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Distance Based Topological Indices of Double graphs and Strong Double graphs

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Abstract

Topological index is a numerical representation of structure of graph. They are mainly classified as Distance and Degree based topological indices. In this article Distance based topological indices of Double graphs and Strong Double graphs are calculated. Let G be a graph of order n with the vertex set $V(G)$ containing vertices $v_1, v_2, ..., v_n$. Double graph of graph G is constructed by taking two copies of G in which a vertex v_i in one copy is adjacent to a vertex v_j in the another copy if v_i and v_j are adjacent in G. Strong Double graph is a double graph in which a vertex v_i in one copy is adjacent to a vertex v_j in the another copy if $i = j$.

Keywords: Topological Indices; Double graphs; Strong Double graphs
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1 Introduction

Numerical representation of structure of a graphs is known as topological indices. They are prominently subdivided into Degree and Distance based topological indices. H. Wiener in 1947 introduced the Wiener index to study the variation of boiling points according to the structure of alkanes known as paraffin [13]. Zagreb indices are also most significant molecular descriptor introduced in 1972 by Gutman and Trinajstić [3, 4]. Many topological indices were introduced. Some of the indices are Hyper-Wiener index, reciprocal complementary Wiener index, Status connectivity indices. Here simple, connected, finite, undirected graphs are considered. For undefined terminologies refer [5]. The total number of edges connected to a vertex u in G is Degree of a vertex u in G and is represented by $d_G(u)$. The length of the shortest path between the vertices u and v in G is distance of u and v and is represented by $d_G(u, v)$. For details on indices see [1, 3, 4, 6, 7, 8, 13]. The Wiener index of G is,

$$W(G) = \frac{1}{2} \sum_{u,v \in V(G)} d_G(u, v).$$

First Zagreb index $M_1(G)$ is,

$$M_1(G) = \sum_{u \in V(G)} (d_G(u))^2.$$

Second Zagreb index $M_2(G)$ is,

$$M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v).$$

In 1993, Randić proposed the Hyper-Wiener index of G is [12],

$$WW(D[G]) = \frac{1}{2} \sum_{u,v \in V(G)} (d_G(u, v) + (d_G(u, v))^2). \quad (1)$$

The reciprocal complementary Wiener index of a graph G is introduced by O. Ivanciuc and is defined as [7, 8],

$$RCW(G) = \sum_{1 \leqslant i \leqslant j \leqslant n} \frac{1}{1 + diam(G) - d_{[G]}(u, v)}. \quad (2)$$

The largest distance between any pair of vertices in G is Diameter of G and is represented by diam(G). The sum of the distances from the vertex $u \in V(G)$ to all other vertices in V(G) is known as Status [6] of a vertex u and is represented by $\sigma(u)$

$$\sigma(u) \quad = \sum_{u \in V(G)} d(u, v). \tag{3}$$

H. S. Ramane and A. S. Yalnaik introduced the Status connectivity indices [11]. First Status connectivity index $S_1(G)$ of G is,

$$S_1(G) \quad = \sum_{uv \in E(G)} (\sigma(u) + \sigma(v)). \tag{4}$$

Second Status connectivity index $S_2(G)$ of G is,

$$S_2(G) \quad = \sum_{uv \in E(G)} (\sigma(u)\sigma(v)). \tag{5}$$

The double graph of G is defined by Munarini et al. [10]. It is represented as D[G]. Let G be a graph containing $v_1, v_2, \dots, v_n$ as vertices. Double graph of G is constructed by considering two copies of G in which a vertex v_i in first copy is adjacent to a vertex v_j in the second copy if v_i and v_j are adjacent in G.
Strong Double graph is a double graph in which a vertex v_i in first copy is adjacent to a vertex v_j in the second copy if $i = j$. It is denoted as SD[G] [2]. The

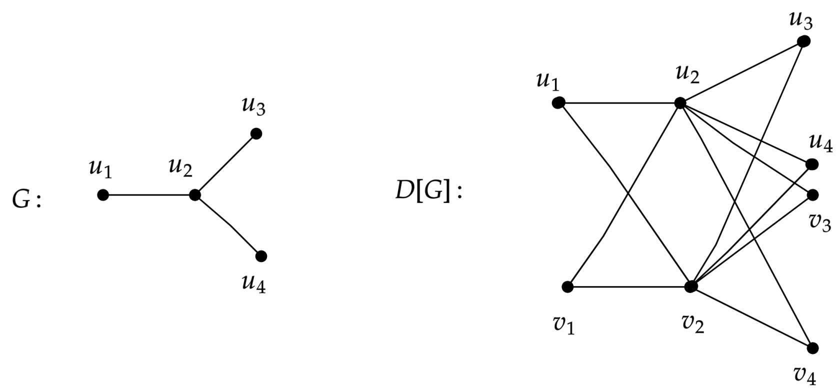

Figure 1: Double graph of $K_{1,3}$

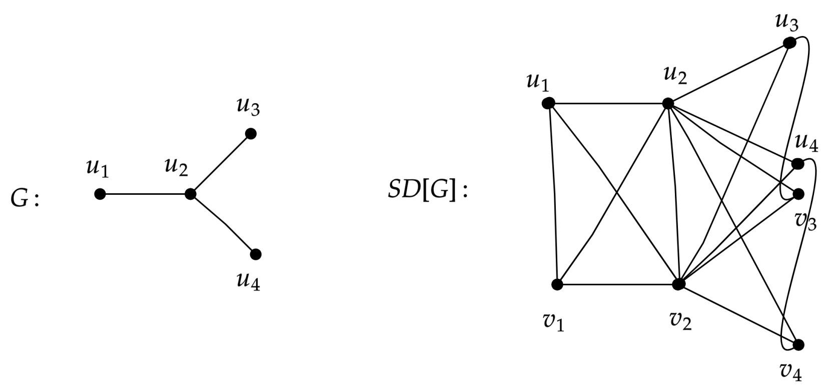

Figure 2: Strong Double graph of $K_{1,3}$

following are used for calculating topological indices.

Remark 1: [9] The Wiener index of Double graph is,

$$W(D[G]) = 4W[G] + 2n. \qquad (6)$$

Remark 2: [1] The Wiener index of Strong Double graph is,

$$W(SD[G]) = 4W[G] + n. \qquad (7)$$

Remark 3:

$$\sum_{1 \leqslant i \leqslant j \leqslant n} \frac{1}{1 + diam(D[G]) - d_{D[G]}(u_i, v_i)} = \frac{n}{diam(D[G]) - 1}$$

Remark 4:

$$\sum_{1 \leqslant i \leqslant j \leqslant n} \frac{1}{1 + diam(SD[G]) - d_{SD[G]}(u_i, v_i)} = \frac{n}{diam(SD[G]) - 1}$$

Remark 5:

$$\sum_{u_i, v_i \in V(D(G))} (d_G(u_i, v_i))^2 = 8n$$

2 Main results

Here, distance based topological indices of Double and Strong Double graphs are established. let G_1 and G_2 be two same copies of a graph G. They are of order n and size m. G_1 contains $u_1, u_2, ..., u_n$ as vertices and G_2 contains $v_1, v_2, ..., v_n$ as vertices.

Theorem 2.1. *Hyper Wiener index of Double graph is*

$$WW(D[G]) \quad = \quad 4WW(G) + 6n.$$

Proof. By definition (1) we have,

$$\begin{aligned} WW(D[G]) &= \frac{1}{2}\sum_{u,v\in V(D[G])}(d_{D[G]}(u,v) + (d_{D[G]}(u,v))^2) \\ &= \frac{1}{2}\sum_{u,v\in V(D[G])}(d_{D[G]}(u,v)) + \frac{1}{2}\sum_{u,v\in V(D[G])}(d_{D[G]}(u,v))^2. \end{aligned}$$

From remark(1) we get,

$$WW(D[G]) \quad = \quad [4W(G) + 2n] + \frac{1}{2}\sum_{u,v\in V(D[G])}(d_{D[G]}(u,v))^2. \tag{8}$$

Consider,

$$\begin{aligned} \frac{1}{2}\sum_{u,v\in V(D[G])}(d_{D[G]}(u,v))^2 &= \frac{1}{2}[\sum_{u_i,u_j\in V(D(G))}(d_G(u_i,u_j))^2 + \\ &\sum_{v_i,v_j\in V(D(G))}(d_G(v_i,v_j))^2 + \\ &\sum_{u_i,v_j\in V(D(G))}(d_G(u_i,v_j))^2 + \\ &\sum_{v_i,u_j\in V(D(G))}(d_G(v_i,u_j))^2 + 8n] \end{aligned} \tag{9}$$

Using (9) in (8) we get,

$$\begin{aligned}
WW(D[G]) &= [4W(G)+2n]+\frac{1}{2}[\sum_{u_i,u_j\in V(D(G))}(d_G(u_i,u_j))^2] \\
&\quad +[4W(G)+2n]+\frac{1}{2}[\sum_{v_i,v_j\in V(D(G))}(d_G(v_i,v_j))^2] \\
&\quad +[4W(G)+2n]+\frac{1}{2}[\sum_{u_i,v_j\in V(D(G))}(d_G(u_i,v_j))^2] \\
&\quad +[4W(G)+2n]+\frac{1}{2}[\sum_{v_i,u_j\in V(D(G))}(d_G(v_i,u_j))^2+8n] \\
WW(D[G]) &= WW(G)+WW(G)+WW(G)+WW(G)+6n \\
WW(D[G]) &= 4WW(G)+6n.
\end{aligned}$$

□

Theorem 2.2. *The reciprocal complementary Wiener index of a Double graph is*

$$RCW(D[G]) = 4RCW(G)+\frac{n}{diam(D[G])-1}.$$

Proof. By definition (2) we have,

$$RCW(D[G]) = \sum_{1\leqslant i\leqslant j\leqslant n}\frac{1}{1+diam(D[G])-d_{D[G]}(u,v)}$$

From remark (3) we have,

$$\begin{aligned}
&= \sum_{1\leqslant i\leqslant j\leqslant n}\frac{1}{1+diam(D[G])-d_G(u_i,u_j)}+ \\
&\quad \sum_{1\leqslant i\leqslant j\leqslant n}\frac{1}{1+diam(D[G])-d_G(v_i,v_j)}+ \\
&\quad 2\sum_{1\leqslant i\leqslant j\leqslant n}\frac{1}{1+diam(D[G])-d_G(u_i,v_j)}+\frac{n}{1+diam(D[G])-2} \\
&= \frac{4RCW(D[G])(diam(D[G])-1)+n}{diam(D[G])-1} \\
&= 4RCW(G)+\frac{n}{diam(D[G])-1}.
\end{aligned}$$

□

Theorem 2.3. *The reciprocal complementary Wiener index of a Strong Double graph is*

$$RCW(SD[G]) = 4RCW(G) + \frac{n}{diam(SD[G])}.$$

Proof. By definition (2) we have,

$$RCW(SD[G]) = \sum_{1\leqslant i\leqslant j\leqslant n} \frac{1}{1 + diam(SD[G]) - d_{SD[G]}(u,v)}$$

From remark (4) we have,

$$\begin{aligned} &= \sum_{1\leqslant i\leqslant j\leqslant n} \frac{1}{1 + diam(SD[G]) - d_G(u_i,u_j)} + \\ &\quad \sum_{1\leqslant i\leqslant j\leqslant n} \frac{1}{1 + diam(SD[G]) - d_G(v_i,v_j)} + \\ &\quad 2\sum_{1\leqslant i\leqslant j\leqslant n} \frac{1}{1 + diam(SD[G]) - d_G(u_i,v_j)} + \frac{n}{1 + diam(SD[G]) - 1} \\ &= \frac{4RCW(SD[G])(diam(SD[G])) + n}{diam(SD[G])} \\ &= 4RCW(G) + \frac{n}{diam(SD[G])}. \end{aligned}$$

□

Theorem 2.4. *The first Status connectivity index of Double graph is*

$$S_1(D[G]) = 8S_1(G) + 16m.$$

Proof. By using definition of Double graph we have,

$$\sigma(u_i) = \sigma(v_i). \tag{10}$$

$$\begin{aligned} \sigma(u_n) &= \sum_{i=1}^{n}(d(u_n,u_i) + d(u_n,v_i)) + \sum_{i=n} d(u_n,v_n) \\ &= 2\sum_{i=1}^{n}(d(u_n,u_i)) + 2. \end{aligned} \tag{11}$$

$$\begin{aligned}\sigma(v_n) &= \sum_{i=1}^{n}(d(v_n,v_i)+d(v_n,u_i))+\sum_{i=n}d(v_n,u_n)\\ &= 2\sum_{i=1}^{n}(d(v_n,v_i))+2. \qquad (12)\end{aligned}$$

By definitions (3) and (4) we have,

$$\begin{aligned}S_1(D[G]) &= \sum_{uv\in E(D[G])}(\sigma(u)+\sigma(v))\\ &= \sum_{u_iu_j\in E(D[G])}(\sigma(u_i)+\sigma(u_j))+\sum_{v_iv_j\in E(D[G])}(\sigma(v_i)+\sigma(v_j))+\\ &\quad \sum_{u_iv_j\in E(D[G])}(\sigma(u_i)+\sigma(v_j))+\sum_{v_iu_j\in E(D[G])}(\sigma(v_i)+\sigma(u_j)). \qquad (13)\end{aligned}$$

Consider first part of equation (13) using (10), (11), (12) we have,

$$\begin{aligned}\sum_{u_iu_j\in E(D[G])}(\sigma(u_i)+\sigma(u_j)) &= \sum_{u_iu_j\in E(D[G])}[2[\sum_{i\neq j}(d(u_i,u_j))+\\ &\quad \sum_{i\neq j}(d(u_j,u_i))]]+4\\ &= \sum_{u_iu_j\in E(D[G])}[2\sum_{i\neq j}(d(u_i,u_j))+2]+\\ &\quad [2\sum_{i\neq j}(d(u_j,u_i))+2]\\ &= 2[(\sigma(u_i)+\sigma(u_j))]+4\\ &= 2S_1(G)+4m. \qquad (14)\end{aligned}$$

Using equation (14) in equation (13) we get,

$$\begin{aligned}S_1(D[G]) &= (2S_1(G)+4m)+(2S_1(G)+4m)+(2S_1(G)+4m)+\\ &\quad (2S_1(G)+4m)\\ &= 8S_1(G)+16m.\end{aligned}$$

□

Theorem 2.5. *The first Status connectivity index of Strong Double graph SD[G] is*

$$S_1(SD[G]) = 8S_1(G)+8W(G)+2n+8m.$$

Proof. By using definition of Strong Double graph we have,

$$\sigma(u_i) = \sigma(v_i). \tag{15}$$

$$\begin{aligned}\sigma(u_n) &= \sum_{i=1}^{n}(d(u_n,u_i)+d(u_n,v_i))+\sum_{i=n} d(u_n,v_n)\\ &= 2\sum_{i=1}^{n}(d(u_n,u_i))+1.\end{aligned} \tag{16}$$

$$\begin{aligned}\sigma(v_n) &= \sum_{i=1}^{n}(d(v_n,v_i)+d(v_n,u_i))+\sum_{i=n} d(v_n,u_n)\\ &= 2\sum_{i=1}^{n}(d(v_n,v_i))+1.\end{aligned} \tag{17}$$

By definitions (3) and (4) we have,

$$\begin{aligned}S_1(SD[G]) &= \sum_{uv\in E(SD[G])}(\sigma(u)+\sigma(v))\\ &= \sum_{u_iu_j\in E(SD[G])}(\sigma(u_i)+\sigma(u_j))+\sum_{v_iv_j\in E(SD[G])}(\sigma(v_i)+\sigma(v_j))+\\ &\quad \sum_{u_iv_j\in E(SD[G])}(\sigma(u_i)+\sigma(v_j))+\sum_{v_iu_j\in E(SD[G])}(\sigma(v_i)+\sigma(u_j)).+\\ &\quad \sum_{u_iv_i\in E(SD[G])}(\sigma(u_i)+\sigma(v_i)).\end{aligned} \tag{18}$$

Consider first part of equation (18) using (15), (16), (17) we have,

$$\begin{aligned}\sum_{u_iu_j\in E(SD[G])}(\sigma(u_i)+\sigma(u_j)) &= \sum_{u_iu_j\in E(SD[G])}[2\sum_{i\neq j}(d(u_i,u_j))+1]+\\ &\quad [2\sum_{i\neq j}(d(u_j,u_j))+1]\\ &= \sum_{u_iu_j\in E(SD[G])}[2[\sum_{i\neq j}(d(u_i,u_j))+\\ &\quad \sum_{i\neq j}(d(u_j,u_i))]]+2\\ &= 2[(\sigma(u_i)+\sigma(u_j))]+2m\\ &= 2S_1(G)+2m.\end{aligned} \tag{19}$$

Using equation(19) in equation (18) and from Remark 2 we have,

$$\begin{aligned} S_1(SD[G]) &= 4(2S_1(G)+2m)+2[W(SD(G))] \\ &= 8S_1(G)+8m+2[4W(G)+n] \\ &= 8S_1(G)+8W(G)+2n+8m. \end{aligned}$$

□

Theorem 2.6. *The second Status connectivity index of Double graph is*

$$S_2(D[G]) = 16S_2(G)+16S_1(G)+16m.$$

Proof. By using definition of Double graph we have,

$$\sigma(u_i)=\sigma(u_j). \tag{20}$$

$$\begin{aligned} \sigma(u_n) &= \sum_{i=1}^{n}(d(u_n,u_i)+d(u_n,v_i))+\sum_{i=n} d(u_n,v_n) \\ &= 2\sum_{i=1}^{n}(d(u_n,u_i))+2. \end{aligned} \tag{21}$$

$$\begin{aligned} \sigma(v_n) &= \sum_{i=1}^{n}(d(v_n,u_i)+d(v_n,v_i))+\sum_{i=n} d(v_n,u_n) \\ &= 2\sum_{i=1}^{n}(d(v_n,v_i))+2. \end{aligned} \tag{22}$$

By definitions (3) and (5) we have,

$$\begin{aligned} S_2(D[G]) &= \sum_{uv\in E(D[G])}(\sigma(u)\sigma(v)) \\ &= \sum_{u_iu_j\in E(D[G])}(\sigma(u_i)\sigma(u_j))+\sum_{v_iv_j\in E(D[G])}(\sigma(v_i)\sigma(v_j))+ \\ &\quad \sum_{u_iv_j\in E(D[G])}(\sigma(u_i)\sigma(v_j))+\sum_{v_iu_j\in E(D[G])}(\sigma(v_i)\sigma(u_j)). \end{aligned} \tag{23}$$

Consider first part of (23) using (20), (21), (22) we have,

$$\begin{aligned}\sum_{u_iu_j\in E(D[G])}(\sigma(u_i)\sigma(u_j)) &= \sum_{u_iu_j\in E(D[G])}[2\sum_{i\neq j}(d(u_i,u_j)+2][2\sum_{i\neq j}(d(u_j,u_i)+2]\\ &= \sum_{u_iu_j\in E(D[G])}[2\sum_{i\neq j}(d(u_i,u_j))2\sum_{i\neq j}(d(u_j,u_i))]+\\ &\quad 4\sum_{i\neq j}(d(u_i,u_j))+4\sum_{i\neq j}(d(u_j,u_i))+4\\ &= 4[(\sigma(u_i)\sigma(u_j))]+4[\sigma(u_i)+\sigma(u_j)]+4m\\ &= 4S_2(G)+4S_1(G)+4m. \qquad (24)\end{aligned}$$

By using (24) in (23) we have,

$$\begin{aligned}S_2(D[G]) &= (4S_2(G)+4S_1(G)+4m)4\\ &= 16S_2(G)+16S_1(G)+16m.\end{aligned}$$

□

Theorem 2.7. *The second Status connectivity index of Strong Double graph is*

$$S_2(SD[G]) = 16S_2(G)+8S_1(G)+4m+\sum_{u_i\in V(SD[G])}(\sigma(u_i))^2.$$

Proof. By definition Strong Double graph we have,

$$\sigma(u_i)=\sigma(v_i). \qquad (25)$$

$$\begin{aligned}\sigma(u_n) &= \sum_{i=1}^{n}(d(u_n,u_i)+d(u_n,v_i))+\sum_{i=n}d(u_n,v_n)\\ &= 2\sum_{i=1}^{n}(d(u_n,u_i))+2. \qquad (26)\end{aligned}$$

$$\begin{aligned}\sigma(v_n) &= \sum_{i=1}^{n}(d(v_n,v_i)+d(v_n,u_i))+\sum_{i=n}d(v_n,u_n)\\ &= 2\sum_{i=1}^{n}(d(v_n,v_i))+2. \qquad (27)\end{aligned}$$

By using definitions (3) and (5) we have,

$$\begin{aligned} S_2(SD[G]) &= \sum_{uv \in E(SD[G])} (\sigma(u)\sigma(v)) \\ &= \sum_{u_iu_j \in E(SD[G])} (\sigma(u_i)\sigma(u_j)) + \sum_{v_iv_j \in E(SD[G])} (\sigma(v_i)\sigma(v_j)) + \\ & \quad \sum_{u_iv_j \in E(SD[G])} (\sigma(u_i)\sigma(v_j)) + \sum_{v_iu_j \in E(SD[G])} (\sigma(v_i)\sigma(u_j)) + \\ & \quad \sum_{u_iv_i \in E(SD[G])} (\sigma(u_i)\sigma(v_i)). \qquad (28) \end{aligned}$$

From (25), (26), (27), (28) we have,

$$S_2(SD[G]) = 16S_2(G) + 8S_1(G) + 4m + \sum_{u_i \in V(SD[G])} (\sigma(u_i))^2.$$

□

3 Application with calculation examples

Distance based topological indices has applications in Chemistry, Biology, Material science, Computer science.
Calculation examples:

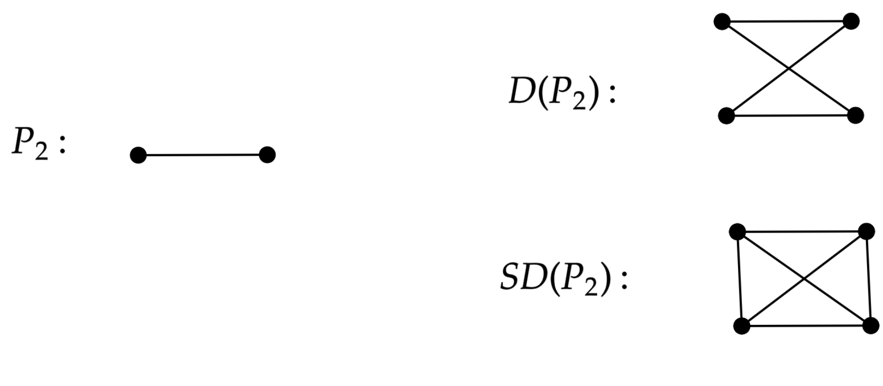

Figure 3: Double and Strong Double graph of P_2

Let $G = P_2$.
1) From Theorem 2.1,
Hyper Wiener index of Double graph is,

$$WW(D[G]) = 4WW(G) + 6n.$$

Calculation:
By definition of Hyper Wiener index,
$WW(P_2) = 2$, $n = 2$.
From Equation 1,

$$\begin{aligned} WW(D[P_2]) &= \frac{1}{2}\sum_{u,v\in V(D[P_2])}(d_{D[P_2]}(u,v)) + \frac{1}{2}\sum_{u,v\in V(D[P_2])}(d_{D[P_2]}(u,v))^2. \\ &= \frac{1}{2}[1+1+2+1+2+1+1+1+2+1+2+1] + \frac{1}{2}[1+ \\ &\quad 1+4+1+4+1+1+1+4+1+4+1] \\ &= \frac{1}{2}[16] + \frac{1}{2}[24] \\ &= 20. \end{aligned} \quad (29)$$

From Theorem 2.1,

$$\begin{aligned} WW(D[P_2]) &= 4WW(P_2) + 6n. \\ &= 4(2) + 6(2) \\ &= 8 + 12 \\ &= 20. \end{aligned} \quad (30)$$

Hence, from equations (29) and (30) values are equal.

2) From Theorem 2.2,
The reciprocal complementary Wiener index of a Double graph is,

$$RCW(D[G]) = 4RCW(G) + \frac{n}{diam(D[G]) - 1}.$$

Calculation:
By definition of reciprocal complementary Wiener index,
$RCW(P_2) = 1$, $diam(D[P_2]) = 2$.

From equation 2,

$$
\begin{aligned}
RCW(D[P_2]) &= \sum_{1\leqslant i\leqslant j\leqslant n} \frac{1}{1+diam(D[P_2])-d_{D[P_2]}(u,v)} \\
&= \frac{1}{1+2-1}+\frac{1}{1+2-1}+\frac{1}{1+2-2}+\frac{1}{1+2-1}+ \\
&\quad \frac{1}{1+2-2}+\frac{1}{1+2-1} \\
&= \frac{1}{2}+\frac{1}{2}+1+\frac{1}{2}+1+\frac{1}{2} \\
&= 4. \qquad (31)
\end{aligned}
$$

From Theorem 2.2,

$$
\begin{aligned}
RCW(D[P_2]) &= 4RCW(P_2)+\frac{n}{diam(D[P_2])-1}. \\
&= 4.[1/2]+\frac{2}{2-1} \\
&= 4. \qquad (32)
\end{aligned}
$$

Hence, from equations (31) and (32) values are equal.

3) From Theorem 2.3,
The reciprocal complementary Wiener index of a Strong Double graph is,

$$
RCW(SD[G]) = 4RCW(G)+\frac{n}{diam(D[G])}.
$$

Calculation:
By definition of reciprocal complementary Wiener index,
$RCW(P_2)=1$, $diam(SD[P_2])=1$.
From equation 2,

$$
\begin{aligned}
RCW(SD[P_2]) &= \sum_{1\leqslant i\leqslant j\leqslant n} \frac{1}{1+diam(SD[P_2])-d_{SD[P_2]}(u,v)} \\
&= \frac{1}{1+1-1}+\frac{1}{1+1-1}+\frac{1}{1+1-1}+\frac{1}{1+1-1}+ \\
&\quad \frac{1}{1+1-1}+\frac{1}{1+1-1} \\
&= 6. \qquad (33)
\end{aligned}
$$

From Theorem 2.3,

$$\begin{aligned} RCW(SD[P_2]) &= 4RCW(P_2) + \frac{n}{diam(D[P_2])}. \\ &= 4(1) + 2 \\ &= 6. \end{aligned} \tag{34}$$

Hence, from equations (33) and (34) values are equal.

4) From Theorems 2.4 and 2.5,
The first Status connectivity index of Double graph is,

$$S_1(D[G]) = 8S_1(G) + 16m.$$

The first Status connectivity index of Strong Double graph SD[G] is,

$$S_1(SD[G]) = 8S_1(G) + 8W(G) + 2n + 8m.$$

Calculation:
By definition of first Status connectivity index,
$S_1(P_2) = 2$.
For Double graph $\sigma(u_i) = 4$.
For Strong Double graph $\sigma(u_i) = 3$.
From equations 3 and 4,

$$\begin{aligned} S_1(D[P_2]) &= \sum_{uv \in E(D[P_2])} (\sigma(u) + \sigma(v)) \\ &= [4+4] + [4+4] + [4+4] + [4+4] \\ &= 32. \end{aligned} \tag{35}$$

By definition of first Status connectivity index,

$$\begin{aligned} S_1(SD[P_2]) &= \sum_{uv \in E(SD[P_2])} (\sigma(u) + \sigma(v)) \\ &= [3+3] + [3+3] + [3+3] + [3+3] + \\ &\quad [3+3] + [3+3] \\ &= 36. \end{aligned} \tag{36}$$

From Theorem 2.4,

$$\begin{aligned} S_1(D[P_2]) &= 8S_1(P_2) + 16m. \\ &= 8(2) + 16(1) \\ &= 32 \end{aligned} \tag{37}$$

From Theorem 2.5,

$$\begin{aligned} S_1(SD[P_2]) &= 8S_1(P_2) + 8W(P_2) + 2n + 8m. \\ &= 8(2) + 8(1) + 2(2) + 8(1) \\ &= 36. \end{aligned} \tag{38}$$

Hence, from equations (35) and (37) values are equal and from equations (36) and (38) values are equal.

5) From Theorems 2.6 and 2.7,
The second Status connectivity index of Double graph is,

$$S_2(D[G]) = 16S_2(G) + 16S_1(G) + 16m.$$

The second Status connectivity index of Strong Double graph SD[G] is,

$$S_2(SD[G]) = 16S_2(G) + 8S_1(G) + 4m + \sum_{u_i \in V(SD[G])} (\sigma(u_i))^2.$$

Calculation:
By definition of second Status connectivity index,
$S_2(P_2) = 1$.
For Double graph $\sigma(u_i) = 4$.
For Strong Double graph $\sigma(u_i) = 3$.
From equations 3 and 5,

$$\begin{aligned} S_2(D[P_2]) &= \sum_{uv \in E(D[P_2])} (\sigma(u)\sigma(v)) \\ &= [4(4)] + [4(4)] + [4(4)] + [4(4)] \\ &= 64. \end{aligned} \tag{39}$$

By definition of second Status connectivity index,

$$\begin{aligned} S_2(SD[P_2]) &= \sum_{uv \in E(SD[P_2])} (\sigma(u)\sigma(v)) \\ &= [3(3)] + [3(3)] + [3(3)] + [3(3)] + [3(3)] + [3(3)] \\ &= 54. \end{aligned} \tag{40}$$

From Theorem 2.6,

$$\begin{aligned} S_2(D[P_2]) &= 16S_2(P_2) + 16S_1(G) + 16m. \\ &= 16(2) + 16(1) + 16(1) \\ &= 64 \end{aligned} \tag{41}$$

From Theorem 2.7,

$$\begin{aligned} S_2(SD[P_2]) &= 16S_2(P_2) + 8S_1(P_2) + 4m + \sum_{u_i \in V(SD[P_2])} (\sigma(u_i))^2. \\ &= 16(1) + 16(2) + 4(1) + 2 \\ &= 54. \end{aligned} \tag{42}$$

Hence, from equations (39) and (41) values are equal and from equations (40) and (42) values are equal.

4 Discussion

H. Wiener in 1947 introduced the Wiener index to study the variation of boiling points according to the structure of alkanes known as paraffin [13]. After that, many topological indices were introduced. As topological indices has huge applications in Chemistry, calculating topological indices is useful to know about molecules in Chemistry. In this article, distance based topological indices of Double graphs and Strong Double graphs is calculated. In the section Application with calculation examples for the Distance based topological indices is calculated by definition and also it is calculated for all the Theorems from 2.1 to 2.7. Both the calculations are giving same values. By following the calculation techniques of this article other topological indices for different graphs can be calculated.

5 Conclusions

In this article the Hyper Wiener index of Double graph is calculated and also reciprocal complementary Wiener index of Double graph and Strong Double graph is calculated. Further results on first Status and second Status connectivity indices of Double graph and Strong Double graph is computed.

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