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Whose Inflation Rates Matter Most?

A DSGE Model and Machine Learning Approach to Monetary Policy in the Euro Area*

By Daniel Stempel† and Johannes Zahner‡

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In the euro area, monetary policy is conducted by a single central bank for 20 member countries. However, countries are heterogeneous in their economic development, including their inflation rates. This paper combines a New Keynesian model and a neural network to assess whether the European Central Bank (ECB) conducted monetary policy between 2002 and 2022 according to the weighted average of the inflation rates within the European Monetary Union (EMU) or reacted more strongly to the inflation rate developments of certain EMU countries. The New Keynesian model first generates data which is used to train and evaluate several machine learning algorithms. We find that a neural network performs best out-of-sample. We use this algorithm to (i) generally classify historical EMU data, and to (ii) determine the exact weight on the inflation rate of EMU members in each quarter of the past two decades. Our findings suggest disproportional emphasis of the ECB on the inflation rates of EMU members that exhibited high inflation rate volatility for the vast majority of the time frame considered (80%), with a median inflation weight of 67% on these countries. We show that these results stem from a tendency of the ECB to react more strongly to countries whose inflation rates exhibit greater deviations from their long-term trend.

JEL: E58, C45, C53
Keywords: New Keynesian Models, Monetary Policy, European Monetary Union, Neural Networks, Transfer Learning

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I. Introduction

In the European Monetary Union (EMU), monetary policy is conducted by a single central bank for its 20 member countries. The European Central Bank (ECB) aims to stabilize the union-wide price level. This setup harbors an obvious potential issue when countries are heterogeneous in their economic development, including the development of their inflation rates. In particular, we show that EMU members significantly differ in their inflation rate volatility: some countries exhibit below-average inflation rate volatility while others display above-average volatility.

This naturally raises the question of whether the ECB conducts monetary policy in accordance with the EMU's weighted average of the inflation rates (i.e., with changes in the HICP)[1] or reacts more strongly to potential deflationary or inflationary pressure in some member states. This paper aims to shed light on how the ECB conducted monetary policy over the past two decades, especially with regard to the weighting of different inflation rate developments. Since latent variables, such as the ECB's inflation weight on EMU members, cannot be directly observed and may be subject to variation over time, conventional empirical methods fall short in its identification. To circumvent these challenges, we simulate a New Keynesian model of a monetary union to generate a synthetic data set in which we control for variation in this latent variable. The New Keynesian model is then combined with a neural network (NN) to assess the ECB's historical inflation weight on high- and low-volatility countries over the last two decades. We find that the ECB reacted disproportionally to the inflation rates experienced by EMU members that exhibited high inflation rate volatility (in 80% of the quarters).

Our analysis consists of five parts. First, we establish that inflation rate developments structurally differ between EMU countries. Second, we build a two-country New Keynesian model of a monetary union that replicates first and second moments of main macroeconomic variables in low- and high-volatility EMU countries. In the model, the central bank is assumed to react to the union-wide inflation rate or more strongly to the inflation rate experienced by either the low- or the high-volatility country respectively. Third, we use this data set to train and evaluate a multitude of machine learning algorithms. We find that a NN performs best, accurately categorizing over 97% of the simulated data in an out-of-sample exercise. Fourth, using the trained NN, we classify a historical EMU data set between 2002 and 2022. The machine learning algorithm classifies 80% of the last two decades as periods during which the ECB reacted more strongly to the infla-

[1] Specifically, the ECB states that *"the Harmonised Index of Consumer Prices (HICP) is used to measure consumer price inflation"*, which is *"compiled by Eurostat"* (European Central Bank, 2022). Eurostat calculates the European HICP *"as the weighted average of the national HICPs, using the weights of the countries [...] concerned. The weight of a country is based on the share of the HFMCE [household final consumption expenditure that occurs in monetary transactions] in the total"* (Eurostat, 2022).

tion rates of high-volatility countries. Fifth, we use a regression NN to determine the exact inflation weight of the ECB in each quarter. We find that the distribution of historical inflation weights is skewed towards high-volatility countries with a median weight of 67%. We empirically show that this result stems from a tendency of the ECB to react more vigorously to the countries whose inflation rates deviate more strongly from their long-term trend. We infer that the ECB's loss function depends on the individual inflation rate deviations of its members rather than the deviation of the average inflation rate in the EMU.
Our paper contributes to the existing literature in the following ways. The approach relates to the trade-off between the degree of theoretical coherence and empirical validity during the model selection process. Pagan (2003) proposes an illustration of this trade-off, known as the Pagan Frontier, in which Dynamic Stochastic General Equilibrium (DSGE) and vector autoregression (VAR) models are the corresponding specializations. Recently, Genberg and Karagedikli (2021) suggested an extension of the Pagan Frontier in conjunction with the growing trend of machine learning in macroeconomics and monetary policy (e.g., Chakraborty and Joseph, 2017; Hansen et al., 2017; Athey, 2019; Tiffin, 2019; Hinterlang, 2020; Baumgärtner and Zahner, 2021; Doerr et al., 2021; Paranhos, 2021; Fouliard et al., 2021), where the black box algorithms are (at least) equivalent in terms of empirical coherence. Figure 1 illustrates this adjusted Pagan Frontier. The authors then pose the question of how machine learning *"can move towards the middle, and what modifications need to be introduced to enable them to do so"* (Genberg and Karagedikli, 2021, p. 4). In this paper, we propose such a modification by *combining* DSGE and machine learning models to study inflation dynamics in the EMU.

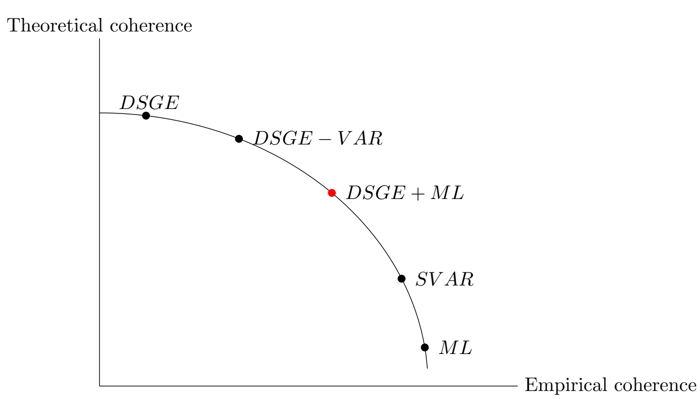

Figure 1 : The Pagan Frontier.

Notes: The illustration resembles Pagan's (2003) original frontier with the extensions by Genberg and Karagedikli (2021). We append the combination of a DSGE model and a machine learning algorithm.

To the best of our knowledge, Hinterlang and Hollmayr (2022) is the only paper using an approach similar to ours. They generate a synthetic data set from a DSGE model to identify monetary and fiscal dominance regimes in the United States. Their approach differs primarily in terms of the applied machine learning models. The authors focus on tree-based models, with the best performing model (AdaBoost) achieving 95% accuracy on a binary classification task with many covariates. We extend this approach by employing NNs, which outperform tree-based models in our case. We further extend the classification approach by Hinterlang and Hollmayr (2022) to a regression approach, which allows us to estimate precise weights.

Lastly, we add to the literature on the assessment of inflation differentials within New Keynesian models (e.g., Canzoneri et al., 2006; Angeloni and Ehrmann, 2007; Andrés et al., 2008; Duarte and Wolman, 2008; Rabanal, 2009).

The remainder of this paper is structured as follows. In Section II, we motivate our research question and provide preliminary descriptive evidence. Section III introduces the New Keynesian model used for the data-generating process, before Section IV assesses first and second moments of the simulation. Section V introduces and evaluates the machine learning algorithms, which are subsequently applied to historical EMU data in Section VI. We present robustness checks in Section VII. Section VIII concludes this paper.

II. Inflation Development in the Euro Area

The primary objective of the ECB is price stability across the euro area. While its monetary policy tools are designed to apply uniformly across its member states, in reality there is substantial heterogeneity across the EMU with respect to inflation rates. This heterogeneity might pose challenges for the ECB when determining the appropriate stance of monetary policy.[2] If some countries consistently fall below the inflation target, expansionary monetary policy would be justified in order to stimulate economic activity and raise inflation in those countries. At the same time, such a policy stance would be inappropriate for the remaining countries, if inflationary pressures there call for contractionary measures.

This is particularly relevant when countries deviate structurally from the euro area average inflation rate. Figure 2 illustrates the inflation rate development of selected EMU countries between 2002 and 2022. Panel b highlights the average deviation of each member when the 2% target is exceeded, as well as in times when the 2% target is missed.

The illustration suggests that Greece (EL), Ireland (IE), Italy (IT), Portugal (PT), and Spain (ES) (highlighted in red in Figure 2) consistently exceed the euro area average in times of high inflation and exhibit below-average inflation rates in times of low inflation. This implies a higher inflation volatility (standard

[2] While the focus of our work is on inflation differentials across countries, we recognize the possibility of heterogeneous inflation developments within countries (e.g., Jaravel, 2018). However, a recent ECB working paper by Consolo et al. (2021) suggests that inflation variation across countries in the EMU is more substantial than inflation variation within countries. We therefore focus on the former.

deviation) in those countries (~ 1.6 on average, shown in panel a) than the average inflation rate of all EMU members (~ 1.4).[3]

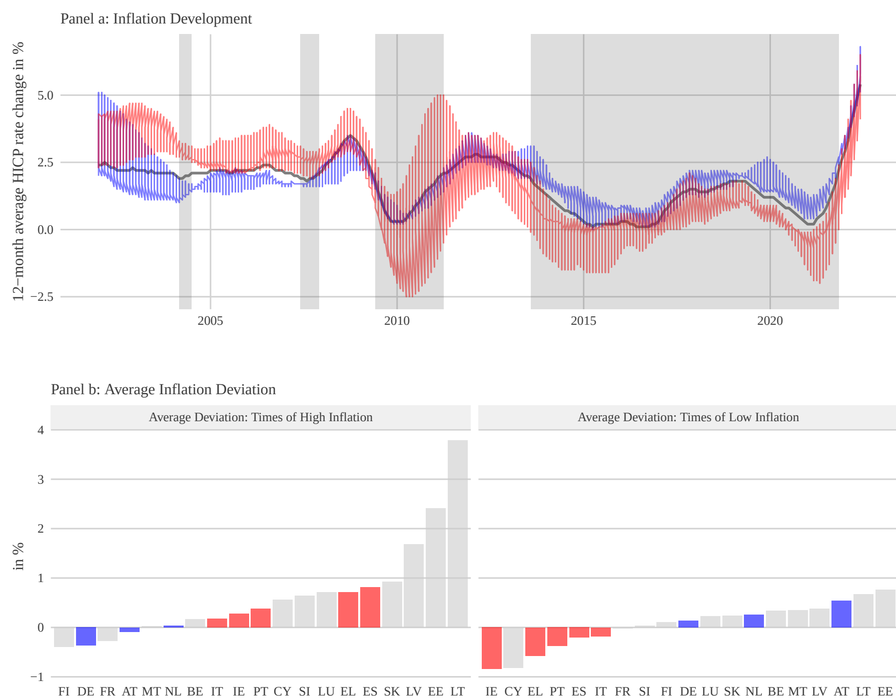

Figure 2 : Inflation Rate Developments and Average Inflation Deviations.

Notes: Panel a: Moving 12-month average inflation rate development of selected high-volatility (red) and low-volatility (blue) countries. Grey-shaded areas indicate periods in which the EMU-wide inflation rate was below 2%. Panel b: Average deviation of the inflation rate of EMU members in periods where the EMU-wide inflation rate was above 2% (left hand side, LHS) and where the EMU-wide inflation rate was below 2% (right hand side, RHS). BE: Belgium, CY: Cyprus, EE: Estonia, FI: Finland, FR: France, LV: Latvia, LT: Lithuania, LU: Luxembourg, MT: Malta, SK: Slovakia, SI: Slovenia. Own calculation, based on years in which countries were members of the EMU. Timeframe: January 2002 - June 2022. Data source: Eurostat.

Conversely, the inflation rates of Austria (AT), Germany (DE), and the Netherlands (NL) (highlighted in blue in Figure 2) display lower volatility (~ 1.0 on average) than the EMU-wide rate. This implies that, on average, their inflation rates deviate positively from the EMU-wide rate in times of low inflation and vice versa (panel b).[4] The difference between high- and low-volatility countries

[3]The average inflation rate volatility of a country group is calculated as the standard deviation of the 12-month moving average rate of change of the HICP of *all* countries in the respective group. The results are qualitatively and quantitatively the same when using the average of the *country-specific* volatilities. The time frame considered is January 2002 to June 2022.

[4]Note that we do not include France in either of the country blocks, as France displays negative deviations in both times of high and of low inflation.

is significant at the 5% level.
This descriptive evidence naturally raises the question of whether the ECB conducts monetary policy in accordance with the weighted average of the inflation rates within the EMU (see Footnote 1) or reacts more strongly to potential deflationary/inflationary pressure in some EMU member states.

Table 1: EMU Taylor Rule

	Dependent variable:				
	Interest Rate				
	(1)	(2)	(3)	(4)	(5)
HICP -2%	2.52***	2.24***	2.44***	2.04***	1.86***
	(0.19)	(0.14)	(0.26)	(0.12)	(0.32)
Constant	−0.12	−0.17	−0.20	−0.21	−0.41*
	(0.19)	(0.17)	(0.22)	(0.16)	(0.24)
Weight on LV countries (ω) =	0.5	0.2	0.8	0	1
Observations	240	240	240	240	240
R^2	0.43	0.53	0.26	0.56	0.12

Note: *p<0.1; **p<0.05; ***p<0.01; HICP is calculated as follows: $HICP := \omega \times CPI_{LV} + (1-\omega) \times CPI_{HV}$.

Although this question may be approached through conventional empirical methods, the constraints of this form of investigation lie in the assumptions underlying the empirical tools. To illustrate, we employ an Ordinary Least Squares (OLS) regression of the Taylor Rule type, wherein we regress weighted averages of the Harmonised Index of Consumer Prices (HICP) of high and low volatility countries on the ECB's short-term interest rates.[5] The results are in Table 1. Assuming equal weights, the inflation response coefficient is estimated at 2.5 and the model can explain 43% of the variation. Assigning higher weights to high-volatility countries leads the inflation response coefficient to decrease but the explained variation to increase substantially.[6]
While this may be interpreted as a bias towards high volatility countries, the regression results do not provide guidance on which weight accurately describes historical EMU monetary policy, leaving it subject to the discretion of the researcher. In lieu of this shortcoming, we adopt a data-driven approach that is predicated upon a simple and tractable theoretical model. This approach permits us to generate simulations for multiple scenarios, and to select the optimal

[5] We rely on the data set presented in Table 3.
[6] It could be (incorrectly) inferred from Table 1 that either a) solely the inflation rate of countries exhibiting high volatility holds relevance to the interest rate setting of the ECB, or b) that the inflation rates of both country blocks are highly correlated to the extent that their distinction is insignificant. To address these issues, we conduct an analysis of variance (ANOVA), verifying that the inflation rate of low-volatility countries contributes to the explained variation even after controlling for the inflation rate of high-volatility countries, indicating the importance of observing both simultaneously.

weight that exhibits superior fit across a range of variables. Additionally, our approach enables us to undertake a dynamic assessment of variations in the inflation weight over time, a task that is infeasible within the confines of a linear modeling framework.

III. Model

This section presents the model that generates the synthetic data set used to train and evaluate the machine learning mechanisms described in Section V. It is purposely kept simple and tractable. However, we test the robustness of our results with a more complex model (see Section VII). The model consists of two countries, $k = L, H$, with $-k$ being the respective other country, in a monetary union. Each country consists of a household and a firm sector. Monetary policy is conducted on the union level.

A. Households

The utility function of a representative household in country k (household k, for simplicity) is given by

$$U_t^k = Z_t^k log\left(C_t^k - \Psi_k C_{t-1}^k\right) - \frac{\left(N_t^k\right)^{1+\varphi_k}}{1+\varphi_k}, \tag{1}$$

with Z_t^k being defined as an AR(1) preference shock, Ψ_k as a habit parameter, N_t^k as labor, φ_k as the inverse Frisch elasticity of labor supply, and C_t^k as a constant elasticity of substitution (CES) index of consumption. Preferences change individually by country

$$log\left(Z_t^k\right) = \rho_Z log\left(Z_{t-1}^k\right) + \eta_Z^k \epsilon_{Z,t},$$

where ρ_Z denotes the persistence, η_Z^k is a scaling parameter that determines the strength of the shock, and $\epsilon_{Z,t} \sim \mathcal{N}\left(0, \sigma_Z^2\right)$ is a normally distributed shock with mean 0 and variance σ_Z^2. The CES consumption index is given by:

$$C_t^k \equiv \left(\gamma_k^{\frac{1}{\vartheta_C^k}} \left(C_{k,t}^k\right)^{\frac{\vartheta_C^k-1}{\vartheta_C^k}} + (1-\gamma_k)^{\frac{1}{\vartheta_C^k}} \left(C_{-k,t}^k\right)^{\frac{\vartheta_C^k-1}{\vartheta_C^k}}\right)^{\frac{\vartheta_C^k}{\vartheta_C^k-1}}. \tag{2}$$

$C_{k,t}^k$ is defined as the consumption of domestically produced goods, $C_{-k,t}^k$ represents foreign consumption. The parameter γ_k indicates the relative weight of domestic goods, ϑ_C^k denotes the elasticity of substitution between the goods. $C_{k,t}^k$ and $C_{-k,t}^k$ are symmetric CES functions given by

$$C_{k,t}^k \equiv \left(\int_0^1 C_{k,t}^k(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{3}$$

$$C_{-k,t}^k \equiv \left(\int_0^1 C_{-k,t}^k(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

with ϵ denoting the elasticity of substitution between domestic and foreign varieties respectively.
Expenditure minimization with respect to the varieties yields

$$C_{k,t}^k(i) = \left(\frac{P_{k,t}(i)}{P_{k,t}} \right)^{-\epsilon} C_{k,t}^k, \tag{5}$$

$$C_{-k,t}^k(i) = \left(\frac{P_{-k,t}(i)}{P_{-k,t}} \right)^{-\epsilon} C_{-k,t}^k, \tag{6}$$

with $P_{k,t} \equiv \left(\int_0^1 P_{k,t}(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ and $P_{-k,t} \equiv \left(\int_0^1 P_{-k,t}(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}$ being the overall price indices of domestic and foreign goods respectively.
Expenditure minimization with respect to the level of domestic and foreign consumption gives

$$C_{k,t}^k = \left(\frac{P_{k,t}}{P_t^{C,k}} \right)^{-\vartheta_C^k} \gamma_k C_t^k, \tag{7}$$

$$C_{-k,t}^k = \left(\frac{P_{-k,t}}{P_t^{C,k}} \right)^{-\vartheta_C^k} (1-\gamma_k) C_t^k, \tag{8}$$

with $P_t^{C,k} \equiv \left(\gamma_k P_{k,t}^{1-\vartheta_C^k} + (1-\gamma_k) P_{-k,t}^{1-\vartheta_C^k}\right)^{\frac{1}{1-\vartheta_C^k}}$ being defined as the consumer price index of household k. The household maximizes its expected discounted lifetime utility given by

$$\mathbb{E}_t \left[\sum_{\iota=0}^{\infty} \beta^{\iota} U_{t+\iota}^k \right], \tag{9}$$

subject to the budget constraint

$$P_t^{C,k} C_t^k + Q_t B_t^k = B_{t-1}^k + W_t^k N_t^k + D_t^k, \tag{10}$$

where B_t^k is defined as one-period, nominally risk-free bonds purchased at price Q_t, W_t^k as the nominal wage, and D_t^k as exogenous dividends from the ownership of firms. The optimality conditions are given by

$$\left(N_t^k\right)^{\varphi_k} = w_{k,t} U_{c,t}^k, \tag{11}$$

$$Q_t = \beta\, \mathbb{E}_t \left[\Lambda_{t,t+1}^k \frac{1}{\Pi_{t+1}^{C,k}} \right] \tag{12}$$

with $U_{c,t}^k \equiv \frac{Z_t}{C_t^k - \Psi_k C_{t-1}^k} - \frac{\mathbb{E}_t[Z_{t+1}]\Psi_k \beta}{\mathbb{E}_t\left[C_{t+1}^k\right] - \Psi_k C_t^k}$ being defined as the marginal utility of consumption, $w_t^k \equiv \frac{W_t^k}{P_t^{C,k}}$ as the real wage, $\beta \Lambda_{t,t+1}^k \equiv \beta\, \mathbb{E}_t \left[\frac{U_{c,t+1}^k}{U_{c,t}^k} \right]$ as the stochastic discount factor, and $\Pi_{t+1}^{C,k} \equiv \frac{P_{t+1}^{C,k}}{P_t^{C,k}}$ as HICP inflation. Due to the shared bond market, we can obtain the following risk sharing condition between the two households by combining the Euler equations for each household

$$U_{c,t}^k = U_{c,t}^{-k} \Phi^k \frac{P_t^{C,k}}{P_t^{C,-k}}, \tag{13}$$

with $\Phi^k \equiv \frac{U_{c,SS}^k}{U_{c,SS}^{-k}}$, where the subscript SS denotes the zero inflation steady state of a variable.

B. Firms

In each country, a representative firm k produces one variety i of goods. The production function is given by

$$Y_{k,t}(i) = \left(N_t^k(i)\right)^{1-\alpha_k}, \tag{14}$$

where $1-\alpha_k$ is the partial factor elasticity of labor. The real cost function is given by

$$TC_{k,t}(i) = A_t^k w_t^k N_t^k(i), \tag{15}$$

where A_t^k is a cost-push shock given by

$$log\left(A_t^k\right) = \rho_A log\left(A_{t-1}^k\right) + \eta_A^k \epsilon_{A,t}.$$

ρ_A denotes the persistence, η_A^k determines the strength of the shock, and $\epsilon_{A,t} \sim \mathcal{N}\left(0, \sigma_A^2\right)$ is a normally distributed shock with mean 0 and variance σ_A^2. The firm maximizes its expected stream of current and future profits given by

$$\mathbb{E}_t\left[\sum_{\iota=0}^{\infty} \beta^\iota \Lambda_{t,t+\iota}^k \lambda_k^\iota \left(\frac{P_{k,t}(i)}{P_{t+\iota}^{C,k}} Y_{k,t+\iota|t}(i) - TC\left(Y_{k,t+\iota|t}(i)\right)\right)\right], \tag{16}$$

subject to

$$Y_{k,t+\iota|t}(i) = \left(\frac{P_{k,t}(i)}{P_{k,t+\iota}}\right)^{-\epsilon} Y_{k,t+\iota}. \tag{17}$$

λ_k is defined as the probability of a firm not being able to reset its price (as in Calvo, 1983), $Y_{k,t+\iota|t}(i)$ as the output of firm i in period $t+\iota$ for a price set in t, and $Y_{k,t+\iota}$ as the overall output produced in country k. Dropping index i due to symmetry, the optimal price is given by

$$\left(p_{k,t}^*\right)^{1+\frac{\epsilon\alpha_k}{1-\alpha_k}} = \mu\left(\frac{P_{k,t}}{P_t^{C,k}}\right)^{-1} \frac{x_{1,t}^k}{x_{2,t}^k}, \tag{18}$$

where the auxiliary variables are defined as

$$x_{1,t}^k \equiv U_{c,t}^k Y_{k,t} mc_{k,t} + \beta\lambda_k \, \mathbb{E}_t \left[\Pi_{k,t+1}^{\frac{\epsilon}{1-\alpha_k}} x_{1,t+1}^k \right],$$

$$x_{2,t}^k \equiv U_{c,t}^k Y_{k,t} + \beta\lambda_k \, \mathbb{E}_t \left[\Pi_{k,t+1}^{\epsilon} \left(\Pi_{t+1}^{C,k} \right)^{-1} x_{2,t+1}^k \right],$$

and $p_{k,t}^* \equiv \frac{P_{k,t}^*}{P_{k,t}}$. The variable $mc_{k,t} = \frac{1}{1-\alpha_k} w_t^k Y_{k,t}^{\frac{\alpha_k}{1-\alpha_k}} A_t^k$ denotes the economy-wide real marginal costs of the good produced in country k and $\Pi_{k,t+1} \equiv \frac{P_{k,t+1}}{P_{k,t}}$ is defined as inflation of domestic goods. Aggregate price dynamics are given by:

$$1 = (1-\lambda_k) \left(p_{k,t}^* \right)^{1-\epsilon} + \lambda_k \left(\frac{1}{\Pi_{k,t}} \right)^{1-\epsilon}. \tag{19}$$

C. Central Bank

The central bank follows a Taylor rule given by

$$i_t = \rho + \phi_\pi \left(\omega_\pi \pi_t^{C,H} + (1-\omega_\pi) \pi_t^{C,L} \right), \tag{20}$$

where $i_t \equiv log\,(1/Q_t)$, $\rho \equiv log\,(1/\beta)$, and $\pi_t^{C,k} \equiv log \left(\Pi_t^{C,k} \right)$. The parameter $\phi_\pi > 1$ denotes the standard reaction coefficient of the central bank to the weighted HICP inflation rates of households from countries H and L. If $\omega_\pi = C_{SS}^H / (C_{SS}^H + C_{SS}^L)$, the central bank reacts to the average (as measured by the ECB, see Footnote 1), economy-wide HICP inflation rate given by:

$$\pi_t^C = \frac{C_{SS}^H}{C_{SS}^H + C_{SS}^L} \pi_t^{C,H} + \left(1 - \frac{C_{SS}^H}{C_{SS}^H + C_{SS}^L} \right) \pi_t^{C,L}. \tag{21}$$

However, if $\omega_\pi \neq C_{SS}^H/(C_{SS}^H + C_{SS}^L)$, the central bank reacts more strongly to the HICP inflation rate of either country H ($\omega_\pi > C_{SS}^H/(C_{SS}^H + C_{SS}^L)$) or L ($\omega_\pi < C_{SS}^H/(C_{SS}^H + C_{SS}^L)$) than suggested by the economy-wide inflation rate.

The Fisher equation holds for each household:

$$i_t = r_t^k + \mathbb{E}_t \left[\pi_{t+1}^{C,k} \right]. \tag{22}$$

D. Market Clearing

Bond markets, labor markets, and goods markets clear:

$$B_t^k = -B_t^{-k}, \tag{23}$$

$$N_t^k = \int_0^1 N_t^k(i)di, \tag{24}$$

$$Y_{k,t} = C_{k,t}^k + C_{k,t}^{-k}. \tag{25}$$

Union-wide output is defined as:

$$Y_t = Y_{k,t} + Y_{-k,t}. \tag{26}$$

E. Calibration

The calibration of the model draws from a multitude of sources. In order to realistically capture the inflation rate developments reported in Section II, we calibrate country H to represent the high-volatility EMU members (in particular, EL, IE, IT, PT, ES) and L according to the EMU members which exhibit low inflation volatility (in particular, AT, DE, and NL).[7] Note that these eight countries account for more than 70% of EMU GDP. For the calibration, we utilize studies that estimate the structural parameters for the countries that we use in our model. In particular, we use Breuss and Rabitsch (2008) for AT, Albonico et al. (2019) for DE, Garcia et al. (2021) for NL, Papageorgiou (2014) for EL, Garcia et al. (2021) for IE, Albonico et al. (2019) for ES and IT, and Almeida (2009) for PT.

We then continue by weighting each country-specific parameter with the country-specific share of GDP in order to calculate the parameter values for H and L.[8] For instance, the consumption habit parameter for country L is calculated in the following way: the values for Germany (0.73), the Netherlands (0.65), and Austria (0.67) are weighted with their relative GDP, leading to an overall value of 0.71 for L. The corresponding calibration is shown in Table A2.1.

We observe that high-volatility EMU members exhibit structurally higher habit formation and a higher Frisch elasticity of labor supply. Interestingly, H and L display a similar level of home bias in consumption as well as a comparable elasticity of substitution between domestic and foreign goods. Importantly, prices are stickier in low-volatility EMU member states, which plays an important role when determining the volatility of the inflation rates of H and L. Note that we

[7] We decide to calibrate rather than to estimate the model for two reasons: (i) an endogeneity problem might arise when estimating parameters with the same EMU data we aim to classify, and (ii) parameter estimates might change depending on the country weights in the Taylor rule.

[8] Note that weighting with consumption shares delivers similar results.

Table 2: Calibration.

	Description	Value	
	Households		
		H	L
Ψ_k	Habit parameter	0.77	0.71
φ_k	Inverse Frisch elasticity	2.01	2.73
η_Z^k	Preference shock strength	1	0.45
γ_k	Weight of domestic goods	0.75	0.75
ϑ_C^k	Elasticity of substitution between domestic and foreign goods	1.42	1.50
ϵ	Price elasticity of demand	6	6
β	Discount rate	0.995	0.995
	Firms		
		H	L
α_k	Output elasticity labor	0.33	0.33
η_A^k	Cost-push shock strength	1	0.45
λ_k	Calvo parameter	0.737	0.852
	Central Bank		
ϕ_π	Taylor rule coefficient	1.5; 2.5	
ω_π	HICP inflation weight	$\frac{C_{SS}^H}{C_{SS}^H+C_{SS}^L}$; $[0.1, 0.9]$	

assume the preference and the cost-push shock to differ in their impacts between the countries. The validity of this assumption is discussed in Section IV. Lastly, we simulate model responses for a variety of different Taylor rule specifications. We simulate three baseline models: one in which the central bank reacts to the union-wide inflation rate ($\omega_C \equiv \omega_\pi = C_{SS}^H/(C_{SS}^H + C_{SS}^L)$), one where the central bank reacts more strongly to country H ($\omega_H \equiv \omega_\pi = 0.8$), and one in which the central bank reacts more strongly to country L ($\omega_L \equiv \omega_\pi = 0.2$).[9] In Section VI.C, we extend ω to a continuous interval ranging from 0.1 to 0.9. The reaction coefficient is set to 1.5 in all cases.[10] We therefore simulate the model for 3 different versions of the Taylor rule, which are then used to train and evaluate a multitude of machine learning mechanisms.

IV. Historical Data

In order to properly assess monetary policy in EMU, the New Keynesian model must accurately match the statistical properties of historical EMU data, as we use the simulated data to train the machine learning algorithms. We briefly describe the historical EMU data in the following.

[9]Note that countries H and L are roughly equal in size. Therefore, the unbiased inflation weight in the steady state is close to parity, i.e., ≈ 0.45.

[10]We test whether our findings depend on the choice of the central bank response parameter ϕ_π. See Section VII for details.

A. Description

We collect data on consumption, employment, price levels, interest rates, output, and population. Detailed information on data availability, frequency, and sources can be found in Table 3.

Table 3: Data Sources.

Data	Countries/Regions	Years	Frequency	Source
Consumption	AT, DE, EL, ES, IE, IT, NL, PT	2002–2022	Quarterly	Eurostat: GDP and main components
Employment	AT, EL, ES, IE, IT, NL, PT	2002–2022	Quarterly	Eurostat: Employment by sex, age, and citizenship
Price levels	AT, DE, EL, ES, IE, IT, NL, PT	2002–2022	Monthly	Eurostat: HICP - monthly data (index)
Interest rates	EA	2002–2022	Monthly	Bundesbank: ECB interest rates for main refinancing operations, shadow rates as in Wu and Xia (2020)
Output	AT, DE, EA, EL, ES, IE, IT, NL, PT	2002–2022	Quarterly	Eurostat: GDP and main components
Population	AT, DE, EA, EL, ES, IE, IT, NL, PT	2002–2021	Annually	Eurostat: Population on 1 January by age and sex

Using population data, consumption, employment, and output values are converted into per capita values.[11] Measures for low- and high-volatility EMU members are constructed as follows.

Consumption: In order to aggregate the country-specific values into a measure for consumption of low- and high-volatility EMU members, we calculate the (consumption-)weighted average per capita consumption of the three low- and five high-volatility countries.

Employment: We weight the per capita employment values with relative GDP in order to calculate the aggregate measures for the low- and high-volatility countries. As data for DE is not available for the entire time period, the employment data for the low-volatility countries is based on AT and NL.

Price levels: We use the monthly HICP index at the beginning of each quarter for each country. Following the ECB, we calculate the aggregate price level of low- and high-volatility countries by weighting the country-specific price levels with the relative consumption of each country respectively (see Footnote 1).

Interest rates: Interest rates apply EMU-wide and are reported on a monthly basis. We use the interest rate at the beginning of each quarter. Until 2004Q3, we use the ECB's interest rate for main refinancing operations (MRO rate) as the policy rate. Starting in 2004Q4 (due to data availability), we utilize the shadow rate, as in Wu and Xia (2020). The shadow rate is useful as it accounts

[11] We use population data from 2021 for the first quarter of 2022, as population data for 2022 is not yet available.

for unconventional monetary policy measures, specifically for quantitative easing. Hence, including the shadow rate allows us to study an uninterrupted time series. Furthermore, it ensures a comparable measure for monetary policy in the data as well as in the simulated model results.

Output: We calculate the aggregate output measure for the low- and high-volatility countries by weighting their individual GDP per capita with their relative total GDP. In addition, we use EMU-wide GDP per capita in our analysis.

Our New Keynesian model reports percentage deviations from steady state. As the data generated from this model is used to train the machine learning algorithms, it is necessary to transform the EMU data set into percentage deviations from steady state as well. Therefore, the entire data set (except for the interest rates) are transformed into logs and we utilize a Hamilton (2018) filter (lag length $p = 4$, forecast horizon $h = 8$) in order to extract the cyclical component of each variable in our data set. Figure 3 provides an overview of the transformed macroeconomic variables. As expected, the macroeconomic indicators

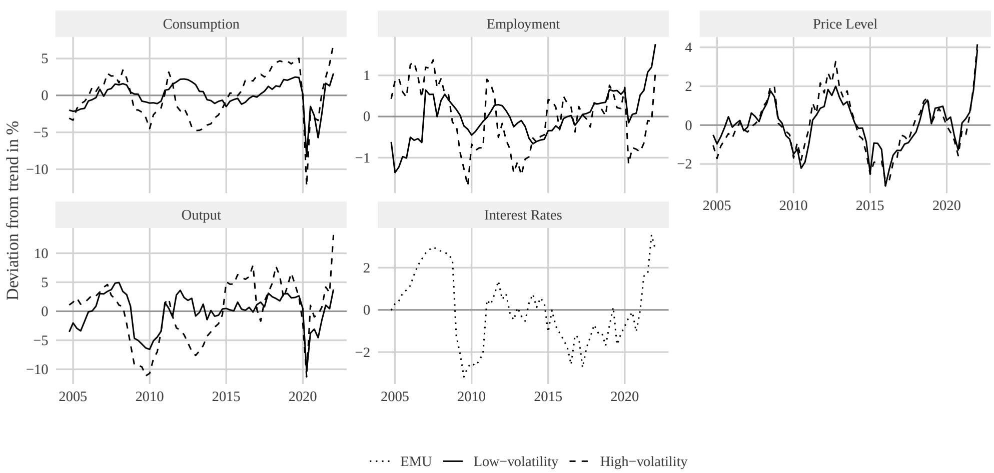

Figure 3 : Hamilton-Filtered Data of Aggregated Low-Volatility, High-Volatility, and Euro Area Variables.

show greater variance in the high-volatility countries than in the low-volatility ones. In particular, the aggregated inflation rate in the high-volatility countries deviates more strongly from its trend than its low-volatility counterpart, as ex-

Table 4: Comparison of Simulated Moments with Data.

Variable	Description	$\omega_\pi = \frac{C^H_{SS}}{C^H_{SS}+C^L_{SS}}$	$\omega_\pi = 0.8$	$\omega_\pi = 0.2$	Data
C^H_{SS}/C^L_{SS}	Relative consumption per capita H, L	0.962	0.962	0.962	0.805
$Y_{H,SS}/Y_{L,SS}$	Relative GDP per capita H, L	0.980	0.980	0.980	0.773
$\sigma(\hat{y}_{L,t})/\sigma(\hat{y}_{H,t})$	Relative volatility GDP L, H	0.779	0.773	0.783	0.587
$\sigma(\hat{y}_t)/\sigma(\hat{y}_{H,t})$	Relative volatility union-wide GDP, H	0.857	0.888	0.862	0.671
$\sigma(\hat{y}_t)/\sigma(\hat{y}_{L,t})$	Relative volatility union-wide GDP, L	1.010	1.149	1.010	1.144
$\sigma(\hat{c}^L_t)/\sigma(\hat{c}^H_t)$	Relative volatility consumption L, H	0.152	0.149	0.158	0.559
$\sigma(\hat{n}^L_t)/\sigma(\hat{n}^H_t)$	Relative volatility labor L, H	0.779	0.773	0.783	0.718
$\sigma(\hat{\pi}^{C,L}_t)/\sigma(\hat{\pi}^{C,H}_t)$	Relative volatility inflation L, H	0.913	0.921	0.904	0.842
$\rho(\hat{y}_{L,t}, \hat{y}_{H,t})$	Correlation GDP L, H	0.859	0.844	0.871	0.591
$\rho(\hat{\pi}^{C,L}_t, \hat{\pi}^{C,H}_t)$	Correlation inflation L, H	0.931	0.990	0.991	0.989
$\rho(\hat{c}^L_t, \hat{c}^H_t)$	Correlation consumption L, H	0.603	0.536	0.640	0.636
$\rho(\hat{n}^L_t, \hat{n}^H_t)$	Correlation labor L, H	0.859	0.844	0.871	0.132
$\rho(\hat{n}^H_t, \hat{c}^H_t)$	Correlation labor, consumption H	0.943	0.942	0.944	0.627
$\rho(\hat{n}^L_t, \hat{c}^L_t)$	Correlation labor, consumption L	0.482	0.437	0.513	0.466

Note: $\hat{x}_t$ denotes the deviation of a variable X from its zero inflation steady state.

pected from the examination of the descriptive data presented in Section II.[12]

B. Model Fit

We simulate 10,000 periods with random demand (preference, Z^k_t) and supply (cost-push, A^k_t) shocks, corresponding to 2,500 years of observations. In order to create perfect counterfactual data, we draw the identical shocks for each of our three baseline models, thus generating a synthetic data set in which only the latent inflation weight varies. Table 4 reports the simulated moments generated by the model compared to the respective moments calculated from the historical data.

Overall, all model specifications match the moments of the actual data reasonably well. While our model understates (steady state) inequality in consumption and GDP, it replicates the fact that low-volatility countries produce and consume more than high-volatility countries do. Furthermore, the model replicates higher volatility across all variables in the high-volatility countries compared to their low-volatility counterparts, i.e., the relative volatility of all model variables is smaller than one. This property can particularly be ascribed to the differences in the impact of the shocks ($\eta^H_Z > \eta^L_Z$ and $\eta^H_A > \eta^L_A$), implying that the assumption

[12] Note that we use deviations from the trend inflation rate instead of deviations from the 2% target of the ECB for the following reasons: (i) the policy rule in the New Keynesian model is defined in deviations from its trend/steady state, (ii) uncertainty remains regarding the exact magnitude of the ECB's inflation target (at least until the announcement of the exact 2% target in July 2021), (iii) it would be difficult to justify a 2% target for each individual country, and (iv) it is unlikely to make a difference since the trend is very close to 2%.

with respect to the values of these parameters seem reasonable.[13] Furthermore, despite focusing on only eight EMU countries, the model replicates the fact that EMU-wide output fluctuates more (less) than the output of low-volatility (high-volatility) members. Finally, we find that the correlations between variables in EMU data and our model are both qualitatively and quantitatively similar. Thus, our model appears to fit both the direction and magnitude of correlations between macroeconomic variables in the EMU. In particular, it replicates the stronger correlation between labor and consumption in high-volatility states. Naturally, the model overstates the strength of this relationship as we abstract from other sources of income apart from work as well as from additional inputs in production (such as capital). We address this question in our robustness checks in Section VII.

V. Machine Learning Algorithms

This section introduces the algorithms considered for our analysis. Specifically, we compare the performance of twelve machine learning algorithms in a horserace-style assessment, subsequently choosing the one with the greatest out-of-sample prediction performance.[14] All models adhere to the following structure

$$y_t = h_\beta(X_t) + \epsilon_t, \tag{27}$$

where $y \in (\omega_H, \omega_L, \omega_C)$ are the categorical inflation weights for low-volatility (L), high-volatility (H) and consumption-weighted average (C) at time t. $h(\cdot)$ is a function with coefficients β that maps the simulated macroeconomic variables X to the inflation weights and ϵ is the residual. We provide only a brief overview of the models, as well as their most essential parameterization. A more comprehensive review can be found in Chakraborty and Joseph (2017).

A. (Quasi-) Linear Model

Multinomial logistic regression: Our benchmark is a linear multinomial logistic regression (MLR) model. MLRs are logistic regressions for a categorical dependent variable with $k \in K$ categories. Explicitly, the following probabilities are being estimated:

$$P(Y_t = k) = \frac{e^{\beta_k \times X_t}}{1 + \sum_{k=1}^{K-1} e^{\beta_k \times X_t}}. \tag{28}$$

[13] We are aware that further structural differences between the countries might cause the reported differences in the development of the macroeconomic variables. However, deciding on which structural differences between the countries to include in the model seems arbitrary due to the sheer amount of possibilities. We therefore decide to rely on differences in the structural parameters of the model.

[14] Note that none of the variables and parameters used in this section coincide with the ones defined in Section III.

Penalized linear regression: An alternative to MLRs are penalized linear regressions, which (in linear models) are primarily used to reduce dimensionality. Although our regression model does not feature many dimensions (10 covariates), introducing constraints to the complexity of the linear model through regularization may nevertheless improve predictive performance. The general form for j covariates can be described as follows

$$L = \sum_{t=1}^{T} \epsilon_t^2 + \lambda \sum_{j=1}^{n} \left[(1-\alpha)|\beta_j| + \alpha|\beta_j|^2\right], \tag{29}$$

where L represents the loss function being optimized, λ is called intensity, and α determines the type of regularization. In particular, we employ a Lasso regression model ($\alpha = 0$), an elastic net ($0 < \alpha < 1$) and a ridge regression model ($\alpha = 1$). For all three regularized regression methods, we optimize $\lambda \in [10^{-2}, 10^{-4}]$ using cross fold validation (e.g., Chakraborty and Joseph, 2017).

KNN: K-Nearest-Neighbor (KNN) is a supervised algorithm that classifies the dependent variable based on a majority vote of the nearest neighbours with respect to the independent variables. In the most basic instance (k = 1), the inflation weight at time t predicted is simply the inflation weight of the single nearest neighbor. We optimize the KNN algorithm over $k \in [1, 100]$.

B. *Tree-Based Model*

Decision tree: Tree-based algorithms are a supervised machine learning methods that divide data into subsets using a series of *if-else* rules. Each additional division (layer) increases the complexity of these models, allowing for more precise and distinct subsets, and thereby predictions. In practice, tree-based models perform very well out-of-box (e.g., Boehmke and Greenwell, 2019) and are straightforward to interpret, given the possibility to provide a schematic representation for a particular tree model. The decision tree *grows* using the following loss function:

$$L_\gamma = -\sum_{y=1}^{Y} p(y|X)\log(p(y|X)) + \gamma|T|. \tag{30}$$

$p(y|X)$ represents the fraction of observations with the specific inflation weight conditioned on the macroeconomic information and T being the number of nodes in the decision tree. γ controls the regularization in a similar manner to λ for penalized regressions, which, in the case of a decision tree, is set to zero. Instead, following the literature, we specify a range of cut-of losses (L_γ), used to determine the complexity of the tree.

Prune tree: Pruning a tree is equivalent to optimizing L_γ over γ. As trees tend to overfit, pruning a tree is a good method of improving the predictive performance.

Random forest: A random forest is a classification algorithm that uses r decision trees to classify data. All decision trees are randomly generated and produce one single prediction for the classification task. The random forest's classification is determined by the class with the most votes. We use 1000 trees ($r = 1000$).

C. Neural Network

A neural network (NN) is a supervised machine learning algorithm modeled on the functioning of the human brain. A network consists of $i \in I$ layers, where each layer has k so-called perceptrons. Every NN consists of - at least - three layers: an input layer (the covariates), a hidden layer and an output layer (the prediction of the NN). An illustrative example is provided in Figure 4. Except for the first layer, the input for each layer is the dot product of a weighting matrix W_i and the output of the previous layer X_{i-1} plus a bias b_i. The output for each layer is then passed through a non-linear activation function $f(\cdot)$:

$$X_i = f(W_i \times X_{i-1} + b_i) \tag{31}$$

The two functional forms of $f(\cdot)$ applied in this paper are rectified linear unit (ReLU) activations and softmax activations. They can be expressed as follows, with x representing the activations input:

$$f(x) = max(0, x) \qquad \text{ReLu} \tag{32}$$

$$f(x) = \frac{e^{x_k}}{\sum_{k=1}^{K} e^{x_k}} \qquad \text{Softmax} \tag{33}$$

During the training process, the NN optimizes W_i and b_i, in order to perform well on a given classification task using an iterative optimization algorithm called stochastic gradient descent (see, e.g., Athey, 2019). We initiate our training with a simple four-layer NN ($I \equiv [1, 4]$), as shown in Figure 4. As a result our inflation weight y is the outcome X_4. With the exception of the output layer where a softmax function is used to obtain a probabilistic distribution over the classification task, we rely on ReLu activation functions. The network is trained

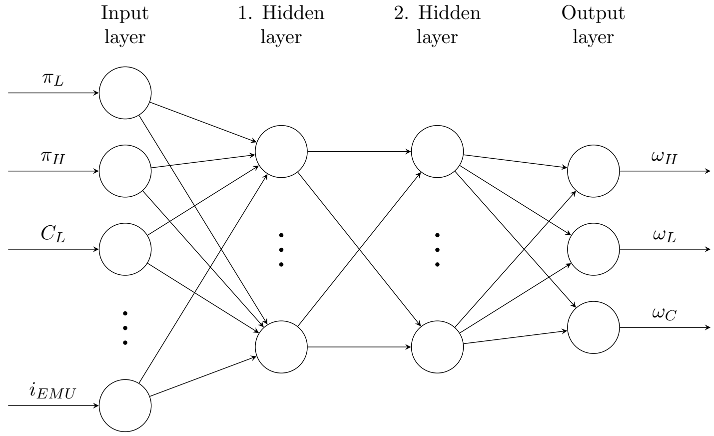

Figure 4 : Illustration of NN.

Notes: This figure illustrates the model architecture of a feed-forward NN with four layers: One input layer, two hidden layers, and an output layer. The connections between the layers represent the weighting matrix W_i and are adjusted during the training process.

on 500 iterations, with 20% of the data set serving as validation.[15]

D. Evaluation

In this subsection, we apply the machine learning models introduced in the previous subsection to our synthetic data set to evaluate their relative performance. The aim is to determine the algorithm that best predicts the underlying inflation weight (ω_π) provided by the New Keynesian model. In order to do so, we simulate the New Keynesian model from Section III for three specifications, namely $\omega_C \equiv \omega_\pi = C^H_{SS}/(C^H_{SS} + C^L_{SS})$ (C), $\omega_H \equiv \omega_\pi = 0.8$ (H), and $\omega_L \equiv \omega_\pi = 0.2$ (L) for 10,000 periods each. We randomly split this data set into a training set (75%) and a test set (25%). In order to maintain a balanced data set, we employ stratification on the training data based on our inflation weight. This ensures that the distribution of classes remains consistent between the training and test sets. The training data is used to parameterize the machine learning algorithms and the test set is used to evaluate the models out-of-sample. The results are presented in Table 5.

There are several noteworthy observations. First, linear models do not perform particularly well, at best marginally better than an uninformed guess. Second, using tree-based models yields an improvement in performance. However, with an accuracy of below 50%, single tree-based models still perform rather poorly.

[15]We choose 32 nodes per hidden layer, which in itself is a relatively small number. It is worth noting that even a simple model like this has over 1500 trainable parameters. In an unreported test, we increase the depth, width, and number of iterations. While we notice a slight improvement, we refrain from evaluating more complicated models due to the strong relative performance of this simple NN.

Table 5: Evaluation on Test Set.

	Accuracy
Uninformed guess	0.33
MLR	0.34
Ridge regression	0.33
Lasso regression	0.33
Elastic net	0.33
K-nearest-neighbor	0.38
Decision tree	0.48
Complex tree	0.48
Prune tree	0.48
Prune complex tree	0.48
Random forest	0.67
Neural network	**0.97**

Note: Accuracy of the machine learning algorithms introduced in Section V in classifying the true inflation weight $\omega_{\pi,t}$ from the simulated data set introduced in section III. Accuracy can be calculated by the fraction of correct predictions over all predictions.

Table 6: Confusion Matrix of Out-of-Sample Prediction by NN.

		True label		
		Consumption	Low-Volatility	High-Volatility
Prediction	Consumption	2405	50	39
	Low-Volatility	48	2443	9
	High-Volatility	47	7	2452

Despite exhibiting relatively superior performance, the random forest model still falls short of our expectations, with an accuracy of only 2/3. [16]

Finally, the NN – despite its simple structure – outperforms the other algorithms by a huge margin. With an accuracy of $> 97\%$, the NN successfully predicts the correct weight around 50% more often than the second best algorithm. Given the black-box nature of machine learning models, it is only possible to speculate as to the underlying causes of the NNs superior performance. Nevertheless, it is important to emphasise that the differences in the simulated data sets depend on the interaction between all variables, a task for which a high-dimensional classifier such as a NN appears to be better equipped than its competitors.

[16] As part of our analysis, we have experimented with diverse model specifications, including variations in the number of trees and the number of variables randomly sampled as potential candidates at each split. We have also attempted to assess the impact of employing different implementations of the random forest model in R, specifically the `caret` and `randomForest` packages. Nonetheless, none of these alterations have yielded an improvement in model accuracy surpassing 67%.

The use of accuracy as an assessment metric involves trade-offs with respect to the informativeness of a model's shortcomings. For example, our model may predict ω_H disproportionately often, revealing a bias that may not be detected in an assessment based only on the accuracy metric. Despite the fact that we have a balanced data set by design, we assess whether our model suffers from biased predictions, which might invalidate the identification of the latent ω_π. As a result, we exhibit the NN's performance through a confusion matrix in Table 6. Each row in the table represents the networks' prediction of ω_π, while the columns reflect the true ω_π. For example, the NN predicted ω_L 2499 times. This forecast was correct 2443 times, while ω_C (ω_H) would have been the correct prediction in 48 (9) cases. Table 6 provides no evidence of bias, which can be further mitigated using alternative performance metrics such as $Recall = 0.97$ and $Precision = 0.97$.[17] Based on the results from our evaluation task and the unbiased predictive performance we are confident in adopting the NN as our primary predictor.

VI. Results

This section presents the results of our machine learning algorithm, applied to historical EMU data. Due to the constraint imposed by the Hamilton filter, the historical inflation weight is classified on a quarterly basis between 2004Q4 and 2022Q1.

A. Monetary Policy Regime Classification

Figure 5 shows the retrieved inflation weight classification and the development of several macroeconomic variables in the EMU.[18] There are several interesting results. First, our findings do not indicate a systematic focus of the ECB on the consumption-weighted, union-wide inflation rate. In fact, we find a disproportional emphasis on inflation rates experienced by high-volatility EMU members (i.e., $\omega_\pi = 0.8$), in 80% of the periods, whereas we find evidence of a balanced stance (ω_C) in only five quarters ($\sim 7\%$).

Second, it appears that the ECB is reacting more strongly to greater deviations of inflation rates from their long-term trend, which would imply a predominant inflation weight on H. This interpretation tallies with the fact that a higher weight on low-volatility countries occurs at times when the inflation rates of low-volatility countries exhibit stronger deviations from their long-term trend, particularly when considering the regime switches around 2010 and 2018. The fact that the (rare) ω_C classification occurs at times when the inflation rate deviations

[17]For this multi-label case, we define the two metrics in the following way with TP := true positive, FP := false positive, and FN := false negative: $Recall := \frac{1}{n}\sum_{i=1}^{n}\frac{TP_i}{TP_i+FP_i}$ and $Precision := \frac{1}{n}\sum_{i=1}^{n}\frac{TP_i}{TP_i+FN_i}$.

[18]We follow Hinterlang (2020) by incorporating a regime change only if it occurs over n quarters, choosing $n = 2$.

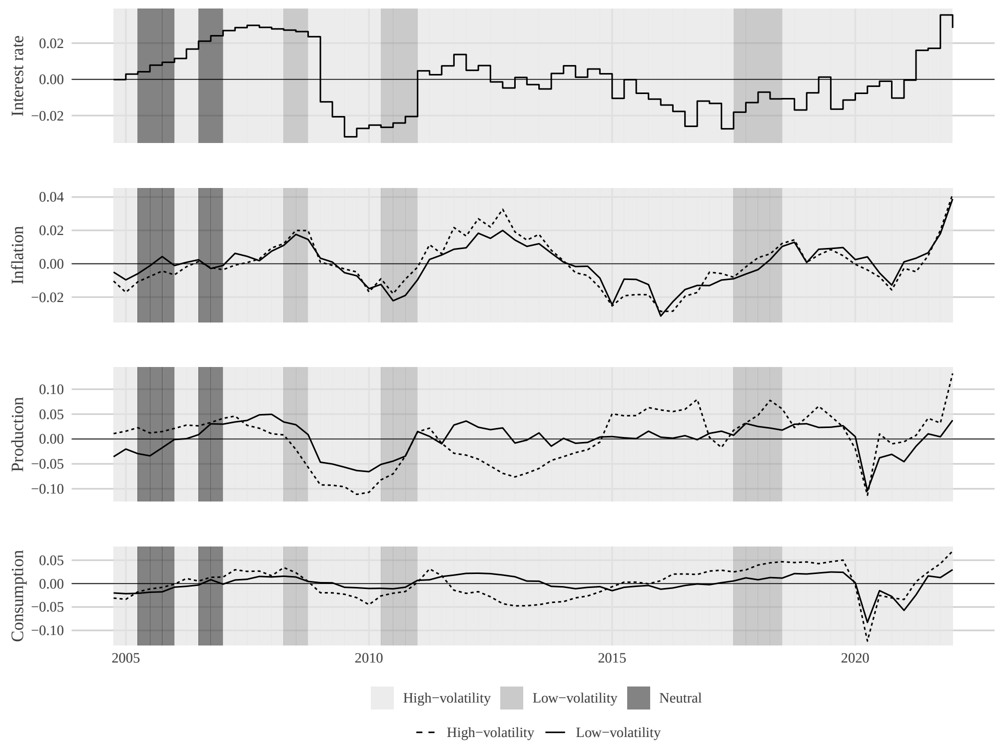

Figure 5 : ω_H, ω_L, ω_C Classification and Macroeconomic Indicators.

Notes: The shaded areas represent the inflation weight at the respective time, given by the NN classifier. The lines illustrate the deviation from the steady state for four macroeconomic variables in the EMU between 2004 and 2022. The deviations are derived as described in Section IV.

of low- and high-volatility countries are almost identical supports this interpretation. The period following the EMU debt crisis in 2012 provides further evidence. The high-volatility economies first experienced a relatively higher trend-adjusted inflation deviation, which prompted the central bank to implement tighter monetary policy. Around 2015, the relationship reverses: high-volatility countries experienced a stronger drop in inflation rates than low-volatility countries at a time when the monetary policy rate decreased substantially.

This interpretation also tallies with the arguments brought forward by Isabel Schnabel (2022) who pleads in favor of *"forceful action[s] by central banks"* in times of high inflation volatility.

B. On the ECB's Taylor Rule and Loss Function

In the following we attempt to align our observations to certain theoretical properties of the ECB's loss function, in particular, with regards to the way heterogeneous inflation rates enter the loss function. Generally, a standard loss function[19] depends on the (consumption-)weighted average of the inflation rates within the EMU:

$$\mathbb{L}_t = -\frac{1}{2}\left(\pi_t^{EMU}\right)^2 . \tag{34}$$

$\pi_t^{EMU} = \sum_{k=1}^{K} \omega^k \pi_t^k$ is defined as the EMU-wide inflation rate, $k \in [1, K]$ denotes the member state of the EMU, and π_t^k its respective inflation rate weighted by $\omega^k = C^k / \sum_{k=1}^{K} C^k$. The corresponding Taylor rule is given by:

$$i_t = \rho + \phi_\pi \pi_t^{EMU} . \tag{35}$$

However, our results indicate that the ECB reacts more strongly to countries whose inflation rates exhibit greater deviations from their long-term trend. Thus, we infer that the ECB's losses arise from individual deviations rather than from aggregated ones, i.e.,[20]

$$\mathbb{L}_t = -\frac{1}{2}\sum_{k=1}^{K} \omega^k \left(\pi_t^k\right)^2 , \tag{36}$$

leading to a Taylor rule given by:

$$i_t = \rho + \phi_\pi \left(\sum_{k=1}^{K} \Omega_t^k \pi_t^k\right) . \tag{37}$$

In particular, the ECB's loss occurs disproportionally from high-volatility members under this specification. A Taylor rule consistent with this loss function considers this fact in Ω_t^k given by

$$\Omega_t^k = \omega^k - \nu_t^k \left(|\pi_t^{EMU}| - |\pi_t^k|\right) . \tag{38}$$

This implies that the weight on the inflation rate of country k is adjusted according to its deviation from the EMU-wide inflation rate, for instance, when $|\pi_t^k| > |\pi_t^{EMU}|$ then $\Omega_t^k > \omega^k$.

[19]See Debortoli et al. (2019) for a comprehensive overview.

[20]Note that, from a theoretical perspective, both ways of aggregation might be consistent with maximizing union-wide utility. Furthermore, both loss functions (Equation (34) and Equation (36)) are consistent with the ECB's mandate.

C. Regression Model

To empirically test Equation (38), we need to account for variations in the inflation weight over time, i.e., Ω_t^k. Since the structure of NNs permits regression models, we proceed with a regression task that allows us to predict the inflation weight on a continuous scale.[21]

Compared to our earlier identification, we make two adjustments, one for the New Keynesian model as specified in Section III and one for the NN as introduced in Section V. First, we redefine the inflation weight in the Taylor rule as $\Omega_\pi \in [0.1, 0.9]$. This continuous scale is then used to recreate the synthetic data set. Practically, we simulate the New Keynesian model with nine Ω_πs in increments of 0.1. Second, to adapt the NN to the regression task, we remove the last softmax layer and adjust the loss function so that the mean square error is minimized.

Repeating the out-of-sample evaluation on 25% of the synthetic data set, the NN predicts the weighting exceptionally well, as evidenced by a mean average error (MAE) of about 4% and mean squared error (MSE) of 0.5%, [22] which makes us confident in the NN's performance. An illustration of the out-of-sample prediction can be found in the appendix in Figure A1.1.

Next, we obtain the weights from the predictions on the historical data set introduced in Section IV. We examine the distribution and the temporal variation of the estimated inflation weight before conducting a regression analysis.

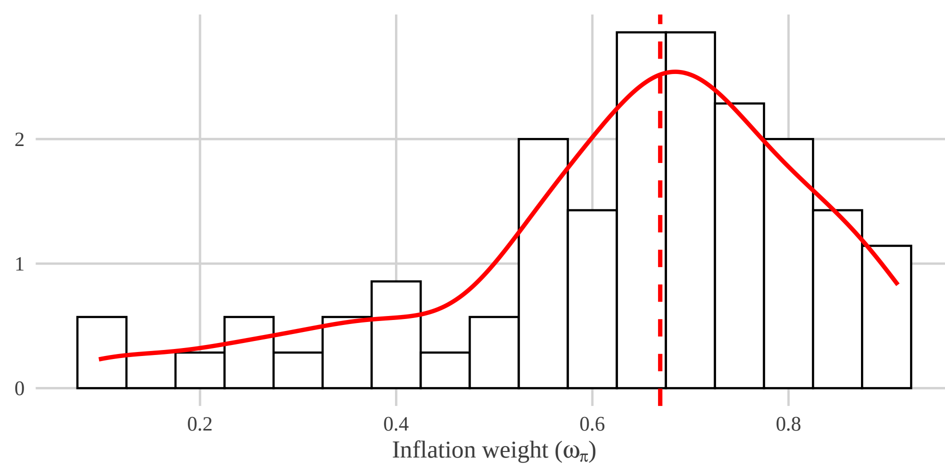

Figure 6 : Density Inflation Weight Prediction.

Figure 6 shows the density distribution of the inflation (solid line), its histogram,

[21] One might also consider this as a robustness check of our methodology.
[22] $MAE = \frac{1}{n}\sum \epsilon_t = 4\%$ and $MSE = \frac{1}{n}\sum \epsilon_t^2 = 0.5\%$

and its median weight over the period (0.67, dotted line). As expected, the inflation weight distribution is skewed to the right, implying that the estimated weight favors the high-volatility countries. This confirms our first finding that the ECB systematically emphasizes the inflation rate development of high-volatility countries.

Next, we examine the inflation weight in relation to the inflation deviation for both country blocks across time in Figure 7. The figure provides further evidence in support of our earlier interpretation that the ECB reacts more strongly to larger deviations of inflation rates from their long-term trend. Stronger deviations in inflation in L countries coincide with periods of higher weights on these countries (early 2005, 2011, 2017/18), while periods of higher weights on high-volatility countries (2012-2014 or 2016) correspond to high inflation deviations in the H countries.

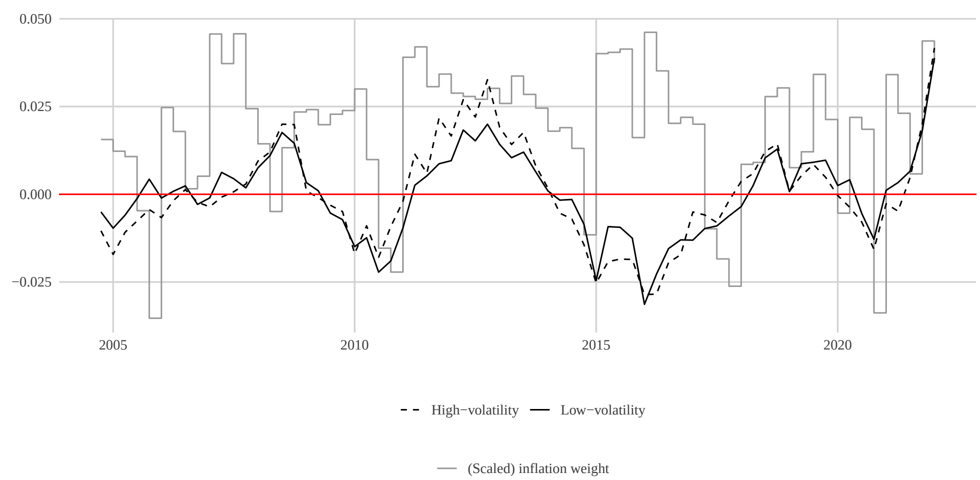

Figure 7 : Inflation Weight from Regression NN 2004Q4 - 2022Q1.

Notes: The deviation of inflation from its long-term trend for H and L countries (black dotted line and black straight line, respectively) from 2004Q4 to 2022Q1 is illustrated. The scaled inflation weight (grey stair step plot) is obtained by subtracting the consumption weight, i.e., 0.45, the result of which is divided by 10.

Finally, we test the aforementioned hypothesis empirically by running an OLS regression of the inflation weight (Ω_t^k) on the difference between the average (EMU) inflation rate and inflation rate in country L ($|\pi_t^{EMU}| - |\pi_t^L|$). This corresponds to the Equation (38). The result can be found in Table 7, column one.

The regression constant (0.62) can be interpreted as ω^k in Equation (38), i.e., as the inflation weight in the absence of inflation deviations. Statistically, the hypothesis of the inflation weight being equal to parity can be rejected with 99% confidence.

Table 7: Main Regression Results.

	Dependent variable:				
	Inflation weight $:= \Omega_t^k$				
	(1)	(2)	(3)	(4)	(5)
HICP ($= \nu_t^k$)	25.09***				24.06**
	(9.41)				(9.56)
Y		3.23**			3.59**
		(1.36)			(1.44)
C			−1.83		−2.90
			(2.60)		(2.73)
L				8.95	7.44
				(6.51)	(6.29)
Constant ($= \omega^k$)	0.62***	0.62***	0.64***	0.63***	0.62***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Observations	70	70	70	70	70
R^2	0.09	0.08	0.01	0.03	0.21
Adjusted R^2	0.08	0.06	−0.01	0.01	0.16

Note: *p<0.1; **p<0.05; ***p<0.01; Regression based on Equation (38); The RHS variables are defined as follows: $\Delta X := |X^{EMU}| - |X^L|$; the EMU average is defined as $X_t^{EMU} := \omega \times X_t^H + (1-\omega) \times X_t^L$ with ω set to 0.45, as this is the correct consumption weight as shown in Table 4. Regression results based on $\omega \neq 0.45$ are available upon request.

Next, the coefficient on HICP can be interpreted as ν_t^k. We find clear evidence that relatively higher deviations in L countries' inflation rates lowers Ω and vice versa. In particular, a 0.1 percentage point increase in the difference between L inflation and EMU inflation decreases the weight on L by 2.5 percentage points. The coefficient on the inflation deviation is statistically significant and economically relevant. A one standard deviation increase in the difference between L inflation and EMU inflation lowers the inflation weight on L by about 0.06 percentage points, which translates to around 1/3 of its standard deviation.[23]

Next, the remaining macroeconomic variables are included in univariate regressions in columns 2-4 and in a multivariate regression together with inflation in the last column. In all specifications we observe uniformity with respect to the intercept, i.e., ω^k. Moreover, only production appears to affect the inflation weight in a significant way. The size of the effect is somewhat smaller than the HICP effect, with a one standard deviation increase in the output difference towards

[23] $sd(HICP) = 0.0023$ and $sd(\omega) = 0.19$.

EMU output shifting the inflation weight by a quarter of a standard deviation.[24] The effect of inflation and output on ω^k persists when both are included into the regression jointly, as highlighted in the last column.

VII. Robustness Checks

In this section, we extend and modify our theoretical framework to test the robustness of our findings with respect to our modeling decisions. Moreover, we examine alternative hypotheses that may explain our empirical results. Across all tests, our findings remain robust and we thus find no reason to deviate from our previous conclusions.

A. Model Extension: Investment and Capital

In our baseline theoretical model, we opted for a small scale tractable model. In this section we test the impact of including additional variables such as investment and capital on our findings. Thus, we extend our model to closely follow a setup such as Gertler and Karadi (2011). While they consider a closed economy, we extend it to a monetary union model of two countries.[25] Consequently, we include capital, intermediate, and final good producers on the firms' side of the model. This allows us to include investment in the set of considered variables. We thus generate synthetic data sets for the classification as well as the regression task. The data set consists of all variables considered in the simple version of the model plus investment in both countries. A detailed overview of the model, its calibration and data fit, as well as the entire set of results can be found in Appendix A.A2.

We proceed by retraining the NN and evaluate the robustness of our outcomes. Notably, our findings remain unaffected by the inclusion of investment and capital. First, we check the robustness of the classification. Figure A2.1 shows that the classification of quarters into neutral, high-, or low-volatility regimes is robust to the model extension and the inclusion of additional variables, with 82% of the quarters being classified in the same way. Furthermore, we assess the prediction of the inflation weight (ω_k), and observe an upward shift in the average weight (from 0.63 to 0.70) and in the median weight (from 0.67 to 0.74), indicating that our main results constitute a lower bound. However, Figure A2.2 shows that the development of the estimated inflation weight in each quarter closely resembles our main findings. Next, we repeat the regression analysis of the inflation weight on the macroeconomic variables in a univariate and multivariate analysis based on Equation (38). We find that our findings exhibit no quantitative or qualitative changes. The comprehensive results are presented in Table A2.3.

[24] $sd(\Delta Y) = 0.0162$ and $sd(\omega) = 0.19$.

[25] A similar open economy model based on Gertler and Karadi (2011) can be found in Horst et al. (2020).

B. Taylor Parameter

Our findings indicate that the ECB reacts more strongly to countries exhibiting larger inflation deviations from their trend. This implies larger overall interest rate adjustments than suggested by the EMU-wide, average inflation rate. In principle, this result could be driven by our choice of the Taylor parameter ϕ_π. We test whether our findings are affected by ϕ_π by re-training the NN on synthetic data with a higher response parameter ($\phi_\pi = 2.5$). The results remain unchanged with the new classification closely resembling the previous classification with an accuracy of 90%.

C. Composition of the board

Previous literature on the ECB decision-making process found it to be both collegial (e.g., Ehrmann and Fratzscher, 2007) and driven by board members pursuing national objectives (e.g., Hayo and Méon, 2013). Hence, we examine whether the composition of the ECB board can explain our results, using the relative share of members from H countries as a proxy.[26] The hypothesis is tested in a regression both by itself and in conjunction with our main inflation variance hypothesis. The results are reported in the Appendix in Table A3.1. We find no significant effect of board composition on the inflation weight. Consequently, our finding indicates that national objectives by board members do not not significantly impact the ECB's inflation weights.

D. Inflation expectations

In our analysis, we adopt the classical Taylor Rule, as stipulated in the existing literature (e.g. Rabanal, 2009; Hinterlang and Hollmayr, 2022) as the basis for our monetary policy rule. However, we acknowledge that in reality the conduct of monetary policy is not solely based on contemporaneous inflation levels, as it is necessary to account for future price level projections in order to disentangle transitory effects on inflation and economic activity. In order to mitigate concerns regarding the selection of our monetary policy framework, we collect inflation data, as well as inflation forecasts from the Organisation for Economic Co-operation and Development (OECD) for all member states within the euro area. We find a correlation of 98% between both series (97% if we focus on the countries in our analysis), underscoring the robustness of our results vis-à-vis variations in the inflation data used.

[26]Considering the low variation across presidents as well as the very different economic conditions during presidencies, we focus on the composition of the board.

VIII. Conclusion

Combining DSGE models with machine learning methods facilitates the examination of various classification and regression exercises. In particular, this paper investigates whether the ECB conducted monetary policy according to the EMU-wide inflation rate between 2002 and 2022. We first show that the inflation rate development of EMU members differs substantially over time, with some countries exhibiting greater volatility in their inflation rates than others.

In order to investigate whether the ECB reacted to the EMU-wide inflation rate or more strongly to low- or high-volatility members, we generate data utilizing a New Keynesian model of a monetary union. We simulate a series of random demand and supply shocks for different monetary policy rules of the union-wide central bank: one where the central bank reacts to the average union-wide inflation rate, and a multitude of rules where the central bank reacts more strongly to the rate experienced in low- or high-volatility countries respectively. This synthetic data set is then used to train and evaluate several machine learning algorithms. We find that a neural network (NN) performs best out-of-sample, with an accuracy of 97%.

Using the NN, we first classify historical EMU data between 2002 and 2022. Our findings suggest a disproportional emphasis on the inflation rates experienced by EMU members that exhibited high inflation rate volatility for the vast majority of the time frame considered (80%). However, we find that there are several instances where a regime switch (from an emphasis on high-volatility countries to the weighted average or to low-volatility countries) takes place. This occurs especially in periods when the inflation rates of high-volatility countries have already moved back to their long-term trend while the inflation rates of low-volatility countries still exhibit deviations from theirs. We then show that these regime switches are related to a tendency of the ECB to react more strongly to the countries whose inflation rates deviate more from their long-term trend by utilizing the trained NN for a regression task. We find a median weight on high-volatility countries of 67%, particularly due to the ECB's stronger reaction to greater inflation deviations. Finally, our results allow us to infer that the loss function of the ECB depends on weighted, individual inflation rate deviations of EMU members rather than on the deviation of the weighted average inflation rate in the EMU.

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Appendix

A1. Regression NN model

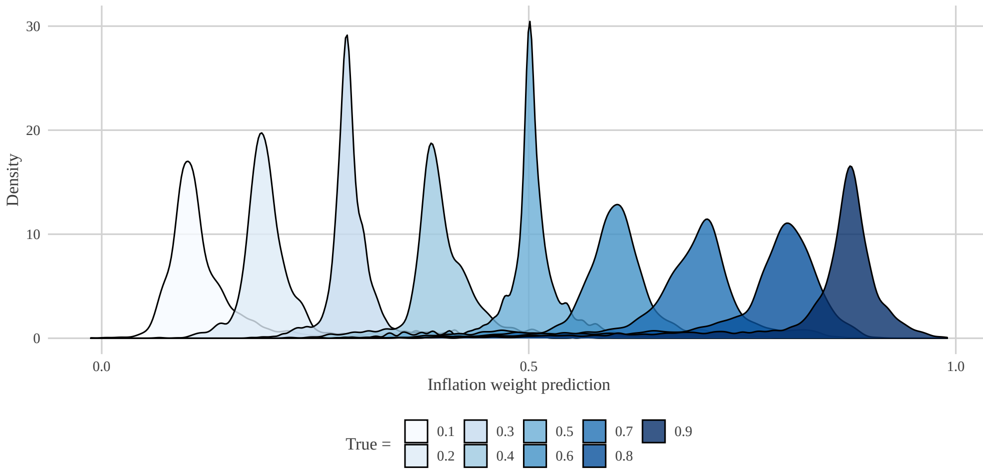

Figure A1.1 : Density Forecasts - Out-of-Sample Predictions.

A2. Model Extension: Investment and Capital

We keep the household section as well as the central banks' reaction function(s) unchanged. If not explicitly defined differently, all variables and parameter definitions from Section III hold. The remaining sections change in the following manner.

Intermediate goods producers

Intermediate goods producing firms are perfectly competitive and supply their goods solely on the domestic market. The production function is

$$Y^k_{m,t} = \left(K^k_{t-1,t}\right)^{\alpha_k} \left(N^k_t\right)^{1-\alpha_k} , \tag{A2.1}$$

with $K^k_{t-1,t}$ denoting capital chosen in $t-1$ and productive in t. The intermediate goods firm buys capital from a capital producer at price $Q_{k,t-1}$ and sells the depreciated capital back to the capital producer at $Q_{k,t} - \delta_k$ for refurbishment, with δ_k denoting the depreciation rate. The cost function is given by

$$TC_{k,t} = A^k_t \left(w_{k,t}N^k_t + Q_{k,t-1}K^k_{t-1,t} - (Q_{k,t} - \delta_k)K^k_{t-1,t}\right) \tag{A2.2}$$

Profit maximization then implies

$$Q_{k,t} = \alpha_k mc_{k,m,t+1} \frac{Y^k_{m,t+1}}{K^k_{t,t+1}} + (Q_{k,t+1} - \delta_k) , \tag{A2.3}$$

$$mc_{k,m,t} = \frac{w_{k,t}A^k_t}{(1-\alpha_k)\frac{Y^k_{m,t}}{N^k_t}} , \tag{A2.4}$$

where $mc_{k,m,t}$ is defined as the marginal costs of the intermediate goods firm.

Capital goods producers

Perfectly competitive capital producers repair depreciated capital and sell the refurbished capital to intermediate goods firms. Gross investment is thus given by

$$I^{gr,k}_t = I^k_t + \delta_k K^k_{t-1,t} , \tag{A2.5}$$

where I^k_t denotes newly created capital, i.e., net investment.
The law of motion for capital is

$$K^k_{t,t+1} = K^k_{t-1,t} + I^k_t . \tag{A2.6}$$

We assume unity capital production costs and capital adjustment costs of the form

$$f\left(\frac{I_t^k + I_{SS}}{I_{t-1}^k + I_{SS}}\right) = \frac{n_k}{2}\left(\frac{I_t^k + I_{SS}}{I_{t-1}^k + I_{SS}} - 1\right)^2 , \tag{A2.7}$$

where n_k captures the degree of capital adjustment costs and I_{SS} is steady state gross investment. From profit maximization follows

$$\begin{aligned} Q_{k,t} = 1 + \frac{n_k}{2}\left(\frac{I_t^k + I_{SS}}{I_{t-1}^k + I_{SS}} - 1\right)^2 + \frac{I_t^k + I_{SS}}{I_{t-1}^k + I_{SS}} n_k \left(\frac{I_t^k + I_{SS}}{I_{t-1}^k + I_{SS}} - 1\right) \\ - \mathbb{E}_t\, \beta \Lambda_{t,t+1}^k \left(\frac{I_{t+1}^k + I_{SS}}{I_t^k + I_{SS}}\right)^2 n_k \left(\frac{I_{t+1}^k + I_{SS}}{I_t^k + I_{SS}} - 1\right) . \end{aligned} \tag{A2.8}$$

FINAL GOODS PRODUCERS

The monopolistically competitive final goods producing firm simply repackages one unit of intermediate goods into one unit of final goods and sells them to households in both countries. Assuming Calvo (1983) price setting, the optimal price of a firm being able to adjust its price in t is

$$p_{k,t}^* = \mu \left(\frac{P_{k,t}}{P_t^{C,k}}\right)^{-1} \frac{x_{k,1,t}}{x_{k,2,t}} , \tag{A2.9}$$

where

$$x_{k,1,t} \equiv U_{c,t}^k Y_{k,t} mc_{k,m,t} + \beta\theta_k\, \mathbb{E}_t \left[\Pi_{k,t,t+1}^{\epsilon_k} x_{k,1,t+1}\right] ,$$
$$x_{k,2,t} \equiv U_{c,t}^k Y_{k,t} + \beta\theta_k\, \mathbb{E}_t \left[\Pi_{k,t,t+1}^{\epsilon_k - 1} x_{k,2,t+1}\right] .$$

EQUILIBRIUM

As before, bonds are in zero net supply and labor markets clear. Goods market clearing implies

$$Y_t^k = C_{k,t}^k + C_{k,t}^{-k} + I_t^{gr,k} + \frac{n_k}{2}\left(\frac{I_t^k + I_{SS}}{I_{t-1}^k + I_{SS}} - 1\right)^2 \left(I_t^k + I_{SS}\right) , \tag{A2.10}$$

as well as

$$Y_{m,t}^k = Y_t^k . \tag{A2.11}$$

Calibration

The calibration of most parameters remains unchanged. Parameters that were added by the extension are either chosen as in Albonico et al. (2019) (i.e., δ_k) or (re-)calibrated to match relative volatilities of output, consumption, inflation, and investment (n_k, η_Z^k, η_A^k). The corresponding model fit can be found in the following.

Table A2.1: Calibration: Model with Investment.

	Description	Value	
	Households		
		H	L
Ψ_k	Habit parameter	0.77	0.71
φ_k	Inverse Frisch elasticity	2.01	2.73
η_Z^k	Preference shock strength	1	0.5
γ_k	Weight of domestic goods	0.75	0.75
ϑ_C^k	Elasticity of substitution between domestic and foreign goods	1.42	1.50
ϵ	Price elasticity of demand	6	6
β	Discount rate	0.995	0.995
	Intermediate Goods Producers		
		H	L
α_k	Output elasticity labor	0.33	0.33
δ_k	Depreciation rate capital	0.0136	0.0143
η_A^k	Cost-push shock strength	1	0.75
	Capital Goods Producers		
		H	L
n_k	Capital adjustment costs	0.75	8
	Final Goods Producers		
		H	L
λ_k	Calvo parameter	0.737	0.852
	Central Bank		
ϕ_π	Taylor rule coefficient	1.5; 2.5	
ω_π	HICP inflation weight	$\frac{C_{SS}^H}{C_{SS}^H+C_{SS}^L}$; $[0.1, 0.9]$	

Investment data description

Quarterly data on investment is provided by Eurostat as gross fixed capital formation within GDP and its main components. We extract values for each of the considered countries within the considered time period and calculate the (GDP-)weighted average per capita investment for country group H and L, respectively.

Model Fit

We include the relative volatility as well as the correlation of investment into the set of compared moments. An overview of the model fit can be found in Table A2.2. Overall, we find that the extended model matches the moments in the data very well in almost all cases. The model fit is, unsurprisingly, better than the one of the simpler model. The moments of the newly added investment data are matched as well. We conclude that the model fit is a satisfactory basis for (re-)training the NN.

Table A2.2: Comparison of Simulated Moments with Data.

Variable	Description	$\omega_\pi = \frac{C_{SS}^H}{C_{SS}^H + C_{SS}^L}$	$\omega_\pi = 0.8$	$\omega_\pi = 0.2$	Data
C_{SS}^H / C_{SS}^L	Relative consumption per capita H, L	0.630	0.630	0.630	0.805
$Y_{H,SS} / Y_{L,SS}$	Relative GDP per capita H, L	0.796	0.796	0.796	0.773
$\sigma(\hat{y}_{L,t}) / \sigma(\hat{y}_{H,t})$	Relative volatility GDP L, H	0.460	0.508	0.446	0.587
$\sigma(\hat{y}_t) / \sigma(\hat{y}_{H,t})$	Relative volatility union-wide GDP, H	0.611	0.610	0.614	0.671
$\sigma(\hat{y}_t) / \sigma(\hat{y}_{L,t})$	Relative volatility union-wide GDP, L	1.330	1.201	1.380	1.144
$\sigma(\hat{c}_t^L) / \sigma(\hat{c}_t^H)$	Relative volatility consumption L, H	0.447	0.476	0.449	0.559
$\sigma(\hat{n}_t^L) / \sigma(\hat{n}_t^H)$	Relative volatility labor L, H	0.441	0.477	0.431	0.718
$\sigma\left(\hat{\pi}_t^{C,L}\right) / \sigma\left(\hat{\pi}_t^{C,H}\right)$	Relative volatility inflation L, H	0.803	0.833	0.790	0.842
$\sigma\left(\hat{i}_t^L\right) / \sigma\left(\hat{i}_t^H\right)$	Relative volatility investment L, H	0.650	0.706	0.623	0.354
$\rho(\hat{y}_{L,t}, \hat{y}_{H,t})$	Correlation GDP L, H	0.492	0.381	0.545	0.591
$\rho\left(\hat{\pi}_t^{C,L}, \hat{\pi}_t^{C,H}\right)$	Correlation inflation L, H	0.977	0.979	0.977	0.989
$\rho(\hat{c}_t^L, \hat{c}_t^H)$	Correlation consumption L, H	0.664	0.567	0.698	0.636
$\rho(\hat{n}_t^L, \hat{n}_t^H)$	Correlation labor L, H	0.480	0.353	0.537	0.132
$\rho(\hat{n}_t^H, \hat{c}_t^H)$	Correlation labor, consumption H	0.903	0.898	0.903	0.627
$\rho(\hat{n}_t^L, \hat{c}_t^L)$	Correlation labor, consumption L	0.04	−0.143	0.141	0.466
$\rho\left(\hat{i}_t^L, \hat{i}_t^H\right)$	Correlation investment L, H	0.508	0.572	0.478	0.408

Note: $\hat{x}_t$ denotes the deviation of a variable X from its zero inflation steady state.

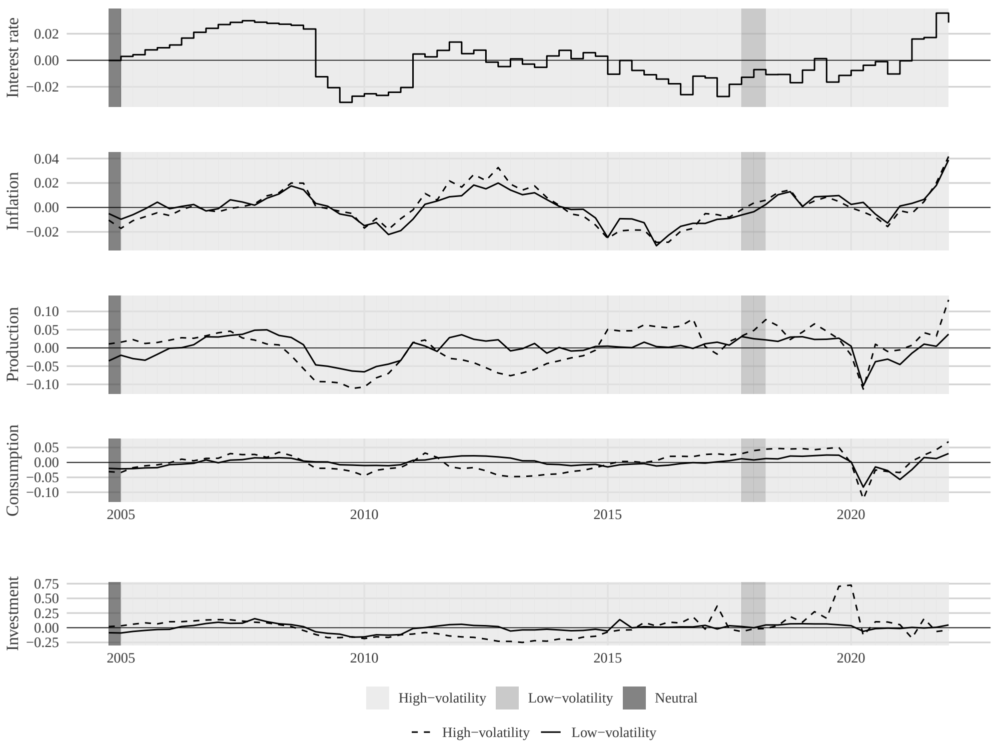

Figure A2.1 : Robustness Check 1: $\omega_H, \omega_L, \omega_C$ Classification and Macroeconomic Indicators.

Notes: The shaded areas represent the inflation weight at the respective time, given by the NN classifier. The lines illustrate the deviation from the steady state for four macroeconomic variables in the EMU between 2004 and 2022. The deviations are derived as described in Section IV.

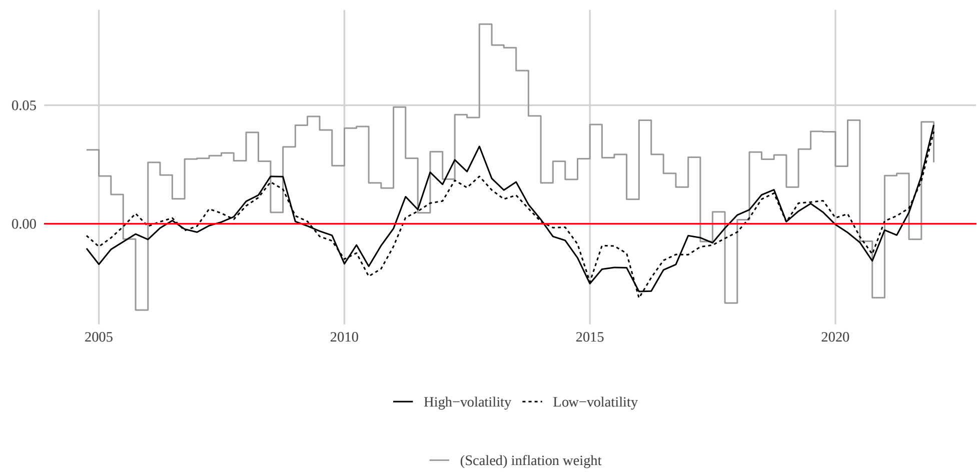

Figure A2.2 : Inflation Weight from Regression NN 2004Q4 - 2022Q1.

Notes: The deviation of inflation from its long-term trend for H and L countries (black dotted line and black straight line, respectively) from 2004Q4 to 2022Q1 is illustrated. The scaled inflation weight (grey stair step plot) is obtained by subtracting the consumption weight, i.e., 0.45, the result of which is divided by 10.

Table A2.3: Robustness Check 1: Investment and Capital

	Dependent variable:					
	Inflation weight := Ω_t^k					
	(1)	(2)	(3)	(4)	(5)	(6)
HICP	22.97**					27.33**
	(10.99)					(10.41)
Y		4.36***				3.56**
		(1.54)				(1.55)
C			−0.14			−2.39
			(2.99)			(2.93)
L				24.24***		20.47***
				(6.97)		(6.89)
I					0.70*	0.53
					(0.42)	(0.38)
Constant (= ω^k)	0.74***	0.74***	0.76***	0.76***	0.74***	0.72***
	(0.03)	(0.03)	(0.03)	(0.02)	(0.03)	(0.03)
Observations	70	70	70	70	70	70
R^2	0.06	0.11	0.0000	0.15	0.04	0.32
Adjusted R^2	0.05	0.09	−0.01	0.14	0.03	0.26

Note: *p<0.1; **p<0.05; ***p<0.01; Regression based on Equation (38); The RHS variables are defined as follows: $\Delta X := |X^{EMU}| - |X^L|$; the EMU average is defined as $X_t^{EMU} := \omega \times X_t^H + (1-\omega) \times X_t^L$ with ω set to 0.45, as this is the correct consumption weight as shown in Table 4. Regression results based on $\omega \neq 0.45$ are available upon request.

A3. Composition of the board

Table A3.1: Regression Results: Political Economy.

	Dependent variable:	
	Inflation weight := Ω_t^k	
	(1)	(2)
HICP ($= \nu_t^k$)		26.2^{***}
		(9.6)
Y		3.2^{**}
		(1.5)
C		-3.3
		(2.7)
L		6.9
		(6.3)
$\frac{HV}{LV+HV}$	-0.5	-0.6
	(0.4)	(0.4)
Constant ($= \omega^k$)	1.0^{***}	1.0^{***}
	(0.3)	(0.3)
Observations	70	70
R^2	0.02	0.2
Adjusted R^2	0.01	0.2

Note: *p<0.1; **p<0.05; ***p<0.01; Regression based on Equation (38); The RHS variables are defined as follows: $\Delta X := |X_{EMU}| - |X_L|$; the EMU average is defined as $X_t^{EMU} := \omega \times X_t^H + (1-\omega) \times X_t^L$ with ω set to 0.45. Regression results based on $\omega \neq 0.45$ are available upon request.