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Neutrosophic Bipolar Vague Binary Topological Space

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Abstract

In this paper, we make an interesting connection between the mathematical approaches to vagueness and the bipolar binary set. The concept of bipolar binary sets over two universal sets. To eliminate ambiguity in a data set, bipolarity is primarily introduced in imprecise binary sets. This article's main objective is to familiarize the reader with the novel concept of a neutrosophic bipolar vague binary set. We also discuss operations like 'union', 'intersection' and 'complement'. The concepts of neutrosophic bipolar vague binary topology, zero, unit, open and closed set, neutrosophic bipolar vague binary '**𝔟**-open' and '**𝔟**-closed' sets are introduced and their properties are also discussed. Furthermore, the neutrosophic bipolar vague binary 'b-interior' and 'b-closure' set are defined. Theorems related to these concepts are stated and proved.
Keywords: Bipolar binary set, Neutrosophic bipolar vague binary 𝔟 -open and 𝔟 -closed, interior, closure
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1. Introduction

The fuzzy set was introduced by Zadeh [24] in 1965 where each element had a degree of membership. The bifuzzy set on a universe was proposed by K. Atanasov [15] in 1983, as a generalization of fuzzy set, which discussed both the degree of membership and the degree of non-membership of each element. The concept of Neutrosophic set was proposed by F. Smarandache [15] as a generalization of the intuitionistic fuzzy set, which straight away proved to be very effective in the domains of decision-making, artificial intelligence, and other areas. The 2020 generalized neutrosophic b-open sets were introduced by Das and Pramanik. 2012 saw the introduction of neutrosophic topological space by Salama and Alblowi [16]. Topological spaces and generalized neutrosophic sets were studied by Salama and Alblowi [17]. Then Deli et al. [8] proposed a bipolar neutrosophic set as an extension of neutrosophic sets. A combination of neutrosophic set and vague set is what Shawkat Alkhazaleh [2] refers to as neutrosophic vague set. The neutrosophic vague theory can be used to practice dealing with vague, biased, and inaccurate information. Neutrosophic bipolar vague sets were first introduced by Satham Hussain [18]. In topological spaces, neutrosophic vague binary sets were presented by P.B.Remya.and A.Francina Shalinl. [14]. According to their definition, Arokiarani et al. [3] developed some relationships between the two neutrosophic semi-open functions. neutrosophic pre-open sets and pre-closed sets were first described by Rao and Srinivasa [13] in neutrosophic topological spaces. Iswaraya and Bageerathi were the first to introduce the ideas of neutrosophic semi-closed sets and neutrosophic semi-open sets [12]. In neutrosophic topological spaces, generalized neutrosophic closed sets were studied by Dhavaseelan and Jafari [9]. Imranet et al. [10] later presented the neutrosophic semi-open sets in neutrosophic topological space. Imranet et al. [11] used neutrosophic topological spaces to define neutrosophic generalized alpha generalized continuity. Pushpalatha and Nandhini [20] established the concept of neutrosophic generalized closed sets in neutrosophic topological spaces. Ebenanjar et al. [21] went on to introduce neutrosophic b-open sets in neutrosophic topological spaces. C. Maheswari, M. Sathyabama and S. Chandrasekar. [23] were the first to introduce the concept of neutrosophic simplistic b-closed sets in neutrosophic topological spaces. Afterward, Das and Pramanik [6] proposed neutrosophic soft open sets in neutrosophic soft topological spaces. Das and Tripathy [7] recently pioneered the idea of pairwise neutrosophic-b-open sets in neutrosophic bitopological spaces. M., Shabir, M., Naz [19] who was the first to identify a bipolar soft set and its operations, such as union, intersection, and complement. Then, Karaaslan and Karata's [22] new approximation redefined bipolar soft sets, giving researchers a chance to examine their topological structures. On two initial universal sets, Binary soft set theory was proposed by Ackgoz and Tas [1]. Then, S.S., Benchalli., [4] related basic properties have appropriate parameters that are defined over two initial universal sets. The concept of neutrosophic bipolar vague binary topological space is discussed in the current work along with its investigation. The basic operation in this paper uses a bipolar binary set. Additionally, this study examines neutrosophic bipolar vague binary b-open, b-closed, b-interior, and b-closure operators in addition to the connections between a number of other operators.

2. Preliminaries

Definition 2.1 [18]

Within the context of discussion S, the word Neutrosophic Bipolar Vague Set (abbreviated as *NBVS*) is expressed as A_{NBVS}. $A_{NBVS} = \{< x, \hat{T}^P{}_{ANBVS}(x), \hat{F}^P{}_{ANBVS}(x), \hat{I}^P{}_{ANBVS}(x), \hat{T}^N{}_{ANBVS}(x), \hat{F}^N{}_{ANBVS}(x), \hat{I}^N{}_{ANBVS}(x), >; x \in S\}$. Their extension pertains to 'truth membership', 'indeterminacy membership', and 'falsity membership, $\{\hat{T}^P{}_{ANBVS}(x) = [(T^-)^P(x), (T^+)^P(x)]$; $\hat{I}^P{}_{ANBVS}(x) = [(I^-)^P(x), (I^+)^P(x)]$; $\hat{F}^P{}_{ANBVS}(x) = [(F^-)^P(x), (F^+)^P(x)]\}$, where $(T^+)^P(x) = 1 - (F^-)^P(x)$, $(F^+)^P(x) = 1 - (T^-)^P(x)$ and given that $0 \leq (T^-)^P(x) + (I^-)^P(x) + (F^-)^P(x) \leq 2$.

Also $\{\hat{T}^N{}_{ANBVS}(x) = [(T^-)^N(x), (T^+)^N(x)]$; $\hat{I}^N{}_{ANBVS}(x) = [(I^-)^N(x), (I^+)^N(x)]$; $\hat{F}^N{}_{ANBVS}(x) = [(F^-)^N(x), (F^+)^N(x)]\}$ where $(T^+)^N(x) = 1 - (F^-)^N(x)$, $(F^+)^N(x) = 1 - (T^-)^N(x)$ and given that $0 \geq (T^-)^N(x) + (I^-)^N(x) + (F^-)^N(x) \geq 2$.

Example 2.1

If $S = \{x_1, x_2, x_3\}$ is a set of universes. We define the NBVS A_{NBV} as follows. $A_{NBV} =$

$$\{\frac{x_1}{[0.4,0.7]^P, [0.6,0.6]^P, [0.5,0.8]^P, [-0.4,-0.7]^N, [-0.5,-0.5]^N, [-0.6,-0.8]^N};$$
$$\frac{x_2}{[0.5,0.7]^P, [0.5,0.7]^P, [0.5,0.7]^P, [-0.5,-0.5]^N, [-0.6,-0.6]^N, [-0.7,-0.7]^N};$$
$$\frac{x_3}{[0.4,0.8]^P, [0.7,0.5]^P, [0.4,0.8]^P, [-0.5,-0.7]^N, [-0.6,-0.7]^N, [-0.5,-0.7]^N}\}$$

Definition 2.2 [14]

A Neutrosophic vague binary set M_{NVB} (NVBS in short) over a standard universe $S_1 = \{x_j, 1 \leq j \leq n\}$; $S_2 = \{y_k, 1 \leq k \leq p\}$ is an object of the form

$$M_{NVB} = \left\{ \begin{array}{l} \langle \frac{\hat{T}_{M_{NVB}}(x_j), \hat{I}_{M_{NVB}}(x_j), \hat{F}_{M_{NVB}}(x_j)}{x_j}; \forall x_j \in S_1 \rangle \\ \langle \frac{\hat{T}_{M_{NVB}}(y_k), \hat{I}_{M_{NVB}}(y_k), \hat{F}_{M_{NVB}}(y_k)}{y_k}; \forall y_k \in S_2 \rangle \end{array} \right\}$$

is defined as $\hat{T}_{M_{NVB}}(x_j) = [T^-(x_j), T^+(x_j)]$; $\hat{I}_{M_{NVB}}(x_j) = [I^-(x_j), I^+(x_j)]$; $\hat{F}_{M_{NVB}}(x_j) = [F^-(x_j), F^+(x_j)]$; $x_j \in S_1$ and $\hat{T}_{M_{NVB}}(y_k) = [T^-(y_k), T^+(y_k)]$; $\hat{I}_{M_{NVB}}(y_k) = [I^-(y_k), I^+(y_k)]$; $\hat{F}_{M_{NVB}}(y_k) = [F^-(y_k), F^+(y_k)]$; $y_k \in S_2$ where

1. $T^+(x_j) = 1 - F^-(x_j)$; $F^+(x_j) = 1 - T^-(x_j)$; $x_j \in S_1$ and $T^+(y_k) = 1 - F^-(y_k)$; $F^+(y_k) = 1 - T^-(y_k)$; $y_k \in S_2$

2. $-0 \leq T^{-}(x_j) + I^{-}(x_j) + F^{-}(x_j) \leq 2^{+};$
   $0 \leq T^{-}(y_k) + I^{-}(y_k) + F^{-}(y_k) \leq 2^{+}$ and
   $-0 \leq T^{+}(x_j) + I^{+}(x_j) + F^{+}(x_j) \leq 2^{+};$
   $-0 \leq T^{+}(y_k) + I^{+}(y_k) + F^{+}(y_k) \leq 2^{+}$
3. $T^{-}(x_j), I^{-}(x_j), F^{-}(x_j): V(S_1) \rightarrow [0,1]$ and $T^{-}(y_k), I^{-}(y_k), F^{-}(y_k): V(S_2) \rightarrow [0,1]$. $T^{+}(x_j), I^{+}(x_j), F^{+}(x_j): V(S_1) \rightarrow [0,1]$ and $T^{+}(y_k), I^{+}(y_k), F^{+}(y_k): V(S_2) \rightarrow [0,1]$.
   Here $V\,(S_1), V\,(S_2)$ represent power set of vague sets on $S_{1,}\ S_2$ respectively.

Example 2.2
Let $S_1 = \{x_1, x_2, x_3\}$, $S_2 = \{y_1, y_2\}$ be the common universe under consideration. A_{NVBS} is given below:

$$M_{NVB} = \left\{ \begin{array}{c} \langle \frac{[0.2,0.3][0.6,0.7][0.7,0.8]}{x_1}; \frac{[0.3,0.7][0.5,0.6][0.3,0.7]}{x_2}; \frac{[0.1,0.9][0.4,0.8][0.1,0.9]}{x_3} \rangle \\ \langle \frac{[0.6,0.8][0.5,0.7][0.2,0.4]}{y_1}; \frac{[0.2,0.7][0.6,0.9][0.3,0.8]}{y_2} \rangle \end{array} \right\}$$

3. Operations on Neutrosophic bipolar vague binary set

Definition 3.1
The bipolar binary set $\widehat{A_B}$ is defined on a conventional universe $S_1 = \{x_j, 1 \leq j \leq n\}$; $S_2 = \{y_k, 1 \leq k \leq p\}$ is an object of the form

$$\widehat{A_B} = \{\langle \frac{\left(T_A(x_j), I_A(x_j), F_A(x_j)\right)^P \left(T_A(x_j), I_A(x_j), F_A(x_j)\right)^N}{x_j} \forall x_j \in S_1 \rangle$$
$$\langle \frac{(T_A(y_k), I_A(y_k), F_A(y_k))^P (T_A(y_k), I_A(y_k), F_A(y_k))^N}{y_k} \forall y_k \in S_2 \rangle\}$$

$\left(T_A(x_j), I_A(x_j), F_A(x_j)\right)^P : S_1 \rightarrow [0,1]$ and $\left(T_A(x_j), I_A(x_j), F_A(x_j)\right)^N : S_1 \rightarrow [-1,0]$ provide the truth, indeterminacy and false membership values of the components $x_j\ in\ S_1$ and $(T_A(y_k), I_A(y_k), F_A(y_k))^P: S_2 \rightarrow [0,1]$ and $(T_A(y_k), I_A(y_k), F_A(y_k))^N: S_2 \rightarrow [-1,0]$ provides the truth, indeterminacy and false membership values of the component y_k in S_2.

Definition 3.2
A Neutrosophic Bipolar Vague Binary Set M_{NBVB} (NBVBS for brevity) over a standard universe $\{S_1 = \{x_j, 1 \leq j \leq n\}; S_2 = \{y_k, 1 \leq k \leq p\}\}$ is conceptual entity that may be represented as

$M_{NBVB} =$

$$= \langle (\frac{\hat{T}_{M_{NBVB}}(x_j), \hat{I}_{M_{NBVB}}(x_j), \hat{F}_{M_{NBVB}}(x_j)}{x_j})^P; \forall x_j \in S_1\rangle \langle (\frac{\hat{T}_{M_{NBVB}}(y_k), \hat{I}_{M_{NBVB}}(y_k), \hat{F}_{M_{NBVB}}(y_k)}{y_k})^P \forall y_k \in S_2\rangle$$

$$U_2$$

$$\langle \left(\frac{\hat{T}_{M_{NBVB}}(x_j), \hat{I}_{M_{NBVB}}(x_j), \hat{F}_{M_{NBVB}}(x_j)}{x_j}\right)^N; \forall x_j \in S_1\rangle \langle \left(\frac{\hat{T}_{M_{NBVB}}(y_k), \hat{I}_{M_{NBVB}}(y_k), \hat{F}_{M_{NBVB}}(y_k)}{y_k}\right)^N \forall y_k \in S_2\rangle$$

is defined as,

$\hat{T}_{M_{NBVB}}(x_j)^P = [T^-(x_j), T^+(x_j)]^P$; $\hat{T}_{M_{NBVB}}(x_j)^N = [T^-(x_j), T^+(x_j)]^N$

$\hat{I}_{M_{NBVB}}(x_j)^P = [I^-(x_j), I^+(x_j)]^P$; $\hat{I}_{M_{NBVB}}(x_j)^N = [I^-(x_j), I^+(x_j)]^N$

$\hat{F}_{M_{NBVB}}(x_j)^P = [F^-(x_j), F^+(x_j)]^P$; $\hat{F}_{M_{NBVB}}(x_j)^N [F^-(x_j), F^+(x_j)]^N$;

$\hat{T}_{M_{NBVB}}(y_k)^P = [T^-(y_k), T^+(y_k)]^P$; $\hat{T}_{M_{NBVB}}(y_k)^N = [T^-(y_k), T^+(y_k)]^N$

$\hat{I}_{M_{NBVB}}(y_k)^P = [I^-(y_k), I^+(y_k)]^P$; $\hat{I}_{M_{NBVB}}(y_k)^N = [I^-(y_k), I^+(y_k)]^N$

$\hat{F}_{M_{NBVB}}(y_k)^P = [F^-(y_k), F^+(y_k)]^P$; $\hat{F}_{M_{NBVB}}(y_k)^N = [F^-(y_k), F^+(y_k)]^N$

$\forall x_j \in S_1$ and $\forall y_k \in S_2$. where, 1. $(T^+)^P(x_j) = 1 - (F^-)^P(x_j)$; $(F^+)^P(x_j) = 1 - (T^-)^P(x_j)$; $(T^+)^N(x_j) = 1 - (F^-)^N(x_j)$; $(F^+)^N(x_j) = 1 - (T^-)^N(x_j)$ *and* $(T^+)^P(y_k) = 1 - (F^-)^P(y_k)$; $(F^+)^P(y_k) = 1 - (T^-)^P(y_k)$ $(T^+)^N(y_k) = 1 - (F^-)^N(y_k)$; $(F^+)^N(y_k) = 1 - (T^-)^N(y_k)$ $\forall x_j \in S_1$ *and* $\forall y_k \in S_2$.

1. $-0 \le (T^-)^P(x_j) + (I^-)^P(x_j) + (F^-)^P(x_j) \le 2^+$
   $0 \ge (T^-)^N(x_j) + (I^-)^N(x_j) + (F^-)^N(x_j) \ge 2^-$;
2. $(T^-)^P(x_j), (I^-)^P(x_j), (F^-)^P(x_j), (T^+)^P(x_j), (I^+)^P(x_j), (F^+)^P(x_j): V(S_1) \to [0,1]$
   $(T^-)^N(x_j), (I^-)^N(x_j), (F^-)^N(x_j), (T^+)^N(x_j), (I^+)^N(x_j), (F^+)^N(x_j): V(S_1) \to [-1,0]$
   $(T^-)^P(y_k), (I^-)^P(y_k), (F^-)^P(y_k), (T^+)^P(y_k), (I^+)^P(y_k), (F^+)^P(y_k): V(S_2) \to [0,1]$
   $(T^-)^N(y_k), (I^-)^N(y_k), (F^-)^N(y_k), (T^+)^N(y_k), (I^+)^N(y_k), (F^+)^N(y_k): V(S_2) \to [-1,0]$

Here $V(S_1), V(S_2)$ denotes power set of bipolar set on S_1 and S_2 respectively.

Example 3.1

Let $S_1 = \{x_1, x_2, x_3\}$ and $S_2 = \{y_1, y_2\}$ be the standard universe under consideration. A NBVBS is given below:

$$M_{NBVBS} = \left\{ \begin{array}{l} \frac{[0.3,0.6]^P, [0.5,0.5]^P, [0.4,0.7]^P, [-0.3,-0.5]^N, [-0.4,-0.4]^N, [-0.5,-0.7]^N}{x_1}; \\ \frac{[0.4,0.6]^P, [0.4,0.6]^P, [0.4,0.6]^P, [-0.4,-0.4]^N, [-0.5,-0.5]^N, [-0.6,-0.6]^N}{x_2}; \\ \frac{[0.3,0.7]^P, [0.6,0.4]^P, [0.3,0.7]^P, [-0.4,-0.6]^N, [-0.5,-0.6]^N, [-0.4,-0.6]^N}{x_3}; \\ \frac{[0.6,0.8]^P, [0.5,0.7]^P, [0.2,0.4]^P, [-0.3,-0.6]^N, [-0.5,-0.7]^N, [-0.4,-0.7]^N}{y_1}; \\ \frac{[0.2,0.7]^P, [0.6,0.9]^P, [0.3,0.8]^P, [-0.5,-0.7]^N, [-0.3,-0.7]^N, [-0.3,-0.5]^N}{y_1} \end{array} \right\}$$

Definition 3.3

Let M_{NBVBS} and $\mathrm{P_{NBVBS}}$ be two NBVBS on a common universe S_1 and S_2 the $\mathrm{M_{NBVBS}}$ is included by P_{NBVBS} denoted by $M_{NBVBS} \subseteq P_{NBVBS}$ if the below condition is true:

If $\forall x_j \in S_1$ *and* $1 \le j \le n$

(1) $\widehat{T}_{M_{NBVB}}(x_j)^P \le \widehat{T}_{P_{NBVB}}(x_j)^P$ *and* $\widehat{T}_{M_{NBVB}}(x_j)^N \ge \widehat{T}_{P_{NBVB}}(x_j)^N$

(2) $\widehat{I}_{M_{NBVB}}(x_j)^P \ge \widehat{I}_{P_{NBVB}}(x_j)^P$ *and* $\widehat{I}_{M_{NBVB}}(x_j)^N \le \widehat{I}_{P_{NBVB}}(x_j)^N$

(3) $\widehat{F}_{M_{NBVB}}(x_j)^P \ge \widehat{F}_{P_{NBVB}}(x_j)^P$ *and* $\widehat{F}_{M_{NBVB}}(x_j)^N \le \widehat{F}_{P_{NBVB}}(x_j)^N$

and $\forall y_k \in S_2$ *and* $1 \le k \le p$

(1) $\widehat{T}_{M_{NBVB}}(y_k)^P \le \widehat{T}_{P_{NBVB}}(y_k)^P$ *and* $\widehat{T}_{M_{NBVB}}(y_k)^N \ge \widehat{T}_{P_{NBVB}}(y_k)^N$

(2) $\widehat{I}_{M_{NBVB}}(y_k)^P \ge \widehat{I}_{P_{NBVB}}(y_k)^P$ *and* $\widehat{I}_{M_{NBVB}}(y_k)^N \le \widehat{I}_{P_{NBVB}}(y_k)^N$

(3) $\widehat{F}_{M_{NBVB}}(y_k)^P \ge \widehat{F}_{P_{NBVB}}(y_k)^P$ *and* $\widehat{F}_{M_{NBVB}}(y_k)^N \le \widehat{F}_{P_{NBVB}}(y_k)^N$

Example 3.2

Let $S_1 = \{x_1, x_2\}$ and $S_2 = \{y_1\}$ be the standard universe. A NBVBS is given below:

$$M_{NBVBS} = \left\{ \begin{array}{l} \frac{[0.1,0.2]^P,[0.6,0.7]^P,[0.8,0.9]^P,[-0.1,-0.2]^N,[-0.6,-0.7]^N,[-0.8,-0.9]^N}{x_1}; \\ \frac{[0.2,0.6]^P,[0.5,0.6]^P,[0.4,0.8]^P,[-0.2,-0.6]^N,[-0.5,-0.6]^N,[-0.4,-0.8]^N}{x_2}; \\ \frac{[0.1,0.3]^P,[0.6,0.7]^P,[0.7,0.9]^P,[-0.1,-0.3]^N,[-0.6,-0.7]^N,[-0.7,-0.9]^{N}"}{y_1} \end{array} \right\}$$

$$P_{NBVBS} = \left\{ \begin{array}{l} \frac{[0.2,0.3]^P, [0.5,0.6]^P, [0.7,0.8]^P, [-0.2,-0.3]^N, [-0.5,-0.6]^N, [-0.7,-0.8]^N}{x_1}; \\ \frac{[0.3,0.7]^P, [0.4,0.5]^P, [0.3,0.7]^P, [-0.3,-0.7]^N, [-0.4,-0.5]^N, [-0.3,-0.7]^N}{x_2}; \\ \frac{[0.2,0.4]^P, [0.5,0.6]^P, [0.6,0.8]^P, [-0.2,-0.4]^N, [-0.5,-0.6]^N, [-0.6,-0.8]^N}{y_1}" \end{array} \right\}$$

Clearly $M_{NBVBS} \subseteq P_{NBVBS}$.

Definition 3.4

Let M_{NBVBS} and P_{NBVBS} are two NBVBS

(i) Union of two NBVBS, M_{NBVBS} and P_{NBVBS} is a NBVBS given as,

$M_{NBVBS} \cup P_{NBVBS} = S_{NBVBS}$

$$= \begin{Bmatrix} \langle (\frac{\hat{T}_{S_{NBVBS}}(x_j), \hat{I}_{S_{NBVBS}}(x_j), \hat{F}_{S_{NBVBS}}(x_j)}{x_j})^P; \forall x_j \in S_1 \rangle \\ \langle (\frac{\hat{T}_{S_{NBVBS}}(y_k), \hat{I}_{S_{NBVBS}}(y_k), \hat{F}_{S_{NBVBS}}(y_k)}{y_k})^P \forall y_k \in S_2 \rangle \\ \langle \left(\frac{\hat{T}_{S_{NBVBS}}(x_j), \hat{I}_{S_{NBVBS}}(x_j), \hat{F}_{S_{NBVBS}}(x_j)}{x_j}\right)^N; \forall x_j \in S_1 \rangle \\ \langle (\frac{\hat{T}_{S_{NBVBS}}(y_k), \hat{I}_{S_{NBVBS}}(y_k), \hat{F}_{S_{NBVBS}}(y_k)}{y_k})^N \forall y_k \in S_2 \rangle \end{Bmatrix}$$

The relationship between the 'truth-membership, indeterminacy-membership, and false-membership functions' of M_{NBVBS} and P_{NBVBS} is provided as

$$\hat{T}_{S_{NBVBS}}(x_j)^P = [max\left(T_{M_{NBVBS}}{}^{-}(x_j), T_{P_{NBVBS}}{}^{-}(x_j)\right), max\left(T_{M_{NBVBS}}{}^{+}(x_j), T_{P_{NBVBS}}{}^{+}(x_j)\right)]$$

$$\hat{I}_{S_{NBVBS}}(x_j)^P = [min\left(I_{M_{NBVBS}}{}^{-}(x_j), I_{P_{NBVBS}}{}^{-}(x_j)\right), min\left(I_{M_{NBVBS}}{}^{+}(x_j), I_{P_{NBVBS}}{}^{+}(x_j)\right)]$$

$$\hat{F}_{S_{NBVBS}}(x_j)^P = [min\left(F_{M_{NBVBS}}{}^{-}(x_j), F_{P_{NBVBS}}{}^{-}(x_j)\right), min\left(F_{M_{NBVBS}}{}^{+}(x_j), F_{P_{NBVBS}}{}^{+}(x_j)\right)]$$

$$\hat{T}_{S_{NBVBS}}(x_j)^N = [min\left(T_{M_{NBVBS}}{}^{-}(x_j), T_{P_{NBVBS}}{}^{-}(x_j)\right), min\left(T_{M_{NBVBS}}{}^{+}(x_j), T_{P_{NBVBS}}{}^{+}(x_j)\right)]$$

$$\hat{I}_{S_{NBVBS}}(x_j)^N = [max\left(I_{M_{NBVBS}}{}^{-}(x_j), I_{P_{NBVBS}}{}^{-}(x_j)\right), max\left(I_{M_{NBVBS}}{}^{+}(x_j), I_{P_{NBVBS}}{}^{+}(x_j)\right)]$$

$$\hat{F}_{S_{NBVBS}}(x_j)^N = [max\left(F_{M_{NBVBS}}{}^{-}(x_j), F_{P_{NBVBS}}{}^{-}(x_j)\right), max\left(F_{M_{NBVBS}}{}^{+}(x_j), F_{P_{NBVBS}}{}^{+}(x_j)\right)]$$

Similarly, for $\forall y_k \in S_2$.

Example 3.3

From above example 3.2,

$$S_{NBVBS}=\left\{\begin{array}{c}\frac{[0.2,0.3],[0.5,0.6][0.7,0.8][-0.1,-0.2][-0.6,-0.7][-0.8,-0.9]}{x_1};\\ \langle\frac{[0.3,0.7][0.4,0.5][0.3,0.7][-0.2,-0.6][-0.5,-0.6][-0.4,-0.8]}{x_2};\rangle\\ \frac{[0.2,0.4][0.5,0.6][0.6,0.8][-0.2,-0.4][-0.5,-0.6][-0.7,-0.9]}{y_1}\end{array}\right\}$$

Definition 3.5

Assume M_{NBVBS} and P_{NBVBS} as two NBVBS

(i) Intersection of two NBVBS, M_{NBVBS} and P_{NBVBS} is a NBVBS given as, $M_{NBVBS}\ \cap P_{NBVBS}=S_{NBVBS}$

$$=\left\{\begin{array}{l}\langle(\frac{\hat{T}_{S_{NBVBS}}(x_j),\hat{I}_{S_{NBVBS}}(x_j),\hat{F}_{S_{NBVBS}}(x_j)}{x_j})^P;\forall x_j\in S_1\rangle\\ \langle(\frac{\hat{T}_{S_{NBVBS}}(y_k),\hat{I}_{S_{NBVBS}}(y_k),\hat{F}_{S_{NBVBS}}(y_k)}{y_k})^P\forall y_k\in S_2\rangle\\ \langle\left(\frac{\hat{T}_{S_{NBVBS}}(x_j),\hat{I}_{S_{NBVBS}}(x_j),\hat{F}_{S_{NBVBS}}(x_j)}{x_j}\right)^N;\forall x_j\in S_1\rangle\\ \langle(\frac{\hat{T}_{S_{NBVBS}}(y_k),\hat{I}_{S_{NBVBS}}(y_k),\hat{F}_{S_{NBVBS}}(y_k)}{y_k})^N\forall y_k\in S_2\rangle\end{array}\right\}$$

It is given that the ‘truth-membership, indeterminacy-membership and false-membership function are similar to that of M_{NBVBS} and P_{NBVBS}.

$$\hat{T}_{S_{NBVBS}}(x_j)^P=[min\left(T_{M_{NBVBS}}{}^{-}(x_j),T_{P_{NBVBS}}{}^{-}(x_j)\right),min\left(T_{M_{NBVBS}}{}^{+}(x_j),T_{P_{NBVBS}}{}^{+}(x_j)\right)]$$

$$\hat{I}_{S_{NBVBS}}(x_j)^P=[max\left(I_{M_{NBVBS}}{}^{-}(x_j),I_{P_{NBVBS}}{}^{-}(x_j)\right),max\left(I_{M_{NBVBS}}{}^{+}(x_j),I_{P_{NBVBS}}{}^{+}(x_j)\right)]$$

$$\hat{F}_{S_{NBVBS}}(x_j)^P=[max\left(F_{M_{NBVBS}}{}^{-}(x_j),F_{P_{NBVBS}}{}^{-}(x_j)\right),max\left(F_{M_{NBVBS}}{}^{+}(x_j),F_{P_{NBVBS}}{}^{+}(x_j)\right)]$$

$$\hat{T}_{S_{NBVBS}}(x_j)^N=[max\left(T_{M_{NBVBS}}{}^{-}(x_j),T_{P_{NBVBS}}{}^{-}(x_j)\right),max\left(T_{M_{NBVBS}}{}^{+}(x_j),T_{P_{NBVBS}}{}^{+}(x_j)\right)]$$

$$\hat{I}_{S_{NBVBS}}(x_j)^N=[min\left(I_{M_{NBVBS}}{}^{-}(x_j),I_{P_{NBVBS}}{}^{-}(x_j)\right),min\left(I_{M_{NBVBS}}{}^{+}(x_j),I_{P_{NBVBS}}{}^{+}(x_j)\right)]$$

$$\hat{F}_{S_{NBVBS}}(x_j)^N=[min\left(F_{M_{NBVBS}}{}^{-}(x_j),F_{P_{NBVBS}}{}^{-}(x_j)\right),min\left(F_{M_{NBVBS}}{}^{+}(x_j),F_{P_{NBVBS}}{}^{+}(x_j)\right)]$$

Similarly, for$\forall y_k\in S_2$.

Example 3.4

From above example 3.2

$$S_{NBVBS} = \left\{ \begin{array}{l} \frac{[0.1,0.2],[0.6,0.7][0.7,0.8][-0.2,-0.3][-0.6,-0.7][-0.8,-0.9]}{x_1}; \\ \langle \frac{[0.2,0.6][0.5,0.6][0.4,0.8][-0.2,-0.6][-0.4,-0.6][-0.3,-0.7]}{x_2}; \rangle \\ \frac{[0.1,0.3][0.6,0.7][0.7,0.9][-0.1,-0.3][-0.5,-0.6][-0.6,-0.8]}{y_1} \end{array} \right\}$$

Definition 3.6

Let ${M_{NBVBS}}^c$ is defined as,

$${M_{NBVBS}}^c = \left\{ \begin{array}{l} \langle \frac{\left(\hat{T}_{M_{NBVBS}}(x_j)^c, \hat{I}_{M_{NBVBS}}(x_j)^c \hat{F}_{M_{NBVBS}}(x_j)^c\right)^P}{x_j}; \frac{\left(\hat{T}_{M_{NBVBS}}(x_j)^c, \hat{I}_{M_{NBVBS}}(x_j)^c \hat{F}_{M_{NBVBS}}(x_j)^c\right)^N}{x_j} \forall x_j \in U_1 \rangle \\ \langle \frac{\left(\hat{T}_{M_{NBVBS}}(y_k)^c, \hat{I}_{M_{NBVBS}}(y_k)^c \hat{F}_{M_{NBVBS}}(y_k)^c\right)^P}{y_k}; \frac{\left(\hat{T}_{M_{NBVBS}}(y_k)^c, \hat{I}_{M_{NBVBS}}(y_k)^c \hat{F}_{M_{NBVBS}}(y_k)^c\right)^N}{y_k} \forall y_k \in U_2 \rangle \end{array} \right.$$

Where

$$\hat{T}_{M_{NBVBS}}(x_j)^{c^P} = \left[1 - T^+(x_j), 1 - T^-(x_j)\right]^P;$$
$$\hat{T}_{M_{NBVBS}}(x_j)^{c^N} = \left[-1 - T^+(x_j), -1 - T^-(x_j)\right]^N$$
$$\hat{I}_{M_{NBVBS}}(x_j)^{c^P} = \left[1 - I^+(x_j),\ 1 - I^-(x_j)\right]^P;$$
$$\hat{I}_{M_{NBVBS}}(x_j)^{c^N} = \left[-1 - I^+(x_j), -1 - I^-(x_j)\right]^N$$
$$\hat{F}_{M_{NBVBS}}(x_j)^{c^P} = \left[1 - F^+(x_j),\ 1 - F^-(x_j)\right]^P;$$
$$\hat{F}_{M_{NBVBS}}(x_j)^{c^N} = \left[-1 - F^+(x_j), -1 - F^-(x_j)\right]^N$$
$$\hat{T}_{M_{NBVBS}}(y_k)^{c^P} = [1 - T^+(y_k),\ 1 - T^-(y_k)]^P;$$
$$\hat{T}_{M_{NBVBS}}(y_k)^{c^N} = [-1 - T^+(y_k), -1 - T^-(y_k)]^N$$
$$\hat{I}_{M_{NBVBS}}(y_k)^{c^P} = [1 - I^+(y_k),\ 1 - I^-(y_k)]^P;$$
$$\hat{I}_{M_{NBVBS}}(y_k)^{c^N} = [-1 - I^+(y_k), -1 - I^-(y_k)]^N$$
$$\hat{F}_{M_{NBVBS}}(y_k)^{c^P} = [1 - F^+(y_k),\ 1 - F^-(y_k)]^P;$$
$$\hat{F}_{M_{NBVBS}}(y_k)^{c^N} = [-1 - F^+(y_k), -1 - F^-(y_k)]^N$$

$\forall x_j \in S_1$ *and* $\forall y_k \in S_2$

Example 3.5

Let M_{NBVB} is defined as in above example. Its complement is given by,

$$M_{NBVBS}{}^{c} = \left\{ \begin{array}{l} \frac{\langle([0.4,0.7][0.5,0.5][0.3,0.6])^{P}([-0.5,-0.7],[-0.6,-0.6],[-0.3,-0.5])^{N}\rangle}{x_1}; \\ \frac{\langle([0.4,0.6][0.4,0.6][0.6,0.4])^{P}([-0.6,-0.6],[-0.5,-0.5],[-0.4,-0.4])^{N}\rangle}{x_2}; \\ \frac{\langle([0.3,0.7][0.6,0.4][0.3,0.7])^{P}([-0.4,-0.6],[-0.4,-0.5],[-0.4,-0.6])^{N}\rangle}{x_3}; \\ \frac{\langle([0.2,0.4][0.3,0.5][0.6,0.8])^{P}([-0.4,-0.7],[-0.3,-0.5],[-0.3,-0.6])^{N}\rangle}{y_1}; \\ \frac{\langle([0.3,0.8][0.1,0.4][0.2,0.7])^{P}([-0.3,-0.75],[-0.3,-0.3],[-0.5,-0.7])^{N}\rangle}{y_2} \end{array} \right\}$$

Proposition 3.1

Let A_{NBVBS}, B_{NBVBS}, and C_{NBVBS} *be NBVBS* then

(a) $A_{NBVBS} \subseteq B_{NBVBS}$ *and* $C_{NBVBS} \subseteq D_{NBVBS} \Rightarrow A_{NBVBS} \cup C_{NBVBS} \subseteq B_{NBVBS} \cup D_{NBVBS}$ and

(b) $A_{NBVBS} \cap C_{NBVBS} \subseteq B_{NBVBS} \cap D_{NBVBS}$

(c) $A_{NBVBS} \subseteq B_{NBVBS}$ *and* $A_{NBVBS} \subseteq C_{NBVBS} \Rightarrow A_{NBVBS} \subseteq B_{NBVBS} \cap C_{NBVBS}$

(d) $A_{NBVBS} \subseteq C_{NBVBS}$ *and* $B_{NBVBS} \subseteq C_{NBVBS} \Rightarrow A_{NBVBS} \cup B_{NBVBS} \subseteq C_{NBVBS}$

(e) $A_{NBVBS} \subseteq B_{NBVBS}$ *and* $B_{NBVBS} \subseteq C_{NBVBS} \Rightarrow A_{NBVBS} \subseteq C_{NBVBS}$

Proof:

Proof is clear.

Theorem 3.2

A bipolar neutrosophic vague binary set is the bipolar fuzzy set.

Proof:

Let *S* is a “bipolar neutrosophic vague binary set.

Then by setting the positive component;$\hat{I}_{M_{NBVB}}(x_j)^{P}, \widehat{F}_{M_{NBVB}}(x_j)^{P}; \forall x_j \in S_1$ and $\hat{I}_{M_{NBVB}}(y_k)^{P}; \hat{F}_{M_{NBVB}}(y_k)^{P} \forall y_k \in S_2$ equal to zero as well as the negative component $\hat{I}_{M_{NBVB}}(x_j)^{N}; \hat{F}_{M_{NBVB}}(x_j)^{N}; \forall x_j \in U_1$ and $\hat{I}_{M_{NBVB}}(y_k)^{N}; \hat{F}_{M_{NBVB}}(y_k)^{N}; \forall y_k \in U_2$ equal to zero reduces the bipolar neutrosophic vague binary set to bipolar fuzzy set.

4. Neutrosophic Bipolar Vague Binary Set Topology

The present paper deals with Neutrosophic bipolar vague binary topology and provides definitions and features for key terminology such as unit, zero, interior, course, ‘ ƀ -open, and ‘ ƀ c-closed sets.

Definition 4.1
A family of 'neutrosophic bipolar vague binary sets' in S_1, S_2 that adhere to the following criteria constitutes a Neutrosophic bipolar vague binary topology in a same universe with S_1, S_2

(i) $\emptyset_{NBVB}, \psi_{NBVB} \in \tau_{\Delta}^{NBVB}$

(ii) For any M_{NBVB}, $P_{NBVB} \in \tau_{\Delta}^{NBVB}$, $M_{NBVB} \cap P_{NBVB} \in \tau_{\Delta}^{NBVB}$

i.e., the finite intersection of a collection of sets belonging to the NBVB topology, denoted as τ_{Δ}^{NBVB}, is also an element of τ_{Δ}^{NBVB}.

(iii) Let $\{M_{NBVB}^{i}; i \in I\} \subseteq \tau_{\Delta}^{NBVB}$ then $\bigcup_{i \in I} \tau_{\Delta}^{NBVB} \subseteq \tau_{\Delta}^{NBVB}$

i.e., the arbitrarily union of a set of NBVB sets in τ_{Δ}^{NBVB} is also belongs to τ_{Δ}^{NBVB}.

Example 4.1
Consider the sets $S_1 = \{x_1, x_2\}, S_2 = \{y_1\}$ The topology under consideration is a "neutrosophic bipolar vague binary topology.

$$
\tau_{\Delta}^{NBVB} = \left\{
\begin{array}{c}
\emptyset_{NBVB} = \langle \frac{[0,0][1,1][1,1],[0,0][-1,-1][-1,-1]}{x_1} \rangle \langle \frac{[0,0][1,1][1,1],[0,0][-1,-1][-1,-1]}{x_2} \rangle \langle \frac{[0,0][1,1][1,1],[0,0][-1,-1][-1,-1]}{y_1} \rangle \\
M_{NBVB} = \langle \frac{[0.2,0.4][0.6,0.8][0.6,0.8][-0.2,-0.4][-0.6,-0.8][-0.6,-0.8]}{x_1} \rangle \langle \frac{[0.3,0.6][0.7,0.8][0.4,0.7][-0.3,-0.6][-0.7,-0.8][-0.4,-0.7]}{x_2} \rangle \\
\langle \frac{[0.6,0.8][0.7,0.9][0.2,0.4][-0.6,-0.8][-0.7,-0.9][-0.2,-0.4]}{y_1} \rangle \\
P_{NBVB} = \langle \frac{[0.6,0.7][0.1,0.9][0.3,0.4][-0.6,-0.7][-0.1,-0.9][-0.3,-0.4]}{x_1} \rangle \langle \frac{[0.7,0.8][0.3,0.7][0.2,0.3][-0.7,-0.8][-0.3,-0.7][-0.2,-0.3]}{x_2} \rangle \\
\langle \frac{[0.6,0.7][0.2,0.5][0.3,0.4][-0.6,-0.7][-0.2,-0.5][-0.3,-0.4]}{y_1} \rangle \\
K_{NBVB} = M_{NBVB} \cap P_{NBVB} = \langle \frac{[0.2,0.4][0.6,0.9][0.6,0.8][-0.2,-0.4][-0.6,-0.8][-0.6,-0.8]}{x_1} \rangle \\
\langle \frac{[0.3,0.6][0.7,0.8][0.4,0.7][-0.7,-0.8][-0.3,-0.7][-0.2,-0.3]}{x_2} \rangle \langle \frac{[0.6,0.7][0.7,0.9][0.3,0.4][-0.6,-0.8][-0.2,-0.5][-0.2,-0.4]}{y_1} \rangle \\
H_{NBVB} = M_{NBVB} \cup P_{NBVB} = \langle \frac{[0.6,0.7][0.1,0.8][0.3,0.4][-0.2,-0.4][-0.6,-0.9][-0.6,-0.8]}{x_1} \rangle \\
\langle \frac{[0.7,0.8][0.3,0.7][0.2,0.3][-0.3,-0.6][-0.7,-0.8][-0.4,-0.7]}{x_2} \rangle \langle \frac{[0.6,0.8][0.2,0.5][0.2,0.4][-0.6,-0.7][-0.7,-0.9][-0.3,-0.4]}{y_1} \rangle \\
\psi_{NBVB} = \langle \frac{[1,1][0,0],[0,0][-1,-1],[0,0],[0,0]}{x_1} \rangle \langle \frac{[1,1][0,0],[0,0][-1,-1],[0,0],[0,0]}{x_2} \rangle \langle \frac{[1,1][0,0],[0,0][-1,-1],[0,0],[0,0]}{y_1} \rangle
\end{array}
\right\}
$$

Definition 4.2
Neutrosophic bipolar vague binary open set is defined as an element of neutrosophic bipolar vague binary topology.

Example 4.2
$\emptyset_{NBVB}, \psi_{NBVB}$, M_{NBVB}, P_{NBVB}, K_{NBVB}, H_{NBVB} are as a NBVBoS.

Definition 4.3
A NBVBCS is the complement of an NBVBoS.

Example 4.3

$$\emptyset_{\mathrm{NBVB}}{}^{\mathrm{C}}=\left\{\begin{array}{l}\langle\frac{[1,1][0,0],[0,0][-1,-1],[0,0],[0,0]}{x_1}\rangle\\ \langle\frac{[1,1][0,0],[0,0][-1,-1],[0,0],[0,0]}{x_2}\rangle\\ \langle\frac{[1,1][0,0],[0,0][-1,-1],[0,0],[0,0]}{y_1}\rangle\end{array}\right\}=\psi_{\mathrm{NBVB}}$$

$$M_{NBVB}{}^{C}=\left\{\begin{array}{l}\langle\frac{[0.6,0.8][0.2,0.4][0.2,0.4][-0.6,-0.8][-0.2,-0.4][-0.2,-0.4]}{x_1}\rangle\\ \langle\frac{[0.4,0.7][0.2,0.3][0.3,0.6][-0.4,-0.7][-0.2,-0.3][-0.3,-0.6]}{x_2}\rangle\\ \langle\frac{[0.2,0.4][0.1,0.3][0.6,0.8][-0.2,-0.4][-0.1,-0.3][-0.6,-0.8]}{y_1}\rangle\end{array}\right\}$$

$$P_{NBVB}{}^{C}=\left\{\begin{array}{l}\langle\frac{[0.3,0.4][0.1,0.9][0.6,0.7][-0.3,-0.3][-0.1,-0.9][-0.6,-0.7]}{x_1}\rangle\\ \langle\frac{[0.2,0.3][0.3,0.7][0.7,0.8][-0.2,-0.3][-0.3,-0.7][-0.7,-0.8]}{x_2}\rangle\\ \langle\frac{[0.3,0.4][0.5,0.8][0.6,0.7][-0.3,-0.4][-0.5,-0.8][-0.6,-0.7]}{y_1}\rangle\end{array}\right\}$$

$$\psi_{NBVB}{}^{C}=\left\{\begin{array}{l}\langle\frac{[0,0][1,1][1,1],[0,0][-1,-1][-1,-1]}{x_1}\rangle\\ \langle\frac{[0,0][1,1][1,1],[0,0][-1,-1][-1,-1]}{x_2}\rangle\\ \langle\frac{[0,0][1,1][1,1],[0,0][-1,-1][-1,-1]}{y_1}\rangle\end{array}\right\}=\emptyset_{NBVB}$$

Definition 4.4

The triple $(S_1, S_2, \tau_{\Delta}^{NBVB})$ is known as a NBVB topological space where τ_{Δ}^{NBVB} is a NBVB topology.

Example 4.4

If $S_1=\{x_1, x_2\}$ and $S_2=\{y_1\}$ $\tau_{\Delta}^{NBVB}=\{\emptyset_{NBVB}, \psi_{NBVB}, M_{NBVB}, P_{NBVB}, K_{NBVB}, H_{NBVB}\}$ then the triplet $(S_1, S_2, \tau_{\Delta}^{NBVB})$ is clearly a NBVBTS.

Definition 4.5

Let $S_1=\{x_j, \leq j \leq n\}, S_2=\{y_k, 1 \leq k \leq p\}$ be the two universes under study.
Over this common universe, a zero NBVBS is represented as ψ_{NBVB} and is given by

$$\psi_{NBVB}=\left\{\begin{array}{l}\langle\frac{\langle\{[1,1][0,0][0,0]\}^{P}\rangle\langle\{[-1,-1][0,0][0,0]\}^{N}\rangle}{x_j}\forall x_j\in S_1\rangle\\ \langle\frac{\langle\{[1,1][0,0][0,0]\}^{P}\rangle\langle\{[-1,-1][0,0][0,0]\}^{N}\rangle}{y_k}\forall y_k\in S_2\rangle\end{array}\right\}$$

Definition 4.6

consider $S_1 = \{x_j, 1 \le j \le n\}, S_2 = \{y_k, 1 \le k \le p\}$ as the two universes. Over this common universe, a Unit NBVBS defined as $\emptyset_{\mathrm{NBVB}}$ is given by

$$\emptyset_{NBVB} = \left\{\begin{matrix} \langle\frac{\langle\{[0,0][1,1][1,1]\}^P\rangle\langle\{[0,0][-1,-1][-1,-1]\}^N\rangle}{x_j}\forall x_j \in S_1\rangle \\ \langle\frac{\langle\{[0,0][1,1][1,1]\}^P\rangle\langle\{[0,0][-1,-1][-1,-1]\}^N\rangle}{y_k}\forall y_k \in S_2\rangle \end{matrix}\right\}$$

Definition 4.7

Let $(S_1, S_2, \tau_\Delta^{NBVB})$ be a NBVBTS and also let *MNBVB*=

$$\left\{\begin{matrix} \langle(\frac{\hat{T}_{M_{NBVB}}(x_j),\hat{I}_{M_{NBVB}}(x_j),\hat{F}_{M_{NBVB}}(x_j)}{x_j})^P; \forall x_j \in S_1\rangle\,\langle(\frac{\hat{T}_{M_{NBVB}}(y_k),\hat{I}_{M_{NBVB}}(y_k),\hat{F}_{M_{NBVB}}(y_k)}{y_k})^P \forall y_k \in S_2\rangle \\ \langle(\frac{\hat{T}_{M_{NBVB}}(x_j),\hat{I}_{M_{NBVB}}(x_j),\hat{F}_{M_{NBVB}}(x_j)}{x_j})^N; \forall x_j \in S_1\rangle\,\langle(\frac{\hat{T}_{M_{NBVB}}(y_k),\hat{I}_{M_{NBVB}}(y_k),\hat{F}_{M_{NBVB}}(y_k)}{y_k})^N \forall y_k \in S_2\rangle \end{matrix}\right\}$$

A NBVBS defined over a common universe S_1, S_2 based on definition 3.2. The neutrosophic bipolar vague binary interior (designated as M_{NBVB}^0) and closure (designated as a $\overline{M_{NBVB}}$) are then defined as follows:

$M_{NBVB}^0 = \cup \{M_{NBVB}^i; i \in I \backslash M_{NBVB}^i \text{ is a } NBVBOS \text{ over } S_1, S_2 \text{ with } M_{NBVB}^i \subseteq M_{NBVB}; \forall i\}$

$\overline{M_{NBVB}} = \cap \{M_{NBVB}^i; i \in I \backslash M_{NBVB}^i \text{ is a } NBVBCS \text{ over } S_1, S_2 \text{ with } M_{NBVB} \subseteq M_{NBVB}^i; \forall i\}$

Example 4.5 In example 4.1,

$$H_{NBVB}^0 = \left(\left\{\begin{matrix} \langle\frac{[0.6,0.7][0.1,0.8][0.3,0.4][-0.2,-0.4][-0.6,-0.9][-0.6,-0.8]}{x_1}\rangle \\ \langle\frac{[0.7,0.8][0.3,0.7][0.2,0.3][-0.3,-0.6][-0.7,-0.8][-0.4,-0.7]}{x_2}\rangle \\ \langle\frac{[0.6,0.8][0.2,0.5][0.2,0.4][-0.6,-0.7][-0.7,-0.9][-0.3,-0.4]}{y_1}\rangle \end{matrix}\right\}\right) = H_{NBVB}$$

From example 4.1,

$$\overline{M_{NBVB}^C} = \left(\left\{\begin{matrix} \langle\frac{[0.6,0.8][0.2,0.4][0.2,0.4][-0.6,-0.8][-0.2,-0.4][-0.2,-0.4]}{x_1}\rangle \\ \langle\frac{[0.4,0.7][0.2,0.3][0.3,0.6][-0.4,-0.7][-0.2,-0.3][-0.3,-0.6]}{x_2}\rangle \\ \langle\frac{[0.2,0.4][0.1,0.3][0.6,0.8][-0.2,-0.4][-0.1,-0.3][-0.6,-0.8]}{y_1}\rangle \end{matrix}\right\}\right) = M_{NBVB}^C$$

Note that NBVBC (M_{NBVB}) is a NBVBCS and NBVBint (M_{NBVB}) is a NBVBOS in S_1 and S_2. Further,

1. *M_{NBVB} is a NBVBCS* $\Leftrightarrow \overline{M_{NBVB}} = M_{NBVB}$
2. *M_{NBVB} is a NBVBOS* $\Leftrightarrow M_{NBVB}^0 = M_{NBVB}$

Definition 4.8

Let τ_Δ^{NBVB} be NBVBT and M_{NBVB} is defined as

(i) 'Neutrosophic $\alpha -$ open set$[N_\alpha O]$ iff $M_{NBVB} \subseteq M_{NBVB}^0\overline{M_{NBVB}}M_{NBVB}^0(M_{NBVB})$

(ii) 'Neutrosophic semi open set[NSO] iff $M_{NBVB} \subseteq \overline{M_{NBVB}}M_{NBVB}^0(M_{NBVB})$

(iii) ‘Neutrosophic pre open set[NPO] iff $M_{NBVB} \subseteq M^{0}_{NBVB}\overline{M_{NBVB}}(M_{NBVB})$

Definition 4.9

In an NBVBT τ_{Δ}^{NBVB}, an NBVB M_{NBVB} is said to be,

(i) Neutrosophic Bipolar Vague Binary ‘𝔟 -Open (NBVBbO) set is defined if and only if $M_{NBVB} \subseteq M^{0}_{NBVB}\overline{(M_{NBVB})} \cup \overline{M_{NBVB}}(M^{0}_{NBVB})$

(ii) Neutrosophic bipolar vague binary’𝔟 -closed (NBVBbC) set is defined if and only if $M_{NBVB} \supseteq M^{0}_{NBVB}\overline{(M_{NBVB})} \cap \overline{M_{NBVB}}(M^{0}_{NBVB})$

It is obvious that (NBVBPO ∪ NBVBSO) ⊆NBVBbO. Inequalities cannot be substituted for the inclusion.

For an NBVBS M_{NBVB} in a NBVBT τ_{Δ}^{NBVB}

(i) M_{NBVB} is an NBVBbO set iff $\overline{M_{NBVB}}$ is an NBVBbC set.

(ii) M_{NBVB} is an NBVBbC set iff $\overline{M_{NBVB}}$ is an NBVBbO set.

Proof: evident from the definition.

Definition 4.10

Let $(S_1, S_2, \tau_{\Delta}^{NBVB})$ be a NBVBTS and M_{NBVB} be NBVBS over S_1, S_2.

(i) Briefly $[M^{bo}_{NBVB}]$,Neutro 𝔟 – interior of M_{NBVB} is the set of all union of “neutrosophic bipolar vague binary ‘𝔟 – open sets that occur in M_{NBVB}. (ie), $M^{bo}_{NBVB} = \cup \{M^{i}_{NBVB}; i \in I \backslash M^{i}_{NBVB}$ is a NBVbOS over S_1, S_2 with $M^{i}_{NBVB} \subseteq M_{NBVB}; \forall i\}$

(ii) Briefly $[\overline{M^{b}_{NBVB}}]$, Neutro 𝔟 -closure of M_{NBVB} is the set of all intersection of neutrosophic bipolar vague ‘𝔟 -closed set of S_1, S_2 that occur in M_{NBVB}. (ie), $\overline{M^{b}_{NBVB}} = \cap \{M^{i}_{NBVB}; i \in I \backslash M^{i}_{NBVB}$ is a $NBVBbCS$ S_1, S_2 with $M_{NBVB} \subseteq M^{i}_{NBVB}; \forall i\}$

It is evident that the complement of the set $\overline{M^{b}_{NBVB}}$ is the smallest ‘𝔟 -closed neutrosophic bipolar vague binary set over S_1 and S_2 that includes M_{NBVB}”. Additionally, $M^{bo}_{NBVB}$is the largest ‘𝔟 -open neutrosophic bipolar vague binary set over S_1 and S_2 that is contained inside M_{NBVB}.

Theorem 4.1

Let M_{NBVB} neutrosophic bipolar vague binary set in a NBVBT τ_{Δ}^{NBVB} then

(i) $(M^{bo}_{NBVB})^{c} = \overline{M^{c}_{NBVB}}^{\,b}$

(ii) $(\overline{M^{b}_{NBVB}})^{c} = M^{c}_{NBVB}{}^{bo}$

Proof:

Let M_{NBVB} be a NBVBS in NBVBT.

Now $M^{b}_{NBVB}{}^{o} = [\cup \{'D; D$ is a NBVBbO set in S_1, S_2 and $D \subseteq M_{NBVB}{}'\}]^{c}$

$= \cap \{'D^{C}; D^{C}$ is a NBVBbC set in $C\ M^{C}_{NBVB} \subseteq D^{C\prime}\}$

Replacing D^{C} *by ‘M’*, we get,

$(M^{bo}_{NBVB})^{c} = \cap \{'M; M$ *is a* NBVBbC set in S_1, S_2 and $M \supseteq M^{C}_{NBVB}{}'\}$

$(M^{bo}_{NBVB})^{c} = \overline{M^{C}_{NBVB}}^{\,b}$.

This shows how (i) equivalent (ii) can be shown.

Theorem 4.2

In the topological space τ^{NBVB} ,"every neutrosophic bipolar vague binary set of neutrosophic bipolar vague binary pre-opens is equivalent to a set of neutrosophic bipolar vague binary $\mathfrak{b}$ -opens".

Proof:

Assuming $'M_{NBVB}'$ as a 'neutrosophic bipolar vague binary pre-open set' in an NBVBTS τ^{NBVB}.then

$$M_{NBVB} \subseteq (M^{O}_{NBVB})(\overline{M_{NBVB}})$$
$$M_{NBVB} \subseteq (M^{O}_{NBVB})(\overline{M_{NBVB}}) \cup (M^{O}_{NBVB})$$
$$\subseteq (M^{O}_{NBVB})\overline{M_{NBVB}}(M_{NBVB}) \cup \overline{M_{NBVB}}(M_{NBVB})$$
$$\subseteq (M^{O}_{NBVB})[\overline{M_{NBVB}}(M_{NBVB})] \cup \overline{M_{NBVB}}[M^{O}_{NBVB}(M_{NBVB})]$$

This M_{NBVB} is a NBVBbO set.

(iii) Let M_{NBVB} be a NBVBO set in a NBVBT τ^{NBVB}.then

$$M_{NBVB} \subseteq \overline{M_{NBVB}}M^{O}_{NBVB}(M_{NBVB})$$

$$M_{NBVB} \subseteq \overline{M_{NBVB}}M^{O}_{NBVB}(M_{NBVB}) \cup M^{O}_{NBVB}(M_{NBVB})$$

$$M_{NBVB} \subseteq \overline{M_{NBVB}}M^{O}_{NBVB}(M_{NBVB}) \cup M^{O}_{NBVB}\overline{M_{NBVB}}(M_{NBVB})$$

M_{NBVB} is therefore an NBVBbO set.

Theorem 4.3

Assuming M_{NBVB} is an NBVBO set in a NBVBT τ^{NBVB}.then

(i) $\overline{M^{b}_{NBVB}} = \overline{M_{NBVB}} \cap \overline{M^{p}_{NBVB}}$

(ii) $M^{O}_{NBVB} = M^{O}_{NBVB} \cup {M^{p}_{NBVB}}^{o}$

Proof:

(i) Given that $\overline{M^{b}_{NBVB}}$ is an open NBVB $\mathfrak{b}$ set

There is *a* $\overline{M^{b}_{NBVB}} \supseteq M^{O}_{NBVB}M_{NBVB}[\overline{M^{b}_{NBVB}}] \cap \overline{M_{NBVB}}M^{O}_{NBVB}(\overline{M^{b}_{NBVB}}) \supseteq M^{O}_{NBVB}[\overline{M_{NBVB}}] \cap \overline{M_{NBVB}}(M^{O}_{NBVB})$

And also $\overline{M^{b}_{NBVB}} \supseteq M_{NBVB} \cup M^{O}_{NBVB}\overline{M_{NBVB}} \cap \overline{M_{NBVB}}M^{O}_{NBVB} = \overline{M^{s}_{NBVB}} \cap \overline{M^{p}_{NBVB}}$

The inclusion in reverse is evident.

Hence, $\overline{M^{b}_{NBVB}} = \overline{M^{s}_{NBVB}} \cap \overline{M^{p}_{NBVB}}$

In the same way, (ii) can be shown

Theorem 4.4

If M_{NBVB} is an NBVB set contained in an NBVTS τ^{NBVB}.then

(i) $\overline{M^{s}_{NBVB}} = M_{NBVB} \cup M^{O}_{NBVB}\overline{M_{NBVB}}$ *and*

$${M^{s}_{NBVB}}^{o} = M_{NBVB} \cap \overline{M_{NBVB}}M^{O}_{NBVB}$$

(ii) $\overline{M^{p}_{NBVB}} = M_{NBVB} \cup \overline{M_{NBVB}}M^{O}_{NBVB}$

$${M^{p}_{NBVB}}^{o} = M_{NBVB} \cap M^{O}_{NBVB}\overline{M_{NBVB}}$$

Proof:

$\overline{M^{s}_{NBVB}} \supseteq M^{O}_{NBVB}\overline{M_{NBVB}}\,\overline{M^{s}_{NBVB}} \supseteq M^{O}_{NBVB}\overline{M_{NBVB}}$

$$M_{NBVB} \cup \overline{M^{s}_{NBVB}} = \overline{M^{s}_{NBVB}} \supseteq M_{NBVB} \cup M^{O}_{NBVB}\overline{M_{NBVB}}$$

So $M_{NBVB} \cup M^{O}_{NBVB}\overline{M_{NBVB}} \subseteq \overline{M^{s}_{NBVB}}$-----------*(1)*

Also $M_{NBVB} \subseteq \overline{M^{s}_{NBVB}}$,

$$M^{O}_{NBVB}\overline{M_{NBVB}} \subseteq M^{O}_{NBVB}\overline{M_{NBVB}}\,\overline{M^{s}_{NBVB}} \subseteq \overline{M^{s}_{NBVB}}$$

$M_{NBVB} \cup M^{O}_{NBVB}\overline{M_{NBVB}} \subseteq \overline{M^{s}_{NBVB}} \cup M_{NBVB} \subseteq \overline{M^{s}_{NBVB}}$-----------*(2)*

From (1) and (2) we get,

$$\overline{M^{s}_{NBVB}} = M_{NBVB} \cup M^{O}_{NBVB}\overline{M_{NBVB}}$$

$${M^{s}_{NBVB}}^{o} = M_{NBVB} \cap \overline{M_{NBVB}}\,M^{O}_{NBVB}$$

The hypothesis can be illustrated by using the complement of $\overline{M^{s}_{NBVB}} = M_{NBVB} \cup M^{O}_{NBVB}$

This shows that (i)

The evidence for (ii) is similar.

Theorem 4.5

If M_{NBVB} is an NBVB set in a NBVTS τ^{NBVB}then

(i) $\overline{M^{s}_{NBVB}}{M^{s}_{NBVB}}^{o} = {M^{s}_{NBVB}}^{o} \cup M^{O}_{NBVB}\overline{M_{NBVB}}\,M^{O}_{NBVB}$

(ii) ${M^{s}_{NBVB}}^{o}\overline{M^{s}_{NBVB}} = \overline{M^{s}_{NBVB}} \cap \overline{M_{NBVB}}\,M^{O}_{NBVB}\overline{M_{NBVB}}$

Proof:

We have $\overline{M^{s}_{NBVB}}{M^{s}_{NBVB}}^{o} = {M^{s}_{NBVB}}^{o} \cup M^{O}_{NBVB}\overline{M_{NBVB}}\left({M^{s}_{NBVB}}^{o}\right)$

$$= {M^{s}_{NBVB}}^{o} \cup M^{O}_{NBVB}(\overline{M_{NBVB}})[M_{NBVB} \cap \overline{M_{NBVB}}\,M^{O}_{NBVB}]$$

$$\subseteq {M^{s}_{NBVB}}^{o} \cup M^{O}_{NBVB}[\overline{M_{NBVB}} \cap \overline{M_{NBVB}}\,(M^{O}_{NBVB})]$$

$$={M^{s}_{NBVB}}^{o} \cup M^{O}_{NBVB}[\overline{M_{NBVB}}\,(M^{O}_{NBVB})]$$

In order to establish the converse inclusion, it is observed that,

$\overline{M^{s}_{NBVB}}\left({M^{s}_{NBVB}}^{o}\right) = {M^{s}_{NBVB}}^{o} \cup M^{O}_{NBVB}\overline{M_{NBVB}}\,({M^{s}_{NBVB}}^{o}) \supseteq {M^{s}_{NBVB}}^{o} \cup M^{O}_{NBVB}\overline{M_{NBVB}}\,(M^{O}_{NBVB})$

We have $\overline{M^{s}_{NBVB}}\left({M^{s}_{NBVB}}^{o}\right)={M^{s}_{NBVB}}^{o} \cup \overline{M_{NBVB}}\,M^{O}_{NBVB}$

This provides evidence for (i)

The proof for (ii) can be considered equivalent.

Theorem4.6

If M_{NBVB} is an NBVB set in a NBVTS τ^{NBVB}then

(i) $\overline{M^{p}_{NBVB}}{M^{p}_{NBVB}}^{o} = {M^{p}_{NBVB}}^{o} \cup \overline{M_{NBVB}}\,M^{O}_{NBVB}$

(ii) ${M^{p}_{NBVB}}^{o}\overline{M^{p}_{NBVB}} = \overline{M^{p}_{NBVB}} \cap M^{O}_{NBVB}\overline{M_{NBVB}}$

Proof:

We have $\overline{M^{p}_{NBVB}}{M^{p}_{NBVB}}^{o} = {M^{p}_{NBVB}}^{o} \cup \overline{M_{NBVB}}\,M^{O}_{NBVB}\left({M^{p}_{NBVB}}^{o}\right)$

$= {M^{p}_{NBVB}}^{o} \cup \overline{(M_{NBVB}}\,M^{O}_{NBVB}[M_{NBVB} \cap M^{O}_{NBVB}\overline{M_{NBVB}}]) = {M^{p}_{NBVB}}^{o} \cup \overline{M_{NBVB}}\,[M^{O}_{NBVB}]$

In order to establish the converse inclusion, it is observed that,

$\overline{M^{s}_{NBVB}}\left({M^{s}_{NBVB}}^{o}\right)={M^{s}_{NBVB}}^{o} \cup M^{O}_{NBVB}\overline{M_{NBVB}}\,({M^{s}_{NBVB}}^{o}) \supseteq {M^{s}_{NBVB}}^{o} \cup M^{O}_{NBVB}\overline{M_{NBVB}}\,(M^{O}_{NBVB})$

Therefore, we have $\overline{M^{s}_{NBVB}}M^{s}_{NBVB}{}^{o} = M^{s}_{NBVB}{}^{o} \cup M^{O}_{NBVB}\overline{M_{NBVB}}M^{O}_{NBVB}$
This provides evidence for (i)
The proof for (ii) can be considered equivalent.

5. Conclusions

In this paper, we have proposed the idea of neutrosophic bipolar vague binary topological spaces with suitable examples. Also, we have discussed neutrosophic bipolar vague binary 𝔟 -open and 𝔟 -closed set and neutrosophic bipolar vague binary interior and closure with respect to neutrosophic theory and we have given their properties also. In future, we will investigate the relationship between NBVBPOS and NBVBS separation theorem.

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