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# Inventory model with preservation technology and exponential holding cost in fuzzy scenario

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**Abstract**

Inventories are ubiquitous in the business sector. Since inventory is most frequently incurring expense, stock control is critical for an organization and it must be scrimping and saving in contemplation of function the merchandising fruitfully. In this paper, an inventory model for a deteriorating item under exponential holding cost with collaborative preservation technology investment under carbon policy is considered. Also, this study is developed in a fuzzy scenario by employing triangular fuzzy numbers. Signed distance method is utilized to enhance decision making and optimization. Further the convexity of the total cost function for both the crisp and the fuzzy case is established. The objective is to determine the optimal investment in preservation technology and the optimal cycle length so as to minimize the total cost. Moreover, some managerial results are obtained by using sensitivity analysis and graphical representation is also carried out. The applications of the proposed model is used in the fields of constructing machinery or heavy duty construction equipment, specific chemicals and processed food.

**Keywords:** carbon emission; preservation technology; signed distance method; stock dependent demand; triangular fuzzy number.
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# 1. Introduction

The displayed inventory level is a promotional device in today's globalized technology to boost income. According to numerous researches, having a significant number of products on display may attract more consumers. This implies a positive relationship between demand and stock levels. As a result, demand in this model is considered as stock-dependent, which is more practical. Bardhan et al. [1] investigated an optimal replenishment policy and preservation technology investment for a non-instantaneous deteriorating item with stock-dependent demand. Preservation technology is a critical component in reducing deterioration. Giri et al. [7] have explored a supply chain model for time-dependent deteriorating item with preservation technology investment. For a single-vendor multi-buyer model, Setiawan and Endrayanto [11] implemented a coordination strategy and synchronization in the production flow, including adjustable lead time. Khanna et al. [9] adopted an optimizing preservation strategy for deteriorating items with time-varying holding cost and stock dependent demand.

Global warming poses a significant hazard to our planet. The world's attention is currently focused on reducing carbon emissions. Dye and Yang [6] anticipated that ordering and storing inventory causes carbon emissions. They looked at sustainability in the context of a collaborative trade credit arrangement, where demand is tied to the credit period. Daryanto and Wee [4] considered a production lot size decision of a manufacturer incorporating environmental impact of carbon emission. Tao and Xu [14] developed an inventory model concerning emission-regulation policies with consumer's low carbon awareness, providing decision support. Shen et al. [13] developed a production inventory model for deteriorating items with collaborative preservation technology investment under carbon tax. Yu et al. [3] presented an inventory model of a deteriorating product considering carbon emissions. Patel et al. [5] decided optimal order quantity for the industries especially chemical industries with trended demand under trade credit with existence of cap and trade structure to reduce carbon emissions. Tripathi and Mishra [12] investigated an EOQ model with linear time dependent demand and different holding cost functions.

In the above analysis, it is presumed that all parameters are precisely known. But in real world, parameters are imprecise in nature and one has to deal with approximation of numbers that are close to real numbers. Fuzzy number provides a way to model this epistemic uncertainty and its propagation. Bjork [2] analysed an EOQ model in a fuzzy environment. Alrefaei and Tuffaha [10] studied an intuitionistic polygonal fuzzy numbers. Hemalatha and Annadurai [8] proposed an integrated production-distribution inventory system for deteriorating products in fuzzy environment by ensuring extra investment thereby reducing setup cost.

We develop the model including some points which highlight the novelty of our model. In our model, the deterioration effect of the product is considered and preservation technology is addressed to regulate the deterioration rate. Demand is cogitated as stock-dependent and the holding cost is ruminated as an exponential. Under carbon emission regulations, the goal is to resolute the optimal investment in preservation technology and cycle time (Dye & Yang, [6]).

As the path of developing EOQ models with uncertainty expressed as fuzzy numbers are quite profitable, the fuzzy model is discussed in this study. The cost parameters are considered as a triangular fuzzy numbers. The total cost function is defuzzified and proven to be convex using the signed distance method.

Following an introduction, the remainder of the article is organized as follows: The second section is devoted to the related preliminary definitions. In Section 3, notations and assumptions are shown. In Section 4, a mathematical model for the crisp model is developed and another mathematical model for a fuzzy model is developed in Section 5. Numerical example is provided to illustrate the crisp and fuzzy models in Section 6. In Section 7, sensitivity analysis and managerial insights are provided to validate the concept. Comparative study is given in Section 8. Finally, conclusion and future research direction are given in the last section.

# 2. Preliminaries

The following definitions of fuzzy sets are relevant to the method used in the proposed model.

**Definition 2.1 Triangular Fuzzy Numbers**: Let $\tilde{D} = (p_1, p_2, p_3), p_1 < p_2 < p_3$ be a triangular fuzzy number with membership function:

$$\mu_{\tilde{B}}(x) = \begin{cases} \dfrac{x - p_1}{p_2 - p_1}, & p_1 \le x \le p_2 \\ \dfrac{p_3 - x}{p_3 - p_2}, & p_2 \le x \le p_3 \\ 0, & otherwise\ . \end{cases}$$

**Definition 2.2 Signed distance method** (Bjork [2]): The signed distance of $[L_\alpha, R_\alpha]$ measured from 0 is $\mathrm{d}_0\left([L_\alpha, R_\alpha], \tilde{0}\right) = \dfrac{\tilde{D}_L(\alpha) + \tilde{D}_R(\alpha)}{2}$. For the triangular fuzzy number $\tilde{B} \in R^{-}$, the distance from $\tilde{D}$ to 0 is written as $d\ (\tilde{D}, \tilde{0}) = \frac{1}{4}\left(p_1 + 2p_2 + p_3\right)$.

# 3. Notations and assumptions

To develop the proposed model, we adopt the following notations and assumptions.

**Notations:**

The notations used in our model are listed as follows:

| | |
|---|---|
| $T_e$ | Length of cycle |
| $\tau_e$ | Investment in preservation technology per unit time |
| $A_{ece}$ | Fixed carbon emission per order |
| $c_{ece}$ | Carbon emission per unit per order |
| $C_{eoc}$ | Cost of ordering (per order) |

$C_{edc}$ Unit cost due to deterioration

$C_{ehc}$ Holding cost per unit time $t$ $(C_{ehc}(t) = h_{ehc}\, \mathrm{e}^{rt})$

$C_{epc}$ Purchasing cost per unit

$h_{ece}$ Carbon emission for inventory per unit time

$I_{eh}(t)$ Level of inventory at time $t, 0 \le t \le T_e$

$Q$ Size of order

$TC_{ep}(T_e, \tau_e)$ Total cost of the system

$y_0$ Rate of deterioration in the absence of preservation technologies

$y(\tau)$ Investment in preservation technologies reduces the rate of deterioration

**Assumptions:**

The following assumptions are made in the model:

1) The rate of demand is directly proportional to the stock level. i.e.,

$$D(I_{eh}(t)) = a + bI_{eh}(t), \quad a > 0,\ 0 < b < 1.$$

2) The time horizon is infinite with negligible lead time.
3) Shortages are not allowed.
4) Preservation technology investment reduces the rate of deterioration gradually. The reduced deterioration rate $y(\tau_e)$, is a function of preservation technology cost $\tau_e$ such that $y(\tau_e) = y_0 e^{-u\tau_e}$, which satisfies the conditions $\partial TC_{ep} / \partial \tau_e < 0,\ \partial^2 TC_{ep} / \partial \tau_e{}^2 > 0$ and $y(0) = y_0$, where $u$ is sensitivity parameter of investment $0 < u < 1$.
5) The holding cost is considered to be dependent on time as $C_{ehc}(t) = h_{ehc}\, \mathrm{e}^{rt},\ 0 < r < 1.$
6) The total amount of carbon emissions includes emissions from ordering, holding and purchasing inventory.

# 4. Mathematical model

In this section, a mathematical model is developed to determine the cycle time and optimal investment in preservation technology. Deteriorating items are likewise regarded with low carbon emission cost and preservation technology in our inventory system. The equation governing the inventory level (Khanna et al. [9]) can be expressed as follows:

$$\frac{dI_{eh}(t)}{dt} + y(\tau)I_{eh}(t) = -(a + bI_{eh}(t)), \quad 0 \le t \le \mathrm{T}_e. \tag{1}$$

The solution of equation (1) using the boundary condition $I_{eh}(\mathrm{T}_e) = 0$, is given by

$$I_{eh}(t) = a\Big[(T_e - t) + \Big((T_e - t)^2 \big(y(\tau_e) + b\big)/2\Big)\Big], \tag{2}$$

and the initial inventory level is

$$Q = I_{eh}(0) = a\Big[T_e + \Big(\big(T_e^2 \big(y(\tau_e) + b\big)\big)/2\Big)\Big]. \tag{3}$$

The total cost of the system is calculated by adding the following costs.

Ordering cost: $C_{eoc}$,

Preservation technology investment: $PT = \tau_e T_e$,

Holding cost: $EHC = \int_0^{T_e} h_{ehc} e^{rt} I_{eh}(t)dt$

$$= ah_{ehc}\left[\left(-T_e / r\right) + \left(e^{rT_e} / r^2\right) - \left(1/r^2\right) + \left(\left(y(\tau_e)+b\right)/2\right)\left(\left(-T_e^2/r\right) - \left(2T_e/r^2\right) + \left(2e^{-rT_e}/r^3\right) + \left(2/r^3\right)\right)\right],$$

Deterioration cost: $EDC = C_{edc}\left(Q - \int_0^{T_e} D\left(I_{eh}(t)\right)dt\right)$

$$= C_{edc}\left[\left(T_e^2 y(\tau_e)\left(3a - abT_e\right) - ab^2 T_e^3\right)/6\right],$$

and Purchasing cost: $EPC = C_{epc}Q = aC_{epc}\left[T_e + \left(\left(T_e^2\left(y(\tau_e)+b\right)\right)/2\right)\right].$

Then total carbon emissions $TEC$ in a finite time horizon $T$ (Dye & Yang, [6]) is

$$TC_{ep}\left(T_e, \tau_e\right) = A_{ece} + c_{ece}Q + h_{ece}\int_0^{T_e} I_{eh}(t)dt$$

$$= A_{ece} + ac_{ece}\left(T_e + \left(T_e^2\left(y(\tau_e)+b\right)/2\right)\right) + ah_{ece}\left(\left(T_e^2/2\right) + \left(T_e^3\left(y(\tau_e)+b\right)/6\right)\right).$$

$$\begin{aligned} \text{i.e., } TC_{ep}\left(T_e, \tau_e\right) &= \left(C_{eoc}/T_e\right) + \left(ah_{ehc}/T\right)\Big[\left(-T_e/r\right) + \left(e^{rT_e}/r^2\right) - \left(1/r^2\right) + \left(\left(y(\tau_e)+b\right)/2\right) \\ &\quad \left(\left(-T_e^2/r\right) - \left(2T_e/r^2\right) + \left(2e^{-rT_e}/r^3\right) + \left(2/r^3\right)\right)\Big] + \left(aC_{epc}/T\right)\left[T_e + \left(\left(T_e^2\left(y(\tau_e)+b\right)\right)/2\right)\right] \\ &\quad + \left(C_{edc}/T_e\right)\left[\left(T_e^2 y(\tau_e)\left(3a - abT_e\right) - ab^2T_e^3\right)/6\right] + \left(\tau_e T_e / T_e\right) \\ &\quad + \frac{1}{T_e}\left[A_{ece} + ac_{ece}\left(T_e + \left(T_e^2\left(y(\tau_e)+b\right)/2\right)\right) + ah_{ece}\left(\left(T_e^2/2\right) + \left(T_e^3\left(y(\tau_e)+b\right)/6\right)\right)\right]. \end{aligned} \quad (4)$$

The objective is to minimize the total cost by jointly optimizing the cycle time $T_e$ and the investment in preservation technology $\tau_e$. To establish optimality, taking the first order partial derivative and equate it into zero, we get

$$\partial TC_{ep} / \partial T_e = 0 \text{ and } \partial TC_{ep} / \partial \tau_e = 0, \quad (5)$$

That is

$$\begin{aligned} &\left(-C_{eoc}/T_e^2\right) + \left(ah_{ehc}e^{rT_e}/r^3T_e^2\right)\left(r + \left(y_0e^{-u\tau_e} + b\right)\right)\left(rT_e - 1\right) + \left(ah_{ehc}/r^2T_e^2\right) + \left(y_0e^{-u\tau_e} + b\right) \\ &\left(\left(-ah_{ehc}/2r\right) - \left(ah_{ehc}/r^3T_e^2\right)\right) + \left(aC_{epc}/2\right)\left(y_0e^{-u\tau_e} + b\right) + \left(C_{edc}/6\right)\left(3ay_0e^{-u\tau_e} - 2abT_ey_0e^{-u\tau_e} - 2ab^2T_e\right) \\ &+ \left(ac_{ece}/2\right)\left(y_0e^{-u\tau_e} + b\right) - \left(A_{ece}/T_e^2\right) + \left(ah_{ece}/6\right)\left[3 + 2T_e\left(y_0e^{-u\tau_e} + b\right)\right] = 0, \end{aligned} \quad (6)$$

and

$$\begin{aligned} &1 - \left(h_{ehc}auy_0e^{-(u\tau_e - rT_e)}/r^3T_e\right) + uy_0e^{-u\tau_e}\left(\left(ah_{ehc}T_e/2r\right) + \left(ah_{ehc}/r^2\right) - \left(ah_{ehc}/r^3T_e\right)\right) - \left(C_{dc}uy_0e^{-u\tau_e}T_e(3a - abT_e)/6\right) \\ &- \left(ah_{ece}T_e^2uy_0e^{-u\tau_e}/6\right) - \left(a\left(C_{epc} + c_{ece}\right)T_euy_0e^{-u\tau_e}/2\right) = 0. \end{aligned} \quad (7)$$

We now derive the optimal values of $T_e$ and $\tau_e$ as $T_e^*$ and $\tau_e^*$ by simultaneously solving equations (6) and (7). We derive the total cost of the system by replacement these values into equation (4). Equation (3) is used to determine the optimal order quantity. By examining the second order sufficient conditions, for the total cost equation (4) to be minimum are

$$\frac{\partial^2 TC_{ep}}{\partial T_e^{\,2}} = \frac{2C_{eoc}}{T_e^{\,3}} + \frac{ah_{ehc}e^{-rT_e}}{r^3T_e^{\,3}}\left(r-\left(y_0e^{-u\tau_e}+b\right)\right)\left(2rT_e + r^2T_e^{\,2}+2\right) - \frac{2ah_{ehc}}{r^2T_e^{\,3}}$$
$$+\frac{2ah_{ehc}}{r^3T_e^{\,3}}\left(y_0e^{-u\tau_e}+b\right) - \frac{C_{edc}ab}{3}\left(y_0e^{-u\tau_e}+b\right) + \frac{2A_{ece}}{T_e^{\,3}} + \frac{2ah_{ece}\left(y_0e^{-u\tau_e}+b\right)}{3} > 0,$$
$$\frac{\partial^2 TC_{ep}}{\partial \tau_e^{\,2}} = -\frac{h_{ehc}au^2y_0e^{-(u\tau_e+rT_e)}}{r^3T_e} + u^2y_0e^{-u\tau_e}\left(\frac{ah_{ehc}T_e}{2r} - \frac{ah_{ehc}}{r^2} + \frac{ah_{ehc}}{r^3T}\right)$$
$$+\frac{C_{edc}au^2y_0e^{-u\tau_e}T_e(3-bT_e)}{6} + \frac{ah_{ece}T_eu^2y_0e^{-u\tau_e}}{3} + \frac{a\left(C_{epc}+c_{ece}\right)T_eu^2y_0e^{-u\tau_e}}{2} > 0,$$
and
$$\frac{\partial^2 TC_{ep}}{\partial \tau_e \partial T_e} = \frac{\partial^2 TC_{ep}}{\partial T_e \partial \tau_e} = -\frac{h_{hc}auy_0e^{-(u\tau_e+rT_e)}}{r^3T_e^{\,3}}\left(rT_e+1\right) - ah_{hc}uy_0e^{-u\tau_e}\left(\frac{1}{2r} - \frac{1}{r^3T_e^{\,2}}\right)$$
$$-\frac{C_{edc}auy_0e^{-u\tau_e}(3-2bT_e)}{6} - \frac{a\left(C_{epc}+c_{ece}\right)uy_0e^{-u\tau_e}}{2} - \frac{2ah_{ece}T_euy_0e^{-u\tau_e}}{3}.$$

Then, we have

$$\left(\left(\left(\partial^2 TC_{ep}\left(T_e,\,\tau_e\right)/\,\partial T_e^{\,2}\right).\left(\partial^2 TC_{ep}\left(T_e,\,\tau_e\right)/\,\partial \tau_e^{\,2}\right)\right) - \left(\left(\partial^2 TC_{ep}\left(T_e,\,\tau_e\right)/\,\partial T_e\partial\tau_e\right).\left(\partial^2 TC_{ep}\left(T_e,\,\tau_e\right)/\,\partial T_e\partial\tau_e\right)\right)\right) > 0. \quad (8)$$

Since all the second order derivatives are highly nonlinear, the optimality is determined graphically (Figure 1).

# 5. Fuzzy model

The fuzzy inventory model, including the fuzzification and defuzzification processes, is described in this section..

The method of fuzzification involves transforming crisp parameters into fuzzy parameters. Fuzzy variables can be represented by the membership function given in the preliminary section for triangular fuzzy numbers. Here, we consider the ordering cost, holding cost, holding cost component, demand parameters, and purchasing cost as uncertain. They are represented as triangular fuzzy numbers as follows:

$$\begin{aligned}
&\tilde{C}_{eoc} = (C_{eoc}-\Delta_1,\, C_{eoc},\, C_{eoc}+\Delta_2), && 0<\Delta_1<C_{eoc},\ \Delta_2>0,\\
&\tilde{C}_{edc} = (C_{edc}-\Delta_3,\, C_{edc},\, C_{edc}+\Delta_4), && 0<\Delta_3<C_{edc},\ \Delta_4>0,\\
&\tilde{a} = (a-\Delta_5,\, a,\, a+\Delta_6), && 0<\Delta_5<a,\ \Delta_6>0,\\
&\tilde{b} = (b-\Delta_7,\, b,\, b+\Delta_8), && 0<\Delta_7<b,\ \Delta_8>0,\\
&\tilde{h}_{ehc} = (h_{ehc}-\Delta_9,\, h_{ehc},\, h_{ehc}+\Delta_{10}), && 0<\Delta_9<h_{ehc},\ \Delta_{10}>0,\\
&\tilde{r}_{hc} = (\mathrm{r}_{hc}-\Delta_{11},\, r_{hc},\, r_{hc}+\Delta_{12}), && 0<\Delta_{11}<r_{hc},\ \Delta_{12}>0,\\
&\tilde{C}_{epc} = (C_{epc}-\Delta_{13},\, C_{epc},\, C_{epc}+\Delta_{14}), && 0<\Delta_{13}<C_{epc},\ \Delta_{14}>0.
\end{aligned} \quad (9)$$

Then the left and right $\alpha$ cuts of the various parameters $C_{eoc}$, $C_{edc}$, $h_{ehc}$, $r$, $a$, $b$ and $C_{epc}$ are given by

$$\tilde{C}_{eoc_L}\left(\alpha\right) = C_{eoc} - \Delta_1 + \alpha\Delta_1 > 0; \qquad \tilde{C}_{eoc_R}\left(\alpha\right) = C_{eoc} - \Delta_2 + \alpha\Delta_2 > 0,$$

$$\begin{array}{ll}\tilde{C}_{edc_L}(\alpha)=C_{edc}-\Delta_3+\alpha\Delta_3>0; & \tilde{C}_{edc_R}(\alpha)=C_{edc}-\Delta_4+\alpha\Delta_4>0,\\ \tilde{a}_L(\alpha)=a-\Delta_5+\alpha\Delta_5>0; & \tilde{a}_R(\alpha)=a-\Delta_6+\alpha\Delta_6>0,\\ \tilde{b}_L(\alpha)=b-\Delta_7+\alpha\Delta_7>0; & \tilde{b}_R(\alpha)=b-\Delta_8+\alpha\Delta_8>0,\\ \tilde{h}_{ehc_L}(\alpha)=h_{ehc}-\Delta_9+\alpha\Delta_9>0; & \tilde{h}_{ehc_R}(\alpha)=h_{ehc}-\Delta_{10}+\alpha\Delta_{10}>0,\\ \tilde{r}_L(\alpha)=r-\Delta_{11}+\alpha\Delta_{11}>0; & \tilde{r}_R(\alpha)=r-\Delta_{12}+\alpha\Delta_{12}>0,\\ \tilde{C}_{epc_L}(\alpha)=C_{epc}-\Delta_{13}+\alpha\Delta_{13}>0; & \tilde{C}_{epc_R}(\alpha)=C_{epc}-\Delta_{14}+\alpha\Delta_{14}>0.\end{array} \tag{10}$$

Hence, when the costs $C_{eoc}$, $C_{edc}$, $h_{ehc}$, $r$ , $a$, $b$ and $C_{epc}$ in equation (4) are fuzzified with triangular fuzzy numbers $\tilde{C}_{eoc}$, $\tilde{C}_{edc}$, $\tilde{h}_{ehc}$, $\tilde{r}$ , $\tilde{a}$, $\tilde{b}$ and $\tilde{C}_{epc}$ as expressed in equation (10). Thus, the total cost is obtained in fuzzy sense is given by

$$\begin{aligned}\tilde{\prod}TC_{ep}(T_e,\tau_e)&=\frac{\tilde{C}_{eoc}}{T_e}+\frac{\tilde{a}\tilde{h}_{ehc}}{T_e}\left[\frac{-T_e}{\tilde{r}}+\frac{e^{\tilde{r}T_e}}{\tilde{r}^2}-\frac{1}{\tilde{r}^2}+\frac{\left(y(\tau_e)+\tilde{b}\right)}{2}\left(\frac{-T_e^2}{\tilde{r}}-\frac{2T_e}{\tilde{r}^2}+\frac{2e^{-\tilde{r}T_e}}{\tilde{r}^3}+\frac{2}{\tilde{r}^3}\right)\right]\\&+\frac{\tilde{a}\tilde{C}_{epc}}{T_e}\left[T_e+\frac{T_e^2\left(y(\tau_e)+\tilde{b}\right)}{2}\right]+\frac{\tilde{C}_{edc}}{T_e}\left[\frac{T_e^2y(\tau_e)\left(3\tilde{a}-\tilde{a}\tilde{b}T_e\right)-\tilde{a}\tilde{b}^2T_e^3}{6}\right]+\frac{\tau_eT_e}{T_e}\\&+\frac{1}{T_e}\left[A_{ece}+\tilde{a}c_{ece}\left(T_e+\frac{T_e^2\left(y(\tau_e)+\tilde{b}\right)}{2}\right)+\tilde{a}h_{ece}\left(\frac{T_e^2}{2}+\frac{T_e^3\left(y(\tau_e)+\tilde{b}\right)}{6}\right)\right].\end{aligned} \tag{11}$$

From equation (11), we obtain the left and right $\alpha$ cuts of $\tilde{\prod}TC_{ep}(T_e,\tau_e)$ is as follows:

$$\begin{aligned}\tilde{\prod}TC_{ep_L}(T_e,\tau_e)&=\frac{\tilde{C}_{eoc_L}(\alpha)}{T_e}+\frac{\tilde{a}_L(\alpha)\tilde{h}_{ehc_L}(\alpha)}{T_e}\left[\frac{-T_e}{\tilde{r}_L(\alpha)}+\frac{e^{\tilde{r}_L(\alpha)T_e}}{\tilde{r}_L(\alpha)^2}-\frac{1}{\tilde{r}_L(\alpha)^2}+\frac{\left(y(\tau_e)+\tilde{b}_L(\alpha)\right)}{2}\right.\\&\left.\left(\frac{-T_e^2}{\tilde{r}_L(\alpha)}-\frac{2T_e}{\tilde{r}_L(\alpha)^2}+\frac{2e^{-\tilde{r}_L(\alpha)T_e}}{\tilde{r}_L(\alpha)^3}+\frac{2}{\tilde{r}_L(\alpha)^3}\right)\right]+\frac{\tilde{a}_L(\alpha)\tilde{C}_{epc_L}(\alpha)}{T_e}\left[T_e+\frac{T_e^2\left(y(\tau_e)+\tilde{b}_L(\alpha)\right)}{2}\right]\\&+\frac{\tilde{C}_{edc_L}(\alpha)}{T_e}\left[\frac{T_e^2y(\tau_e)\left(3\tilde{a}_L(\alpha)-\tilde{a}_L(\alpha)\tilde{b}_L(\alpha)T_e\right)-\tilde{a}_L(\alpha)\tilde{b}_L(\alpha)^2T_e^3}{6}\right]+\frac{\tau_eT_e}{T_e}\\&+\frac{1}{T_e}\left[A_{ece}+\tilde{a}_L(\alpha)c_{ece}\left(T_e+\frac{T_e^2\left(y(\tau_e)+\tilde{b}_L(\alpha)\right)}{2}\right)+\tilde{a}_L(\alpha)h_{ece}\left(\frac{T_e^2}{2}+\frac{T_e^3\left(y(\tau_e)+\tilde{b}_L(\alpha)\right)}{6}\right)\right],\end{aligned}$$

$$\begin{aligned}\tilde{\Pi} TC_{ep_R}(T_e,\ \tau_e) &= \frac{\tilde{C}_{eoc_R}(\alpha)}{T_e} + \frac{\tilde{a}_R(\alpha)\tilde{h}_{ehc_R}(\alpha)}{T_e}\left[\frac{-T_e}{\tilde{r}_R(\alpha)} + \frac{e^{-\tilde{r}_R(\alpha)T_e}}{\tilde{r}_R(\alpha)^2} - \frac{1}{\tilde{r}_R(\alpha)^2} + \frac{\left(y(\tau_e)+\tilde{b}_R(\alpha)\right)}{2}\right.\\ &\left.\left(\frac{-T_e^2}{\tilde{r}_R(\alpha)} - \frac{2T_e}{\tilde{r}_R(\alpha)^2} + \frac{2e^{-\tilde{r}_R(\alpha)T_e}}{\tilde{r}_R(\alpha)^3} + \frac{2}{\tilde{r}_R(\alpha)^3}\right)\right]\frac{\tilde{a}_R(\alpha)\tilde{C}_{epc_R}(\alpha)}{T_e}\left[T_e + \frac{T_e^2\left(y(\tau_e)+\tilde{b}_R(\alpha)\right)}{2}\right]\\ &+ \frac{\tilde{C}_{edc_R}(\alpha)}{T_e}\left[\frac{T_e^2 y(\tau_e)\left(3\tilde{a}_R(\alpha) - \tilde{a}_R(\alpha)\tilde{b}_R(\alpha)T_e\right) - \tilde{a}_R(\alpha)\tilde{b}_R(\alpha)^2 T_e^3}{6}\right] + \frac{\tau_e T_e}{T_e}\\ &+ \frac{1}{T_e}\left[A_{ece} + \tilde{a}_R(\alpha)c_{ece}\left(T_e + \frac{T_e^2\left(y(\tau_e)+\tilde{b}_R(\alpha)\right)}{2}\right) + \tilde{a}_R(\alpha)h_{ece}\left(\frac{T_e^2}{2} + \frac{T_e^3\left(y(\tau_e)+\tilde{b}_R(\alpha)\right)}{6}\right)\right].\end{aligned}$$

The process of defuzzifying involves turning the fuzzified results into quantifiable quantities. Hence, the fuzzified total cost equation (11) narrated with triangular fuzzy number is transformed into the crisp function by utilizing signed distance formula. Then the defuzzified total cost is calculated and is given by

$$\begin{aligned}d(\tilde{\Pi} TC_{ep}(T_e,\ \tau_e),\tilde{0}) &= \frac{H_1}{T_e} + \frac{H_3H_5}{T_e}\left[\frac{-T_e}{H_6} + \frac{e^{-H_6T_e}}{H_6^2} - \frac{1}{H_6^2} + \frac{\left(y(\tau_e)+H_4\right)}{2}\left(\frac{-T_e^2}{H_6} - \frac{2T_e}{H_6^2} + \frac{2e^{-H_6T_e}}{H_6^3} + \frac{2}{H_6^3}\right)\right]\\ &+\frac{H_3H_7}{T_e}\left[T_e + \frac{T_e^2\left(y(\tau_e)+H_4\right)}{2}\right] + \frac{H_2}{T_e}\left[\frac{T_e^2 y(\tau_e)\left(3H_3 - H_3H_4T_e\right) - H_3H_4^2T_e^3}{6}\right] + \frac{\tau_eT_e}{T_e}\\ &+\frac{1}{T_e}\left[A_{ece} + H_3c_{ece}\left(T_e + \frac{T_e^2\left(y(\tau_e)+H_4\right)}{2}\right) + H_3h_{ece}\left(\frac{T_e^2}{2} + \frac{T_e^3\left(y(\tau_e)+H_4\right)}{6}\right)\right].\end{aligned} \qquad (12)$$

Where,

$$H_1 = C_{eoc} + \frac{\Delta_2 - \Delta_1}{4} > 0,\ \ H_2 = C_{edc} + \frac{1}{4}(\Delta_4 - \Delta_3),\ H_3 = a + \frac{1}{4}(\Delta_6 - \Delta_5),$$

$$H_4 = b + \frac{1}{4}(\Delta_8 - \Delta_7),\ H_5 = h_{ehc} + \frac{1}{4}(\Delta_{10} - \Delta_9),\ H_6 = r_{hc} + \frac{1}{4}(\Delta_{12} - \Delta_{11}),$$

and $H_7 = C_{epc} + \frac{1}{4}(\Delta_{14} - \Delta_{13})$

Since all the second order derivatives are highly nonlinear, the optimality is determined graphically (Figure 2).

# 6. Numerical example

In this section, a suitable example is given to illustrate the model. Here, we consider an inventory system with the same data as in Khanna et al. [9] and Yu et al. [3]. $C_{eoc}$ = 40/order, $C_{edc}$ = 50/year, $u$ = 0.05, $y_0$ = 0.09, $h_{ehc}$ = 0.7 per unit per year, $r$ = 0.5, $a$ = 25, $b$ = 0.07, $C_{epc}$ = 90 per unit, $A_{ece} = 0.02$, $c_{ece} = 0.1$, $h_{ece} = 0.1$. Moreover, we summarize the input parameters as fuzzy triangular values and defuzzified values in Table 1. The total cost for the crisp model is $TC_{ep}(T_e, t_e)$ = 2460, the cycle time is $T_e$ = 0.8 year and investment in preservation technology $t_e$ = 40.52. The total cost of the fuzzy model is $\tilde{\prod} TC_{ep}(T_e, \tau_e)$ = 1631, the cycle time is $\tilde{T}_e$ = 0.92 year and investment in preservation technology $\tilde{t}_e$ =37.59.

# 7. Sensitivity analysis

We inspect the effects of variations in the system variables $C_{eoc}$, $a$, $b$, $h_{ehc}$, $r$, and $C_{epc}$ on the optimal ordering quantity $Q$ the cycle time is $T_e$ and investment in preservation technology $\tau_e$ with minimum total expected cost. The optimal values of $Q$, $T_e$, $\tau_e$ and $TC_{ep}(T_e, t_e)$ are derived, when one of the parameters changes (increases or decreases) by 25% and all other parameters remain unchanged. The results of sensitivity analysis are presented for both the cases in Table 2 and are graphically shown in Figures 3 – 8. On the basis of the results of Table 2 and Figures 3 – 8, we see that fuzzy model provides best optimal solution as compared to crisp model.

# 7. 1 Managerial insights

In this section, we study the effect of changes in the cost components of the system on the optimal length of the cycle $T_e^*$, the optimal ordering quantity $Q^*$, the optimal investment in preservation technology $\tau_e^*$ and the minimum total cost for crisp model as $\prod TC_{ep}$ and for fuzzy model as $\tilde{\prod} TC_{ep}$. A sensitivity analysis is carried out by considering the same numerical example and computed results are shown in Table 2. Based on the computational results, we obtain the following managerial insights.

(1) It's interesting to note that increasing the holding costs components $r_{hc}$ has a positive effect. This will lead to a decrease in $Q$, $T_e$, $\tau_e$ and $TC_{ep}(T_e, t_e)$. But increase in the values of the holding costs components $h_{hc}$ will lead to an increase in $Q$, $T_e$, $\tau_e$ and $TC_{ep}(T_e, t_e)$.

(2) The optimal solution for several values of $D$, increase in demand parameter $a$ results increase in $Q$ and $\tau_e$. This result has implication on the holding cost, ordering cost as well as delivery cost. Therefore, an increase in $a$ will lead to an increase of $TC_{ep}(T_e, t_e)$ and decrease in $T_e$.

(3) From Table 2, the values of $Q$, $T_e$, and $\tau_e$ decrease with decrease in the values of parameter $b$ but increases of $TC_{ep}(T_e, t_e)$

(4) It is foreseeable that if the buyer's ordering cost $C_{eoc}$ rises, $TC_{ep}(T_e, t_e)$ and $Q$ will increases. This is because, for high values of ordering cost, departing from the

optimal solution has a substantial effect on $T_e$ and $\tau_e$ respectively. As a result, an increase in $C_{eoc}$ will result in an increase in $T_e$ and $\tau_e$ in both circumstances.

(5) In Table 2, with an increase in purchasing cost $C_{epc}$, $\tau_e$ and $TC_{ep}(T_e, t_e)$ increases but decrease of $T_e$ and $Q$.

| Fuzzy Input parameters | Triangular fuzzy numbers | Defuzzified values |
|---|---|---|
| $\tilde{C}_{eoc}$ | (30, 40, 50) | 37.5 |
| $\tilde{C}_{edc}$ | (40, 50, 60) | 45 |
| $\tilde{a}$ | (20, 25, 30) | 22.5 |
| $\tilde{b}$ | (0.06, 0.07, 0.08) | 0.06 |
| $\tilde{h}_{ehc}$ | (0.6, 0.7, 0.8) | 0.6 |
| $\tilde{r}$ | (0.4, 0.5, 0.6) | 0.45 |
| $\tilde{C}_{epc}$ | (80, 90, 100) | 65 |

Table 1. Fuzzy input parameters as triangular values

## 8. Comparative study

From Table 2, it is observed that triangular fuzzy number gives the best optimum solution. The fuzzy model with triangular fuzzy numbers generates a better result than the crisp model with the total cost with 33.70% savings. In this paper, it is shown that the knowledge of the crisp model is gradually improved to a fuzzy model with triangular fuzzy and fine-tuned our model into more specific knowledge with minimum total cost. The main reason for this situation is the low carbon emission cost and exponential holding cost which helps to increase the sales and a positive impact on customer preference.

The triangular fuzzy model finds lower values of $\tau_e =$ 37.59 and total cost $\tilde{\prod} TC_{ep}(T_e, \tau_e)$ = 1631 (better) at each performance criterion than the crisp model $t_e$ = 40.52 and total cost $TC_{ep}(T_e, t_e)$ = 2460 indicating that total cost is higher than the fuzzy model with triangular fuzzy numbers. Thus fuzzy model gives a better result than the crisp model. Hence, fuzzy model gives the advantages of the application of fuzzy in real-world environment on Supply Chain management.

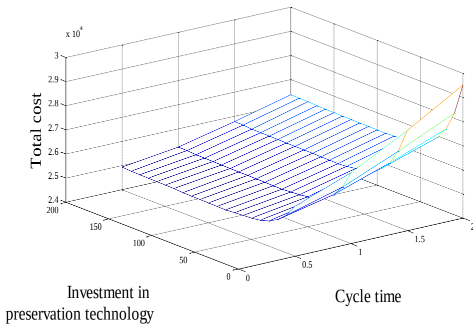


Figure 1. Graphical representation for crisp model

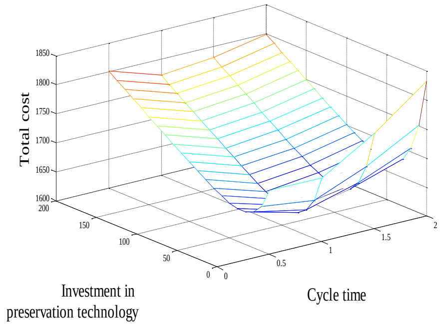


Figure 2. Graphical representation for fuzzy model

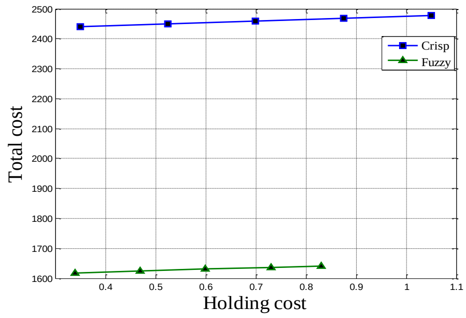


Figure 3. Effect of holding cost $h_{ehc}$

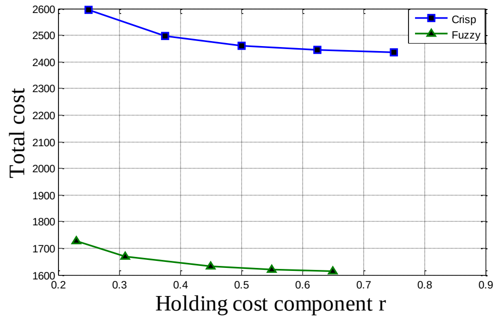


Figure 4. Effect of holding cost component $r$

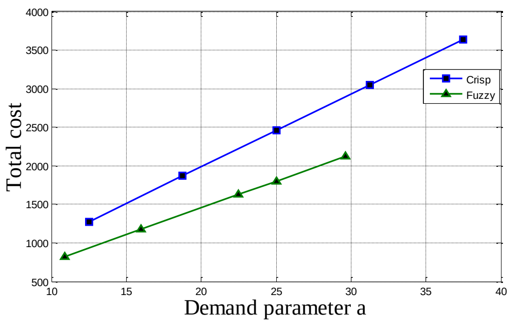


Figure 5. Effect of demand parameter $a$

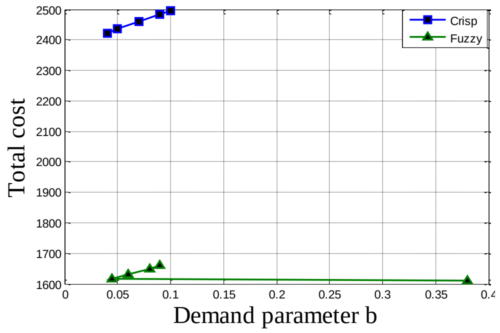


Figure 6. Effect of demand parameter $b$

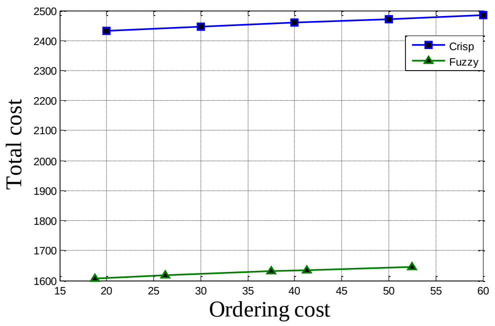


Figure 7. Effect of ordering cost $C_{eoc}$

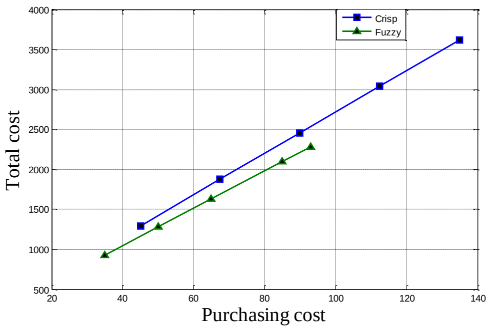


Figure 8. Effect of purchasing cost $C_{epc}$

| Parameter | % | Crisp Para-meter value | Fuzzy Para-meter value | Crisp Optimal values | | | | Fuzzy Optimal values | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $T_e^*$ | $\tau_e^*$ | $Q^*$ | $\prod TC_{ep}$ | $T_e^*$ | $\tau_e^*$ | $Q^*$ | $\tilde{\prod} TC_{ep}$ |
| $C_{eoc}$ | -50 | 20 | 18.75 | 0.6 | 38.63 | 15.37 | 2431 | 0.76 | 35.72 | 17.59 | 1608 |
| | -25 | 30 | 26.25 | 0.7 | 39.59 | 18.01 | 2446 | 0.83 | 36.48 | 19.25 | 1618 |
| | 0 | 40 | 37.50 | 0.75 | 40.52 | 19.33 | 2460 | 0.92 | 37.59 | 21.40 | 1631 |
| | +25 | 50 | 41.25 | 0.81 | 41.40 | 20.92 | 2472 | 0.95 | 37.95 | 22.12 | 1635 |
| | +50 | 60 | 52.50 | 0.87 | 42.24 | 22.52 | 2484 | 1.04 | 38.98 | 24.29 | 1646 |
| $a$ | -50 | 12.5 | 10.9 | 0.91 | 28.99 | 11.85 | 1271 | 1.10 | 25.29 | 12.55 | 824.8 |
| | -25 | 18.75 | 16 | 0.81 | 35.65 | 15.71 | 1867 | 1.00 | 31.74 | 16.63 | 1181 |
| | 0 | 25 | 22.5 | 0.75 | 40.52 | 19.33 | 2460 | 0.92 | 37.59 | 21.40 | 1631 |
| | +25 | 31.25 | 25 | 0.71 | 44.38 | 22.82 | 3050 | 0.90 | 39.43 | 23.23 | 1803 |
| | +50 | 37.5 | 29.63 | 0.68 | 47.59 | 26.18 | 3640 | 0.86 | 42.42 | 26.26 | 2122 |
| $b$ | -50 | 0.04 | 0.038 | 0.84 | 41.85 | 21.45 | 2423 | 0.99 | 38.45 | 22.84 | 1609 |
| | -25 | 0.05 | 0.045 | 0.8 | 41.30 | 20.49 | 2435 | 0.96 | 38.13 | 22.21 | 1616 |
| | 0 | 0.07 | 0.06 | 0.75 | 40.52 | 19.33 | 2460 | 0.92 | 37.59 | 21.40 | 1631 |
| | +25 | 0.09 | 0.08 | 0.72 | 39.98 | 18.66 | 2483 | 0.88 | 37.08 | 20.62 | 1650 |
| | +50 | 0.1 | 0.09 | 0.70 | 39.77 | 18.19 | 2495 | 0.87 | 36.89 | 20.46 | 1660 |
| $h_{ehc}$ | -50 | 0.35 | 0.34 | 0.69 | 36.87 | 17.75 | 2440 | 0.86 | 34.53 | 19.98 | 1617 |
| | -25 | 0.525 | 0.47 | 0.72 | 38.86 | 18.54 | 2450 | 0.89 | 36.19 | 20.69 | 1624 |
| | 0 | 0.70 | 0.60 | 0.75 | 40.52 | 19.33 | 2460 | 0.92 | 37.59 | 21.40 | 1631 |
| | +25 | 0.875 | 0.73 | 0.78 | 41.93 | 20.12 | 2469 | 0.95 | 38.82 | 22.12 | 1637 |
| | +50 | 1.05 | 0.83 | 0.80 | 43.17 | 20.64 | 2478 | 0.97 | 39.66 | 22.59 | 1642 |
| $r$ | -50 | 0.25 | 0.23 | 1.46 | 58.14 | 38.50 | 2595 | 1.76 | 54.65 | 41.89 | 1728 |
| | -25 | 0.375 | 0.31 | 0.94 | 47.10 | 24.37 | 2496 | 1.25 | 46.43 | 29.33 | 1670 |
| | 0 | 0.5 | 0.45 | 0.75 | 40.52 | 19.33 | 2460 | 0.92 | 37.59 | 21.40 | 1631 |
| | +25 | 0.625 | 0.55 | 0.67 | 36.58 | 17.22 | 2444 | 0.82 | 33.91 | 19.03 | 1619 |
| | +50 | 0.75 | 0.65 | 0.63 | 34.19 | 16.18 | 2436 | 0.77 | 31.53 | 17.85 | 1613 |
| $C_{epc}$ | -50 | 45 | 35 | 0.97 | 37.24 | 25.24 | 1295 | 1.13 | 34.83 | 26.51 | 930.1 |
| | -25 | 67.5 | 50 | 0.84 | 38.99 | 21.73 | 1879 | 1.01 | 36.30 | 23.58 | 1281 |
| | 0 | 90 | 65 | 0.75 | 40.52 | 19.33 | 2460 | 0.92 | 37.59 | 21.40 | 1631 |
| | +25 | 112.5 | 85 | 0.69 | 41.85 | 17.73 | 3039 | 0.83 | 39.11 | 19.24 | 2095 |
| | +50 | 135 | 93 | 0.64 | 43.04 | 16.41 | 3616 | 0.81 | 39.66 | 18.76 | 2281 |

Table 2. Effects of parameters on optimal solution

# 9. Conclusions

In this study, we examined the impacts of cycle time and investment in preservation technology on an inventory model with exponential holding cost in a fuzzy scenario. Our research revealed that various integrated inventory models would be beneficial for both the seller and buyer in cases where the cost parameters take the form of a triangular fuzzy number. We obtained more information about the cost parameters in relation to the decision variables and total fuzzy profit from the managerial insights. The model's viability is investigated using numerical analysis and sensitivity analysis. The present study can be extended with the trade credit financing policy, seasonal and expiry products, inflation and multi items.

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