CHEMICAL ENGINEERING TRANSACTIONS

VOL. 107, 2023

A publication of AIDIC

The Italian Association of Chemical Engineering
Online at www.cetjournal.it

Guest Editors: Petar S. Varbanov, Bohong Wang, Petro Kapustenko

ISBN 979-12-81206-07-6; **ISSN** 2283-9216

DOI: 10.3303/CET23107103

Optimizing Topology of Structures Considering Fatigue-Resistance

Muayad Habashneh, Majid Movahedi Rad*

Faculty of Architecture, Civil Engineering and Transport Sciences, Department of Structural and Geotechnical Engineering, Széchenyi István University, H-9026 Győr, Hungary
majidmr@sze.hu

The pursuit of reliable design has become increasingly crucial in various engineering disciplines, aiming to minimize environmental impact and enhance resource efficiency. In this context, this study explores the integration of topology optimization techniques with fatigue analysis to develop reliable designs for structural components. Fatigue failure is a critical concern in engineering applications, as it significantly affects the lifespan and reliability of structures. The proposed methodology combines mathematical optimization algorithms, computational modeling, and fatigue analysis techniques. The primary objective of this study is to minimize structural weight by determining the optimal material arrangement within the design domain while also considering fatigue as a constraint within the optimization problem. The bi-directional evolutionary structural optimization (BESO) method is developed to meet the goal of this research. Furthermore, topology optimization of L-shape and U-plate problems are considered as numerical examples to demonstrate the effectiveness of the suggested method. By considering fatigue behavior in topology optimization, engineers can develop lightweight and durable structures that effectively utilize materials while minimizing resource utilization. The integration of these two fields opens up new avenues for reliable design, promoting resource efficiency and contributing to the overall reliability of engineering practices.

1. Introduction

Numerous researchers have conducted extensive research on topology optimization (TO) of structures over the past few decades. This discipline aims to find the most efficient arrangement of material within a given design domain while ensuring that all design constraints are satisfied. Several approaches to topology optimization have been proposed, including the Solid Isotropic Material Penalization (SIMP) method (Bendsøe, 1989), evolutionary structural optimization (ESO) method (Xie and Steven, 1993), the developed approach, known as the Bi-Directional Evolutionary Structural Optimization (BESO) method, which focuses on restoring elements that were removed in the previous iteration (Querin et al., 1998), level-set methods (Sethian, 1999), and moving morphable component (MMC) method (Guo et al., 2014).

As a pioneering numerical and design technique, topology optimization has undergone significant development and attracted considerable scientific interest. This potent instrument has been exhaustively investigated and implemented in numerous problem domains, such as topology optimization of elasto-plastic materials (Movahedi Rad et al., 2021), geometrically nonlinear problems (Habashneh and Movahedi Rad, 2022) and other developments in the field of TO such as considering thermoelastic analysis (Habashneh and Rad, 2023).

Fatigue is a significant concern within the field of structural engineering, as shown by prior studies. An investigation conducted by (Wang et al., 2016) examined the impact of corrosive deterioration on the structural integrity of reinforced concrete highway bridges. By treating fatigue failure as a constraint in the optimization process, optimal designs can be achieved, effectively addressing the challenges posed by real-life applications and ensuring the long-term structural integrity and reliability of engineered systems (Jeong et al., 2015).

In light of the growing emphasis on sustainability, it is imperative to establish a strong connection between topology optimization, fatigue considerations, and reliable design principles. (Gao et al., 2021) proposed topological fatigue optimization method, taking into account the effect of defects on fatigue strength and integrating stress constraints. The proposed topology optimization approach by (Desmorat and Desmorat,

Paper Received: 02 May 2023; Revised: 02 August 2023; Accepted: 03 November 2023
Please cite this article as: Habashneh M., Movahedi Rad M., 2023, Optimizing Topology of Structures Considering Fatigue-Resistance, Chemical Engineering Transactions, 107, 613-618 DOI:10.3303/CET23107103

2008), which focuses on maximizing fatigue lifetime through optimization algorithm considering cyclic plasticity and the Lemaitre damage law, aligns with the objective of reliable design. Additionally, the algorithm developed by (Nabaki et al., 2019), specifically tailored to mitigate the risk of high cycle fatigue failure, offers a promising avenue for integrating sustainability considerations into topology optimization practices. By linking these advancements in topology optimization with sustainability principles, engineers and designers can foster the development of structurally robust, fatigue-resistant, and environmentally responsible solutions for a wide range of applications.

This study serves as an extension of the authors' previous work, where they successfully addressed various aspects of topology optimization, including stiffness optimization and other relevant considerations (Movahedi Rad et al., 2021). Therefore, the current study aims to develop a structural optimization algorithm that specifically incorporates fatigue failure as a critical constraint besides the volume constraint. To accomplish this objective, the developed BESO method is utilized within the topology optimization process. Two numerical examples of L-shape and U-plate problems are solved to show the efficiency of the proposed methodology.

2. Theoretical background

2.1 Fatigue analysis

In this study, our focus is on the fatigue damage caused by cyclic loadings without considering crack propagation. While our primary objective is not to model crack propagation explicitly, we capture the cumulative effect of these cyclic loadings through a scalar stress state coefficient. As a result, the stress state at every location in the structure and at any point in the cyclic loading may be determined as a scalar factor of the stress state corresponding to the reference loading. Using Basquin's equation, the number of cycles until failure at various effective stress amplitudes is calculated (Dowling, 2004). The cumulative damage from the loading history's stress reversals is then calculated linearly using Palmgren-Miner's rule. Basquin's equation gives the number of cycles N_{f_i} before failure due to repeated applications of reversal i as:

$$N_{f_i}(x) = \frac{1}{2}\left(\frac{\sigma_{ar_i}}{\sigma_f'}\right)^{\frac{1}{b}} \tag{1}$$

where σ_{ar_i} is the completely reversed equivalent stress amplitude, σ_f' is the coefficient of material's fatigue strength, b is the exponent of material's fatigue strength. Using Palmgren-Miner's rule, the accumulated fatigue at a position is calculated as the sum of the harm inflicted at that position by every reversal.

$$D(x) = \sum_{i=1}^{N_r} \frac{n_i}{N_{f_i}(x)} \tag{2}$$

Here, we define n_i as the count of occurrences of reversals during the cyclic loading, while N_r represents the overall count of reversals throughout the loading process.

2.2 Definition of the topology optimization problem

This section focuses on the analysis of a topology optimization problem aimed at minimizing the mean compliance while integrating constraints, including fatigue and volume.

$$Minimize: C = u^T K u \tag{3.a}$$

$$Subject\ to: \ V^* - \sum_{i=1}^{N} V_i x_i = 0 \tag{3.b}$$

$$\frac{V^*}{V_0} - V_f \leq 0 \tag{3.c}$$

$$D(x) - \sum_{i=1}^{N_r} \frac{n_i}{N_{f_i}(x)} = 0 \tag{3.d}$$

$$x_i \in \{0,1\} \tag{3.e}$$

The topology optimization problem aims to minimize the mean compliance (C) while incorporating constraints related to volume and fatigue. The mean compliance depends on several factors, such as displacement vectors (u), and the overall stiffness matrix (K), which are crucial in determining the compliance. To provide further clarification, N represents the total number of elements, V_i signifies the volume of an individual element, and V^* denotes the approved volume of the entire structure. Furthermore, the binary design variable x_i indicates whether an element is present (1) or absent (0) in the design. The total volume of the design domain is denoted as V_0, and V_f stands for the volume fraction ratio, indicating the proportion of volume occupied in the optimized design. It is important to mention that Eq(3.d) represents the constraint related to fatigue safety. It ensures that the accumulated fatigue damage does not exceed allowable limits. Otherwise, the alternating stress would be negative, leading to undesirable outcomes. Moreover, it is crucial to avoid stresses that are high enough to induce plastic deformation, as this renders the high-cycle fatigue model ineffective. It should be mentioned that the BESO method achieves the optimal design for the structure by iteratively adjusting the number of elements while considering their sensitivity.

3. Numerical examples

In this section, the topology optimization problem with constraints associated with fatigue and volume is addressed by utilizing the developed BESO algorithm. As previously mentioned, the proposed work is an extension of our prior work (Movahedi Rad et al., 2021) to propose a structural optimization algorithm that specifically incorporates fatigue failure as a critical constraint. It should be noted that in the figures of the resulting optimized shapes, the black color represents the solid elements of the model. Therefore, the optimized shapes exhibit a reduction in the proportion of solid (black) elements according to the desired volume fraction values.

3.1 L-shape problem

The paper's first numerical example takes into account the L-shaped beam optimization problem. The example's geometry, boundary, and loading conditions are shown in Figure 1. The considered material properties are density of $2{,}800\ \mathrm{kg/m^3}$, Young's modulus of $70\ \mathrm{GPa}$, and Poisson's ratio of 0.3. $\sigma_f^{'}$ and b are assumed $580\ \mathrm{MPa}$ and -0.262. Furthermore, the values of ER, AR_{max}, r_{min}, and τ for BESO parameters are 1 %, 1 %, $18\ \mathrm{mm}$ and 1 %. V_f is set to $40\ \%$. In the case of linear and geometrically nonlinear optimization, the applied load is represented by $F = 12\ \mathrm{kN}$. However, for fatigue constraint optimization, the value of F is treated as a displacement with a magnitude of $2\ \mathrm{mm}$.

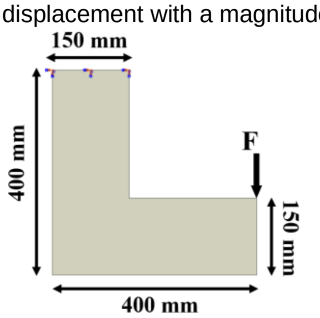

Figure 1: Considered L-shaped example

A comprehensive comparison of resulting topological shapes is provided, considering different models: linear, geometrically nonlinear, and the proposed fatigue constraint optimization model proposed in the manuscript. The findings presented in Figures from 2 to 4, indicate that when fatigue constraints are taken into consideration, the resulting optimized shape diverges from the shapes obtained through linear and geometrically nonlinear optimization approaches.

(a) *(b)* *(c)*

Figure 2: Topological results in the case of linear designs (a) $V_f = 100\ \%$ (b) $V_f = 65\ \%$ (c) $V_f = 40\ \%$

(a) (b) (c)

Figure 3: Topological results of geometrically nonlinear designs (a) $V_f = 100\ \%$ (b) $V_f = 65\ \%$ (c) $V_f = 40\ \%$

(a) (b) (c)

Figure 4: Topological results of fatigue constrained designs (a) $V_f = 100\ \%$ (b) $V_f = 65\ \%$ (c) $V_f = 40\ \%$

Table 1 represents another comparison according to the maximal Huber-Mises-Hencky stress (σ_{HMH}^{max}) values for each design case. It can be noticed that in the fatigue constraint topology optimization the value of σ_{HMH}^{max} decreased dramatically from the obtained values in the cases of linear and geometrically nonlinear designs.

Table 1: Resulted σ_{HMH}^{max} for each design

Model	$\sigma_{HMH}^{max}(MPa)$
Linear design	314
Geometrically nonlinear design	310
Fatigue constraint design	12.24

3.2 U-plate problem

The paper's second numerical example takes into account the U-shaped plate optimization problem. The example's loading conditions, geometry, and boundaries are shown in Figure 5. The material properties are density of $7{,}800\ \mathrm{kg/m^3}$, Young's modulus of $160\ \mathrm{GPa}$, and Poisson's ratio of 0.3. $\sigma_f^{'}$ and b are assumed 749 MPa and -0.090 respectively. Furthermore, the values of ER, AR_{max}, r_{min}, and τ for BESO parameters are 1 %, 1 %, $6\ \mathrm{mm}$ and 1 %, respectively. V_f is set to $50\ \%$. The applied load in the cases of linear and geometrically nonlinear optimization is considered to be $F = 1\ \mathrm{kN}$ while in the case of fatigue constraint optimization, F is considered as displacement with magnitude of $1\ \mathrm{mm}$.

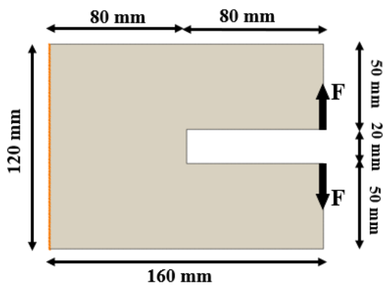

Figure 5: considered U plate example

A comprehensive comparison of the resulting topological morphologies is presented, taking into account various models, including linear, geometrically nonlinear, and the fatigue constraint optimization model. Figures 6 through 8 demonstrate that the incorporation of fatigue constraints has a significant impact on the optimal design, resulting in a geometry that is specifically tailored to enhance fatigue resistance and durability.

(a) *(b)* *(c)*

Figure 6: Topological results in the case of linear designs (a) $V_f = 100\ \%$ *(b)* $V_f = 65\ \%$ *(c)* $V_f = 40\ \%$

(a) *(b)* *(c)*

Figure 7: Topological results of geometrically nonlinear designs (a) $V_f = 100\ \%$ *(b)* $V_f = 65\ \%$ *(c)* $V_f = 40\ \%$

(a) *(b)* *(c)*

Figure 8: Topological results of fatigue constrained designs(a) $V_f = 100\ \%$ *(b)* $V_f = 65\ \%$ *(c)* $V_f = 40\ \%$

σ_{HMH}^{max} values for each design case are compared in Table 2. Similar to the findings of the previous example, during fatigue constraint topology optimization, the value of σ_{HMH}^{max} decreased significantly in comparison to the values obtained for linear and geometrically nonlinear designs. It suggests the fatigue performance of the optimized structure is enhanced.

Table 2: Resulted σ_{HMH}^{max} *for each design*

Model	$\sigma_{HMH}^{max}(MPa)$
Linear design	289
Geometrically nonlinear design	287
Fatigue constraint design	96

4. Conclusions

This study explored the integration of topology optimization techniques with fatigue analysis to develop reliable designs for structural components which minimize fatigue damage by integrating the developed BESO method. Numerical examples show that our technique successfully obtains lightweight structural designs which fulfil fatigue and volume constraints. The results of our investigation have provided compelling evidence for the significance of incorporating fatigue constraints into the topology optimization process. When fatigue considerations were integrated, the resulting optimized shapes exhibited a remarkable departure from those achieved through linear and geometrically nonlinear optimization approaches which was done by (Movahedi

Rad et al., 2021). Furthermore, one of the most significant observations was the dramatic decrease in the maximal Huber-Mises-Hencky stress (σ_{HMH}^{max}) values for designs subjected to fatigue constraint optimization compared to their linear and geometrically nonlinear counterparts. This quantitative finding highlights the improved fatigue performance and durability of the optimized structures. The findings of this study contribute to the advancement of reliable design principles by incorporating fatigue failure as a critical constraint. Engineers can develop lightweight and durable structures that effectively utilize materials while minimizing resource utilization. Future research could delve into the incorporation of advanced materials, such as composites materials within the topology optimization process to Investigate how these materials can further enhance fatigue resistance, or multi-objective optimization that balance not only fatigue performance but also other critical factors, such as cost, environmental impact, and manufacturability.

Nomenclature

N_{f_i} – number of cycles, -
$\sigma_f^{'}$ – coefficient of material's fatigue strength, MPa
b – exponent of material's fatigue strength, -
N_r – overall count of reversals throughout the loading process, -
n_i – the count of occurrences of reversals during the cyclic loading, -
C – mean compliance, Nmm
u – displacement, mm
K – stiffness matrix, $\mathrm{N/mm}$
N – total number of elements, -
V_i – element volume, mm^3
V^*– approved volume of the entire structure, mm^3
x_i – binary design variable, -
V_f – volume fraction, -
V_0 – total volume of design domain, mm^3

References

Bendsøe M.P., 1989, Optimal shape design as a material distribution problem, Structural Optimization, 1, 193–202, DOI: 10.1007/BF01650949.
Desmorat B., Desmorat R., 2008, Topology optimization in damage governed low cycle fatigue, Comptes Rendus Mécanique, 336, 448–453.
Dowling N.E., 2004, Mean Stress Effects in Stress-Life and Strain-Life Fatigue, SAE Technical Papers, DOI: 10.4271/2004-01-2227.
Gao X., Caivano R., Tridello A., Chiandussi G., Ma H., Paolino D., Berto F., 2021, Innovative formulation for topological fatigue optimisation based on material defects distribution and TopFat algorithm, Int J Fatigue, 147, 106176.
Guo X., Zhang W., Zhong W., 2014, Doing Topology Optimization Explicitly and Geometrically—A New Moving Morphable Components Based Framework, J Appl Mech, 81(8), 081009, DOI: 10.1115/1.4027609.
Habashneh M., Movahedi Rad M., 2022, Reliability based geometrically nonlinear bi-directional evolutionary structural optimization of elasto-plastic material, Scientific Reports, 12, 5989, DOI: 10.1038/s41598-022-09612-z.
Habashneh M., Rad M.M., 2023, Reliability based topology optimization of thermoelastic structures using bi-directional evolutionary structural optimization method, International Journal of Mechanics sand Materials in Design, 19, 605–620.
Jeong S.H., Choi D.H., Yoon G.H., 2015, Fatigue and static failure considerations using a topology optimization method, Appl Math Model 39, 1137–1162.
Movahedi Rad M., Habashneh M., Lógó J., 2021, Elasto-Plastic limit analysis of reliability based geometrically nonlinear bi-directional evolutionary topology optimization, Structures, 34, 1720–1733.
Nabaki K., Shen J., Huang X., 2019, Evolutionary topology optimization of continuum structures considering fatigue failure, Mater Des, 166, 107586.
Querin O.M., Steven G.P., Xie Y.M., 1998, Evolutionary structural optimisation (ESO) using a bidirectional algorithm, Eng Comput, 15, 1031–1048, DOI: 10.1108/02644409810244129.
Sethian J.A., 1999, Level set methods and fast marching methods: evolving interfaces in computational geometry, fluid mechanics, computer vision, and materials science, Cambridge University Press, Cambridge, United Kingdom.
Wang T., Zhang H., Tian L., Yin C., Yu F., 2016, Study on Fatigue Performance of Concrete Highway Bridge under Corrosive Environment, Chemical Engineering Transactions, 55, 373–378, DOI: 10.3303/CET1655063.
Xie Y.M., Steven G.P., 1993, A simple evolutionary procedure for structural optimization, Comput Struct, 49, 885–896.