to prove that $(p, A) \in PJNF$. Let $M \supseteq A$, $M \supseteq dom(p)$, $\left|M\right| > 2^{\rho \cdot |T| \cdot \mu}$ and M is $\Sigma_{\alpha+1}^{+}$-saturated model of the theory, T^{*}, $t \in S^{PM}(M)$, $p \subseteq t, t$ does not forks over A. By Axiom 7 $(t \upharpoonright A, A) \in$ $\in PJNF$, and by axiom 5 there is $q \in SPJ(M)$ that $q \supseteq t \upharpoonright A$ and $(q, A) \in PJNF$. As shown above $(q, A) \in PJNF$ implies that q does not forks over A. By Lemma 1 there is automorphisms f of model M identical on A such that $y = f(q)$. Then, by Axiom 1 $(t, A) \in PJNF$ and Axiom 2 $(p, A) \in PJNF$. Therefore, 1 ⇒ 2 is proved.

2 ⇒ 1. Since the center of the theory T, namely, T^{*} is a complete theory, then you can apply forking properties in the sense of Shelah. For example, as inthe proof of Theorem 19.1 (2 ⇒ 1) [10]. The results (analogs of axioms 1–7 forcomplete theory) can be easily limited to generalizations of the correspondingconcepts in α-Jonsson sense.

§2 Stable properties of a central-type for Δ-PM-theory

In this section we give a proof of the fact that the properties of central stable types as the stability in the usual sense for centers with additional predicate coincides with the stability in the sense of PM with additional predicate.

D well on the fact that the predicate is highlighted. At one time a Frenchmathematician B.Poizat [12] defined the concept of elementary pair of models. In fact it is a model in which as an elementary submodel describes the implementation of a single predicate symbol. Later Mustafin T.G. introduced theconcept of T-stability [13], which generalizes the notion of an elementary pairabove. The latest achievement in this issue is the notion of E-stability [14] introduced and considered Palyutin E.A. Concept of an E-stability differs from theconcept of T-stability, in the sense that it is stable with respect to definability. Recall that in the classical case if the theory is stable, then any type definable.

We introduce the following notation:

Let T is an arbitrary $\Delta-PM$ theory in the language of the signature σ. Let *C* semantic model of theory *T*. $A \subseteq C$. Let $\sigma_\Gamma(A) = \sigma \bigcup \{c_a | a \in A\} \bigcup \Gamma$ where $\Gamma = \{P\} \bigcup \{c\}$. Consider the following theory $T_\Gamma^{PM}(A) = Th_{\Pi^+_{\alpha+2}}(C,a)_{a\in A} \bigcup \{P(c)\} \bigcup \{"P \subseteq"\}$, where $\{"P \subseteq"\}$ there are infinite number of sentences, which says that the interpretation of characters *P* have positively existentially closed submodel in the signature σ. This theory to necessarily complete. Therefore it may be the finite model. The requirement of existential isolation submodel is not accidental. This is due to the fact that the sub-model in our reasoning is bound to be endless. And any existentially closed model is infinite by definition.

Through S_Γ^{PM} the set of all $\Sigma^+_{\alpha+1}$-completions, theory $T - P - \lambda$-stable if $|S_\Gamma^{PM}| \le \lambda$ for any *A*. Such that $|A| \le \lambda$.

Let us consider all completions of a center T^* theory *T* in the new signature σ_Γ where $\Gamma = \{c\}$. Due to the fact that this theory *T* satisfied of a condition $\Delta-PM$ theory, that enrich the language does not change. Further, due to the fact that the condition *T* quite as α-Jonsson theory, that T^* — is a $\Delta - PM$ theory.

Then there is its center, and it is one of the completions of the theory T^* in the rich language. This center we denote it as T^c. When restricted T^c to the signature σ, the theory T^c becomes a complete type. This type we call as thecentral type theory *T*.

Under the above definitions, we obtain the following.

Theorem. Let *T* is $\Sigma_{\alpha+1}$-complete, perfect $\Delta - PM$ theory. Then thefollowing conditions are equivalent:

1) theory $T^c - P - \lambda$-stable in the sense of [1];

2) theory $T^* - PM - \lambda$-stable.

Proof. From 1) ⇒ 2) the proof is trivial, if the completions are not more than λ, then $\Sigma^+_{\alpha+1}$-completions obviously are not more than λ.

We prove from 2) to 1). Let theory $T^* - PM - \lambda$-stable. This is equivalentto saying that $T_\Gamma^{PM}(A)$ in the signature $\sigma_p(A) = \sigma_A \bigcup \{P\}$ is equal to the thecorresponding hull of Kaiser, denoting by *T0*. By the perfectness of the theory *T*, we have that $T_0 = T^*$ and $\sum^+_{\alpha+1} T = ModT^*$ ([4]) and then $T_\Gamma^{PM}(A) = T_0$ is a perfect Jonsson theory. Let the theory T_0 has no more than $\lambda \sum^+_{\alpha+1}$-completions. The center of theory T in the new signature $\sigma_p(A) = \sigma_A \bigcup \{P\}$ will be equals to $Th(C,a)_{a\in A} \bigcup \{P(c_a) | a \in A\} \{"P \le"\}$. We have show thatall completions of T^* are no more λ. There by $T^* P - \lambda$ stable (in the sense of [9]). Understand what is due of T^* is not complete in the new signature. Adding constants gives only inessential extension that will does not change thenumber of types of existentially closed submodels of *C*. The significant roll play simplementation predicate *P*. In this case, implementation of the predicate *P* is an elementary submodel *M* of model *C*. Since semantic model *C* of α−Jonssontheory of *T* is existentially closed [4], in view of the predicate *P* in *C* ($M \le C$) follows that $M \in \sum^+_{\alpha+1} T$. Let us consider an arbitrary completion *T′* of thetheory T^* in a new signature. By the definition T^*, there exists a model $Mof \sum^+_{\alpha+1} T$ such that $T' = Th(C, M, a)_{a\in A}$ where *M* — interpretation of the predicate *P* in the semantic model *C*. We have that $T' = Th(C, M, a)_{a\in A}$ is an Jonsson theory. In this case, by model completeness of *T′* any formula in *T′* is equivalent to some an existential formula in the *T′*. Then by $\sum^+_{\alpha+1}$-completeness of the theory of *T*, the number of the completions by 2) not more than λ. Thisproves the assertion.

Conclusion. Note that since the theory, complete for existential sentences satisfies the joint embedding property (*JEP*), but the converse is not true one conclude that $\sum^+_{\alpha+1}$-completeness condition in Theorem can not be removed. Due to thefact that there is a continuum non-elementary equivalent between themselves existentially closed groups, and group theory is Jonsson, it can be concluded that the condition of the perfectness in the theorem can not remove.

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А.Р.Ешкеев

Позитивті йонсондық теориялардың стабильдік қасиеттері

Мақалада $\Delta - PM$-теориялар мен олардың орталықтарының байланыстары зерттелді. Осы $\Delta - PM$-теорияның және оның неше түрлі компаньондарының қасиеттері позитивті йонсондық теориялардың орталық типтердің тілінде қарастырылды.

А.Р.Ешкеев

Стабильные свойства позитивных йонсоновских теорий

В статье изучены связи $\Delta - PM$-теорий и их центров в обогащенной сигнатуре. На языке центральных типов позитивной йонсоновской теории рассмотрены свойства различных компаньонов $\Delta - PM$-теории с самой теорией.