Inventory Model for Quadratic Demand and Deteriorating Items Following Weibull Distribution with Trade Credit Policy

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Abstract

In this paper, an inventory model for deteriorating items following two parameter Weibull distribution with trade credit policy is developed, while demand is viewed as quadratic function of time. The supplier gives the retailer a trade credit period. Trade credit is a frequently used method of payment implemented by suppliers, and it generally leads to greater revenue and ultimately, higher income. The suggested inventory model seeks to calculate the ideal replenishment cycle duration in order to maximize the overall profit per unit of time. Shortages are permitted and partially backlogged. Two categories are applied to the mathematical model. Case I: When the payment to settle the account is made on or before the positive inventory. Case II: When the payment to settle the account is made after the inventory reaches to zero. The model is illustrated through numerical experiments, sensitivity analysis, and graphical depiction.

Keywords: Inventory; Quadratic Demand; Deterioration; Weibull Distribution; Trade Credit.‡

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1. Introduction

Inventory system management is a critical component in controlling corporate costs. Extensive research in this topic has looked at several models that take into account issues including item scarcity, deterioration, demand trends, order cycles, and their interactions. Notably, efficiently monitoring and regulating ageing products is a big difficulty in any inventory management system. Fruits, vegetables, electronics, and chemicals deteriorate with time, necessitating a comprehensive inventory plan that accounts for potential losses due to deterioration. One of the primary goals of inventory models for depreciating commodities is to determine appropriate inventory levels and procurement techniques that maximise overall profitability while accommodating consumer demand while accounting for the costs associated with product depreciation. Inventory demand, which denotes the number of items consumers intend to purchase within a particular timeframe, plays an important role in inventory management, assisting organisations in keeping acceptable stock levels while avoiding surplus inventory.

The study of inventory systems has seen significant development, as evidenced by a series of models proposed by various researchers in recent years. R. Roy Chowdhury (2015) focused on devising an inventory model catering to seasonal items, considering the degradation rate to determine optimal ordering quantities and pricing strategies for maximizing predicted profits. Similarly, K. Karthikeyan (2015) delved into the implications of the degradation factor, accounting for a cubic function of time in demand and time-dependent holding costs. Umakanta Mishra (2015) concentrated on Weibull degrading items, emphasizing the determination of the most suitable procurement quantity. Sumit Saha (2017) developed an inventory model for goods with negative exponential demand and time-proportional deterioration, allowing partial backlogging. Sandeep Kumar (2017) studied perishable goods with exponentially declining demand, incorporating partial backordering and considering the impact of replenishment timing on backlog rate. Furthermore, the incorporation of credit risk in inventory management was explored by Gour Chandra Mahata (2017), which provided insightful guidelines for retailers selling depreciating goods. Bidyadhara Bishi (2018) investigated a deterministic inventory model for decaying goods, allowing shortages and partial backlogging, with a quadratic demand rate over time. Dr. D. Chitra (2019) designed an inventory model for non-instantaneous decaying items, considering a parabolic backlog pattern in a dynamic demand scenario. Biswaranjan Mandal (2020) examined a time-varying degrading item EOQ inventory model with cubic demand. P. D. Khatri (2020) proposed an EPQ model for deteriorating goods with a ramp-type demand rate and time-varying holding costs, accommodating shortages and backlogs. Cardenas-Barron (2020) introduced a trade credit EOQ inventory model, highlighting the influence of nonlinear stock-dependent holding costs and demand on inventory management. Additionally, Ashish Sharma

(2021) and Maryam Esmaeili (2021) explored partially backlogged inventory models with ramp-type demand, taking into account time, price, and availability constraints, and inflation for decaying items, respectively. To handle the complexities of inventory systems with degrading products, recent studies by Khyati and A. K. Saxena (2022), A. K. Saxena and R. K. Yadav (2011), Shaikh, A. A., Khakzad, A. (2020), and Duary, A. (2022) proposed various models considering different aspects of inventory management, such as trade credit, stock-dependent demand, inspection times, and capacity constraints, each contributing valuable insights into the field. Kumari, M. (2023) introduced a comprehensive mathematical model considering the influence of both selling price and inventory levels on consumer demand, emphasizing the benefits of a complete trade credit period for both vendors and buyers.

This research provides a quadratic demand inventory model for degradable products that takes into account trade credit rules. Employing a quadratic function of time demand, this model simplifies the estimation of optimal ordering quantities and pricing methods over defined periods. This approach is especially beneficial for businesses dealing with products with fluctuating demand levels, such as seasonal goods or items subject to changing consumer preferences, as it allows for more informed inventory and pricing decisions, boosting profitability and market competitiveness. Furthermore, incorporating a trade credit policy into the inventory model digs into a thorough review of credit costs, which includes monitoring expenses and revenue creation from credit sales. This strategy tries to reduce overall inventory costs, match customer demand, and maximise credit sales revenues by optimising order quantities and reorder points. Taking into account numerous characteristics such as loan period length, interest rates on outstanding accounts, and default risk, the model tries to find an optimal balance between the costs and benefits of providing credit, assuring successful credit management practises.

2. Notations and Assumptions

There are some notations and assumptions which are necessary for mathematical formulation of model. These are as follows:

Notations

$D(t) = a + bt + ct^2$: quadratic demand.
$\theta(t) = \alpha\beta t^{\beta-1}$: The Weibull distribution deterioration rate where $0 < \alpha < 1$the scale parameter and $\beta > 0$ is the shape parameter.
t_0 = The period of time after which interest is payable.
t_1 = Length of time with a positive item stock.

T= Cycle Length.
Q= Ordered Quantity per Cycle.
t_0= Earlier Period of Deterioration.
M = Period of trade credit provided by supplier.
Z_e= Earned interest per dollar per unit of time.
Z_{t_0}= Interest charges per dollar per unit item by the supplier for the period (M, t_0).
Z_{t1}= Interest charges per dollar per unit item by the supplier for the period (t_0, T).
C_1 = Ordering Price per unit.
C_2 = Cost of inventory deterioration per item.
C_3 = Holding cost per item
C_4 = Backordering cost per item
C_5 = Sales revenue cost per item
C_p= The constant purchase cost.

Assumptions

- The demand rate is linearly time dependent.
- A single warehouse and a single item are considered.
- Shortages are allowed and partially backlogged.
- Lead time is assumed to be zero.
- The deterioration rate follows the two parameter Weibull distribution.
- There is no repair or replenishment of the deteriorated items during the inventory cycle.
- The inventory is replenished only one in each cycle.
- The retailer can increase returns and produce interest, which will be followed by customer payments in purchasing costs from the retailer in anticipation of the dealer's permissible payment time being completed.

3. Mathematical Formulation of Model

Let Q represent the greatest amount of inventory, including backorders, that can be filled. During time period $(0, t_1)$, the inventory depletion occurs due to deterioration and demand. The inventory level becomes zero at $t_1$and shortages are allowed in interval (t_1, T). And finally, inventory replenished at time T. The quadratic function of time demand is typically of the form:

$\mathrm{D(t)} = \mathrm{a} + \mathrm{bt} + ct^2$ where D(t) represents the demand for the item at time t, a representing the baseline demand, b reflecting the rate of change in demand over time, and c expressing the acceleration in the rate of change in demand over time.

The following differential equations describes the states of inventory level $Z(t)$ at time t:

$$\frac{dZ(t)}{dt} + \theta(t)Z(t) = -D(t) \qquad 0 \le t \le T \tag{1}$$

Where $\theta(t) = \alpha\beta e^{\beta-1}$ and $\mathrm{D(t)} = \mathrm{a} + \mathrm{bt} + ct^2$

$$\frac{dZ(t)}{dt} + \alpha\beta e^{\beta-1}Z(t) = -(\mathrm{a} + \mathrm{bt} + ct^2) \qquad 0 \le t \le T \tag{2}$$

From equation (2), we get.

$$\mathrm{Z(t)} = -e^{-\alpha t^{\beta}}\left[at + \frac{bt^2}{2} + \frac{ct^3}{3} + \alpha\left(\frac{at^{\beta+1}}{\beta+1} + \frac{bt^{\beta+2}}{\beta+2} + \frac{ct^{\beta+2}}{\beta+3}\right) + \mathrm{c}_1\right] \tag{3}$$

Showing the boundary conditions $Z(t) = Q$ at $t = 0$, we get

$$\mathrm{c}_1 = Q$$

Putting the value of c_1 in (3), we get,

$$\mathrm{Z(t)} = -e^{-\alpha t^{\beta}}\left[Q - \left[at + \frac{bt^2}{2} + \frac{ct^3}{3} + \alpha\left(\frac{at^{\beta+1}}{\beta+1} + \frac{bt^{\beta+2}}{\beta+2} + \frac{ct^{\beta+3}}{\beta+3}\right)\right]\right] 0 \le t \le T \tag{4}$$

Since $\mathrm{Z(t)} = 0$ at $t = t_1$

We get

$$Q = at_1 + \frac{bt_1{}^2}{2} + \frac{ct_1{}^3}{3} + \alpha\left(\frac{at_1{}^{\beta+1}}{\beta+1} + \frac{bt_1{}^{\beta+2}}{\beta+2} + \frac{ct_1{}^{\beta+3}}{\beta+3}\right) \tag{5}$$

Putting the value of Qin (4)

$$\mathrm{Z(t)} = -e^{-\alpha t^{\beta}}\left[a(t_1 - t) + \frac{b(t_1-t)^2}{2} + \frac{c(t_1-t)^3}{3} + \alpha\left(\frac{a(t_1-t)^{\beta+1}}{\beta+1} + \frac{b(t_1-t)^{\beta+2}}{\beta+2} + \frac{c(t_1-t)^{\beta+3}}{\beta+3}\right)\right]$$

(6)

Also $\mathrm{Z(t)} = 0$ in $t_1 \le t \le T$

1. Ordering Cost

$$\mathrm{O_c} = \frac{C_1}{T}, \tag{7}$$

2. Deterioration Cost

$$D_c = \frac{C_2}{T}\left[Q - \int_0^{t_1} Z(t)dt\right]$$

$$D_c = \frac{C_2}{T}\left[\alpha\left(\frac{at_1^{\beta+1}}{\beta+1} + \frac{bt_1^{\beta+2}}{\beta+2} + \frac{ct_1^{\beta+3}}{\beta+3}\right)\right] \tag{8}$$

3. Holding cost:

$$H_c = \frac{C_3}{T}\left[\int_0^{t_1} Z(t)dt\right]$$

$$H_c = \frac{C_3}{T}\left[\frac{at_1^{\,2}}{2} + \frac{bt_1^{\,3}}{3} + \frac{ct_1^{\,4}}{4} + \alpha\left(1 - \frac{1}{\beta+1}\right)\left(\frac{at_1^{\beta+2}}{\beta+2} + \frac{bt_1^{\beta+3}}{\beta+3} + \frac{ct_1^{\beta+4}}{\beta+4}\right)\right] \tag{9}$$

4. Backorder cost:

$$B_c = \frac{C_4}{T}\left[\int_0^{T-t_1} D(t)tdt\right]$$

$$B_c = \frac{C_4}{T}\left[\frac{a(T-t_1)^2}{2} + \frac{b(T-t_1)^3}{3} + \frac{c(T-t_1)^4}{4}\right] \tag{10}$$

5. Sales Revenue

$$SR = \frac{C_p}{T}\left[\int_0^{t_1} D(t)dt\right]$$

$$SR = \frac{C_p}{T}\left[\frac{at_1^{\,2}}{2} + \frac{bt_1^{\,3}}{3} + \frac{ct_1^{\,4}}{4}\right] \tag{11}$$

In this paper, we examined two periods of permissible payment delay: (on the basis of the length of T and M):

Case-1: When the payment to settle the account is made on or before the time when all of the inventory has been utilized up i.e., when $M \le t_0 \le t_1 \le T$

$$ZC_1 = \frac{\mathrm{C_p}}{T}\left[Z_{t_0}\int_M^{t_0} Z(t)dt + Z_{t_1}\int_{t_0}^{t_1} Z(t)dt + Z_{t_1}\int_{t_1}^{T} Z(t)dt\right]$$

$$ZC_1 = \frac{C_p}{T}\left[(Z_{t_0} - Z_{t_1})\left\{a\left(t_1 t_0 - \frac{{t_0}^2}{2}\right) + \frac{b}{2}\left({t_1}^2 t_0 - \frac{{t_0}^3}{3}\right) + \frac{c}{3}\left({t_1}^3 t_0 - \frac{{t_0}^4}{4}\right)\right.\right.$$
$$+ \alpha\left\{\frac{a}{\beta+1}\left({t_1}^{\beta+1}t_0 - \frac{{t_0}^{\beta+2}}{\beta+2}\right) + \frac{b}{\beta+2}\left({t_1}^{\beta+2}t_0 - \frac{{t_0}^{\beta+3}}{\beta+3}\right)\right.$$
$$\left.\left.+ \frac{c}{\beta+3}\left({t_1}^{\beta+3}t_0 - \frac{{t_0}^{\beta+4}}{\beta+4}\right)\right\}\right\}$$
$$- Z_{t_0}\left[a\left(t_1 M - \frac{M^2}{2}\right) + \frac{b}{2}\left({t_1}^2 M - \frac{M^3}{3}\right) + \frac{c}{3}\left({t_1}^3 M - \frac{M^4}{4}\right)\right.$$
$$+ \alpha\left\{\frac{a}{\beta+1}\left({t_1}^{\beta+1}M - \frac{M^{\beta+2}}{\beta+2}\right) + \frac{b}{\beta+2}\left({t_1}^{\beta+2}M - \frac{M^{\beta+3}}{\beta+3}\right)\right.$$
$$\left.\left.+ \frac{c}{\beta+3}\left({t_1}^{\beta+1}M - \frac{M^{\beta+4}}{\beta+4}\right)\right\}\right]$$
$$\left.+ Z_{t_1}\left[\frac{a{t_1}^2}{2} + \frac{b{t_1}^3}{3} + \frac{c{t_1}^4}{4} + \alpha\left(1 - \frac{1}{\beta+1}\right)\left(\frac{a{t_1}^{\beta+2}}{\beta+2} + \frac{b{t_1}^{\beta+3}}{\beta+3} + \frac{c{t_1}^{\beta+4}}{\beta+4}\right)\right]\right] \tag{12}$$

The interest earned is:

$$ZE_1 = \frac{C_p}{T} Z_e \int_0^{t_1} D(t)dt$$
$$ZE_1 = \frac{C_p}{T} Z_e\left[a t_1 + \frac{b{t_1}^2}{2} + \frac{c{t_1}^3}{3}\right] \tag{13}$$

The total cost will be the sum of ordering cost, deteriorating cost, holding cost, backorder cost, the interest charged on a cycle-by-cycle basis for unsold inventory after the due date M (i.e., Z_{c1}) and the interest earned per cycle during the positive inventory level ZE_1.

$$TP_1 = \mathrm{SR} - \mathrm{O_c} - D_c - H_c - B_c - Z_{c1} + ZE_1$$

$$TP_1 = \frac{1}{T}\left\{C_p\left[\frac{a{t_1}^2}{2} + \frac{b{t_1}^3}{3} + \frac{c{t_1}^4}{4}\right] + C_p Z_e\left[a t_1 + \frac{b{t_1}^2}{2} + \frac{c{t_1}^3}{3}\right] - C_1 - C_2\left[\alpha\left(\frac{a{t_1}^{\beta+1}}{\beta+1} + \frac{b{t_1}^{\beta+2}}{\beta+2} + \frac{c{t_1}^{\beta+3}}{\beta+3}\right)\right] - \right.$$
$$C_3\left[\frac{a{t_1}^2}{2} + \frac{b{t_1}^3}{3} + \frac{c{t_1}^4}{4} + \alpha\left(1 - \frac{1}{\beta+1}\right)\left(\frac{a{t_1}^{\beta+2}}{\beta+2} + \frac{b{t_1}^{\beta+3}}{\beta+3} + \frac{c{t_1}^{\beta+4}}{\beta+4}\right)\right] - C_4\left[\frac{a(T-t_1)^2}{2} + \frac{b(T-t_1)^3}{3} + \frac{c(T-t_1)^4}{4}\right] -$$
$$C_p\left[(Z_{t_0} - Z_{t_1})\left\{a\left(t_1 t_0 - \frac{{t_0}^2}{2}\right) + \frac{b}{2}\left({t_1}^2 t_0 - \frac{{t_0}^3}{3}\right) + \frac{c}{3}\left({t_1}^3 t_0 - \frac{{t_0}^4}{4}\right) + \alpha\left\{\frac{a}{\beta+1}\left({t_1}^{\beta+1}t_0 - \frac{{t_0}^{\beta+2}}{\beta+2}\right) + \right.\right.\right.$$
$$\left.\left.\frac{b}{\beta+2}\left({t_1}^{\beta+2}t_0 - \frac{{t_0}^{\beta+3}}{\beta+3}\right) + \frac{c}{\beta+3}\left({t_1}^{\beta+3}t_0 - \frac{{t_0}^{\beta+4}}{\beta+4}\right)\right\}\right\} - Z_{t_0}\left[a\left(t_1 M - \frac{M^2}{2}\right) + \frac{b}{2}\left({t_1}^2 M - \frac{M^3}{3}\right) + \right.$$
$$\left.\frac{c}{3}\left({t_1}^3 M - \frac{M^4}{4}\right) + \alpha\left\{\frac{a}{\beta+1}\left({t_1}^{\beta+1}M - \frac{M^{\beta+2}}{\beta+2}\right) + \frac{b}{\beta+2}\left({t_1}^{\beta+2}M - \frac{M^{\beta+3}}{\beta+3}\right) + \frac{c}{\beta+3}\left({t_1}^{\beta+1}M - \frac{M^{\beta+4}}{\beta+4}\right)\right\}\right] +$$
$$\left.\left.Z_{t_1}\left[\frac{a{t_1}^2}{2} + \frac{b{t_1}^3}{3} + \frac{c{t_1}^4}{4} + \alpha\left(1 - \frac{1}{\beta+1}\right)\left(\frac{a{t_1}^{\beta+2}}{\beta+2} + \frac{b{t_1}^{\beta+3}}{\beta+3} + \frac{c{t_1}^{\beta+4}}{\beta+4}\right)\right]\right]\right\} \tag{14}$$

Case-2: The permissible trade credit period greater than the period with positive inventory of the item i.e., when $t_1 \leq M$:-

$$Z_{c2} = 0 \tag{15}$$

Since Interest earned = Interest earned during $t_1 \leq M$+ Interest earned during time (per cycle)

Hence the total interest earned per unit time (ZE_2) is given by.

$$ZE_2 = \frac{\mathrm{C_p}}{T} Z_e \left[\int_0^{t_1} D(t)tdt + D(t_1)t_1(M - t_1)\right]$$

$$ZE_2 = \frac{\mathrm{C_p}}{T} Z_e \left[\frac{at_1{}^2}{2} + \frac{bt_1{}^3}{3} + + \frac{ct_1{}^4}{4} + (at_1 + bt_1{}^2 + ct_1{}^3)(M - t_1)\right] \tag{16}$$

$$TP_2 = \mathrm{SR} - \mathrm{O_c} - D_c - H_c - B_c - Z_{c2} + ZE_2$$

$$\mathbf{TP_2} = \frac{1}{\mathrm{T}}\left\{\mathrm{C_p}\left[\frac{\mathrm{at_1}^2}{2} + \frac{\mathrm{bt_1}^3}{3} + \frac{\mathrm{ct_1}^4}{4}\right] + \mathrm{C_p Z_e}\left[\frac{\mathrm{at_1}^2}{2} + \frac{\mathrm{bt_1}^3}{3} + + \frac{\mathrm{ct_1}^4}{4} + (\mathrm{at_1} + \mathrm{bt_1}^2 + \mathrm{ct_1}^3)(\mathrm{M} - \mathrm{t_1})\right] - \mathrm{C_1} - \mathrm{C_2}\left[\alpha\left(\frac{\mathrm{at_1}^{\beta+1}}{\beta+1} + \frac{\mathrm{bt_1}^{\beta+2}}{\beta+2} + \frac{\mathrm{ct_1}^{\beta+3}}{\beta+3}\right)\right] - \mathrm{C_3}\left[\frac{\mathrm{at_1}^2}{2} + \frac{\mathrm{bt_1}^3}{3} + \frac{\mathrm{ct_1}^4}{4} + \alpha\left(1 - \frac{1}{\beta+1}\right)\left(\frac{\mathrm{at_1}^{\beta+2}}{\beta+2} + \frac{\mathrm{bt_1}^{\beta+3}}{\beta+3} + \frac{\mathrm{ct_1}^{\beta+4}}{\beta+4}\right)\right] - \mathrm{C_4}\left[\frac{\mathrm{a(T-t_1)^2}}{2} + \frac{\mathrm{b(T-t_1)^3}}{3} + \frac{\mathrm{c(T-t_1)^4}}{4}\right] - \mathrm{C_p}\left[\left(\mathrm{Z_{t_0}} - \mathrm{Z_{t_1}}\right)\left\{\mathrm{a}\left(\mathrm{t_1 t_0} - \frac{\mathrm{t_0}^2}{2}\right) + \frac{\mathrm{b}}{2}\left(\mathrm{t_1}^2\mathrm{t_0} - \frac{\mathrm{t_0}^3}{3}\right) + \frac{\mathrm{c}}{3}\left(\mathrm{t_1}^3\mathrm{t_0} - \frac{\mathrm{t_0}^4}{4}\right) + \alpha\left\{\frac{\mathrm{a}}{\beta+1}\left(\mathrm{t_1}^{\beta+1}\mathrm{t_0} - \frac{\mathrm{t_0}^{\beta+2}}{\beta+2}\right) + \frac{\mathrm{b}}{\beta+2}\left(\mathrm{t_1}^{\beta+2}\mathrm{t_0} - \frac{\mathrm{t_0}^{\beta+3}}{\beta+3}\right) + \frac{\mathrm{c}}{\beta+3}\left(\mathrm{t_1}^{\beta+3}\mathrm{t_0} - \frac{\mathrm{t_0}^{\beta+4}}{\beta+4}\right)\right\}\right\}\right]\right\} \tag{17}$$

4. Theoretical Results and Optimality of Mathematical Model

Our aim is to find the optimal values of t_1 and T to obtain maximize TP_1. The necessary condition for maximization of total cost is as following:

If $\frac{\partial TP_1}{\partial t_1} = 0$ and $\frac{\partial TP_1}{\partial T} = 0$,

Providing that $\frac{\partial^2 TP_1}{\partial^2 t_1} < 0$ and $\frac{\partial^2 TP_1}{\partial^2 T} < 0$. Then the objective function will be maximum for some values of t_1 and T.

Now we will use the theorems to prove the optimality of objective function in mathematically.

Theorem-1: Objective function TP_1 achieves its global maximum with respect to t_1 if $C_p[at_1 + bt_1{}^2 + ct_1{}^3 + Z_e(a + bt_1 + ct_1{}^2)] < 0$, providing decision variable T is fixed.

Proof: See the Appendix A. We can easily see that $\frac{\partial^2 TP_1}{\partial^2 t_1} < 0$. So, the objective function TC_1attains its global maximisation with respect to $t_1$while decision variable T is fixed.

Theorem -2: Objective function TP_1 achieves its global maximum with respect to T if $\frac{2(X-Y(T))-T^2\frac{\partial^2 Y(T)}{\partial T}+2T\frac{\partial Y(T)}{\partial T}}{T^3} < 0$, providing decision variable t_1 is fixed.

Proof: See the Appendix B. We can easily see that $\frac{\partial^2 TP_1}{\partial^2 T} < 0$. So, the objective function TP_1attains its global maximization with respect to Twhile decision variable t_1 is fixed.

Similarly, we can prove the optimality of second objective function TP_2. find the optimal values of t_1 and T to obtain maximize TP_2. The necessary condition for maximization of total cost is as following:

If $\frac{\partial TP_2}{\partial t_1} = 0$ and $\frac{\partial TP_2}{\partial T} = 0$,

Providing that $\frac{\partial^2 TP_2}{\partial^2 t_1} < 0$ and $\frac{\partial^2 TP_2}{\partial^2 T} < 0$. Then the objective function will be maximum for some values of t_1 and T.

Now we will use the theorems to prove the optimality of objective function in mathematically.

Theorem-3: Objective function TP_2 achieves its global maximum with respect to t_1 if $\mathrm{C_p} Z_e\big((2b+6ct_1)(M-t_1)\big) < 0$, providing decision variable T is fixed.

Proof: See the Appendix A. We can easily see that $\frac{\partial^2 TP_2}{\partial^2 t_1} < 0$. So, the objective function TP_2attains its global maximisation with respect to $t_1$while decision variable T is fixed.

Theorem -4: Objective function TP_1 achieves its global maximum with respect to T if $\frac{2(U-V(T))-T^2\frac{\partial^2 V(T)}{\partial T}+T\frac{\partial V(T)}{\partial T}}{T^3} < 0$, providing decision variable t_1 is fixed.

Proof: See the Appendix B. We can easily see that $\frac{\partial^2 TP_2}{\partial^2 T} < 0$. So, the objective function TP_2attains its global maximization with respect to Twhile decision variable t_1 is fixed.

5. Numerical Example and Sensitivity Analysis

Let

$$a = 100, b = 50\,, c = 20, \alpha = 0.5, \beta = 1\,, \mathrm{C}_1 = 200\, per\ order,\ \mathrm{C}_2 = 0.6\, per\ unit\,,\ \mathrm{C}_3 = 2\, per\ unit\ per\ year, \mathrm{C}_4 = 3\, per\ unit, Z_{t_0} = 0.12\, per\ year, Z_{t_1} = 0.15\, per\ year, \mathrm{C_p} = 10\, per\ unit, Z_e = 0.07\, per\ year, t_0 = 0.1$$

For case-I, $, M = 0.25\, year$ and for case-II, $M = 0.5\, year$ By putting all these values of parameter in total cost equations and solving in MATLAB software, we find the optimum values $t_1 = 1.036541, T = 1.628414\ and\ TC = 2596$

In order to explore the impact of under or overestimating the inventory system characteristics on the ideal values of the starting time period, cycle duration, as well as the maximum profit of the system, we did a sensitivity analysis on the numerical data mentioned above. The percentage variations from the previously mentioned ideal values are used as sensitivity indicators. Changes (increases and decreases) of -20% to +20% are made to the parameters to do the analysis. One parameter is changed at a time while the other parameters are left unchanged to produce the results. For both models, the findings of these analysis are presented in Tables-1 and Table-2.

5.1 Sensitivity analysis for Case-I: $\boldsymbol{M \le t_0 \le t_1 \le T}$

Table-1. Variation in Total Profit, t_1, T, D and Q on changing the value of one parameter while other parameters are constant:

Parameter	% Change in Parameter	% Change in			
		TP^*	$t_1{}^*$	T^*	Q^*
C_p	-20	3560.4507	1.2091	1.6228	678.1169
	-10	2890.5695	1.0795	1.5901	495.2035
	10	1976.7853	0.8976	1.4896	381.1964
	20	1031.9881	0.6973	1.4001	114.9168
C_1	-20	2819.9600	1.0859	1.5078	468.1512
	-10	2801.0034	1.0985	1.5090	470.3921
	10	2790.1133	1.1009	1.5102	472.7648
	20	2784.7814	1.1178	1.5125	474.4517
C_2	-20	1750.5130	0.7819	1.5351	169.1951
	-10	2510.9015	0.8617	1.5789	193.7615
	10	3018.8106	0.9901	1.5336	217.0015
	20	3981.7098	0.8181	1.4903	206.1109
C_3	-20	2567.1176	1.1581	1.5950	421.5214
	-10	2501.3940	1.0770	1.5894	375.8917
	10	2480.7890	1.0195	1.5476	301.9150
	20	2336.7861	1.0081	1.5001	285.7981

C_4	-20	1145.8761	1.0410	1.5601	380.9510
	-10	1131.7321	1.0434	1.5581	382.8150
	10	1113.8911	1.0479	1.5553	387.1506
	20	1107.5214	1.0509	1.5521	390.0015
M	-20	2771.0159	1.0685	1.5104	220.1513
	-10	2849.0078	1.0631	1.4975	218.9504
	10	2990.0195	1.0567	1.4515	210.1511
	20	3013.5330	1.0511	1.4017	206.8769
a	-20	2634.8526	1.0655	1.4967	390.2816
	-10	2965.3281	1.0601	1.4863	439.0867
	10	3156.2880	1.0593	1.4800	483.5671
	20	3320.6501	1.0586	1.4773	519.2041
b	-20	2529.2541	1.0642	1.5069	387.2657
	-10	2603.5963	1.0629	1.5132	389.2811
	10	2697.3214	1.0600	1.5201	390.0840
	20	2703.2824	1.0593	1.5293	391.2900
c	-20	2492.1280	1.0613	1.5179	385.2014
	-10	2505.1413	1.0609	1.5038	384.0371
	10	2581.6941	1.0600	1.4900	383.9869
	20	2597.0054	1.0596	1.4890	383.1084

5.2 Sensitivity analysis for Case-II: $t_1 \leq M$

Table-2. Variation in Total Profit, t_1, T, D and Q on changing the value of one parameter while other parameters are constant:

Parameter	% Change in Parameter	% Change in			
		TP^*	$t_1{}^*$	T^*	Q^*
C_p	-20	4015.6954	1.3206	1.7036	683.2654
	-10	3380.2148	1.1984	1.6925	502.3624
	10	2465.3584	0.9652	1.6825	396.3541
	20	1403.4367	0.7956	1.6726	204.9587
C_1	-20	2984.3645	1.0765	1.6354	490.6853
	-10	2978.2565	1.0864	1.6379	491.3687

	10	2953.6842	1.0965	1.6396	493.5298
	20	2921.3657	1.0836	1.6403	495.2150
C_2	-20	2065.8495	0.8652	1.5532	180.6965
	-10	2632.2451	0.9654	1.5436	201.3670
	10	3294.3256	1.0023	1.5395	223.6041
	20	4065.9451	0.9984	1.5203	228.9520
C_3	-20	2631.5468	1.2036	1.6132	425.5214
	-10	2536.2546	1.1362	1.6069	379.5241
	10	2498.3654	1.0365	1.5932	305.2146
	20	2331.3254	1.0036	1.5520	290.3894
C_4	-20	1250.5423	1.0568	1.5732	381.0261
	-10	1239.3256	1.0573	1.5701	383.2549
	10	1214.3256	1.0596	1.5635	389.5249
	20	1101.9556	1.0615	1.5596	392.6940
M	-20	2832.5436	1.0765	1.5536	223.6370
	-10	2910.3566	1.0712	1.5336	220.1856
	10	3050.6565	1.0651	1.5031	215.1025
	20	3089.2586	1.0592	1.4930	209.2367
a	-20	2736.5012	1.0764	1.5638	289.6216
	-10	2928.2691	1.0738	1.5328	297.3548
	10	3091.2800	1.0703	1.5036	301.9434
	20	3215.3652	1.0684	1.4979	305.5622
b	-20	2631.6902	1.0641	1.5152	295.6817
	-10	2770.0521	1.0621	1.5035	298.3640
	10	2836.2894	1.0603	1.4863	300.5122
	20	2964.3950	1.0583	1.4794	301.2819
c	-20	2694.3601	1.0651	1.5700	299.3617
	-10	2706.2891	1.0642	1.5682	300.2811
	10	2797.6318	1.0640	1.5602	301.5717
	20	2836.9525	1.0632	1.5574	302.6945

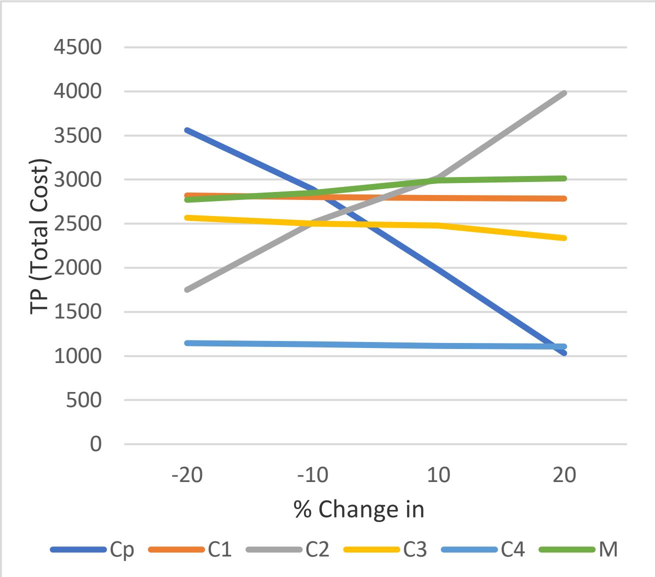

Figure:1 Variation in TP (Total Profit) with respect to C_p, C_1, C_2, C_3, C_4 and M

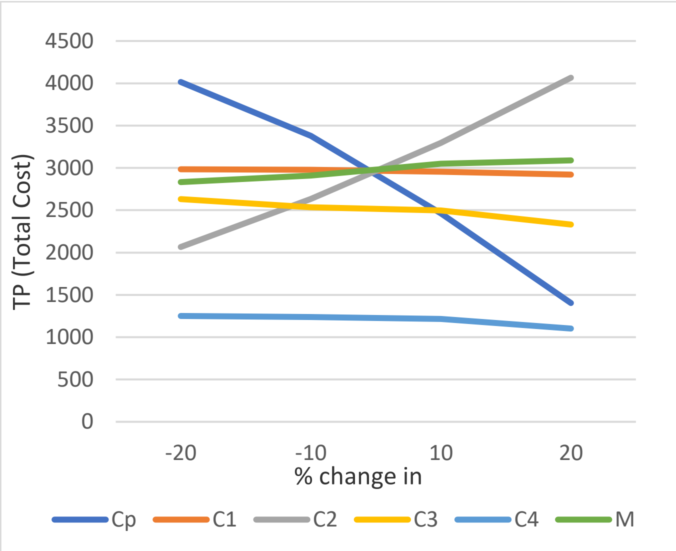

Figure:2 Variation in TP (Total Profit) with respect to C_p, C_1, C_2, C_3, C_4 and M

From Table-1 and Table-2 we can observe that:

1. If purchase cost C_p increases, the profit TP^* decreases and Q^*, $t_1{}^*$and T^* also decrease or if purchase cost C_p decreases, the profit TP^* increases and Q^*, $t_1{}^*$and T^* also increase.

2. If ordering cost C_1 increases, the profit TP^* decreases and Q^*, $t_1{}^*$and T^* increase or if ordering cost C_1 decreases, the profit TP^* increases and Q^*, $t_1{}^*$and T^* decrease.
3. If deterioration cost C_2 increases, the profit TP^*, Q^*increase and $t_1{}^*$, T^* decrease or if deterioration cost C_2 decreases, the profit TP^*, Q^*decrease and $t_1{}^*$, T^* increase.
4. If holding cost C_3 increases, the profit TP^* decreases and Q^*, $t_1{}^*$and T^* also decrease or if holding cost C_3 decreases, the profit TP^* increases and Q^*, $t_1{}^*$and T^* also increase.
5. If backlogging cost C_4 increases, the profit TP^*, T^* decreases and Q^*, $t_1{}^*$ increase or if backlogging cost C_4 decreases, the profit TP^*, T^* increases and Q^*, $t_1{}^*$ decrease.
6. If trade credit period M increases, the profit TP^* increases and Q^*, $t_1{}^*$and T^* decrease or if trade credit period M decreases, the profit TP^* decreases and Q^*, $t_1{}^*$and T^* increase.
7. If demand parameter a, b and c increases, the profit $TP^*$and Q^*, increases and $t_1{}^*$and T^*decreases or if demand parameter a, b and c decreases, the profit $TP^*$and Q^*, decreases and $t_1{}^*$and T^*increases.
8. Purchase cost C_p and deterioration cost C_2 are more sensitive to total profit TP^*, $T^*$and Q^*than other parameters.

6. Discussion

The decision-maker or management can be advised of the following findings based on the results of the sensitivity studies that were conducted for model:

From here, we derive our findings that higher purchasing costs result in reduced profit margins, a longer cycle time between inventory replenishments, and a drop in replenishment quantity. Higher ordering costs diminish profitability, but they also imply a shorter cycle length between inventory replenishments and an increase in replenishment quantities. Higher product deterioration costs result in higher profits and a greater quantity of replenishment. However, in order to reduce the negative consequences of deterioration, the cycle length must be reduced. Higher holding costs result in lower profitability, as well as a longer cycle length between inventory replenishments and a drop in replenishment quantity. Higher backlog expenses reduce profitability while also implying an increase in the quantity of replenishment. One significant finding is that profit will drop when the credit period supplied by the supplier to the retailer is shorter since the retailer is unable to receive interest on his profit during that time. Increased demand parameters result in higher profitability and a greater amount of replenishment. This results in a reduced cycle duration and lower holding costs. Retailers must boost inventory levels because as purchasing costs rise, overall profit also rises. Additionally, this yields a longer replenishing cycle. Additionally, this benefits shops in avoiding the issue of a shortage.

7. Conclusion

We have developed an inventory model for quadratic demand and deteriorating items that follows the Weibull distribution with a trade credit policy involves incorporating various factors such as the rate of demand, rate of deterioration, credit terms, and holding costs into the decision-making process for stocking and selling goods. The Weibull distribution is used to model the probability of demand and deterioration over time, while the trade credit policy can be used to determine the optimal time to sell goods and receive payment. This can help to balance the trade-off between the benefits of holding inventory and the costs of carrying it, and can lead to more effective inventory management. This model can further be extended for the quadratic demand function of price.

References

[1] R. R., Chowdhury, S. K., Ghosh, K. S. Chaudhuri. An inventory model for deteriorating items with stock and price sensitive demand. *International Journal of Applied and Computational Mathematics*, *1*, 187-201, 2015

[2] K. Karthikeyan, G. Santhi. An inventory model for constant deteriorating items with cubic demand and salvage value. *International Journal of Applied Engineering Research*, *10*(55), 2015

[3] P. K. Lakshmidevi, M. Maragatham. An inventory model with three rates of production and time dependent deterioration rate for quadratic demand rate. International Journal of Fuzzy Mathematical Archive, 6(1), 99-103, 2015.

[4] S. Saha, N. Sen. A study on inventory model with negative exponential demand and probabilistic deterioration under backlogging. Uncertain supply chain management, 5(2), 77-88, 2017.

[5] S. Kumar. Development of an EOQ Inventory Model for Perishable Items with Exponentially Decaying Demand, Shortages and Partial Backlogging. International Journal for Research in Applied Science & Engineering Technology, 2017.

[6] P. Aggrawal, T. J. Singh. An EOQ model with ramp type demand rate, time dependent deterioration rate and shortages. Global Journal of Pure and Applied Mathematics, 13(7), 3381-3393, 2017.

[7] G. C. Mahata, S. K. De. Supply chain inventory model for deteriorating items with maximum lifetime and partial trade credit to credit-risk customers. International Journal of Management Science and Engineering Management, 12(1), 21-32, 2017.

[8] U. Mishra, C. K. Tripathy. An inventory model for Weibull deteriorating items with salvage value. *International Journal of Logistics Systems and Management*, *22*(1), 67-76, 2015.

[9] R. Udayakumar, K. V. Geetha. An EOQ model for non-instantaneous deteriorating items with two levels of storage under trade credit policy. Journal of Industrial Engineering International, 14, 343-365, 2018.

[10] B. Bishi, S. K. Sahu. An inventory model for deteriorating items with quadratic demand and partial backlogging. Journal of computer and Mathematical Sciences, 9(12), 2188-2198, 2018.

[11] D. Chitra. Replenishment Policy for Non instantaneous Deteriorating Items with Parabolic demand Pattern and partial backlogging for Queued Customers: Computational Approach. International Journal of Applied Engineering Research, 14(8), 1818-1827, 2019.

[12] B. Mandal. An EOQ Inventory Model for Time-varying Deteriorating Items with Cubic Demand under Salvage Value and Shortages. International Journal of Systems Science and Applied Mathematics, 5(4), 36-42, 2020.

[13] P. D. Khatri, U. B. Gothi. An EPQ Model for Non-Instantaneous Weibully Decaying Items with Ramp Type Demand and Partially Backlogged Shortages, 2020.

[14] L. E. Cárdenas-Barrón, A. A. Shaikh, S. Tiwari, G. Treviño-Garza. An EOQ inventory model with nonlinear stock dependent holding cost, nonlinear stock dependent demand and trade credit. Computers & Industrial Engineering, 139, 2020.

[15] A. Sharma, J. Kaushik. Inventory model for deteriorating items with ramp type demand under permissible delay in payment. International Journal of Procurement Management. 14. 578, 2021.

[16] M. Esmaeili, M. Nasrabadi. An inventory model for single - vendor multi - retailer supply chain under inflationary conditions and trade credit, Journal of Industrial and Production Engineering, 38:2, 75-88, 2021.

[17] Khyati, A. K. Saxena. Review on EOQ Models for Instanteneous and Non-Instanteneous Deteriorating Items, 2021 10th International Conference on System Modeling and Advancement in Research Trends (SMART), MORADABAD, India, pp. 312–315, 2021.

[18] Khyati, A. K. Saxena. "An EOQ Model for Deteriorating Items with Time Dependent Demand," 2022 11th International Conference on System Modeling & Advancement in Research Trends (SMART), Moradabad, India, pp. 926-930, 2022.
[19] A. K. Saxena, R. K. Yadav. Ordering Policy For Non Instantaneous Decaying Items With Stock Dependent Demand and Shortages. In Proceedings of the International Conference on Soft Computing for Problem Solving (SocProS 2011), 497–513. 2011.
[20] A. A. Shaikh, .C. Panda, M. A. A. Khan, A. H. M. Mashud, A. Biswas. An inventory model for deteriorating items with preservation facility of ramp type demand and trade credit. International Journal of Mathematics in Operational Research, 17(4), 514-551, 2020.
[21] A. Khakzad, A., M. R. Gholamian. The effect of inspection on deterioration rate: An inventory model for deteriorating items with advanced payment. Journal of cleaner production, 254, 2020.
[22] A. Duary, S. Das, M.G. Arif, K.M. Abualnaja, M. A. A. Khan, M. Zakarya, M., A.A. Shaikh. Advance and delay in payments with the price-discount inventory model for deteriorating items under capacity constraint and partially backlogged shortages. Alexandria Engineering Journal, 61(2), 1735-1745,2022.
[23] M. Kumari, P. Narang, P.K. De. Optimization of an Inventory Model with Selling Price and Stock Sensitive Demand Along with Trade Credit Policy. In Soft Computing: Theories and Applications: Proceedings of SoCTA 2022 (pp. 47-57), 2023.

Appendix A

On differentiating the objective function with respect to t_1, we get

$$\frac{\partial TP_1}{\partial t_1} = \frac{1}{T}\Bigg[C_p(at_1 + bt_1{}^2 + ct_1{}^3) + C_pZ_e(a + bt_1 + ct_1{}^2) - C_1 - C_2\left(\alpha\left(at_1{}^{\beta} + bt_1{}^{\beta+1} + ct_1{}^{\beta+2}\right)\right) - (C_3 - Z_{t_1})\left[at_1 + bt_1{}^2 + ct_1{}^3 + \alpha\left(1 - \frac{1}{\beta+1}\right)\left(at_1{}^{\beta+1} + bt_1{}^{\beta+2} + ct_1{}^{\beta+3}\right)\right] + C_4[a(T - t_1) + b(T - t_1)^2 + c(T - t_1)^3] - \mathrm{C_p}\left[\left(Z_{t_0} - Z_{t_1}\right)\left\{t_0(a + bt_1 + ct_1{}^2) + \alpha\left\{t_0\left(at_1{}^{\beta} + bt_1{}^{\beta+1} + ct_1{}^{\beta+2}\right)\right\}\right\} - Z_{t_0}M\left[a + bt_1 + ct_1{}^2 + \alpha\left\{at_1{}^{\beta} + bt_1{}^{\beta+1} + ct_1{}^{\beta+2}\right\}\right]\right]\Bigg] \quad \text{(A.1)}$$

Differentiating the objective function again with respect to t_1, we have

$$\frac{\partial^2 TP_1}{\partial^2 t_1} = \frac{1}{T}\Bigg[C_p\left(a + 2bt_1 + 3ct_1{}^2\right) + C_pZ_e[b + 2ct_1] - C_2\left[\alpha\left(a\beta t_1{}^{\beta-1} + b(\beta + 1)t_1{}^{\beta} + c(\beta + 2)t_1{}^{\beta+1}\right)\right] - (C_3 - Z_{t_1})\left[a + 2bt_1 + 3ct_1{}^2 + \alpha\left(1 - \frac{1}{\beta+1}\right)\left(a(\beta + 1)t_1{}^{\beta} + b(\beta + 2)t_1{}^{\beta+1} + c(\beta + \right.\right.$$

$3)t_1^{\beta+2})\Big] - C_4[a + 2b(T - t_1) + 3c(T - t_1)^2] - \mathrm{C_p}\Big[\big(Z_{t_0} - Z_{t_1}\big)\Big\{t_0(b + 2ct_1) + \alpha\big\{t_0(a\beta t_1^{\beta-1} + b(\beta + 1)t_1^{\beta} + c(\beta + 2)t_1^{\beta+1})\big\}\Big\} - Z_{t_0}M\big[b + 2ct_1 + \alpha(a\beta t_1^{\beta-1} + b(\beta + 1)t_1^{\beta} + c(\beta + 2)t_1^{\beta+1})\big]\Big]\Big]$

(A.2)

From equation (A.2) we can observe that

$\frac{\partial^2 TP_1}{\partial^2 t_1} < 0$

if

$C_p[at_1 + bt_1^2 + ct_1^3 + Z_e(a + bt_1 + ct_1^2)] < 0$

This demonstrates *Theorem-1.*

Appendix B

Differentiate the objective function $TP_1$with respect to T, we get

$$\frac{\partial TP_1}{\partial T} = -\frac{X}{T^2} - \frac{\frac{\partial Y(T)}{\partial T}}{T} + \frac{Y(T)}{T^2} \tag{B.1}$$

Again, differentiate the objective function $TP_1$with respect to T, we have

$$\frac{\partial^2 TP_1}{\partial^2 T} = \frac{2\big(X - Y(T)\big) - T^2\frac{\partial^2 Y(T)}{\partial T} + 2T\frac{\partial Y(T)}{\partial T}}{T^3} \tag{B.2}$$

From equation (B.2) we can observe that

$\frac{\partial^2 TC_1}{\partial^2 T} < 0$

If

$\frac{2\big(X - Y(T)\big) - T^2\frac{\partial^2 Y(T)}{\partial T} + T\frac{\partial Y(T)}{\partial T}}{T^3} < 0$, provided that $T > 0$

This demonstrates *Theorem- 2.*

Appendix C

On differentiating the objective function TP_2 with respect to t_1, we get

$$\frac{\partial TP_1}{\partial t_1} = \frac{1}{T}\Big[C_p[at_1 + bt_1^2 + ct_1^3] + \mathrm{C_p}Z_e\big[at_1 + bt_1^2 + ct_1^3 + \big(a + 2bt_1 + 3ct_1^2\big)(M - t_1) - (at_1 + bt_1^2 + ct_1^3)\big] - C_2\big[\alpha\big(at_1^{\beta} + bt_1^{\beta+1} + ct_1^{\beta+2}\big)\big] - C_3\Big[at_1 + bt_1^2 + ct_1^3 + \alpha\Big(1 - \frac{1}{\beta+1}\Big)\big(at_1^{\beta+1} + bt_1^{\beta+2} + ct_1^{\beta+3}\big)\Big] + C_4[a(T - t_1) + b(T - t_1)^2 + c(T - t_1)^3] - \mathrm{C_p}\Big[\big(Z_{t_0} - Z_{t_1}\big)\Big\{t_0(a + bt_1 + ct_1^2) + \alpha\big\{t_0\big(at_1^{\beta} + bt_1^{\beta+1} + ct_1^{\beta+2}\big)\big\}\Big\}\Big]\Big] \tag{C.1}$$

Differentiating the objective function TP_2again with respect to t_1, we have,

$\frac{\partial^2 TP_1}{\partial^2 t_1} = \frac{1}{T}\Big[\big[C_p(1 - Z_e)\big(a + 2bt_1 + 3ct_1^2\big) + \mathrm{C_p}Z_e\big((2b + 6ct_1)(M - t_1)\big)\big] - C_2\big[\alpha\big(a\beta t_1^{\beta-1} + b(\beta + 1)t_1^{\beta} + c(\beta + 2)t_1^{\beta+1}\big)\big] - C_3\Big[a + 2bt_1 + 3ct_1^2 + \alpha\Big(1 - \frac{1}{\beta+1}\Big)\big(a(\beta + 1)t_1^{\beta} + b(\beta +$

$2)t_1^{\beta+1} + c(\beta+3)t_1^{\beta+2})\Big] - C_4[a + 2b(T-t_1) + 3c(T-t_1)^2] - \mathrm{C_p}\Big[(Z_{t_0} - Z_{t_1})\Big\{t_0(b + 2ct_1) + \alpha\{t_0(a\beta t_1^{\beta-1} + b(\beta+1)t_1^{\beta} + c(\beta+2)t_1^{\beta+1})\}\Big\}\Big]\Big]$ (C.2)

From equation (C.2) we can observe that

$\frac{\partial^2 TP_1}{\partial^2 t_1} < 0$ If

$$\mathrm{C_p} Z_e((2b + 6ct_1)(M - t_1)) < 0 \quad \text{(C.3)}$$

This demonstrates *Theorem-3*

Appendix D

Differentiate the objective function TP_2 with respect to T, we get

$$\frac{\partial TP_2}{\partial T} = -\frac{U}{T^2} - \frac{\frac{\partial V(T)}{\partial T}}{T} + \frac{V(T)}{T^2} \quad \text{(D.1)}$$

Again, differentiate the objective function TP_1 with respect to T, we have

$$\frac{\partial^2 TP_2}{\partial^2 T} = \frac{2(U - V(T)) - T^2\frac{\partial^2 V(T)}{\partial T} + 2T\frac{\partial V(T)}{\partial T}}{T^3} \quad \text{(D.2)}$$

From equation (D.2) we can observe that

$\frac{\partial^2 TC_2}{\partial^2 T} < 0$

If

$$\frac{2(U - V(T)) - T^2\frac{\partial^2 V(T)}{\partial T} + T\frac{\partial V(T)}{\partial T}}{T^3} < 0 \text{, provided that } T > 0 \quad \text{(D.3)}$$

This demonstrates *Theorem- 4.*