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Monophonic Distance Laplacian Energy of Transformation Graphs S_n^{++-}, S_n^{+-+}, S_n^{+++}

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Abstract

Let G be a simple connected graph of order n, v_i its vertex. Let $\delta_1^L, \delta_2^L, \ldots, \delta_n^L$ be the eigenvalues of the distance Laplacian matrix D^L of G. The distance Laplacian energy is denoted by $LE_D(G)$. This motivated us to defined the new graph energy monophonic distance Laplacian energy of graphs. The eigenvalues of monophonic distance Laplacian matrix $M^L(G)$ are denoted by $\mu_1^L, \mu_2^L, \ldots, \mu_n^L$ and are said to be M^L- eigenvalues of G and to form the M^L-spectrum of G, denoted by $Spec_{M^L}(G)$. Here $MT_G(v_j)$ is the j^{th} row sum of monophonic distance matrix of $M(G)$ and $\mu_1^L \leq \mu_2^L \leq, \ldots, \leq \mu_n^L$ be the eigenvalues of the monophonic distance Laplacian matrix is $M^L(G)$. The monophonic distance Laplacian energy is defined as $LE_M(G)$. In this paper we computed the monophonic distance Laplacian energy of S_n^{++-}, S_n^{+-+}, S_n^{+++} graphs based on its spectrum values.

Keywords: distance Laplacian energy, monophonic distance matrix, monophonic distance Laplacian energy, transformation graphs.

2020 AMS subject classifications: 05C12,05C50 [1]

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1 Introduction

The concept of energy stems from chemistry to approximate the total π electron energy of a molecule of a conjugated hydrocarbon. Conjugated hydrocarbon can be represented by a graph called molecular graph according to the rule: every carbon atom is represented by a vertex and every carbon-carbon bond by an edge, hydrogen atoms are ignored. The eigenvalues of the molecular graph represent the energy level of the electron in a molecule. An interesting quantity in H¨uckel theory is the sum of the energies of all the electrons in a molecule, the so called total π electron energy $E\pi$. The first results on energy of a graph was obtained as early as 1940's. The same concept of energy is extended to simple graphs.

Spectral graph theory studies the relation between graph properties and the spectra of certain matrices associated to it. It finds applications in other areas of graph theory such as Chemistry, Biology, Physics, Computer Science, Statistics, etc. Here we defind the monophonic distance Laplacian energy of graphs. The concept of energy of a graph was introduced by I.Gutman in 1978[4]. The energy of a graph G, indicated by $E(G)$, is defined as the absolute sum of the eigenvalues μ_i, $(1 \leq i \leq n)$ of the adjacency matrix of the graph $E(G) = \sum_{i=1}^{n} |\mu_i|$.

In 2006, Ivan Gutman and Bo Zhou[7] found the Laplacian energy of graphs. Let $\mu_1, \mu_2 \ldots \mu_n$ be the Laplacian eigenvalues of G. The Laplacian energy of G has defined as $LE(G) = \sum_{i=1}^{n} \left| \mu_i - \frac{2m}{n} \right|$. Also, In 2008, I.Gutman et.al.,[3] have introduced the distance energy of graphs. The distance matrix or D- matrix of G is defined as $D = [d_{ij}]$, where d_{ij} is the distance between the vertices v_i and v_j in G. The eigenvalues $\mu_1, \mu_2, \ldots, \mu_p$ of the D- matrix of G are said to be the D-eigenvalues of G and to form the D-spectrum of G, denoted by $Spec_D(G)$.

The distance Laplacian energy of graphs was introduced by Jieshan yang et.al., in 2013[8]. The distance degree of the vertex v_i, denoted by D_i is given by $D_i = \sum_{j=1}^{n} d_{ij}$. The distance Laplacian matrix of a connected graph G is $D^L = D^L(G) = diag(D_i) - D(G)$, where $diag(D_i)$ denotes the diagonal matrix of the distance degrees. The distance Laplacian energy of G is denoted by $LE_D(G)$ is defined as $LE_D(G) = \sum_{i=1}^{n} \left| \delta_i^L - \frac{1}{n} \sum_{j=1}^{n} D_j \right|$. In 2015, V.S Shigehalli and Kenchappa S Betageri[11] determined the color Laplacian energy. The seidal Laplacian energy of graphs was initiated by H.S. Ramane and others in 2017[9]. The seidal signless Laplacian energy of graphs was derived by S. Harishchandra and Ivan Gutman in 2017[6]. Path Laplacian energy was introduced by Shridhar Chandrakant Patekar and Maruti Mukinda Shikare in 2018[12]. The monophonic distance in graphs was introduced by A.P. Santhakumaran and P.Titus in 2011[10]. Based on these we introduce a new concept monophonic distance Laplacian energy of graphs.

2 Definitions

Definition 2.1. Let G be a connected graph with vertex set $\{v_1, v_2, \ldots, v_n\}$. The monophonic distance matrix G is defined as

$$M = M(G) = (d_{m_{ij}})_{n\times n}, where\ d_{m_{ij}} = \begin{cases} d_m(v_i, v_j) & \text{if } \ i \neq j \\ 0 & \text{otherwise.} \end{cases}$$

Here $d_m\left(v_i, v_j\right)$ is the monophonic distance of v_i to v_j.

Definition 2.2. The monophonic transmission $MT_G\left(v\right)$ of a vertex v as $\sum\limits_{u\in V(G)} d_m(u, v)$ and monophonic transmission matrix $MT\left(G\right)$ is the diagonal matrix $diag\left[MT_G\left(v_1\right), MT_G\left(v_2\right), \ldots, MT_G\left(v_n\right)\right]$. For $1 \leq i \leq n$, $MT_G\left(v_i\right)$ is the i^{th} row sum of $M\left(G\right)$. The monophonic distance Laplacian matrix of a connected graph G is defined as $M^L\left(G\right) = MT\left(G\right) - M\left(G\right)$. The eigen values of monophonic distance Laplacian matrix $M^L\left(G\right)$ are denoted by $\mu_1^L, \mu_2^L, \ldots, \mu_n^L$ and are said to be M^L- eigen values of G and to form the M^L-spectrum of G, denoted by $Spec_{M^L}(G)$. Since the monophonic distance Laplacian matrix is symmetric and its eigen values are real,it can be ordered as $\mu_1^L \leq \mu_2^L \leq, \ldots, \leq \mu_n^L$.

Definition 2.3. [2] The monophonic distance Laplacian energy of a graph is defined as $LE_M\left(G\right) = \sum_{i=1}^n \left|\mu_i^L - \dfrac{1}{n}\sum_{j=1}^n MT_G\left(v_j\right)\right|$.

3 Main Results

Definition 3.1. [1]Let $G = (V(G), E(G))$ be a graph and x, y, z be three variables taking values $+$ or $-$. The transformation graph G^{xyz} is the graph having $V(G)\cup E(G)$ as the vertex set and for $\alpha, \beta \in V(G) \cup E(G)$, α and β are adjacent in G^{xyz} if and only if one of the following holds:
(i) $\alpha, \beta \in V(G)$, α and β are adjacent in G if $x = +$; α and β are not adjacent in

G if $x = -$.
(ii) $\alpha, \beta \in E(G)$, α and β are adjacent in G if $y = +$; α and β are not adjacent in G if $y = -$.
(iii) $\alpha \in V(G)$, $\beta \in E(G)$, α and β are incident in G if $z = +$; α and β are not incident in G if $z = -$.
Thus,we may obtain eight kinds of transformation graphs, in which G^{+++} is the total graph of G and G^{---} is its complement. Also G^{--+}, G^{-+-} and G^{-++} are the complements of G^{++-}, G^{+-+} and G^{+--} respectively.

Theorem 3.1. Let S_n be a star graph with $n \geq 3$ vertices, S_n^{++-} be the transformation graph and $M(S_n^{++-})$ be the monophonic distance matrix and its dimension is $(2n-1) \times (2n-1)$. Then
(a) the spectrum of S_n^{++-} is given by
$Spec_{M^L}(S_n^{++-}) = \begin{pmatrix} 0 & 3n + \sqrt{n^2-4n+8} & 3n - \sqrt{n^2-4n+8} \\ 1 & n-1 & n-1 \end{pmatrix}$
(b)the monophonic distance Laplacian energy of S_n^{++-} is
$LE_M\,(S_n^{++-}) = \dfrac{2}{2n-1}\left[(3n^2-2n-1) + (2n^2-3n+1)\sqrt{n^2-4n+8}\right]$, for $n \geq 3$.

Proof. The graph S_n^{++-} as shown in Figure: 1

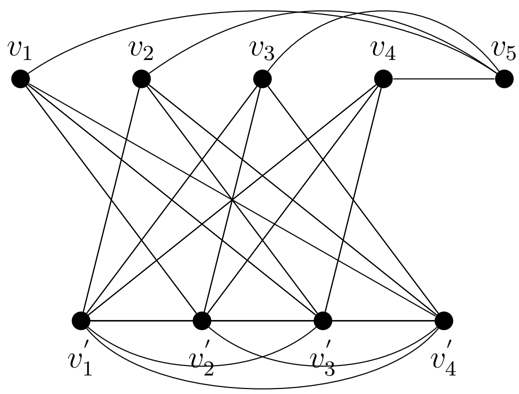

Figure 1: The graph S_n^{++-}, $n = 5$

We have $M(S_n^{++-})$=$(d_{m_{ij}})_{(2n-1)\times(2n-1)}$

$$where\ d_{m_{ij}} = \begin{cases} 0 & \text{if } \ v_i = v_j \\ 1 & \text{if } \ v_i\ and\ v_j \text{ are adjacent} \\ 3 & \text{if } \ v_i\ and\ v_j \text{ are non adjacent} \end{cases}$$

The monophonic distance Laplacian matrix of S_n^{++-} is
$M^L\left(S_n^{++-}\right) = MT\left(S_n^{++-}\right) - M\left(S_n^{++-}\right)$

$$= \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \\ v_1^{'} \\ v_2^{'} \\ v_3^{'} \\ \vdots \\ v_n^{'} \end{array} \begin{array}{c} \begin{array}{cccccccccc} v_1 & v_2 & v_3 & \dots & v_n & v_1^{'} & v_2{'} & v_3^{'} & \dots & v_n{'} \end{array} \\ \begin{pmatrix} 4n-4 & -3 & -3 & \dots & -1 & -3 & -1 & -1 & \dots & -1 \\ -3 & 4n-4 & -3 & \dots & -1 & -1 & -3 & -1 & \dots & -1 \\ -3 & -3 & 4n-4 & \dots & -1 & -1 & -1 & -3 & \dots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \dots & 4n-4 & -3 & -3 & -3 & \dots & -3 \\ -3 & -1 & -1 & \dots & -3 & 2n+2 & -1 & -1 & \dots & -1 \\ -1 & -3 & -1 & \dots & -3 & -1 & 2n+2 & -1 & \dots & -1 \\ -1 & -1 & -3 & \dots & -3 & -1 & 1 & 2n+2 & \dots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \dots & -3 & -1 & -1 & -1 & \dots & 2n+2 \end{pmatrix} \end{array}$$

The M^L-eigenvalues of S_n^{++-} are $\begin{pmatrix} \mu_1^L \\ \mu_2^L \\ \vdots \\ \mu_n^L \\ \mu_{n+1}^L \\ \vdots \\ \mu_{2n-1}^L \end{pmatrix} = \begin{pmatrix} 0 \\ 3n-\sqrt{n^2-4n+8} \\ \vdots \\ 3n-\sqrt{n^2-4n+8} \\ 3n+\sqrt{n^2-4n+8} \\ \vdots \\ 3n+\sqrt{n^2-4n+8} \end{pmatrix}$.

The M^L- spectrum of S_n^{++-} is
$Spec_{M^L}(S_n^{++-}) = \begin{pmatrix} 0 & 3n-\sqrt{n^2-4n+8} & 3n+\sqrt{n^2-4n+8} \\ 1 & n-1 & n-1 \end{pmatrix}$.

Let $n \geq 4$

$$\begin{aligned} LE_M\left(S_n^{++-}\right) &= \sum_{i=1}^{2n-1} \left|\mu_i^L - \frac{6n^2-4n-2}{2n-1}\right| \\ &= \left|0 - \frac{6n^2-4n-2}{2n-1}\right| + \\ &\quad (n-1)\left|3n+1+\sqrt{n^2-4n+8} - \frac{6n^2-4n-2}{2n-1}\right| + \\ &\quad (n-1)\left|(3n+1)-\sqrt{n^2-4n+8} - \frac{6n^2-4n-2}{2n-1}\right| \end{aligned}$$

Since $\mu_i^L - \frac{1}{2n-1}\sum_{j=1}^{2n-1} MT_G\left(v_j\right) < 0$, where $i = 2, 3, \dots, n$, we have

$LE_M\left(S_n^{++-}\right) = \left[\frac{6n^2-4n-2}{2n-1}\right] +$

$$(n-1)\left[\frac{(6n^2-4n-2)-(2n-1)\left[(3n+1)+\sqrt{n^2-4n+8}\right]}{2n-1}\right]+$$
$$(n-1)\left[\frac{(3n+1)(2n-1)+(2n-1)\sqrt{n^2-4n+8}-6n^2+4n+2}{2n-1}\right]$$
$$=\frac{1}{2n-1}\left[(6n^2-4n-2)+2(n-1)(2n-1)\sqrt{n^2-4n+8}\right]$$
$$=\frac{2}{2n-1}\left[(3n^2-2n-1)+(2n^2-3n+1)\sqrt{n^2-4n+8}\right].$$

Theorem 3.2. The monophonic distance Laplacian energy of S_n^{+-+} is
$LE_M\left(S_n^{+-+}\right)=\dfrac{2}{2n-1}[12n^2-32n+19]$.

Proof. The graph S_n^{+-+} as shown in Figure: 2

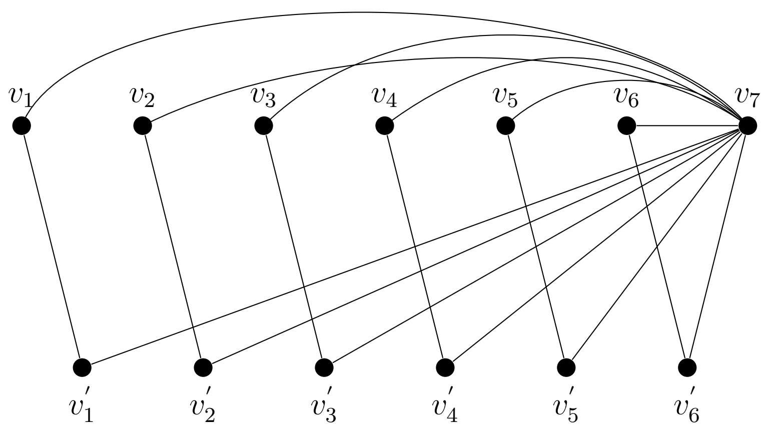

Figure 2: The graph S_n^{+-+}, $n=7$

We have $M(S_n^{+-+})$=$(d_{m_{ij}})_{(2n-1)\times(2n-1)}$

$$where\ d_{m_{ij}}=\begin{cases}0 & \text{if } \ v_i=v_j\\ 1 & \text{if } \ v_i\ and\ v_j \text{ are adjacent}\\ 2 & \text{if } \ v_i\ and\ v_j \text{ are non adjacent}\end{cases}$$

The monophonic distance Laplacian matrix of S_n^{+-+} is
$M^L\left(S_n^{+-+}\right)=MT\left(S_n^{+-+}\right)-M\left(S_n^{+-+}\right)$

$$M^L\left(S_n^{+-+}\right) = \begin{array}{c c} & \begin{array}{ccccccccc} v_1 & v_2 & v_3 & \dots & v_n & v_1^{'} & v_2{'} & \dots & v_n{'} \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \\ v_1^{'} \\ v_2^{'} \\ \vdots \\ v_n^{'} \end{array} & \begin{pmatrix} 4n-6 & -2 & -2 & \dots & -1 & -1 & -2 & \dots & -2 \\ -2 & 4n-6 & -2 & \dots & -1 & -2 & -1 & \dots & -2 \\ -2 & -2 & 4n-6 & \dots & -1 & -2 & -2 & \dots & -2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \dots & 2n-2 & -1 & -1 & \dots & -1 \\ -1 & -2 & -2 & \dots & -1 & 4n-6 & -2 & \dots & -2 \\ -2 & -1 & -2 & \dots & -1 & -2 & 4n-6 & \dots & -2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -2 & -2 & -2 & \dots & -1 & -2 & -2 & \dots & 4n-6 \end{pmatrix} \end{array}$$

The M^L-eigenvalues of S_n^{+-+} are $\begin{pmatrix} \mu_1^L \\ \mu_2^L \\ \mu_3^L \\ \vdots \\ \mu_{n+1}^L \\ \mu_{n+2}^L \\ \vdots \\ \mu_{2n-1}^L \end{pmatrix} = \begin{pmatrix} 0 \\ 2n-1 \\ 4n-5 \\ \vdots \\ 4n-5 \\ 4n-3 \\ \vdots \\ 4n-3 \end{pmatrix}$.

The M^L- spectrum of S_n^{+-+} is
$Spec_{M^L}(S_n^{+-+}) = \begin{pmatrix} 0 & 2n-1 & 4n-5 & 4n-3 \\ 1 & 1 & n-1 & n-2 \end{pmatrix}$.

$$\begin{aligned} \text{Then,} LE_M\left(S_n^{+-+}\right) &= \sum_{i=1}^{2n-1} \left|\mu_i^L - \frac{8n^2-18n+10}{2n-1}\right| \\ &= \left|0 - \frac{8n^2-18n+10}{2n-1}\right| + \left|(2n-1) - \frac{8n^2-18n+10}{2n-1}\right| + \\ &\quad (n-1)\left|(4n-5) - \frac{8n^2-18n+10}{2n-1}\right| + \\ &\quad (n-2)\left|(4n-3) - \frac{8n^2-18n+10}{2n-1}\right| \end{aligned}$$

Since $\mu_i^L - \dfrac{1}{2n-1}\displaystyle\sum_{j=1}^{2n-1} MT_G\left(v_j\right) < 0$, where $i = 2$, we have

$$\begin{aligned} LE_M\left(S_n^{+-+}\right) &= \left[\frac{8n^2-18n+10}{2n-1}\right] + \left[\frac{(8n^2-18n+10)-(2n-1)^2}{2n-1}\right] + \\ &\quad (n-1)\left[\frac{(4n-5)(2n-1)-(8n^2-18n+10)}{2n-1}\right] + \\ &\quad (n-2)\left[\frac{(2n-1)(4n-3)-(8n^2-18n+10)}{2n-1}\right] \end{aligned}$$

$$= \frac{1}{2n-1}\left[12n^2 - 32n + 19 + (n-1)(4n-5) + (n-2)(8n-7)\right]$$
$$= \frac{2}{2n-1}\left[12n^2 - 32n + 19\right].$$

Theorem 3.3. Let S_n^{+++} be the transformation graph of order 2n-1. Then the monophonic distance Laplacian energy of S_n^{+++} is
$LE_M\left(S_n^{+++}\right) = \dfrac{2}{2n-1}[(6n^2 - 14n + 7) + (2n^2 - 5n + 2)\sqrt{n^2 - 2n + 2}$, for $n \geq 4$.

Proof. The graph S_n^{+++} as shown in Figure: 3

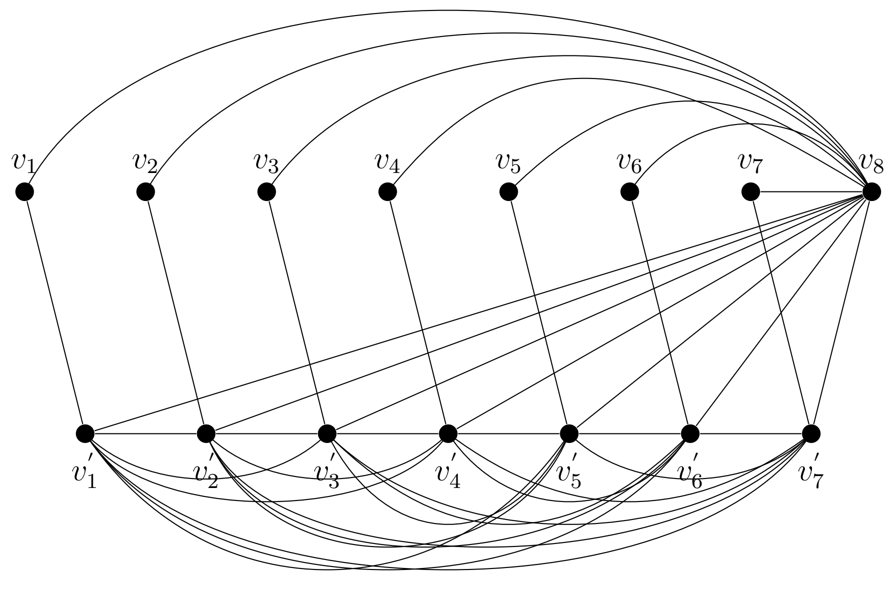

Figure 3: The graph S_n^{+++}, $n = 8$

The monophonic distance matrix of S_n^{+++} is

$$M\left(S_n^{+++}\right) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \\ v_1^{'} \\ v_2^{'} \\ v_3^{'} \\ \vdots \\ v_n^{'} \end{array} \begin{array}{c} \begin{array}{cccccccccc} v_1 & v_2 & v_3 & \dots & v_n & v_1^{'} & v_2{'} & v_3^{'} & \dots & v_n{'} \end{array} \\ \begin{pmatrix} 0 & 3 & 3 & \dots & 1 & 1 & 2 & 2 & \dots & 2 \\ 3 & 0 & 3 & \dots & 1 & 2 & 1 & 2 & \dots & 2 \\ 3 & 3 & 0 & \dots & 1 & 2 & 2 & 1 & \dots & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 0 & 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 2 & \dots & 1 & 0 & 1 & 1 & \dots & 1 \\ 2 & 1 & 2 & \dots & 1 & 1 & 0 & 1 & \dots & 1 \\ 2 & 2 & 1 & \dots & 1 & 1 & 1 & 0 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & 2 & \dots & 1 & 1 & 1 & 1 & \dots & 0 \end{pmatrix} \end{array}$$

The monophonic distance Laplacian matrix of S_n^{+++} is
$M^L\left(S_n^{+++}\right) = MT\left(S_n^{+++}\right) - M\left(S_n^{+++}\right)$

$$M^L\left(S_n^{+++}\right) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \\ v_1^{'} \\ v_2^{'} \\ \vdots \\ v_n^{'} \end{array} \begin{array}{c} \begin{array}{ccccccccc} v_1 & v_2 & v_3 & \dots & v_n & v_1^{'} & v_2{'} & \dots & v_n{'} \end{array} \\ \begin{pmatrix} 5n-8 & -3 & -3 & \dots & -1 & -1 & -2 & \dots & -2 \\ -3 & 5n-8 & -3 & \dots & -1 & -2 & -1 & \dots & -2 \\ -3 & -3 & 5n-8 & \dots & -1 & -2 & -2 & \dots & -2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \dots & 2n-2 & -1 & -1 & \dots & -1 \\ -1 & -2 & -2 & \dots & -1 & 3n-4 & -1 & \dots & -1 \\ -2 & -1 & -2 & \dots & -1 & -1 & 3n-4 & \dots & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -2 & -2 & -2 & \dots & -1 & -1 & -1 & \dots & 3n-4 \end{pmatrix} \end{array}$$

The M^L-eigenvalues of S_n^{+++} are $\begin{pmatrix} \mu_1^L \\ \mu_2^L \\ \mu_3^L \\ \vdots \\ \mu_n^L \\ \mu_{n+1}^L \\ \mu_{n+2}^L \\ \vdots \\ \mu_{2n-1}^L \end{pmatrix} = \begin{pmatrix} 0 \\ 2n-1 \\ (4n-4)-\sqrt{n^2-2n+2} \\ \vdots \\ (4n-4)-\sqrt{n^2-2n+2} \\ 4n-5 \\ (4n-4)+\sqrt{n^2-2n+2} \\ \vdots \\ (4n-4)+\sqrt{n^2-2n+2} \end{pmatrix}$

The M^L-spectrum of S_n^{+++} is

$$Spec_M^L(S_n^{+++}) = \begin{pmatrix} 0 & 2n-1 & (4n-4)-\sqrt{n^2-2n+2} & 4n-5 & (4n-4)+\sqrt{n^2-2n+2} \\ 1 & 1 & n-2 & 1 & n-2 \end{pmatrix}$$

Let $n \geq 4$

$$
\begin{aligned}
LE_M\left(S_n^{+++}\right) &= \sum_{i=1}^{2n-1}\left|\mu_i^L - \frac{8n^2-18n+10}{2n-1}\right| \\
&= \left|0 - \frac{8n^2-18n+10}{2n-1}\right| + (n-1)\left|(2n-1) - \frac{8n^2-18n+10}{2n-1}\right| + \\
&\quad (n-1)\left|(4n-5) - \frac{8n^2-18n+10}{2n-1}\right| + \\
&\quad (n-2)\left|(4n-4) + \sqrt{n^2-2n+2} - \frac{8n^2-18n+10}{2n-1}\right| + \\
&\quad (n-2)\left|(4n-4) - \sqrt{n^2-2n+2} - \frac{8n^2-18n+10}{2n-1}\right|
\end{aligned}
$$

Since $\mu_i^L - \frac{1}{2n-1}\sum_{j=1}^{2n-1} MT_G\left(v_j\right) < 0$, where $i = 2, 3, \ldots, n$, we have

$$
\begin{aligned}
LE_M\left(S_n^{+++}\right) &= \left[\frac{8n^2-18n+10}{2n-1}\right] + (n-1)\left[\frac{(8n^2-18n+10)-(2n-1)^2}{2n-1}\right] + \\
&\quad (n-1)\left[\frac{(4n-5)(2n-1)-(8n^2-18n+10)}{2n-1}\right] + \\
&\quad (n-2)\left[\frac{((4n-4)+\sqrt{n^2-2n+2})(2n-1)-(8n^2-18n+10)}{2n-1}\right] + \\
&\quad (n-2)\left[\frac{((8n^2-18n+10)-(4n-4)-\sqrt{n^2-2n+2})}{2n-1}\right] \\
&= \frac{1}{2n-1}\left[12n^2-28n+14+2(n-2)(2n-1)\sqrt{n^2-2n+2}\right] \\
&= \frac{2}{2n-1}[(6n^2-14n+7)+(2n^2-5n+2)\sqrt{n^2-2n+2}].
\end{aligned}
$$

4 Conclusion

In this paper we derived a new graph energy called monophonic distance Laplacian energy. We obtained monophonic distance Laplacian energy of transformation graphs S_n^{++-}, S_n^{+-+}, S_n^{+++}. In this study we computed monophonic distance Laplacian spectrum based on these eigenvalues. Future scope of this study is comparitive study of monophonic distance Laplacian energy of graphs with other graphs. We can also extend this concept monophonic distance Laplacian energy of graph to various graph distance parameters.

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