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On NS $^{s}(\mathscr{I})$ Semi Open Sets

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Abstract

In this paper we present Nano soft(NS) $^{s}(\mathscr{I})$ semi open(SO) sets and NS $^{s}(\mathscr{I})$ semi closed(SC) sets in nano soft ideal topological space. Further we investigate NS $^{s}(\mathscr{I})$ semi interior and NS $^{s}(\mathscr{I})$ semi closure with theorems and examples. Also their properties and characterization are discussed.
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1 Introduction

Molodstov[9] created the soft set (SS) theory in 1999 to address the issue of modelling uncertainty mathematically. The soft topological spaces (STS) were first described by M. Shabir and M. Naz[12]. Many other authors contributed to this area. Soft Semi Open sets and Soft Semi Closed sets were developed by Binchen [2] and their characteristics in soft topological spaces were examined. As for the nano topological spaces, it seems that their study was initiated by Lellis Thivagar et al.[6] in 2013. Since then, various mathematicians proved many important results on nano topologies. In 1990, Jankovic and Hamlett[3] developed the ideal topological space. The notion of soft ideal was initiated by Kandil et al.[4]. S. P. R. Priyalatha et al.[10] invented the nano soft ideal topology. In this paper we investigate NS $^{s}(\mathscr{I})$ Semi Open sets, NS $^{s}(\mathscr{I})$ Semi Closed sets, NS $^{s}(\mathscr{I})$ Semi Interior sets and NS $^{s}(\mathscr{I})$ Semi Closure sets in nano soft ideal topological space based on the concept of nano topology with soft ideal sets. Further, the characterization and properties are discussed with theorems and examples.

2 Preliminaries

Definition 2.1. *Let $\tilde{U}$ be a non empty finite set of objects called the universe, R be an equivalence relation on $\tilde{U}$ named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\tilde{U}, R)$ is said to be approximation space. Let $X \subseteq \tilde{U}$.*

(i) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \left\{ \bigcup\limits_{x \in \tilde{\mathcal{U}}} \{R(x) : R(x) \subseteq X\} \right\}$, where $R(x)$ denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects, which can be for possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \left\{ \bigcup\limits_{x \in \tilde{\mathcal{U}}} \{R(x) : R(x) \cap X \neq \emptyset\} \right\}$.

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$".

Definition 2.2. *Let $\tilde{U}$ be the universe, R be an equivalence relation on $\tilde{U}$ and $\tau_R(x) = \left\{ \tilde{U}, \tilde{\emptyset}, L_R(X), U_R(X), B_R(X) \right\}$ where $X \subseteq \tilde{U}$ and $\tau_R(X)$ satisfies the* *following axioms.*

(i) $\tilde{U}$ *and* $\tilde{\emptyset} \in \tau_R(X)$.

(ii) The union of the elements of any subcollection $\tau_R(X)$ *is in* $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection $\tau_R(X)$ *is in* $\tau_R(X)$.

That is, $\tau_R(X)$ *forms a topology on* $\tilde{U}$ *and it is called as the nano topology on* $\tilde{U}$ *with respect to* X*. The elements of* $\tau_R(X)$ *are called as nano open sets".*

Definition 2.3. *If* $(\tilde{U}, \tilde{\tau}_R(X))$ *is a nano topological space with respect to* X *where* $X \subseteq \tilde{U}$ *and if* $A \subseteq \tilde{U}$*, then the nano interior of* A *is defined as the union of all nano open subsets of* A *and it is denoted by* $NInt(A)$

Definition 2.4. *If* $(\tilde{U}, \tilde{\tau}_R(X))$ *is a nano topological space with respect to* X *where* $X \subseteq \tilde{U}$ *and if* $A \subseteq \tilde{U}$*, then the nano closure of* A *is defined as the intersection of all nano closed sets containing* A *and it is denoted by* $NCl\,(A)$*.*

Definition 2.5. *If* $(\tilde{U}, \tilde{\tau}_R(X))$ *is a nano topological space and* $A \subseteq \tilde{U}$*. Then A is said to be Nano SO if* $A \subseteq NCl(NInt(A))$*.*

Definition 2.6. *A SS* $\mathscr{F}_{\mathscr{A}}$ *on the universe* $\tilde{U}$ *is defined by the set of ordered pairs* $\mathscr{F}_{\mathscr{A}} = \Big\{(e, F(e)) : e \in E, F(e) \in P(\tilde{U})\Big\}$*, where* $F : E \to P(\tilde{U})$ *such that* $F(e) = \emptyset$*, if* $e \notin \mathscr{A}$ *and* $\mathscr{A} \subseteq E$

Definition 2.7. *Let* $\tilde{\tau}$ *be the collection of SSs over* $\tilde{U}$*, then* $\tilde{\tau}$ *is said to be soft topology on* $\tilde{U}$ *if*

(i) $\tilde{U}, \tilde{\emptyset} \in \tilde{\tau}$

(ii) Union of any number of SSs in $\tilde{\tau}$ *belongs to* $\tilde{\tau}$*.*

(iii) Intersection of any two SSs in $\tilde{\tau}$ *belongs to* $\tilde{\tau}$*.*

The triplet $(\tilde{U}, \tilde{\tau}, E)$ *is called soft topological space over* $\tilde{U}$*. The members in* $\tilde{\tau}$ *are said to be soft open sets in* $\tilde{U}$*."*

Definition 2.8. *A soft subset* $\mathscr{F}_{\mathscr{A}}$ *of a soft toplogical space* $\tilde{U}$ *is said to be soft closed if* $\tilde{U} - \mathscr{F}_{\mathscr{A}}$ *is soft open.*

Definition 2.9. *Let $(\tilde{U}, \tilde{\tau}, E)$ be a soft topological space over $\tilde{U}$ and $\mathscr{F}_{\mathscr{A}}$ be a SS over $\tilde{U}$. Then the soft closure of $\mathscr{F}_{\mathscr{A}}$, denoted by $\bar{\mathscr{F}}_{\mathscr{A}}$ is the intersection of all soft closed supersets of $\mathscr{F}_{\mathscr{A}}$."*

Definition 2.10. *Let $(\tilde{U}, \tilde{\tau}, E)$ be a soft topological space over $\tilde{U}$, $\mathscr{F}_{\mathscr{A}}$ be a SS over $\tilde{U}$ and $u \in \tilde{U}$. Then u is said soft interior of $\mathscr{F}_{\mathscr{A}}$ if there exists a soft open set $\mathscr{G}_{\mathscr{A}}$ such that $u \in \mathscr{G}_{\mathscr{A}} \subset \mathscr{F}_{\mathscr{A}}$.*

Definition 2.11. *A SS $\mathscr{F}_{\mathscr{A}}$ in a soft topological space $(\tilde{U}, \tilde{\tau}, E)$ is said to be soft SO if there exists a soft open set $\mathscr{G}_{\mathscr{A}}$ such that $\mathscr{G}_{\mathscr{A}} \subset \mathscr{F}_{\mathscr{A}} \subset Cl(\mathscr{G}_{\mathscr{A}})$.*

Definition 2.12. *An ideal $\mathscr{I}$ on a topological space $(\tilde{U}, \tilde{\tau})$ is a non empty collection of subsets of $\tilde{U}$ which satisfies*

(i) $A \in \mathscr{I}$ and $B \subseteq A$ imply $B \in \mathscr{I}$ and

(ii) $A \in \mathscr{I}$ and $B \in \mathscr{I}$ imply $A \cup B \in \mathscr{I}$.

Definition 2.13. *Let $\mathscr{I}$ be the non empty collection of SSs over $\tilde{U}$, with the same set of parameters E. Then $\mathscr{I} \subseteq SS(\tilde{U})_E$ is called a soft ideal on $\tilde{U}$ with the same set E if,*

(i) $\mathscr{F}_E \in \mathscr{I}$ and $\mathscr{G}_E \in \mathscr{I}$ implies $\mathscr{F}_E \cup \mathscr{G}_E \in \mathscr{I}$.

(ii) $\mathscr{F}_E \in \mathscr{I}$ and $\mathscr{G}_E \subseteq \mathscr{F}_E$ implies $\mathscr{G}_E \in \mathscr{I}$."

Definition 2.14. *Let $\tilde{U}$ be a non empty finite set of objects called the universe, $\mathscr{F}_{\mathscr{A}} \subseteq \mathscr{G}_{\mathscr{A}}$ is an SS over $\tilde{U}$ and $\mathscr{I}$ is an ideal on $\mathscr{G}_{\mathscr{A}}$. Then $(\tilde{U}, \mathscr{F}_{\mathscr{A}}, \mathscr{I})$ is an triplet ordered pair of soft ideal approximation space and $\tilde{\tau}_R(\mathscr{I}) = \left\{\tilde{U}, \emptyset, L_R(\mathscr{I}), U_R(\mathscr{I}), B_R(\mathscr{I})\right\}$ where $\mathscr{I} \subseteq \mathscr{G}_{\mathscr{A}}$ and $\tilde{\tau}_R(\mathscr{I})$ satisfies the following axiom*

(i) $\tilde{U}, \emptyset \in \tilde{\tau}_R(\mathscr{I})$

(ii) The union of the elements of any subcollection of soft ideal $\tilde{\tau}_R(\mathscr{I})$is in $\tilde{\tau}_R(\mathscr{I})$.

(iii) The intersection of the elements of any finite subcollection of soft ideal $\tilde{\tau}_R(\mathscr{I})$is in $\tilde{\tau}_R(\mathscr{I})$.

That is, $\tilde{\tau}_R(\mathscr{I})$ forms a soft ideal topology on $\tilde{U}$ having atmost five elements of soft ideal and four ordered pair $(\tilde{U}, \tilde{\tau}_R, E, \mathscr{I})$ is called a NS ideal topological space over $\tilde{U}$ with respect to $\mathscr{I}$, then the members of $\tilde{\tau}_R$ are said to be NS ideal open sets in $\tilde{U}$.

Definition 2.15. *Let $(\tilde{U}, \tilde{\tau}, E, \mathscr{I})$ be a NS ideal topological space over $\tilde{U}$. Then NS ideal closure of SS $\mathscr{F}_E$ over $\tilde{U}$ is denoted by $\mathscr{N}_{SI}Cl(\mathscr{F}_E)$. Thus $\mathscr{N}_{SI}Cl(\mathscr{F}_E)$ is the smallest NS ideal closed set which containing $\mathscr{F}_E$ and is defined as the intersection of all NS ideal closed supersets of $\mathscr{F}_E$.*

Definition 2.16. *Let $\mathscr{F}_{\mathscr{B}}$ be a soft subset of a soft topological space $(\mathscr{F}_A, \tilde{\tau})$. $\mathscr{F}_B$ is said to be a soft SO set if,$\mathscr{F}_B \subseteq cl(int(\mathscr{F}_B))$*

Definition 2.17. *Let $\mathscr{F}_{\mathscr{B}}$ be a soft subset of a soft topological space $(\mathscr{F}_A, \tilde{\tau})$. $\mathscr{F}_B$ is said to be a soft SC set if its relative complement is soft SO.*

Definition 2.18. *Let $(\mathscr{F}_A, \tilde{\tau})$ be a soft topological space and let $\mathscr{F}_B$ be a SS in $\mathscr{F}_A$. The soft semi- interior of $\mathscr{F}_B$ is the SS $\cup\{\mathscr{F}_C : \mathscr{F}_C$ is soft SO and $\mathscr{F}_C \subseteq \mathscr{F}_B\}$ and is denoted by S- int$(\mathscr{F}_B)$.*

Definition 2.19. *Let $(\mathscr{F}_A, \tilde{\tau})$ be a soft topological space and let $\mathscr{F}_B$ be a SS in $\mathscr{F}_A$. The soft semi- closure of $\mathscr{F}_B$ is the SS $\cap\{\mathscr{F}_C : \mathscr{F}_C$ is soft SC and $\mathscr{F}_B \subseteq \mathscr{F}_C\}$ and is denoted by S- cl$(\mathscr{F}_B)$.*

3 NS $^s(\mathscr{I})$ SO Sets

In this section we present NS $^s(\mathscr{I})$ SO sets, NS $^s(\mathscr{I})$ SC sets, NS $^s(\mathscr{I})$ semi interior and NS $^s(\mathscr{I})$ semi closure with theorems and examples. Throughout this paper we represent nano soft ideal semi open sets by NS $^s(\mathscr{I})$ SO sets, nano soft ideal semi closed sets by NS $^s(\mathscr{I})$ SC sets, nano soft ideal topological space by NSITS.

Definition 3.1. *"Let $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ be a NS ideal topological space over $\tilde{Y}$ and soft ideal $\mathscr{F}_A$ is said to be NS $^s(\mathscr{I})$ SO set if $\mathscr{F}_A \subseteq \mathscr{N}_{SI}Cl(\mathscr{N}_{SI}Int(\mathscr{F}_A)$.*

Example 3.1. *Let $\tilde{Y} = \{y_1, y_2\}$ and $E = \{e_1, e_2\}$ Let $A = \{e_1\}$. Then $\mathscr{F}_A = \{(e_1, \tilde{Y})\}$ and $^s(\mathscr{I}) \subseteq \mathscr{F}_E$ where $^s(\mathscr{I}) = \{\emptyset, (e_1, \{y_1\})\}$ and $R = \{\mathscr{F}(y_1) \times \mathscr{F}(y_1)\}$. Here $\iota^s_R(\mathscr{I}) = \{\tilde{Y}, \emptyset, (e_1, \{y_1, y_2\})\}$ and $(\iota^s_R(\mathscr{I}))^c = \{\tilde{Y}, \emptyset, (e_2, \{y_1, y_2\})\}$. Then the family of all NS$^s(\mathscr{I})$ SO sets are $\emptyset, \tilde{Y}, \{e_1, \{y_1, y_2\}\}, \{(e_1, \{y_1, y_2\}), (e_2, \{y_1\})\}, \{(e_1, \{y_1, y_2\}), (e_2, \{y_2\})\}$ and the family of all NS$^s(\mathscr{I})$ SC sets are $\emptyset, \tilde{Y}, \{e_2, \{y_1\}\}, \{(e_2, \{y_1, y_2\})\}, \{e_2, \{y_2\}\}$.*

Theorem 3.1. *Every NS $^s(\mathscr{I})$ open set in NS ideal space, $\mathscr{F}_A$ is a NS $^s(\mathscr{I})$ SO set.*

Proof. This proof is deduced from definition 3.1. □

Remark 3.1. *The above theorem does not hold true in its opposite implication. Therefore, not all NS $^s(\mathscr{I})$ SO sets are NS $^s(\mathscr{I})$ open sets.*

Example 3.2. *Consider the NS $^s(\mathscr{I})$ SO set in the example 3.1 are* $\{(e_1, \{y_1, y_2\}), (e_2, \{y_1\})\}, \{(e_1, \{y_1, y_2\}), (e_2, \{y_2\})\}$ *but these are not NS $^s(\mathscr{I})$ open sets.*

Remark 3.2. *$\mathscr{F}_\emptyset, \mathscr{F}_A$ are always NS $^s(\mathscr{I})$ S- closed and NS$^s(\mathscr{I})$ S- open sets.*

Theorem 3.2. *A soft ideal $\mathscr{F}_B$ in a nano soft ideal topological space is a NS $^s(\mathscr{I})$ S- open set iff there exists a NS $^s(\mathscr{I})$ open set $\mathscr{F}_C \ni \mathscr{F}_C \subset \mathscr{F}_B \subset N_{SI}cl(\mathscr{F}_C)$.*
Proof. *Suppose that $\mathscr{F}_B \subseteq N_{SI}cl(N_{SI}int(\mathscr{F}_B))$.Then for $\mathscr{F}_C = N_{SI}int(\mathscr{F}_B)$, we have $\mathscr{F}_C \subseteq \mathscr{F}_B \subseteq N_{SI}cl(\mathscr{F}_C)$. Therefore the condition holds. Conversely suppose that $\mathscr{F}_C \subset \mathscr{F}_B \subset N_{SI}cl(\mathscr{F}_C))$ for some NS $^s(\mathscr{I})$ open set $\mathscr{F}_C$. Since $\mathscr{F}_C \subseteq N_{SI}int(\mathscr{F}_B)$ and so $N_{SI}cl(\mathscr{F}_C) \subseteq N_{SI}cl(N_{SI}int(\mathscr{F}_B))$. Hence $\mathscr{F}_B \subseteq N_{SI}cl(\mathscr{F}_C) \subseteq N_{SI}cl(N_{SI}int(\mathscr{F}_B))$. Hence $\mathscr{F}_B$ is NS $^s(\mathscr{I})$ SO set.* □

Theorem 3.3. *Let $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ be a NS ideal topological space over $\tilde{Y}$ and $\{(\mathscr{F}_{B_\alpha}) : \alpha \in \Delta\}$ be a set of NS $^s(\mathscr{I})$ S- open sets in $\tilde{Y}$. Then $\cup_{\alpha\in\Delta}\mathscr{F}_{B_\alpha}$ is also a NS $^s(\mathscr{I})$ S- open set.*
Proof. *Let $\{(\mathscr{F}_{B_\alpha}) : \alpha \in \Delta\}$ be a collection of NS $^s(\mathscr{I})$ SO sets in $\tilde{Y}$. Then for each $\alpha \in \Delta$, we have a NS $^s(\mathscr{I})$ open set $\mathscr{F}_{C_\alpha} \subseteq \mathscr{F}_{B_\alpha}$ such that $\mathscr{F}_{C_\alpha} \subseteq \mathscr{F}_{B_\alpha} \subseteq N_{SI}cl(\mathscr{F}_{C_\alpha})$. Then $\cup_{\alpha\in\Delta}\mathscr{F}_{C_\alpha} \subseteq \mathscr{F}_{B_\alpha} \subseteq \cup_{\alpha\in\Delta}N_{SI}cl(\mathscr{F}_{C_\alpha}) \subseteq N_{SI}cl(\cup_{\alpha\in\Delta}\mathscr{F}_{C_\alpha})$.*
Hence $\cup_{\alpha\in\Delta}\mathscr{F}_{B_\alpha}$ is a NS $^s(\mathscr{I})$ SO set. □

Definition 3.2. *A soft ideal $\mathscr{F}_B$ in a NS ideal space $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is called a NS $^s(\mathscr{I})$ SC set, if its complement is a NS $^s(\mathscr{I})$ SO set.*

Theorem 3.4. *For each NSI closed set in a NS ideal TS $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is NS $^s(\mathscr{I})$ SC set.*
Proof. *Assume that $\mathscr{F}_B$ in a NSITS $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ is a NS $^s(\mathscr{I})$ closed set. By the definition of NS $^s(\mathscr{I})$ closed set, its complement is NS $^s(\mathscr{I})$ open set. Then $(\mathscr{F}_B)^c$ is a NS $^s(\mathscr{I})$ SO set. Hence $\mathscr{F}_B$ is a NS $^s(\mathscr{I})$ SC set.* □

Remark 3.3. *The converse of the above theorem is not true. In other words, not every NS $^s(\mathscr{I})$ SC set is an NS $^s(\mathscr{I})$ closed set.*

Example 3.3. *From the example 3.2, consider the NS $^s(\mathscr{I})$ SC sets $\{(e_1, y_1)\}$, $\{(e_2, y_2)\}$ which are not NS $^s(\mathscr{I})$ closed sets.*

Theorem 3.5. *Let $\mathscr{F}_C$ be a NS $^s(\mathscr{I})$ SC set in a NS ideal topological space $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ iff $N_{SI}int\mathscr{F}_E \subseteq \mathscr{F}_C \subseteq \mathscr{F}_E$ for some NS $^s(\mathscr{I})$ closed set $\mathscr{F}_E$.*
Proof : *Since $\mathscr{F}_C$ is NS $^s(\mathscr{I})$ SC*

$\iff$ *$(\mathscr{F}_C)^c$ is NS $^s(\mathscr{I})$ SO*

$\iff$ *there exists $\mathscr{F}_D \ni \mathscr{F}_D \subseteq (\mathscr{F}_C)^c \subseteq N_{SI}cl(\mathscr{F}_D)$*
by theorem 3.2

$\iff$ *there is a NS* $^{s}(\mathscr{I})$ *open set* $\mathscr{F}_D \ni (N_{SI}cl(\mathscr{F}_D))^c \subseteq \mathscr{F}_C \subseteq (\mathscr{F}_D)^c$.
$\iff$ *a NS* $^{s}(\mathscr{I})$ *open set* $\mathscr{F}_D \ni N_{SI}int(\mathscr{F}_D^c) \subseteq \mathscr{F}_C \subseteq (\mathscr{F}_D)^c$.
$\iff$ *there exists* $\mathscr{F}_D \ni N_{SI}int(\mathscr{F}_E) \subseteq \mathscr{F}_C \subseteq \mathscr{F}_E$
where $\mathscr{F}_E = \mathscr{F}_D^c$. □

Theorem 3.6. *A soft ideal* $\mathscr{F}_B$ *in a NSITS* $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ *is NS* $^{s}(\mathscr{I})$ *SC iff* $N_{SI}int(N_{SI}cl(\mathscr{F}_B)) \subseteq \mathscr{F}_B$.
Proof : $\mathscr{F}_B$ *is NS* $^{s}(\mathscr{I})$ *SC*
$\iff (\mathscr{F}_B)^c \subseteq N_{SI}cl(N_{SI}int(\mathscr{F}_B)^c)$
$\iff (\mathscr{F}_B)^c \subseteq N_{SI}cl(N_{SI}cl(\mathscr{F}_B)^c)$, *by definition*
$\iff (\mathscr{F}_B)^c \subseteq (N_{SI}int(N_{SI}cl(\mathscr{F}_B)))^c$
$\iff (\mathscr{F}_B)^c \subseteq N_{SI}int(N_{SI}cl(\mathscr{F}_B)) \subseteq \mathscr{F}_B$
This completes the proof. □

Theorem 3.7. *Let* $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ *be a NSITS and* $\{(\mathscr{F}_{B_\alpha}) : \alpha \in \Delta\}$ *be a collection of NS* $^{s}(\mathscr{I})$ *SC sets in* $\tilde{Y}$. *Then* $\cap_{\alpha\in\Delta}\mathscr{F}_{B_\alpha}$ *is also NS* $^{s}(\mathscr{I})$ *SC set.*
Proof : *Let* $\{(\mathscr{F}_{B_\alpha}) : \alpha \in \Delta\}$ *be a collection of NS* $^{s}(\mathscr{I})$ *SC sets in* $\tilde{Y}$. *Then for each* $\alpha \in \Delta$, *we have a NS* $^{s}(\mathscr{I})$ *closed set* $\mathscr{F}_{C_\alpha}$ *such that* $N_{SI}int\mathscr{F}_{C_\alpha} \subseteq \mathscr{F}_{B_\alpha} \subseteq \mathscr{F}_{C_\alpha}$. *Then* $N_{SI}int(\cap_{\alpha\in\Delta}\mathscr{F}_{C_\alpha}) \subseteq \cap_{\alpha\in\Delta}N_{SI}int(\mathscr{F}_{C_\alpha}) \subseteq \cap_{\alpha\in\Delta}\mathscr{F}_{B_\alpha} \subseteq \cap_{\alpha\in\Delta}\mathscr{F}_{C_\alpha}$. *Because* $\cap_{\alpha\in\Delta}\mathscr{F}_{C_\alpha} = \mathscr{F}_C$ *is NS* $^{s}(\mathscr{I})$ *closed set then* $\cap_{\alpha\in\Delta}\mathscr{F}_{B_\alpha}$ *is NS* $^{s}(\mathscr{I})$ *SC set.* □

Definition 3.3. *Let* $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ *be a NSITS and let* $\mathscr{F}_B$ *be a soft ideal in* $\tilde{Y}$.

(i) The NS $^{s}(\mathscr{I})$ *semi- interior of* $\mathscr{F}_B$ *is the* $\cup\{\mathscr{F}_C : \mathscr{F}_C$ *NS*$^{s}(\mathscr{I})$ *SO and* $\mathscr{F}_C \subseteq \mathscr{F}_B\}$ *and is represented by NS*$^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B)$.

(ii) The NS $^{s}(\mathscr{I})$ *semi- closure of* $\mathscr{F}_B$ *is the* $\cap\{\mathscr{F}_C : \mathscr{F}_C$ *NS*$^{s}(\mathscr{I})$ *SC and* $\mathscr{F}_B \subseteq \mathscr{F}_C\}$ *and is represented by NS* $^{s}(\mathscr{I})$ *S- cl*$(\mathscr{F}_B)$.

Clearly NSI $^{s}(\mathscr{I})$ *S- cl*$(\mathscr{F}_B)$ *is the smallest NS*$^{s}(\mathscr{I})$ *SC set containing* $\mathscr{F}_B$ *and NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B)$ *is the largest NS*$^{s}(\mathscr{I})$ *SO set contained in* $\mathscr{F}_B$.
By theorem 3.8 and 3.15 we have NSI S-int$(\mathscr{F}_B)$ *is NS*$^{s}(\mathscr{I})$ *SO and NS* $^{s}(\mathscr{I})$ *S-cl*$(\mathscr{F}_B)$ *is NS* $^{s}(\mathscr{I})$ *SC sets.*

Example 3.4. *Let* $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ *be a NSITS and the soft ideal* $\mathscr{F}_B = \{(e_1, y_1), (e_2, y_1, y_2)\}$ *as in example 3.2 we get NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B) = \mathscr{F}_B$.

Example 3.5. *Let* $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ *be a NSITS and the soft ideal* $\mathscr{F}_B = \{(e_1, y_1), (e_2, y_2)\}$ *as in example 3.2 we get NS* $^{s}(\mathscr{I})$ *S-cl*$(\mathscr{F}_B) = \mathscr{F}_B$.

Theorem 3.8. *Let* $(\tilde{Y}, \tilde{\iota}_R(^{s}(\mathscr{I})))$ *be a NSITS and* $\mathscr{F}_B$ *be a soft ideal in* $\tilde{Y}$. *We have* $N_{SI}int(\mathscr{F}_B) \subseteq$*NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B) \subseteq \mathscr{F}_B \subseteq$*NS* $^{s}(\mathscr{I})$ *S-cl*$(\mathscr{F}_B) \subseteq N_{SI}cl(\mathscr{F}_B)$.
Proof: *This proof follows from the theorem 3.1, theorem 3.4 and definition 3.3.*

Theorem 3.9. *Let $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ be a NSITS and $\mathscr{F}_B$ be a soft ideal in $\mathscr{F}_A$. Then the following hold.*

(i) (NS $^s(\mathscr{I})S - cl(\mathscr{F}_B))^c$ =NS $^s(\mathscr{I})S - int(\mathscr{F}_B^c)$

(ii) (NS $^s(\mathscr{I})S - int(\mathscr{F}_B))^c$ =NS $^s(\mathscr{I})S - cl(\mathscr{F}_B^c)$

Proof :

(i) (NS $^s(\mathscr{I})$ S-cl$(\mathscr{F}_B))^c = (\cap\{\mathscr{F}_C : \mathscr{F}_C$ is NS $^s(\mathscr{I})$ semi closed and $\mathscr{F}_C \subseteq \mathscr{F}_B\})^c$.
$= \cup\{\mathscr{F}_C^c : \mathscr{F}_C$ is NS$^s(\mathscr{I})$ SC and $\mathscr{F}_C \subseteq \mathscr{F}_B\}$.

$= \cup\{\mathscr{F}_C^c : \mathscr{F}_C^c$ is NS $^s(\mathscr{I})$ SO and $\mathscr{F}_C^c \subseteq \mathscr{F}_B^c\}$.

=NS $^s(\mathscr{I})$ int $(\mathscr{F}_B)^c$.

(ii) (NS $^s(\mathscr{I})$ S-int$(\mathscr{F}_B))^c = (\cup\{\mathscr{F}_C : \mathscr{F}_C$ is NS $^s(\mathscr{I})$ semi open and $\mathscr{F}_C \subseteq \mathscr{F}_B\})^c$.

$= \cap\{\mathscr{F}_C^c : \mathscr{F}_C$ is NS $^s(\mathscr{I})$ SO and $\mathscr{F}_C \subseteq \mathscr{F}_B\}$.

$= \cap\{\mathscr{F}_C^c : \mathscr{F}_C^c$ is NS $^s(\mathscr{I})$ SC and $\mathscr{F}_C^c \subseteq \mathscr{F}_B^c\}$.

=NS $^s(\mathscr{I})$ cl $(\mathscr{F}_B)^c$.

Theorem 3.10. *Let $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ be a NSITS and let $\mathscr{F}_B$ and $\mathscr{F}_C$ are soft ideals in $\tilde{Y}$. Then the next holds.*

(i) NS $^s(\mathscr{I})$ S-cl$(\mathscr{F}_\emptyset) = \mathscr{F}_\emptyset$ and NS $^s(\mathscr{I})$ S- cl$(\mathscr{F}_A) = \mathscr{F}_A$.

(ii) $\mathscr{F}_B$ is NS $^s(\mathscr{I})$ SC set iff $\mathscr{F}_B =$ NS $^s(\mathscr{I})$ S-cl$(\mathscr{F}_B)$.

(iii) NS $^s(\mathscr{I})$ S-cl(NS $^s(\mathscr{I})$ S-cl$(\mathscr{F}_B)$) =NS $^s(\mathscr{I})$ S-cl$(\mathscr{F}_B)$.

(iv) $\mathscr{F}_B \subset \mathscr{F}_C$ implies NS $^s(\mathscr{I})$ S-cl $(\mathscr{F}_B) \subseteq$ NS $^s(\mathscr{I})$ S-cl$(\mathscr{F}_C)$.

(v) NS $^s(\mathscr{I})$ S-cl$(\mathscr{F}_B \cap \mathscr{F}_C) \subseteq$NS $^s(\mathscr{I})$ S-cl$(\mathscr{F}_B)\cap$NS $^s(\mathscr{I})$ S-cl$(\mathscr{F}_C)$.

(vi) NS $^s(\mathscr{I})$ S-cl$(\mathscr{F}_B \cup \mathscr{F}_C) \subseteq$NS $^s(\mathscr{I})$ S-cl$(\mathscr{F}_B)\cup$NS $^s(\mathscr{I})$ S-cl$(\mathscr{F}_C)$.

Proof:

(i) It is trivial.

(ii) *If $\mathscr{F}_B$ is NS $^s(\mathscr{I})$ S- closed set, then $\mathscr{F}_B$ is itself a NS $^s(\mathscr{I})$ S-closed in $\mathscr{F}_A$ containing $\mathscr{F}_B$. So NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_B$) is the smallest NS $^s(\mathscr{I})$ SC set containing $\mathscr{F}_B$ and $\mathscr{F}_B$ =NS $^s(\mathscr{I})$ S-cl(F_B). Conversely, suppose that $\mathscr{F}_B$ =NS$^s(\mathscr{I})$ S-cl($\mathscr{F}_B$). Since NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_B$) is a NS $^s(\mathscr{I})$ SC set, so $\mathscr{F}_B$ is NS $^s(\mathscr{I})$ SC set.*

(iii) *Since NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_B$) is a NS$^s(\mathscr{I})$ SC set therefore by part (ii) we have NS $^s(\mathscr{I})$ S-cl(NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_B$)) =NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_B$).*

(iv) *Suppose that $\mathscr{F}_B \subseteq \mathscr{F}_C$. Then every NS $^s(\mathscr{I})$ SC super set of $\mathscr{F}_C$ will also contain $\mathscr{F}_B$. This means every NS $^s(\mathscr{I})$ SC super set of $\mathscr{F}_C$ is also a NS $^s(\mathscr{I})$ SC super set of $\mathscr{F}_B$. Hence the NS $^s(\mathscr{I})$ intersection of NS $^s(\mathscr{I})$ semi- closd super sets of $\mathscr{F}_B$ is contained in the NS$^s(\mathscr{I})$ intersection of NS $^s(\mathscr{I})$ SC super sets of $\mathscr{F}_C$. Thus NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_B$) $\subseteq$NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_C$).*

(v) *Since $\mathscr{F}_B \cap \mathscr{F}_C \subseteq \mathscr{F}_B$ and $\mathscr{F}_B \cap \mathscr{F}_C \subseteq \mathscr{F}_C$ and so by part(iv) NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_B \cap \mathscr{F}_C$) $\subseteq$NS $^s(\mathscr{I})$ S- cl ($\mathscr{F}_B$) and NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_B \cap \mathscr{F}_C$) $\subseteq$NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_C$).Thus NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_B \cap \mathscr{F}_C$) $\subseteq$NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_B$)$\cap$NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_C$).*

(vi) *Since $\mathscr{F}_B \subseteq \mathscr{F}_B \cup \mathscr{F}_C$ and $\mathscr{F}_C \subseteq \mathscr{F}_B \cup \mathscr{F}_C$, so by part(iv) $\mathscr{F}_B \subseteq \mathscr{F}_C$ implies NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_B$) $\subseteq$NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_C$). Then NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_B$) $\subseteq$NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_B \cup \mathscr{F}_C$) and NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_C$) $\subseteq$NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_B \cup \mathscr{F}_C$), which implies NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_B$)$\cup$NS $^s(\mathscr{I})$ S- int($\mathscr{F}_C$) $\subseteq$NS $^s(\mathscr{I})$ S- cl ($\mathscr{F}_B \cup \mathscr{F}_C$). Now, NS$^s(\mathscr{I})$ S- cl($\mathscr{F}_B$), NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_C$) belongs to NS $^s(\mathscr{I})$ SC set in $\mathscr{F}_A$ which implies that NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_B$)$\cup$NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_C$) belong to NS $^s(\mathscr{I})$ SC set in $\mathscr{F}_A$. Then $\mathscr{F}_B$ $\subseteq$NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_B$) and $\mathscr{F}_C$ $\subseteq$NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_C$) imply $\mathscr{F}_B \cup \mathscr{F}_C \subseteq$ NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_B$)$\cup$NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_C$). That is, NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_B$)$\cup$NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_C$) is a NS $^s(\mathscr{I})$ SC set containing $\mathscr{F}_B \subseteq \mathscr{F}_C$. Hence NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_B \cup \mathscr{F}_C$) $\subseteq$NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_B$)$\cup$NS $^s(\mathscr{I})$ S- cl ($\mathscr{F}_C$). So, NS $^s(\mathscr{I})$ S-cl($\mathscr{F}_B \cup \mathscr{F}_C$) =NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_B$)$\cup$NS $^s(\mathscr{I})$ S- cl($\mathscr{F}_C$).*

Theorem 3.11. *Let $(\tilde{Y}, \tilde{\iota}_R(^s(\mathscr{I})))$ be a NSITS and let $\mathscr{F}_B$ and $\mathscr{F}_C$ are soft ideals in $\tilde{Y}$. Then the following holds.*

(i) *NS $^s(\mathscr{I})$ S-int($\mathscr{F}_\emptyset$) $= \mathscr{F}_\emptyset$ and NS $^s(\mathscr{I})$ S- int($\mathscr{F}_A$) $= \mathscr{F}_A$.*

(ii) *$\mathscr{F}_B$ is NS $^s(\mathscr{I})$ S- open set iff $\mathscr{F}_B =$ NS $^s(\mathscr{I})$ S-int($\mathscr{F}_B$).*

(iii) *NS $^s(\mathscr{I})$ S-int(NS $^s(\mathscr{I})$ S-int($\mathscr{F}_B$)) =NS $^s(\mathscr{I})$ S-int($\mathscr{F}_B$).*

(iv) *$\mathscr{F}_B \subseteq \mathscr{F}_C$ implies NS $^s(\mathscr{I})$ S-int ($\mathscr{F}_B$) $\subseteq$ NS $^s(\mathscr{I})$ S-int($\mathscr{F}_C$).*

(v) *NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B)$∩*NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_C)$ ⊆*NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B \cap \mathscr{F}_C)$.

(vi) *NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B \cup \mathscr{F}_C)$=*NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B)$∪*NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_C)$.

Proof:

(i) *Clear.*

(ii) *First part of the proof is obvious from the definition 3.1. Conversely, suppose that* $\mathscr{F}_B$ =*NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B)$. *Since NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B)$ *is a NS* $^{s}(\mathscr{I})$ *SO set, so* $\mathscr{F}_B$ *is NS* $^{s}(\mathscr{I})$ *S- open set in* $\mathscr{F}_A$.

(iii) *Since NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B)$ *is a NS* $^{s}(\mathscr{I})$ *S- open set therefore by part (ii) we have NS* $^{s}(\mathscr{I})$ *S-int(NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B))$ =*NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B)$.

(iv) *Suppose that* $\mathscr{F}_B \subseteq \mathscr{F}_C$. *Since NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B) \subseteq \mathscr{F}_B \subseteq \mathscr{F}_C$. *NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B)$ *is a NS* $^{s}(\mathscr{I})$ *SO subset of* $\mathscr{F}_C$, *so by the definition of NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_C)$, *NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B)$ ⊆*NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_C)$.

(v) *Since*$\mathscr{F}_B \subseteq \mathscr{F}_B \cap \mathscr{F}_C$ *and* $\mathscr{F}_C \subseteq \mathscr{F}_B \cap \mathscr{F}_C$ *and so by part(iv) NS* $^{s}(\mathscr{I})$ *S- int* $(\mathscr{F}_B)$ ⊆*NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B \cap \mathscr{F}_C)$ *and NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_C)$ ⊆ *NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B \cap \mathscr{F}_C)$. *So that NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B)$∩*NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_C)$ ⊆ *NS* $^{s}(\mathscr{I})$ *S- int*$(F_B \cap \mathscr{F}_C)$, *since NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B \cap \mathscr{F}_C)$ *is a NS* $^{s}(\mathscr{I})$ *SO set.*

(vi) *Since* $\mathscr{F}_B \subseteq \mathscr{F}_B \cup \mathscr{F}_C$ *and* $\mathscr{F}_C \subseteq \mathscr{F}_B \cup \mathscr{F}_C$, *so by part(iv)* $\mathscr{F}_B \subseteq \mathscr{F}_C$ *implies NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B)$ ⊆*NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_C)$. *Then NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B)$ ⊆*NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B \cup \mathscr{F}_C)$ *and NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_C)$ ⊆*NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B \cup \mathscr{F}_C)$, *which implies NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B)$∪*NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_C)$ ⊆*NS* $^{s}(\mathscr{I})$ *S- int* $(\mathscr{F}_B \cup \mathscr{F}_C)$. *Now, NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B)$, *NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_C)$ *belongs to NS* $^{s}(\mathscr{I})$ *SO set in* $\mathscr{F}_A$ *which implies that NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B)$∪*NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_C)$ *belong to NS* $^{s}(\mathscr{I})$ *semi- open set in* $\mathscr{F}_A$. *Then* $\mathscr{F}_B$ ⊆*NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B)$ *and* $\mathscr{F}_C$ ⊆*NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_C)$ *imply* $\mathscr{F}_B \cup \mathscr{F}_C$ ⊆ *NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B)$∪*NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_C)$. *That is, NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B)$∪*NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_C)$ *is a NS* $^{s}(\mathscr{I})$ *SO set containing* $\mathscr{F}_B \cup \mathscr{F}_C$. *Hence NS* $^{s}(\mathscr{I})$ *S-int*$(\mathscr{F}_B \cup \mathscr{F}_C)$ ⊆*NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B)$∪*NS*$^{s}(\mathscr{I})$ *S- int* $(\mathscr{F}_C)$. *So, NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B \cup \mathscr{F}_C)$ =*NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_B)$∪*NS* $^{s}(\mathscr{I})$ *S- int*$(\mathscr{F}_C)$.

4 Conclusion

This paper investigate the concept of NS $^{s}(\mathscr{I})$ SO sets in nano soft ideal topological space. Some of the theorems and examples are discussed. Moreover, we defined NS $^s(\mathscr{I})$ SC sets, NS $^s(\mathscr{I})$ semi interior, NS $^s(\mathscr{I})$ semi closure sets and go through their properties.

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