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# On Pseudo Regular Spherical Fuzzy Graphs

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**Abstract**

In this paper, pseudo regular and totally pseudo regular spherical fuzzy graphs are defined using pseudo degree and total pseudo degree of a vertex. A necessary and sufficient condition for pseudo regular spherical fuzzy graph is given. Some properties of pseudo regular and totally pseudo regular spherical fuzzy graphs are derived. Theorems related to these concepts are stated and proved.

**Keywords:** pseudo degree; pseudo regular; totally pseudo regular; spherical fuzzy graphs

**2010 AMS subject classification**: 05C22, 05C90.‡

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# 1. Introduction

Zadeh [16] introduced the concept of fuzzy set (FS) as a generalization of the crisp set, and it has numerous applications in networking, decision making, telecommunication, artificial intelligence, management sciences, computer science, social science, and the chemical industry. The fuzzy set theory was proposed in order to deal with problems involving a lack of information and impreciseness. It can deal with uncertainty and vague problems whose membership functions lie in the closed interval [0, 1]. Atanassov [2] proposed the intuitionistic fuzzy sets (IFSs) as an extension of Zadeh's [16] fuzzy set theory. Cuong [4, 5] initiated the concept of the picture fuzzy set (PFS) as a direct extension of intuitonistic fuzzy sets, which may be adequate in cases when human opinions are of types: yes, abstain, no, and refusal. Mahmood et al. [10] introduced the concept of spherical fuzzy set which gives an additional strength to the concept of picture fuzzy set by enlarging the space for the grades for all the four parameters. Kifayat et al. [7] studied the geometrical comparison of fuzzy sets, intuitionistic fuzzy sets, Pythagorean fuzzy sets, picture fuzzy sets with spherical fuzzy sets. Cen Zuo, et al. [15] introduced the some new concepts of picture fuzzy graph. Akram et.al [1] introduced the notion of spherical fuzzy graphs and Abhishek Guleria [6] also introduced generalized version spherical fuzzy graphs using T-spherical fuzzy sets. Mordeson and Peng [11] introduced some operations on fuzzy graphs. Mohamed Harif and Nazeera Begam [8] defined regular spherical fuzzy graphs and studied some properties of regular spherical fuzzy graphs. Muhammad Shoaib et al. [9] introduced complex spherical fuzzy graphs. In this paper, pseudo regular and totally pseudo regular spherical fuzzy graphs are defined using pseudo degree and total pseudo degree of a vertex. A necessary and sufficient condition for pseudo regular spherical fuzzy graph is given. Some comparative study between pseudo regular and totally pseudo regular spherical fuzzy graphs are done.

# 2. Preliminaries

**Definition 2.1[10]** A spherical fuzzy set $S$ in $U$ (universe of discourse) is given by $S = \{< \alpha, \mu_S(\alpha), \eta_S(\alpha), \nu_S(\alpha) >: \alpha \in U\}$ where $\mu_S: U \to [0,1]$, $\eta_S: U \to [0,1]$ and $\nu_S: U \to [0,1]$ denote degree of membership, degree of neutral membership and degree of non-membership respectively, and for each $\alpha \in U$ satisfying the condition $0 \leq \mu_S^2(\alpha)+\eta_S^2(\alpha)+ \nu_S^2(\alpha) \leq 1$. The degree of refusal for any spherical fuzzy set $S$ and $\alpha \in U$ is given by $r_S(\alpha) = \sqrt{1 - (\mu_S^2(\alpha) + \eta_S^2(\alpha) + \nu_S^2(\alpha))}$.

**Definition 2.2 [1]** A spherical fuzzy graph (SFG) $\mathcal{G} = (N, L)$ where

- $N = \{v_1, v_2, \ldots, v_n\}$ such that $\sigma_1: N \to [0,1]$, $\sigma_2: N \to [0,1]$ and $\sigma_3: N \to [0,1]$ denote the degree of membership, degree of neutral membership and degree of

non-membership of each element $v_i \in N$ respectively, and
$0 \leq \sigma_1^2(v_i) + \sigma_2^2(v_i) + \sigma_3^2(v_i) \leq 1$, for every $v_i \in N$,$(i = 1,2,3 \ldots, n)$.

- $L \subseteq N \times N$ where $\mu_1: L \to [0,1]$, $\mu_2: L \to [0,1]$ and $\mu_3: L \to [0,1]$ are such that

$$\mu_1(u_i, u_j) \leq min\{\sigma_1(u_i), \sigma_1(u_j)\},$$
$$\mu_2(u_i, u_j) \leq min\{\sigma_2(u_i), \sigma_2(u_j)\},$$
$$\mu_3(u_i, u_j) \leq max\{\sigma_3(u_i), \sigma_3(u_j)\}$$

$0 \leq \mu_1^2(u_i, u_j) + \mu_2^2(u_i, u_j) + \mu_3^2(u_i, u_j) \leq 1$, for every $(u_i, u_j) \in L$, $(i = 1,2,3 \ldots, n)$.

**Definition 2.3 [1]** Let $\mathcal{G} = (N, L)$ be a SFG. The degree of a vertex $u$ of a SFG is

$$d_{\mathcal{G}}(u) = \left(\sum_{u \neq v} \mu_1(u, v), \sum_{u \neq v} \mu_2(u, v), \sum_{u \neq v} \mu_3(u, v)\right).$$

**Definition 2.4[1]** The minimum degree of a SFG $\mathcal{G} = (N, L)$ is defined by $\delta(\mathcal{G}) = (\delta_1(\mathcal{G}), \delta_2(\mathcal{G}), \delta_3(\mathcal{G}))$, where

$$\delta_1(\mathcal{G}) = min\{d_1(u)/\, u \in N\},$$
$$\delta_2(\mathcal{G}) = min\{d_2(u)/\, u \in N\}$$
$$\delta_3(\mathcal{G}) = min\{d_3(u)/\, u \in N\}.$$

**Definition 2.5 [1]** The maximum degree of a SFG $\mathcal{G} = (N, L)$ is defined by $\Delta(\mathcal{G}) = (\Delta_1(\mathcal{G}), \Delta_2(\mathcal{G}), \Delta_3(\mathcal{G}))$ where

$$\Delta_1(\mathcal{G}) = max\{d_1(u)/\, u \in N\}\},$$
$$\Delta_2(\mathcal{G}) = max\{d_2(u)/\, u \in N\},$$
$$\Delta_3(\mathcal{G}) = max\{d_3(u)/\, u \in N\}.$$

**Definition 2.6 [8]** Let $\mathcal{G} = (N, L)$ be a SFG. If $d_{\mathcal{G}}(u) = (c_1, c_2, c_3)$, $\forall\, u \in N$. (i,e) each vertex has same degree $(c_1, c_2, c_3)$, then $\mathcal{G}$ is said to be regular spherical fuzzy graph (RSFG) of degree $(c_1, c_2, c_3)$ or $(c_1, c_2, c_3) -$ regular spherical fuzzy graph.

**Definition 2.7 [8]** Let $\mathcal{G} = (N, L)$ be a SFG, then the order of $\mathcal{G}$ is denoted by $O(\mathcal{G})$ and defined by

$$O(\mathcal{G}) = \left(\sum_{v \in N} \sigma_1(v), \sum_{v \in N} \sigma_2(v), \sum_{v \in N} \sigma_3(v)\right).$$

**Definition 2.8 [8]** Let $\mathcal{G} = (N, L)$ be a SFG, then the size of $\mathcal{G}$ is denoted by $S(\mathcal{G})$ and defined by

$$S(\mathcal{G}) = \left(\sum_{uv \in L} \mu_1(uv), \sum_{uv \in L} \mu_2(uv), \sum_{uv \in L} \mu_3(uv)\right).$$

**Definition 2.9 [3]** Let $G^* = (V, E)$ be a crisp graph. Then the 2-degree of $v$ is defined as the sum of the degrees of the vertices adjacent to $v$ and it is denoted by $t(v)$.

**Definition 2.10 [14]** Let $G^* = (V, E)$ be a crisp graph. The average degree of $v$ is defined as $\frac{t(v)}{d(v)}$ where $t(v)$ is the $2-$degree of $v$ and $d(v)$ is the degree of $v$ and it is denoted by $d_a(v)$.

**Definition 2.11 [14]** A graph $G^* = (V, E)$ is called pseudo regular if every vertex of $\mathcal{G}$ has equal average degree.

# 3. Pseudo regular spherical fuzzy graphs

**Definition 3.1** Let $\mathcal{G} = (N, L)$ be a spherical fuzzy graph on $G^* = (V, E)$. The $2-$degree of a vertex $v$ in $\mathcal{G}$ is defined as the sum of degrees of the vertices adjacent to $v$ and is denoted by $t_{\mathcal{G}}(v) = (t_1(v), t_2(v), t_3(v))$ where $t_i(v) = \sum d_i(u) = \sum_{u \neq w} \mu_i(u, w)$; $i = 1, 2, 3$ and $d_i(u)$ is the degree of the vertex $u$ which is adjacent with the vertex $v$.

**Definition 3.2** Let $\mathcal{G} = (N, L)$ be a SFG on $G^* = (V, E)$. A pseudo (average) degree of a vertex $v$ in spherical fuzzy graph $\mathcal{G}$ is denoted by $d_{\mathcal{G}}^a(v)$ and is defined by $d_{\mathcal{G}}^a(v) = (d_1^a(v), d_2^a(v), d_3^a(v))$ where $d_i^a(v) = \frac{t_i(v)}{d_{G^*}(v)}$, $i = 1, 2, 3$ and $d_{G^*}(v)$ is the number of edges incident at $v$.

**Definition 3.3** Let $\mathcal{G} = (N, L)$ be a SFG on $G^* = (V, E)$. If $d_{\mathcal{G}}^a(v) = (k_1, k_2, k_3)$ for all $v \in N$ then $\mathcal{G}$ is called $(k_1, k_2, k_3)$-pseudo regular spherical fuzzy graph or $(k_1, k_2, k_3)-$PRSFG.

**Example 3.4** Consider a SFG $\mathcal{G}$ on $G^* = (V, E)$ given in figure 3.1.

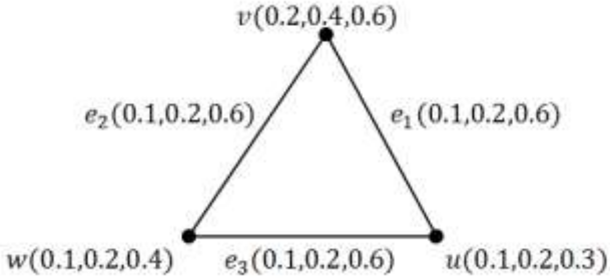


**Figure 3.1:** Pseudo Regular Spherical Fuzzy Graph $\mathcal{G}$

Here, $d(u) = (0.2,0.4,1.2)$, $d(v) = (0.2,0.4,1.2)$, $d(w) = (0.2,0.4,1.0)$ and $d_{G^*}(u) = 2$, for all $u \in N$.

Now $u$ is adjacent to $v$ and $w$.

Then $d_{\mathcal{G}}^a(u) = (0.2,0.4,1.2)$.

Similarly, $d_{\mathcal{G}}^a(v) = d_{\mathcal{G}}^a(w) = (0.2,0.4,1.2)$.

Thus $\mathcal{G}$ is $(0.2,0.4,1.2)-$PRSFG.

**Definition 3.5** Let $\mathcal{G} = (N, L)$ be a spherical fuzzy graph on $G^* = (V, E)$. The total pseudo degree of a vertex $v$ in $\mathcal{G}$ is denoted by $td_{\mathcal{G}}^a(v)$ and is defined by $td_{\mathcal{G}}^a(v) = \left(td_1^a(v), td_2^a(v), td_3^a(v)\right)$ where $td_i^a(v) = d_i^a(v) + \sigma_i(v)$ for all $v \in V$ and $i = 1, 2, 3$.

**Definition 3.6** Let $\mathcal{G} = (N, L)$ be a spherical fuzzy graph on $G^* = (V, E)$. If all the vertices of $G$ have the same total pseudo degree $(k_1, k_2, k_3)$, then $\mathcal{G}$ is said to be a $(k_1, k_2, k_3)$-totally Pseudo regular spherical fuzzy graph $\left((k_1, k_2, k_3) - TPRSFG\right)$.

**Example 3.7** Consider a SFG $\mathcal{G}$ on $G^* = (V, E)$ given in figure 3.2.

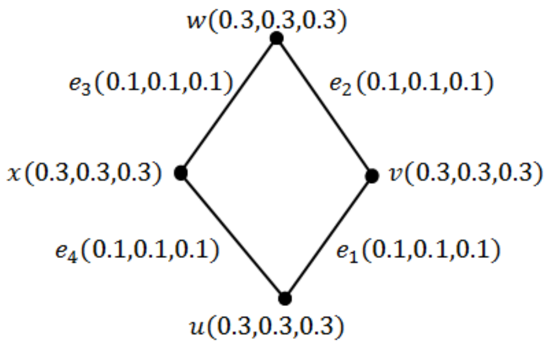


**Figure 3.2:** Totally Pseudo Regular Spherical Fuzzy Graph $\mathcal{G}$

Here $d_{\mathcal{G}}^a(u) = d_{\mathcal{G}}^a(v) = d_{\mathcal{G}}^a(w) = d_{\mathcal{G}}^a(x) = (0.2,0.2,0.2)$ and $td_{\mathcal{G}}^a(u) = (0.5,0.5,0.5)$ for all $u \in V$. Hence $\mathcal{G}$ is $(0.5,0.5,0.5)$ −TPRSFG.

**Remark 3.8** A Pseudo regular spherical fuzzy graph need not be a totally Pseudo regular spherical fuzzy graph. It can be verified by the following example.

**Example 3.9** Consider a SFG $\mathcal{G}$ on $G^* = (V, E)$ given in figure 3.3.

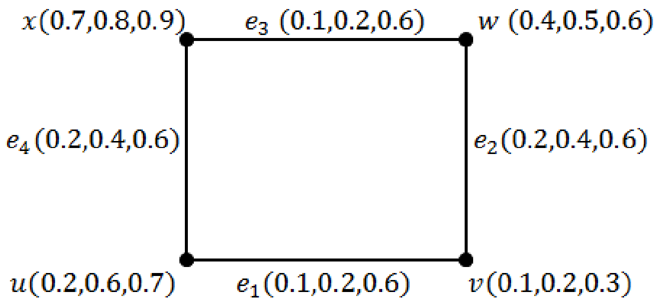


**Figure 3.3:** Illustration of Remark 3.8

Here $d_{\mathcal{G}}^a(u) = d_{\mathcal{G}}^a(v) = d_{\mathcal{G}}^a(w) = d_{\mathcal{G}}^a(x) = (0.3,0.6,1.2)$;

$td_{\mathcal{G}}^a(u) = (0.5,1.2,1.9)$ and $td_{\mathcal{G}}^a(v) = (0.4,1.8,1.5)$

Therefore $\mathcal{G}$ is PRSFG.

But $td_{\mathcal{G}}^{a}(u) \neq td_{\mathcal{G}}^{a}(v)$

Hence $\mathcal{G}$ is not TPRSFG.

**Remark 3.10** A totally pseudo regular spherical fuzzy graph need not be a pseudo regular spherical fuzzy graph.

**Example 3.11** Consider a SFG $\mathcal{G}$ on $G^* = (V, E)$ given in figure 3.4.

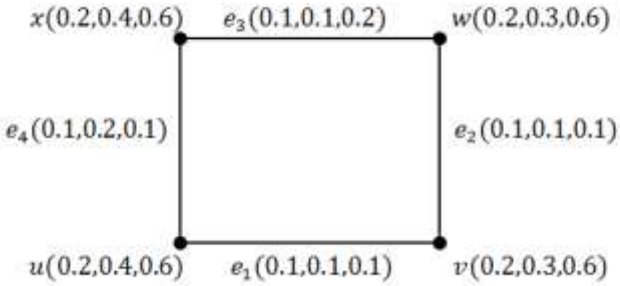


**Figure 3.4:** Illustration of Remark 3.10

Here $td_{\mathcal{G}}^{a}(u) = td_{\mathcal{G}}^{a}(v) = td_{\mathcal{G}}^{a}(w) = td_{\mathcal{G}}^{a}(x) =$ (0.4,0.6,0.8);

$d_{\mathcal{G}}^{a}(u) = (0.2, 0.2, 0.2)$ and $d_{\mathcal{G}}^{a}(v) = (0.2, 0.3, 0.2)$

Therefore $\mathcal{G}$ is TPRSFG.

But $d_{\mathcal{G}}^{a}(u) \neq d_{\mathcal{G}}^{a}(v)$

Hence $\mathcal{G}$ is not PRSFG.

**Note:** From the above examples, it is clear that in general there does not exist any relationship between regular spherical fuzzy graphs and totally regular spherical fuzzy graphs. However, a necessary and sufficient condition under which these two types of spherical fuzzy graphs are equivalent is provided in the following theorem.

**Theorem 3.12** Let $\mathcal{G} = (N, L)$ be a spherical fuzzy graph on $G^* = (V, E)$. Then $N$ is a constant function if the following are equivalent.

1. $\mathcal{G}$ is pseudo regular spherical fuzzy graph.
2. $\mathcal{G}$ is a totally pseudo regular spherical fuzzy graph.

**Proof:**

Assume that $N$ is a constant function.

Let $\sigma_1(u) = c_1, \sigma_2(u) = c_2, \sigma_3(u) = c_3$ for all $u \in N$.

Suppose $\mathcal{G}$ is a pseudo regular spherical fuzzy graph.

Then $d_{\mathcal{G}}^{a}(u) = (k_1, k_2, k_3)$ for all $u \in N$.

Now,

$$\begin{aligned} t\, d_{\mathcal{G}}^{a}(u) &= d_{\mathcal{G}}^{a}(u) + \sigma(u) \\ &= (k_1, k_2, k_3) + (c_1, c_2, c_3) \\ &= (k_1 + c_1,\ k_2 + c_2,\ k_3 + c_3) \end{aligned}$$

Hence, $\mathcal{G}$ is a totally pseudo regular spherical fuzzy graph.

Thus (i) implies (ii) is proved.
Suppose $\mathcal{G}$ is a totally pseudo regular spherical fuzzy graph.
Then,
$td_{\mathcal{G}}^{a}(u) = (k_1, k_2, k_3)$ for all $u \in N$.

$$d_{\mathcal{G}}^{a}(u) + \sigma_i(u) = (k_1, k_2, k_3) \text{ for all } u \in N$$

$$d_{\mathcal{G}}^{a}(u) + (c_1, c_2, c_3) = (k_1, k_2, k_3) \text{ for all } u \in N.$$

$$d_{\mathcal{G}}^{a}(u) = (k_1, k_2, k_3) - (c_1, c_2, c_3) \text{ for all } u \in N$$

Hence $\mathcal{G}$ is a Pseudo regular spherical fuzzy graph.
Thus (ii) implies (i) proved.
Conversely Suppose (i) and (ii) are equivalent.
Let $\mathcal{G}$ be a pseudo regular spherical fuzzy graph and a totally pseudo regular spherical fuzzy graph.
Then, $d_i^a(u) = k_i$ and
$td_i^a(u) = k_j$, for all $u \in N$ and $i, j = 1,2,3$.

$$d_i^a(u) + \sigma_i(u) = k_j \text{ for all } u \in n \text{ and } i, j = 1,2,3.$$

$$k_i + \sigma_i(u) = k_j \text{ and } i, j = 1,2,3.$$

$$\sigma_i(u) = k_j - k_i \text{ for all } u \in V \text{ and } i, j = 1,2,3.$$

Hence $N$ is a constant function.
**Example 3.13** Consider a SFG $\mathcal{G}$ on $G^* = (V, E)$ given in figure 3.5.

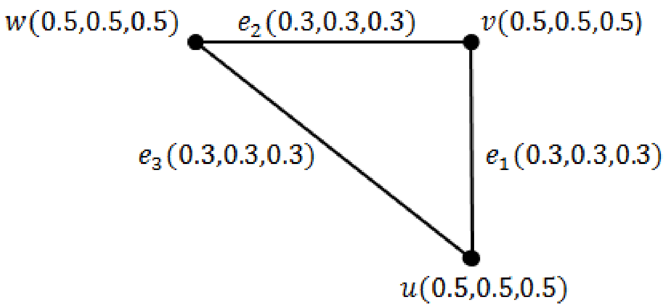


**Figure 3.5:** Spherical Fuzzy Graph $\mathcal{G}$

$d_{\mathcal{G}}^{a}(u) = (0.6,0.6,0.6)$ and $td_{\mathcal{G}}^{a}(u) = (1.1,1.1,1.1)$
The graph is $(0.6,0.6,0.6)$ −PRSFG and $(1.1,1.1,1.1)$ −TPRSFG.

**Theorem 3.14** Let $\mathcal{G} = (N, L)$ be a spherical fuzzy graph on $G^* = (V, E)$. If $\mathcal{G}$ is both pseudo regular and totally pseudo regular spherical fuzzy graph, then $N$ is a constant function.
**Proof:**

Assume that $\mathcal{G}$ is both pseudo regular and totally pseudo regular spherical fuzzy graph. Then $d_i^a(u) = c_i$ and $td_i^a(u) = k_i$, for all $u \in V$.
Now,

$$td_i^a(u) = k_i$$
$$d_i^a(u) + \sigma_i(u) = k_i$$
$$c_i + \sigma_i(u) = k_i$$
$$\sigma_i(u) = k_i + c_i$$
$$= \text{constant}$$

Hence $N$ is a constant function.

**Remark 3.15** The converse of the theorem 3.14 need not be true.

**Example 3.16** Consider a SFG $\mathcal{G}$ on $G^* = (V, E)$ given in figure 3.6.

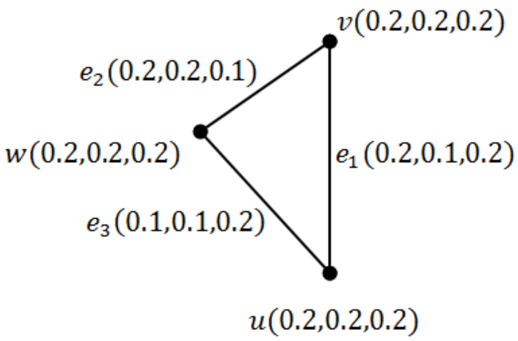


**Figure 3.6:** Illustration of Remark 3.15

Here $N$ is a constant function.
But $\mathcal{G}$ is neither PRSFG nor a totally PRSFG.

**Theorem 3.17** Let $\mathcal{G} = (N, L)$ be a spherical fuzzy graph on $G^*(V, E)$ a cycle of length $n$. If $L$ is $a$ constant function, then $\mathcal{G}$ is a pseudo regular spherical fuzzy graph.

**Proof:**

If $L$ is a constant function then $\mu_1(uv) = c_1$, $\mu_2(uv) = c_2$, $\mu_3(uv) = c_3$ for all $uv \in L$. Then $d_1^a(u) = 2c_1$, $d_2^a(u) = 2c_2$, $d_3^a(u) = 2c_3$ for all $u \in N$.
Hence $\mathcal{G}$ is $(c_1, c_2, c_3)$-Pseudo regular spherical fuzzy graph.

**Remark 3.18** Converse of the above theorem 3.17 need not be true.

**Example 3.19** Consider a SFG $\mathcal{G}$ given in figure 3.3. Here $\mathcal{G}$ is a pseudo regular spherical fuzzy graph but $L$ is not a constant function.

**Theorem 3.20** Let $\mathcal{G} = (N, L)$ be a spherical fuzzy graph on $G^* = (V, E)$, an even cycle of length $n$. If the alternate edges have same membership values, then $\mathcal{G}$ is a pseudo regular spherical fuzzy graph.

**Proof:** If the alternate edges have the same membership values, then

$L(e_j) = \begin{cases} (m_1, m_2, m_3), & if\ j\ is\ odd \\ (n_1, n_2, n_3), & if\ j\ is\ even \end{cases}$ for all $i = 1,2,3$.

If $(m_1, m_2, m_3) = (n_1, n_2, n_3)$, then $L$ is a constant function.

So by the theorem 3.17, $\mathcal{G}$ is a pseudo regular spherical fuzzy graph.

If $(m_1, m_2, m_3) \neq (n_1, n_2, n_3)$, then $d_{\mathcal{G}}(v) = (m_1, m_2, m_3) + (n_1, n_2, n_3)$ for all $v \in V$.

So $t_{\mathcal{G}}(v) = (2m_1 + 2n_1, 2m_2 + 2n_2, 2m_3 + 2n_3)$ and $d_{G^*}(v) = 2$.

Hence $\mathcal{G}$ is a pseudo regular spherical fuzzy graph.

**Remark 3.20** The above theorem 3.20 does not hold for a totally pseudo regular spherical fuzzy graph.

**Example 3.21** Consider a SFG $\mathcal{G}$ on $G^* = (V, E)$ given in figure 3.7.

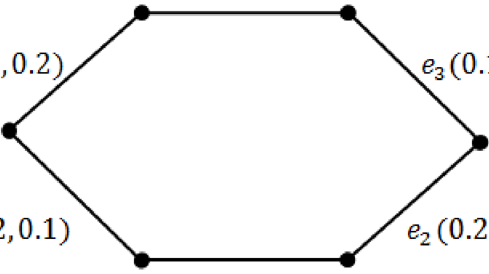


**Figure 3.7:** Illustration of Remark 3.20

Here alternate edges have the same membership values.

$d_{\mathcal{G}}^a(u_1) = d_{\mathcal{G}}^a(u_2) = d_{\mathcal{G}}^a(u_3) = d_{\mathcal{G}}^a(u_4) = d_{\mathcal{G}}^a(u_5) = (0.3,0.4,0.3)$ and

$$td_{\mathcal{G}}^a(u_1) = (0.4,0.6,0.6) \text{ and } td_{\mathcal{G}}^a(u_2) = (0.5,0.7,0.4)$$

$$td_{\mathcal{G}}^a(u_1) \neq td_{\mathcal{G}}^a(u_2)$$

Therefore, $\mathcal{G}$ is not TPRSFG.

**Theorem 3.22** If $\mathcal{G} = (N, L)$ is a regular spherical fuzzy graph on $G^* = (V, E)$, an $r$ −regular graph, then $d_i^a(v) = d_i^{\mathcal{G}}(v)$ for all $v \in N$.

**Proof:**

Let $\mathcal{G} = (N, L)$ is a $(k_1, k_2, k_3)$ regular spherical fuzzy graph on $G^*(V, E)$ an $(r_1, r_2, r_3)$ regular spherical fuzzy graph.

Then $d_1(v) = k_1, d_2(v) = k_2, d_3(v) = k_3$ for all $v \in \mathcal{G}$ and $d_{G^*}(v) = r$ for all $v \in \mathcal{G}$.

So, $t_G(v) = (\sum d_1(u), \sum d_2(u), \sum d_3(u))$ where each $u$ is adjacent with vertex $v$.

Which implies

$$t_{\mathcal{G}}(v) = \left(\sum d_1(u), \sum d_2(u), \sum d_3(u)\right)$$
$$= (rk_1, rk_2, rk_3,)$$

Also $d_i^a(v) = \frac{t_{\mathcal{G}}(v)}{d_{G^*}(v)}$

Which implies $d_i^a(v) = \frac{t_{\mathcal{G}}(v)}{r}$.

Which implies $d_i^a(v) = (\frac{rk_1}{r}, \frac{rk_2}{r}, \frac{rk_3}{r})$

$= (k_1, k_2, k_3)$

Which implies $d_i^a(v) = d_i(v)$ for all $i = 1, 2, 3$.

**Theorem 3.23** Let $\mathcal{G} = (N, L)$ be a spherical fuzzy graph on $G^* = (V, E)$, an $r$ −regular graph. Then $\mathcal{G}$ is pseudo regular spherical fuzzy graph, if $\mathcal{G}$ is a regular spherical fuzzy graph.

**Proof:** Let $G = (N, L)$ be a $(k_1, k_2, k_3)$ regular spherical fuzzy graph on $G^*(V, E)$ an $(r_1, r_2, r_3)$ regular spherical graph.

Then $d_i^a(v) = d_i(v)$ for all $v \in N$.

$d_i^a(v) = k_i$ for all $v \in N$.

Which implies all the vertices have same pseudo degree $(k_1, k_2, k_3)$.

Hence $\mathcal{G}$ is $(k_1, k_2, k_3)$ pseudo regular spherical fuzzy graph.

# 4 Conclusions

Graph theory has become a most powerful conceptual framework for modelling and for solutions of combinatorial problems that arise in various areas, including mathematics, computer science, and engineering. Many real life problems can be represented by graphs. The spherical fuzzy model is a more flexible model because it broadly resolves the ambiguity of actual phenomena. To solve many real-world problems and loosen restrictive constraints, the need for SFGs will arise. In this paper, we have defined pseudo regular and totally pseudo regular spherical fuzzy graphs using pseudo degree and total pseudo degree of a vertex. A necessary and sufficient condition for pseudo regular spherical fuzzy graph is given. Some results between Pseudo regular and totally Pseudo regular spherical fuzzy graphs are derived. In the future, we plan to extend this study to (i) operations on pseudo regular spherical fuzzy graphs; (ii) complex spherical fuzzy graphs.

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