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Stephan Müller and Georg von Wangenheim

The impact of market innovations on the evolution of norms: the sustainability case

Coordination: Bernd Hayo • Philipps-University Marburg
Faculty of Business Administration and Economics • Universitätsstraße 24, D-35032 Marburg
Tel: +49-6421-2823091, Fax: +49-6421-2823088, e-mail: hayo@wiwi.uni-marburg.de

The impact of market innovations on the evolution of norms: the sustainability case.

Stephan Müller, Georg von Wangenheim*

Abstract

That institutions matter is widely accepted among economists and so are social norms as an important category of informal institutions. Social norms matter in many economic situations, but in particular for markets. The economic literature has studied the interrelation between markets and social norms in both directions – how social norms affect markets and how markets affect social norms. Starting from these two perspectives, we add to the literature, by suggesting a new link between product markets and the evolution of social norms: we analyze how the evolution of a social norm may be affected by a product innovation which adds to the variation of products with respect to their level of norm compliance. We derive necessary and sufficient conditions for a) a positive impact of the innovation on the level of norm adoption and b) for multiplicity of norm equilibria. Finally we discuss policy implications.

Keywords: Consumer Behavior – Social Norms – Evolutionary Economics – Sustainability – Innovation

JEL Classifications: A13; D02, D11, Q01, Q55

* Stephan Müller (corresponding author): Göttingen University, Platz der Göttinger Sieben, 3, 37073 Göttingen, Germany and Georg v. Wangenheim: Kassel University, Nora-Platiel-Straße 4, 34109 Kassel, Germany (email stephan.mueller@wiwi.uni-goettingen.de and g.wangenheim@uni-kassel.de)

1. Introduction

That institutions matter is widely accepted among economists and so are social norms as an important category of informal institutions (see e.g. Huck et al. 2012, Saak 2012, Elster 1989, Cole et al. 1992, Lindbeck et al. 1999). Social norms matter in many economic situations, but in particular for markets. The economic literature has studied the interrelation between markets and social norms in both directions – how social norms affect markets and how markets affect social norms.

Not the least because of an increased interest in "sustainable consumption" the impact of social norms on market outcomes has gained attention in national and international agendas (e.g. Heap and Kent 2000, UN 2002). With respect to theory, there are various attempts to incorporate norm-motivated behavior into neoclassical consumer theory (see e.g. Nyborg et al. 2006, Brekke et al. 2003). Another branch of the literature treats social norms as a prerequisite for working market systems (e.g. Platteau 1994)[1]. However there is no general or partial equilibrium theory based on norm-motivated behavior[2]. This may be the reason for why most research in the field is empirical. Hong and Kacperczyk (2009) and Johnson (2004) study the impact of norms on financial markets. Kim (2007) finds support for the relevance of norms for the market pricing of private property rights. A series of competitive-market and bilateral-bargaining experiments carried out by Fehr et al. (1998) indicate that competition has a rather limited effect on market outcomes if the norm of reciprocity is operative. The impact of a preference to keep a positive self-image as a morally responsible person on the demand for "green" electricity is studied by Ek and Soderholm (2008). Using evidence from Central Kenya, Johnson (2004) develops a framework for the relation of gender norms and financial markets, i.e. the demand and access to financial services.

From the opposite perspective, the research on the impact of markets on the (evolution of) norms primarily deals with the analysis of the relationship of norm-driven intrinsic motives and market- or price-driven extrinsic motives. Fehr and Gächter (2001) provide empirical support for incentive contracts crowding out reciprocity-driven voluntary cooperation. In a similar vein, Gneezy and Rustichini (2000) present results of a field study that contradict any deterrence hypothesis. A first survey of this stream of empirical literature on motivation crowding-out effects is given by Frey and Jegen (2001). With respect to theory, Benabou and Tirole (2006) provide a theory of pro-social behavior where rewards or punishments create doubt about the true motives for which good deeds are performed and hence may lead to partial or even total crowding-out of pro-social behavior. Huck et al. (2012) provide a model of the interplay of social norms and economic incentives in a firm in which crowding-out of social incentives may occur. Bohnet et al. (2001) study the connection between contract enforceability and individual performance, both theoretically and in the laboratory. They find that trustworthiness is "crowded in" with weak and "crowded out" with medium enforcement. All approaches are limited to

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[1] For a normative theory of social norms in market economies see Bergsten (1985).

[2] For a discussion of an extension of Walrasian economics by social norms and psychological dispositions see Bowles and Gintis (2000). For a multi-agent simulation model on the psychological factors like need for identity on market dynamics see Janssen and Jager (2001).

monetary incentives provided by markets. The interplay od social norms and another important dimension of markets, that of product variety generated in the course of innovation, is missing.

To see the potential interdependence consider a market where at the pre-innovation stage the individual characteristic of having adopted a specific norm is not observable, neither by observation of the individual itself or its general behavior, nor by observation of its consumption behavior. Obviously, the latter presupposes that products or services fail to differ with respect to their norm compliance. If a new product or service which is characterized by a relatively high degree of norm compliance enters this scene, this has two effects on the process of norm adoption. First, the innovation allows an individual to consume in accordance to its norm. Thereby it directly facilitates the adoption of the norm by reducing potential cognitive dissonances that would occur if a norm adopter consumes in contradiction to his norm. We call this the *cognitive bias*. Second, although the innovation enables an individual to consume in a norm-compatible way, it will also expose him to social influence (Cialdini and Goldstein 2004), in particular to the *conformity bias* (Boyd and Richerson, 1985). The consumption of the (old) norm-violating product and of the (new) norm-complying product will hence become the more attractive, the more other individuals still, or already consume the respective product. We therefore address the link between product innovations and the evolution of social norms. More precisely we analyze how the evolution of a social norm may be affected by a product innovation which adds to the variation of products with respect to their level of norm compliance.

This link between the process of norm adoption and the market may only be relevant, if the product or service of concern is sufficiently important for individuals in terms of time spent with it, money spent on it, utility drawn from it, social status connected to it etc., since otherwise cognitive dissonances would be too weak to have a major impact. For our analysis, we therefore take *e-mobility* as the innovation and *sustainable transportation* as the norm. In 2010 German private households spent around two third of their income on the following four categories: accommodation, water, electricity, gas and other fuels (30.8%), transportation (13.2%[3]), leisure, entertainment and culture (11.6%) and food including non-alcoholic beverages (10.4%). Of these four categories essentially only expenditures for transportation and food can reflect the attitude towards sustainable consumption in an observable way for others.[4] According to an extensive study on the mobility in Germany conducted by the *infas* Institute for Applied Social Sciences and the *DLR* German Aerospace Centre in 2008 (MiD 2008, p.21) a mobile person on average spent 1,5 h a day on traveling excluding regular travel time associated with the job, e.g. bus-driver. Almost 60 % of that time, i.e. about 54 minutes are assigned to private individual transportation. In summary, the car is expensive, important, omnipresent, relevant for sustainable consumption and therefore a product with a high potential for a conformity bias and cognitive dissonances for norm adopters. However our analysis is not limited to this case. Three other examples shall illustrate the wider relevance of our approach, two in which the innovation already took place and one where it hasn't yet. Consider first the technological innovation of *social networks* based on internet services like Facebook or Twitter etc. and the norm *share yourself* (opinions, activities etc) in opposition to the norm *protect your privacy*. Prior to the innovation, individuals willing to share

[3] More than 85% of these expenditures are spent on private transportation.
[4] Exceptions are things like solar panels for the accommodation category or the attendance of a pro-environmental concert.

their lives with a wide public and gather information about others could not live in accordance to their norm. However, privacy-loving individuals were able to conceal most information about themselves. *Protect your privacy* was the prevalent norm in many countries. When internet services like Facebook or Twitter etc. entered the market, individuals who did never share this norm became able to, and actually started to live according to their norm *share yourself*. The innovation has entailed a complete reversal of the social norm. The second example is the innovation of *ecological food* and the norm of *sustainable and healthy consumption*. Today almost all big supermarket chains include ecological food in their shelves, most of them even with own brands. With this innovation people concerned with sustainability, health but also with the conditions of livestock breeding can live in accordance to this norm and have become a large minority. Finally, consider a not yet invented *weapon* that can only be used for defensive purposes and the norm of (bourgeois) *pacifism*.

To make our argument precise, we proceed as follows in the remainder of the paper. In Section 2 we introduce the model. Assumptions and notation are presented in 2.1. In 2.2 we first derive the market equilibrium for a given share of norm adopters and a given number of firms operating on the innovative and on the traditional markets and then deduce the equilibrium number of firms supplying on the innovative market. We turn to studying the dynamics of norm adoption in 2.3. Results are summarized in Section 3. Policy implications are discussed in Section 4 and Section 5 concludes.

2. The Model

We consider a market where the demand side is characterized by a large number of consumers, who differ only with respect to their having adopted a particular consumption-related norm. The commodity traded on the market may occur in two specifications, one in compliance with the norm and one in violation thereof. We base our argument on a specific example, the market for automobiles and the norm of sustainable transportation, with which electrically powered cars as the norm-compliant variant and gasoline-powered cars as the norm-violating variant. However, as we have already argued in the introduction, the argument extends to other examples as well.

To make identification of the two consumer groups easy, we call those consumers who have adopted the norm *adopters* and those who did not, *hedonists*. $t \in \{a, h\}$ identifies the type of consumers in the natural way, while $v \in \{e, g\}$ identifies the variant of the norm-compliant (electrically powered) and, respectively, the norm-violating (gasoline-powered) variant of the commodity automobiles. Both variants of the commodity are imperfect substitutes to each other and the slopes of demand curves as well as substitutability are assumed to be independent of the type of the consumer for simplicity. With the simplification of linearity, and p^e and p^g denoting the prices of electrically powered and gasoline powered cars, respectively, demand per consumer may therefore be written as

$$x_t^v\left(p^e, p^g\right) = \chi_t^v - \kappa p^v + \lambda p^{\neg v} \text{ with } v \neq \neg v \in \{e, g\},\ \chi_t^v > 0 \text{ and } \kappa > \lambda > 0, \tag{1}$$

for those price combinations which induce strictly positive quantities. To keep the analysis simple, we concentrate on these combinations and leave other cases to further research:

Assumption 1: $\min\left(x_a^e\left(p^e,p^g\right),x_a^g\left(p^e,p^g\right),x_h^e\left(p^e,p^g\right),x_h^g\left(p^e,p^g\right)\right)>0$.

We refer to χ_t^v as the zero-price consumption of variant v by type t. To reflect that electrically powered cars comply with the norm of sustainable transportation to a larger degree than gasoline-powered cars, we state the following

Assumption 2: If prices of the two variants of the commodity are identical ($p^e = p^g$), then the difference between consumption of the norm-compliant variant and of the norm-violating variant will be larger for the norm adopters than for the hedonists: $x_a^e\left(\tilde{p},\tilde{p}\right)-x_a^g\left(\tilde{p},\tilde{p}\right)>$ $x_h^e\left(\tilde{p},\tilde{p}\right)-x_h^g\left(\tilde{p},\tilde{p}\right)$.

Corollary 1: $\chi_a^e-\chi_a^g>\chi_h^e-\chi_h^g$.

We will later make use of the *effect of norm adoption on individual demand* for electric cars and for gasoline-driven cars, $\Delta^e \equiv \chi_a^e-\chi_h^e$ and $\Delta^g \equiv \chi_a^g-\chi_h^g$, respectively, where the former is obviously larger than the latter due to Corollary 1.

If we normalize the number of consumers to unity and write q as the proportion of consumers who have adopted the norm, market demands for the two product variants is:

$$\begin{aligned} X^e &= qx_a^e+\left(1-q\right)x_h^e = q\chi_a^e+\left(1-q\right)\chi_h^e-\kappa p^e+\lambda p^g \\ X^g &= qx_a^g+\left(1-q\right)x_h^g = q\chi_a^g+\left(1-q\right)\chi_h^g-\kappa p^g+\lambda p^e \end{aligned} \tag{2}$$

or equivalently the system of inverse demand functions:

$$\begin{aligned} p^e &= \frac{1}{\kappa^2-\lambda^2}\left(\left(q\chi_a^e+\left(1-q\right)\chi_h^e\right)\kappa+\left(q\chi_a^g+\left(1-q\right)\chi_h^g\right)\lambda-\kappa X^e-\lambda X^g\right) \\ p^g &= \frac{1}{\kappa^2-\lambda^2}\left(\left(q\chi_a^g+\left(1-q\right)\chi_h^g\right)\kappa+\left(q\chi_a^e+\left(1-q\right)\chi_h^e\right)\lambda-\kappa X^g-\lambda X^e\right) \end{aligned} \tag{3}$$

On the supply side, we assume myopic profit maximization[5] on a simple Cournot oligopoly market for both variants of the commodity with constant marginal production costs of c^g and c^e for the gasoline-powered and electrically powered cars, respectively. We assume that the number of suppliers on the market for gasoline-powered cars is given exogenously by n. The number m of suppliers on the market for electrically powered automobiles is given by the maximum number of producers who can produce for both markets when adding the second production line entails a fixed cost of k. Note that the oligopoly market may well turn into a monopoly market. For consistency with the simplifications on the demand side, we here exclude by assumption the absence of any producer on the market for electrically powered cars.

[5] We believe that especially in large incorporations profits are the main concern of decision makers.

We assume that markets find their equilibrium fast enough to neglect the specific dynamics when investigating the norm dynamics. In other words, we make use of the method of adiabatic elimination[6] which allows us to include markets into the norm dynamics only by their equilibria, which may, of course, depend on the current level of norm adoption.

We finally assume that dynamics of norm adoption and norm abandonment is a Markov process driven by randomly assigned moments in which each individual may adopt or abandon the norm. Whether it does, may depend on the current state of the society with respect to norm adoption and norm-related market behavior. The dynamics of the proportion of individuals having adopted the norm, *q,* is thus given by

$$\dot{q} = (1-q)\pi_{h\to a} - q\pi_{a\to h} \tag{4}$$

where the transition rates $\pi_{h\to a}$ and $\pi_{a\to h}$ are the expected number of adoptions and, respectively of abandonments of the norm per individual and per time unit.[7] This approximate equation of motion is standard in population dynamics[8] and has a simple intuition. The change in the share is simply the difference in the inflow and outflow. The inflow (outflow) is the product of the share of hedonists (norm-adopters) and the rate of transition from hedonists to adopters (adopters to hedonists).

In order to clearly identify the effect of the market innovation on the norm dynamics we assume that norm may not be inferred from consumption behaviour is not observable when no product variant compliant with the norm exists. The transition rates are then independent of the current proportion of norm adoption in society and any parameters relating to the (non-existent) market for the norm compliant variant of the commodity:

$$\pi^{o}_{a\to h} = \sigma_h \text{ and } \pi^{o}_{h\to a} = \sigma_a\text{, where } \sigma_h > 0 \text{ and } \sigma_a > 0 \text{ are constants.} \tag{5}$$

If the norm-compliant variant of the product enters the market, this has two effects on the transition rates. The cognitive dissonance effect and the conformity bias effect. The former is due to the possibility to behave according to the norm. It makes adopting the norm easier and having it less repelling. We capture this idea in the formal presentation of the dynamics by increasing the norm adoption rate by a factor $(1+CB)$ and lowering rate by which norm holders abandon it by a factor $(1-CB)$, where CB is the *reduction* in cognitive dissonances from having the norm but not complying with it. We assume $CB<1$ to ensure that the transition rates remain positive.

The conformity bias has a similar effect on norm adoption and norm abandonment. Once the norm-compliant variant of the product enters the market, individual consumers may observe

[6] The method was introduced under this label by Haken (1977) for the synergetic approach of aggregation of dynamics of micro-data to the dynamics of macro-data. It has been introduced to economics e.g. by Weidlich and Haag (1983). The basic idea of the method may, however, already be found in Samuelson's "*Foundations*" (1947).
[7] Strictly speaking, the transition rates are the limits of the expected number of transitions per second, when we consider ever shorter time intervals (similar to the speed of a car being measured in miles per hour, but measured for a specific point in time, not for an entire hour).
[8] See e.g. Weidlich and Haag (1983).

whether their consumption conforms to the majority of consumers. Acting against the majority implies dissonances, which will be larger the larger the majority is. An individual is more likely to adopt the norm, if norm-compliant behavior reflects the consumption pattern of the majority, i.e. if the ratio of electric cars to gasoline cars exceeds unity, then the transition rate towards norm adoption should increase relative to the pre-innovation level. If the opposite is true with respect to $\frac{X^e}{X^g}$ then the abandonment should be facilitated.[9] If $\alpha \in (0,1)$ measures the relative weight on the conformity bias, the post-innovation rates of transition can be written as follows:

$$\pi_{h\to a} = \sigma_a \left[\alpha(1+CB) + (1-\alpha)\frac{X^e}{X^g} \right] \text{ and } \pi_{a\to h} = \sigma_h \left[\alpha(1-CB) + (1-\alpha)\frac{X^g}{X^e} \right] \tag{6}$$

Thus the dynamics of the proportion of norm adopters become:

$$\dot{q} = \underbrace{\alpha\big((1-q)\sigma_a - q\sigma_h\big)}_{\text{pre-innovation dynamics (linear)}} + \underbrace{\alpha CB\big((1-q)\sigma_a + q\sigma_h\big)}_{\text{cognitive bias (linear)}} + \underbrace{(1-\alpha)\left((1-q)\sigma_a\frac{X^e}{X^g} - q\sigma_h\frac{X^g}{X^e}\right)}_{\text{conformity bias (non-linear)}} \tag{7}$$

The norm-cum-market dynamics described in equation (7) completes the model, the equilibria of which will be discussed in the following sections.

3. Equilibria

3.1 Market equilibrium

To find the equilibria of the norm-cum-market system described in the previous section, we first determine the market equilibrium and then turn to the dynamic part (section 3.2).

As oligopolists, each producer $i \in \{1,2,\ldots,n\}$ maximizes $\max\{\hat{\Pi}_i, \tilde{\Pi}_i\}$, with $\hat{\Pi}_i = p^g\hat{x}_i^g - c^g\hat{x}_i^g$ and $\tilde{\Pi}_i = p^g\tilde{x}_i^g + p^e\tilde{x}_i^e - c^g\tilde{x}_i^g - c^e\tilde{x}_i^e - k$ over his production quantities $\hat{x}_i^g$, $\tilde{x}_i^g$ and $\tilde{x}_i^e$.

Proposition 1: For each share of norm adopters $q \in [0,1]$ and each number $m \in \{0,\ldots,n\}$ of firms producing the innovative product there is a unique equilibrium in the Cournot oligopoly game.

The proof follows Okuguchi and Szidarovszky (1990) and is given in the appendix, as are all other proofs too.

Taking the derivatives of $\tilde{\Pi}_i$ for the *m* producers of both variants with respect to $\tilde{x}_i^g$ and $\tilde{x}_i^e$ yields two first order conditions which entail

$$\tilde{x}_i^g = (p^g - c^g)\kappa - (p^e - c^e)\lambda \text{ and } \tilde{x}_i^e = (p^e - c^e)\kappa - (p^g - c^g)\lambda. \tag{8}$$

[9] We neglect the possibility of having a conformity bias affecting consumption directly. This allows us to concentrate on the effects of the conformity bias on norm adoption and abandonment. We conjecture that this has no qualitative effects because the conformity bias affecting consumption directly should only reinforce the effects of the norm-related conformity bias.

Similarly, the derivative of $\hat{\Pi}_i$ for the *n-m* producers of gasoline cars only with respect to $\hat{x}_i^g$ yields a first order condition which simplifies to

$$\hat{x}_i^g = \left(p^g - c^g\right)\left(\kappa^2 - \lambda^2\right)/\kappa \tag{9}$$

Summing up all x_i^g and all x_i^e yields

$$\begin{aligned} X^e &= \sum_{i=1}^m \tilde{x}_i^e = m\left(\left(p^e - c^e\right)\kappa - \left(p^g - c^g\right)\lambda\right) \\ X^g &= \sum_{i=1}^m \tilde{x}_i^g + \sum_{i=1}^{n-m} \hat{x}_i^g = \frac{n\kappa^2 - (n-m)\lambda^2}{\kappa}\left(p^g - c^g\right) - m\left(p^e - c^e\right)\lambda \end{aligned} \tag{10}$$

Inserting p^e and p^g from equation (3) and solving for X^e and X^g gives the market equilibrium quantities

$$\begin{aligned} X^{e*} &= \frac{m}{m+1}\left(q\chi_a^e + (1-q)\chi_h^e - \kappa c^e + \lambda c^g\right) \\ X^{g*} &= \frac{n}{n+1}\left(q\chi_a^g + (1-q)\chi_h^g - \kappa c^g + \lambda c^e\right) + \left(\frac{1}{m+1} - \frac{1}{n+1}\right)\left(q\chi_a^e + (1-q)\chi_h^e - \kappa c^e + \lambda c^g\right)\frac{\lambda}{\kappa} \end{aligned} \tag{11}$$

As it is obvious from equations (8) and (9) already, the equilibrium is symmetric in the sense that each firm of the same type (only conventional cars or both variants of cars) produces the same quantities. Indeed from Proposition 1 we know that this equilibrium is unique.

It is noteworthy that the equilibrium price for conventional cars does not depend on *m*, the number of firms serving both markets, nor on c^e, the marginal costs of producing electrically powered cars.

The market entry equilibrium in terms of the equilibrium number of firms operating in both markets is given by the condition of equal payoffs. Due to indivisibility, the equilibrium number of firms active also on the market for e-mobility, m^{eq}, corresponds to the integer part of *m** solving $\tilde{\Pi}_i = \Pi_i$ with $\tilde{x}_i^g, \tilde{x}_i^e, \hat{x}_i^g$ given by (8) and (9) and p^e and p^g by inserting X^{e*}, X^{g*} from (11) into (3). m^{eq} is thus given by:

$$m^{eq} = \min\left\{n, \max\left\{0, \text{integerpart}\left(m^*\right)\right\}\right\} \text{ where } m^* = \frac{q\chi_a^e + (1-q)\chi_h^e - \kappa c^e + \lambda c^g}{\sqrt{k\kappa}} - 1 \tag{12}$$

Note that the condition on m^{eq} to be of integer value will cause discontinuity in equilibrium prices and quantities at levels of *q* that induce a change in the value of m^{eq}. The number of firms serving both markets in equilibrium is increasing in the weighted willingness to pay for e-mobility and in the weighted cost differential between conventional cars and electric cars. It is decreasing in the fixed costs k. Notably the equilibrium number of firms producing both products is independent of the total number of firms *n*. We further note the following:

Lemma 2: The number of firms m^* is monotonically increasing in the share of norm adopters if and only if $\chi_a^e > \chi_h^e$, i.e. if and only if the effect of the norm adoption on individual demand for electric cars is positive ($\Delta^e > 0$).

Hence, if the sustainable-transportation norm goes along with a reduced overall demand for individual mobility, then an increasing share of norm adopters may induce a larger number of producers of electric cars only if the reduction in the demand for transportation exclusively affects the demand for gasoline-powered cars, the demand for which has to be partially substituted by an increased demand for electrically powered cars. Lemma 2 will be helpful in section 3.2.2 when we study the impact of the discontinuity of m^{eq} on the number of stable equilibria.

Having derived the number of firms serving both markets, we can now determine the quantities emerging if the expansion of firms on the e-mobility market is endogenous as $\hat{X}^e = X^{e*}\big|_{m=m^{eq}}$ and $\hat{X}^g = X^{g*}\big|_{m=m^{eq}}$. For expositional simplicity, we will make heavily use of the continuous version of m for the moment:

$$\begin{aligned}
\tilde{X}^e &= X^{e*}\big|_{m=m^*} = \left((1-q)\,\chi_h^e + q\chi_a^e - \kappa c^e + \lambda c^g\right) - \sqrt{k}\sqrt{\kappa} = \theta^e + \Delta^e q \\
\tilde{X}^g &= X^{g*}\big|_{m=m^*} \\
&= \frac{n}{n+1}\left((1-q)\chi_h^g + q\chi_a^g - \kappa c^g + \lambda c^e\right) - \frac{1}{n+1}\left((1-q)\,\chi_h^e + q\chi_a^e - \kappa c^e + \lambda c^g\right)\frac{\lambda}{\kappa} + \frac{\lambda}{\kappa}\sqrt{k}\sqrt{\kappa}, \\
&= \frac{n}{n+1}\left(\theta^g + \Delta^g q\right) - \frac{1}{n+1}\left(\theta^e + \Delta^e q\right)\frac{\lambda}{\kappa}
\end{aligned} \tag{13}$$

where the tilde denotes the simplification of the continuous version of m and the two terms

$$\theta^e = \chi_h^e - \kappa c^e + \lambda c^g - \sqrt{k}\sqrt{\kappa} > 0 \text{ and } \theta^g = \chi_h^g - \kappa c^g + \lambda c^e + \sqrt{k}\sqrt{\kappa}\frac{\lambda}{\kappa} > 0 \tag{14}$$

facilitate notation in the remainder of the paper. Before we turn to the analysis of the norm dynamics we briefly study the total demand for private transportation:

$$\begin{aligned}
\tilde{X}^g + \tilde{X}^e &= \frac{n}{n+1}\left((1-q)\chi_h^g + q\chi_a^g + (1-q)\,\chi_h^e + q\chi_a^e - (\kappa-\lambda)\left(c^e + c^g\right)\right) \\
&\quad + \frac{1}{n+1}\left((1-q)\,\chi_h^e + q\chi_a^e - \kappa c^e + \lambda c^g\right)\left(1-\frac{\lambda}{\kappa}\right) - \sqrt{k}\sqrt{\kappa}\left(1-\frac{\lambda}{\kappa}\right) \\
&= \frac{n}{n+1}\left(\theta^e + \theta^g + \left(\Delta^e + \Delta^g\right)q\right) + \frac{1}{n+1}\left(\theta^e + \Delta^e q\right)\left(1-\frac{\lambda}{\kappa}\right)
\end{aligned} \tag{15}$$

Total demand for individual transportation is a linear function in the share of norm adopters. Neglecting a factor of proportionality close to 1, it increases (decreases) if the effect of norm adoption on the individual demand for electric cars (Δ^e) is larger (smaller) than the opposite effect on the individual demand for conventional cars ($-\Delta^g$).[10] The precise condition is:

$$\frac{\partial\left(\tilde{X}^e+\tilde{X}^g\right)}{\partial q} \gtreqless 0 \Leftrightarrow \Delta^e \gtreqless \frac{n}{n+1-\frac{\lambda}{\kappa}}\left(-\Delta^g\right).$$

3.2 Norm equilibrium

We now turn to the evolution of the share *q* in the population carrying a norm to consume in a sustainable way. In particular we will study the existence, stability and multiplicity of equilibria. We concentrate on this multiplicity of equilibria, because this phenomenon may most substantially affect the consequences of policy measures affecting market parameters only temporarily and of the sequence of their choice.

In the pre-innovation stage where transition rates are given by the constants defined in equation (5) the dynamics of equation (4) has an easy-to-calculate stable and unique equilibrium at $q^o = \sigma_a / \left(\sigma_h + \sigma_a\right)$.

When the innovation enters the market, transition rates change, depending now on the equilibrium quantities of the different product variants and as given in equation (6). In the following paragraphs we study the effects of three phenomena which become relevant in consequence. We first study the interplay of the cognitive dissonance bias and the conformity bias and then turn to the discontinuity resulting from the fact that the number of firms has to be an integer.

3.2.1 Cognitive Bias and Conformity Bias

In order to understand the interplay of the cognitive dissonance bias and the conformity bias we neglect the requirement that the number of firms supplying the norm-compliant variant of the product be an integer and base our argument on the continuous version of the equilibrium number of such firms as defined by m^* in equation (12). Obviously, this requires assuming for the moment that demand for electric vehicles by hedonists is large enough to keep X^e as defined by equation (13) strictly positive. In order to clearly differentiate between the continuous-*m** version of the model from the version with the discrete m^{eq}, we write $\dot{\tilde{q}}$ instead of $\dot{q}$ whenever we use $\tilde{X}^e$ and $\tilde{X}^g$ instead of $\hat{X}^e$ and $\hat{X}^g$ in equation (7). To guarantee differentiability of $\dot{\tilde{q}}$ we will further assume, that $m^* \in [1,n]$. This translates into a pair of inequalities:

$m^* \in [1,n] \Leftrightarrow \frac{1}{\sqrt{k\kappa}}\left(\theta^e + q\Delta^e\right) \in [1,n] \Leftrightarrow \frac{\theta^e}{\sqrt{k\kappa}} \in [1,n] \wedge \frac{1}{\sqrt{k\kappa}}\left(\theta^e + \Delta^e\right) \in [1,n]$, or equivalently: $\sqrt{k\kappa} \le \theta^e \le n\sqrt{k\kappa} \quad \wedge \quad \sqrt{k\kappa} - \theta^e \le \Delta^e \le n\sqrt{k\kappa} - \theta^e$. We will neglect this condition in the

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[10] Note that $\frac{n}{n+1-\frac{\lambda}{\kappa}} \approx 1$ for sufficiently large *n* and if the cross price "elasticity" is sufficiently close to the direct price "elasticity", i.e. if the two types of goods are very close substitutes.

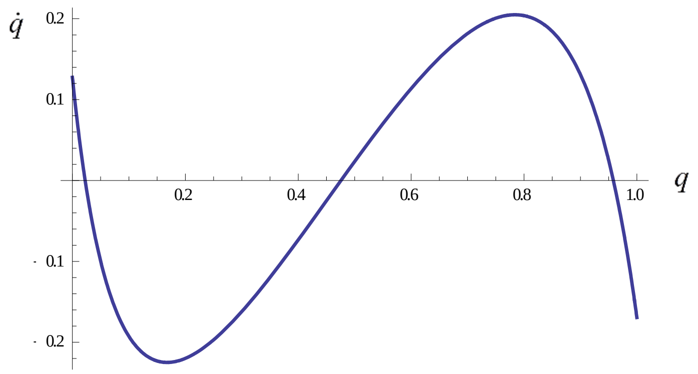

Figure 1: market-norm dynamics.
$\theta^e = 0.1,\ \theta^g = 1,\ \Delta^e = 0.75,\ \Delta^g = -0.65,\ \mathrm{n} = 4,\ \lambda/\kappa = 0.8,\ \delta_a = \delta_h = 1,\ \alpha = 0$

following since its inclusion would be straightforward but unnecessarily complicated notation. So far the reader should keep in mind that the number of firms *n* should be sufficiently high and fixed set up cost *k* sufficiently small. We will return to this issue in the discussion.

Neglecting the conformity bias ($\alpha = 1$), inspection of equation (7) shows that the cognitive bias shift the norm dynamics upwards and turns it counterclockwise, thus increases the equilibrium level of norm adoption. The conformity bias changes the motion of the norm adoption proportion described in equation (7) from a linear function to an s-shaped function with at most one increasing branch in the middle (see Figure 1):

Lemma 3: Assume that χ_h^e and χ_a^g are large enough to guarantee that $\tilde{X}^e$ and $\tilde{X}^g$ as defined by equation (13) are strictly positive for all $q \in [0,1]$. Then:

1. $\left.\dot{\tilde{q}}\right|_{q=0} > \alpha(1+CB)\sigma_a > 0$ and $\left.\dot{\tilde{q}}\right|_{q=1} < \alpha(CB-1)\sigma_h < 0$;
2. Any value of $\dot{\tilde{q}}$ is reached for at most three different $q \in [0,1]$.
3. $\Delta^e \le \dfrac{\theta^e}{\theta^g}\Delta^g$ implies $\dfrac{d\left(\tilde{X}^e/\tilde{X}^g\right)}{dq} \le 0$, which in turn implies $\dfrac{d\dot{\tilde{q}}}{dq} < 0$

The intuition behind claims 1 and 2 is simple: claim 1 is obvious when X^e and X^g are strictly positive. Claim 2 follows from the fact that X^e and X^g are linear in q and thus solving equation (7) for q for any given value of $\dot{\tilde{q}}$ is tantamount to solving a polynomial of degree three. The first implication of Claim 3 follows from the fact that the denominator of the derivative $\dfrac{d\left(\tilde{X}^e/\tilde{X}^g\right)}{dq}$ is strictly positive and the numerator is given by:

$$
\begin{aligned}
&\frac{d\tilde{X}^e}{dq}\tilde{X}^g - \frac{d\tilde{X}^g}{dq}\tilde{X}^e \\
&= \left(\Delta^e\left(\frac{n}{n+1}\left(\theta^g + \Delta^g q\right) - \frac{1}{n+1}\left(\theta^e + \Delta^e q\right)\right)\frac{\lambda}{\kappa}\right) - \left(\frac{n}{n+1}\Delta^g - \frac{1}{n+1}\Delta^e\frac{\lambda}{\kappa}\right)\left(\theta^e + \Delta^e q\right) \qquad (16) \\
&= \frac{n}{n+1}\left[\theta^g\Delta^e - \theta^e\Delta^g\right]
\end{aligned}
$$

The second implication of Claim 3 follows from the observation that all three terms summed up in

$$\frac{d\dot{\tilde{q}}}{dq} = -\alpha\left(\sigma_a\left(1+CB\right)+\sigma_h\left(1-CB\right)\right)-\left(1-\alpha\right)\left(\sigma_a\frac{\tilde{X}^e}{\tilde{X}^g}+\sigma_h\frac{\tilde{X}^g}{\tilde{X}^e}\right) \\ +\left(1-\alpha\right)\left(\frac{\left(1-q\right)\sigma_a}{\left(\tilde{X}^g\right)^2}+\frac{q\sigma_h}{\left(\tilde{X}^e\right)^2}\right)\left(\frac{d\tilde{X}^e}{dq}\tilde{X}^g-\frac{d\tilde{X}^g}{dq}\tilde{X}^e\right) \tag{17}$$

are negative if $\frac{d\left(\tilde{X}^e/\tilde{X}^g\right)}{dq}\leq 0$.

As a consequence of claim 1 of Lemma 3, $\dot{\tilde{q}}$ must have at least one branch declining in q. Claim 2 of the lemma then implies that there is at most one increasing branch. Such an increasing branch is a necessary condition for multiple inner equilibria of the market-norm dynamics. Hence, a direct consequence of Claim 3 is the following

Corollary 4: If the market-norm dynamics has multiple (two) stable inner equilibria then $\tilde{X}^e/\tilde{X}^g$ increases strongly in *q* for all $q\in\left[0,1\right]$, i.e. $\Delta^e > \Delta^g\, \theta^e/\theta^g$.

Figure 1 illustrates the possibility of multiple equilibria. In the following we look at the conditions and thereby at the parameter set that give rise to this phenomenon. With the assumption of strictly positive demand the roots of (7) are equivalent to the roots of (18).

$$\dot{\hat{q}} \equiv X^e X^g \dot{\tilde{q}} = \\ \alpha\tilde{X}^e\tilde{X}^g\left(\left(1+CB\right)\sigma_a - q\left(\left(1+CB\right)\sigma_a+\left(1-CB\right)\sigma_h\right)\right)+\left(1-\alpha\right)\left(\left(1-q\right)\sigma_a\left(\tilde{X}^e\right)^2-q\sigma_h\left(\tilde{X}^g\right)^2\right) \tag{18}$$

The dynamics given by (18) is a polynomial of degree 3 and has two stable inner equilibria in the unit interval if and only if it has two extreme points with negative functional value at the minimum and positive functional value at the maximum. Note that if there are two extreme points $q^{Low} < q^{High}$ then $\dot{q}\left(q^{High}\right)>0$ implies $q^{High}<1$ and $\dot{q}\left(q^{Low}\right)<0$ implies $q^{Low}>0$ by inspection of (7), given strictly positive demand. Given $\dot{q}\left(q^{High}\right)>0$ and $\dot{q}\left(q^{Low}\right)<0$, the fact that $\dot{q}\left(0\right)>0, \dot{q}\left(1\right)<0$ implies that q^{Low} is the minimum and q^{High} is the maximum.

Hence only the two conditions with respect to the existence of two extrema and the sign condition at the extrema points remain. Since demand is linear in the share of norm adopters the conditions of positive demand amount to: $0<\theta^e<\frac{n}{\lambda}\theta^g$ and $-\theta^e<\Delta^e<-\theta^e+\frac{n}{\lambda}\left(\Delta^g+\theta^g\right)$.

Hence the binding constraints are given by: $\dot{q}\left(q^{Low}\right)<0$, $\dot{q}\left(q^{High}\right)>0$, $0<\theta^e<\frac{n}{\lambda}\theta^g$ and $-\theta^e<\Delta^e<-\theta^e+\frac{n}{\lambda}\left(\Delta^g+\theta^g\right)$. It turns out that only $\dot{q}\left(q^{Low}\right)<0$, $\dot{q}\left(q^{High}\right)>0$ and

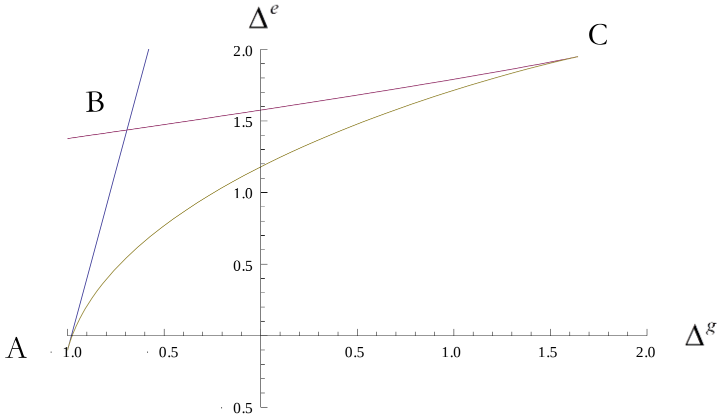

Figure 2: Range of multiple equilibria: blue line: $\tilde{X}^g(1)=0$ $\tilde{X}^g(1)>0$ to the right of the blue line; red line: $\Delta^{e,Min}(\Delta g)$ upper bound of Δ^e allowing for multiple equilibria; yellow line: $\Delta^{e,Max}(\Delta g)$ lower bound of Δ^e allowing for multiple equilibria.

$\theta^e = 0.1,\ \ \theta^g = 1,\ \ \sigma_h/\sigma_a = 1,\ \ n = 4,\ \ \alpha = 0,\ \ \lambda/\kappa = 4/5$

$\Delta^e < -\theta^e + \frac{n}{\lambda}\left(\Delta^g + \theta^g\right)$ depend on Δ^e and Δ^g. Therefore, if we study the parameter region of Δ^e and Δ^g such that multiple equilibria exist, only these three condition are relevant given the values for the other parameters satisfy the remaining inequalities ($0 < \theta^e < \frac{n}{\lambda}\theta^g$). Figure 2 gives an illustrative example.

The intuition behind having an upper and a lower limit for Δ^e is simple. If Δ^e were too large, $X^e(q)$ increases so quickly relative to $X^g(q)$ that $\dot{\tilde{q}}(q)$ increases at $q=0$ or the minimum of $\dot{\tilde{q}}(q)$ is above the $\dot{\tilde{q}}=0$-axis. If Δ^e were too small, $X^g(q)$ declines so quickly relative to $X^e(q)$ that $\dot{\tilde{q}}(q)$ never increases or only has a minimum but no maximum or has a maximum which remains below the $\dot{\tilde{q}}=0$-axis. In our application, a relatively large Δ^e implies that norm adoption has so strong an effect on the market equilibrium amount of norm compliant consumption that the growth in this consumption (possibly at the cost of norm violating consumption) reinforces the norm so quickly that norm adoption is always self-reinforcing until the number of individuals not having adopted the norm becomes very small. If, on the other hand, $\tilde{X}^g(1)>0$ is very small, than norm adoption has to small an effect on norm compliant consumption to become self-reinforcing.

In the next section we will derive sufficient conditions for multiple equilibria to exist. If we look at Figure 2 it appears that these three conditions define a triangular region. In what follows, we will derive the vertices of that region and reformulate the two differential equations $\dot{q}\left(q^{Low}\right)<0$, $\dot{q}\left(q^{High}\right)>0$ as differential equation for $\Delta^e\left(\Delta^g\right)$.

Given strictly positive demand (7) gives rise to a fixed point equation:

$$\dot{q} = \alpha\left((1-q)\sigma_a - q\sigma_h\right) + \alpha CB\left((1-q)\sigma_a + q\sigma_h\right) + (1-\alpha)\left((1-q)\sigma_a \frac{X^e}{X^g} - q\sigma_h \frac{X^g}{X^e}\right) = 0 \overset{\alpha\neq 1}{\Leftrightarrow}$$

$$(1-q)\sigma_a \frac{X^e}{X^g} - q\sigma_h \frac{X^g}{X^e} = \frac{\alpha}{1-\alpha}\sigma_a(1+CB) - q\frac{\alpha}{1-\alpha}\left(\sigma_a(1+CB)+\sigma_h(1-CB)\right) \equiv \gamma + \beta q \Leftrightarrow$$

$$\left(\frac{X^e}{X^g}\right)^2 = \frac{q}{(1-q)}\underbrace{\sigma}_{\equiv\sigma_h/\sigma_a} + \frac{(\gamma+\beta q)}{(1-q)}\frac{X^e}{X^g} \underset{z(q)\equiv\frac{X^e}{X^g}}{\Leftrightarrow} \left(z(q)\right)^2 = \frac{q}{(1-q)}\sigma + \frac{(\gamma+\beta q)}{(1-q)}z(q) \Rightarrow$$

At $q = q^{extr.}$ such that $\dot{q}'\left(q^{extr.}\right)=0$ this gives a fixed point equation in Δ^e,Δ^g :

$$\dot{q}\left(q^{extr.}\left(\Delta^e,\Delta^g\right)\right) = 0$$

We take the total derivative w.r.t. Δ^e,Δ^g and apply the envelope theorem[11].

$$\left[2z\left(q^{extr.}\right)\frac{\partial z(q)}{\partial \Delta^e}\bigg|_{q=q^{extr.}} - \frac{(\gamma+\beta q)}{(1-q)}\frac{\partial z(q)}{\partial\Delta^e}\bigg|_{q=q^{extr.}}\right]d\Delta^e + \left[2z\left(q^{extr.}\right)\frac{\partial z(q)}{\partial \Delta^g}\bigg|_{q=q^{extr.}} - \frac{(\gamma+\beta q)}{(1-q)}\frac{\partial z(q)}{\partial\Delta^g}\bigg|_{q=q^{extr.}}\right]d\Delta^g = 0$$

$\Leftrightarrow$

$$\frac{n+1}{n}\frac{X^g}{X^e} + \frac{\lambda}{n} = \frac{d\Delta^g}{d\Delta^e} \Leftrightarrow \frac{d\Delta^g}{d\Delta^e} = \frac{\Delta^g q^{extr.} + \theta^g}{\Delta^e q^{extr.} + \theta^e} > 0$$

Together with initial conditions: $\left(\Delta\mathrm{e},\Delta g\right)\big|\dot{q}\left(q^{Max}\left(\Delta\mathrm{e},\Delta g\right)\right) = 0$ and $\left(\Delta^e,\Delta^g\right)\big|\dot{q}\left(q^{Min}\left(\Delta^e,\Delta^g\right)\right) = 0$ the differential equation $\frac{d\Delta^g}{d\Delta^e} = \frac{\Delta^g q^{extr.} + \theta^g}{\Delta^e q^{extr.} + \theta^e}$ gives rise to two boundary functions: $\Delta^{e,Min}\left(\Delta^g\right),\Delta^{e,Max}\left(\Delta^g\right)$

Definition: All $\left(\Delta^e,\Delta^g\right)$-pairs that satisfy the following three conditions define the parameter region such that multiple equilibria exist: (1) $\Delta^e < -\theta^e + \frac{n}{\lambda}\left(\Delta^g + \theta^g\right)$, (2) $\Delta^e > \Delta^{e,Min}\left(\Delta^g\right)$, (3) $\Delta^e < \Delta^{e,Max}\left(\Delta^g\right)$. We will refer to this set as the multiple equilibria set (**MES**).

Before we continue, we will state some observations based on $\frac{d\Delta^e}{d\Delta^g} = \frac{1}{\frac{n+1}{n}\frac{X^g}{X^e} + \frac{\lambda}{n}}$ that will be helpful in the course of our argument:

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[11] $\frac{\partial \dot{q}\left(q^{extr.}\left(\Delta^e,\Delta^g\right)\right)}{\partial\text{parameter}} = \frac{\partial\dot{q}(q)}{\partial\text{parameter}}\bigg|_{q=q^{extr.}}$

(1) The slopes of $\Delta^{e,Min}\left(\Delta^{g}\right),\Delta^{e,Max}\left(\Delta^{g}\right)$ are positive and smaller than the slope of the third constraint $\Delta^{e} < -\theta^{e} + \frac{n}{\lambda}\left(\Delta^{g} + \theta^{g}\right)$.

(2) By corollary 4 $\frac{d\Delta^{g}}{d\Delta^{e}} = \frac{\Delta^{g}q + \theta^{g}}{\Delta^{e}q + \theta^{e}}$ is c.p. decreasing in q

(3) In point A the relevant constraints have the same slope.

We are able to determine the coordinates for points A and B (figure 2) analytically. For better readability table 1 below presents the results for $\alpha = 0$. Note that there exist multiple equilibria if and only if $\left(\Delta^{e}\right)^{B} > \left(\Delta^{e}\right)^{A}$. As mentioned before, the dynamics given by (7) consist of a linear and nonlinear term, the latter is weighted with $1-\alpha$. Intuitively one would expect that α, the weight of the linear term, must be sufficiently small so that the nonlinear term dominates the dynamics and for some parameter constellations multiple equilibria might arise. It indeed turns out that there exists a unique threshold value for α, such that multiple equilibria are possible. Its derivation is deferred to the appendix. The value and its properties are summarized in the next lemma.

Lemma 5: For $0 < \theta^{e} < \frac{n}{\lambda}\theta^{g}$ there exists a unique

$$\alpha^{crit.} = \frac{(1-CB)(n+1)\theta^{e} + 2\left(n\theta^{g} - \lambda\theta^{e}\right) - \sqrt{\left((1-CB)(n+1)\theta^{e}\right)^{2} + 4\left(n\theta^{g} - \lambda\theta^{e}\right)^{2}\left(1-CB^{2}\right)}}{2\left(CB^{2}\left(n\theta^{g} - \lambda\theta^{e}\right) + (n+1)(1-CB)\theta^{e}\right)},$$

such that MES is non-empty if and only if $\alpha < \alpha^{crit.}$. Furthermore

$\frac{\partial\alpha^{crit.}}{\partial\theta^{g}} = \frac{\partial\alpha^{crit.}}{\partial n} = -\frac{\theta^{e}}{\theta^{g}}\frac{\partial\alpha^{crit.}}{\partial\theta^{e}} = -\frac{n}{\theta^{e}}\frac{\partial\alpha^{crit.}}{\partial\lambda}\bigg|_{\substack{\theta^{e},\theta^{g}\\ fixed}} > 0; \frac{\partial\alpha^{crit.}}{\partial\sigma_{a}} = \frac{\partial\alpha^{crit.}}{\partial\sigma_{h}} = 0; \frac{\partial\alpha^{crit.}}{\partial CB} > 0$. This implies: $\frac{\partial\alpha^{crit.}}{\partial\chi_{h}^{e}} < 0; \frac{\partial\alpha^{crit.}}{\partial\chi_{h}^{g}} > 0; \frac{\partial\alpha^{crit.}}{\partial c^{e}} > 0; \frac{\partial\alpha^{crit.}}{\partial c^{g}} < 0; \frac{\partial\alpha^{crit.}}{\partial k} > 0$

In other words, as long as the weight for the non-linear term is sufficiently large there will always be $\left(\Delta^{e},\Delta^{g}\right)$-pairs such that multiple equilibria exist. With respect to partial effects Lemma 5 states that the required weight for the non-linear term of the dynamics $1-\alpha$ is increasing in maximum willingness to pay for electric cars by hedonists χ_{h}^{e} and in the marginal cost for gasoline cars c^{g}. The required weight decreasing in the maximum willingness to pay for gasoline cars χ_{h}^{g}, the marginal cost for electric cars c^{e} and the fixed setup cost *k*. The effects with respect to parameters measuring the price sensitivity are ambiguous. The weight also decreases in the number of firms in the market and in CB a measure for the reduction of cognitive dissonances from having the norm but not complying with it.

Point	A	B	C
q	$q^{Max}=1$	$q^{Min}=\frac{4(n+1)^2(\theta^e)^2}{(n\theta^g-\lambda\theta^e)^2+4(n+1)^2(\theta^e)^2}$	$q^{ip}=\frac{(n\theta^g-\lambda\theta^e)}{3(n\Delta^g-\lambda\Delta^e)+4(n\theta^g-\lambda\theta^e)}$
$\begin{pmatrix}\Delta^g\\ \Delta^e\end{pmatrix}$	$\begin{pmatrix}-\theta^g\\ -\theta^e\end{pmatrix}$	$\begin{pmatrix}\frac{\lambda}{n}\left(\frac{n\theta^g-\lambda\theta^e}{2(n+1)}\right)^2\frac{1}{\theta^e}-\theta^g\\ \left(\frac{n\theta^g-\lambda\theta^e}{2(n+1)}\right)^2\frac{1}{\theta^e}-\theta^e\end{pmatrix}$	/

Table 1: Vertices of multiple equilibria set for $\alpha=0$.

The differential equations given by $\frac{d\Delta^g}{d\Delta^e}=\frac{\Delta^g q^{extr.}+\theta^g}{\Delta^e q^{extr.}+\theta^e}$ cannot be solved for analytically. In the following we present our approximation strategy for $\alpha=0$, such that we can state explicit sufficient conditions for multiple equilibria to exist. Again, the general case can be found in the appendix. Note that the values for *q* that correspond to (Δ^e,Δ^g)-pairs that are elements of the graph of $\Delta^{e,Max}(\Delta^g)$ range from $q^C=\frac{(n\theta^g-\lambda\theta^e)}{3(n\Delta^g-\lambda\Delta^e)+4(n\theta^g-\lambda\theta^e)}$ to $q^A=1$. We can use the $(\Delta^g)^B$ as a lower bound for Δ^g and by that can give a lower bound for q independent of Δ^e and Δ^g, i.e. $\underline{q}=\frac{4(n+1)^2\theta^e}{4(n+1)^2\theta^e+3\lambda(n\theta^g-\lambda\theta^e)}$. The system $\dot{q}(q^C)=0$, $\dot{q}'(q^C)=0$ can be solved for Δ^e and Δ^g as a function of *q*. If we plug in $\underline{q}$ we get as point *D* a (Δ^e,Δ^g)-pair on the graph of $\Delta^{e,Max}(\Delta^g)$ that corresponds to a maximum for the dynamics in (7) that equals $\underline{q}$.

$$\begin{pmatrix}\Delta^e\\ \Delta^g\end{pmatrix}^D=\begin{pmatrix}\frac{4\sqrt{3\theta^e\lambda\tau^3}-\theta^e\left(4(n+1)^2\theta^e+9\lambda(n\theta^g-\lambda\theta^e)\right)}{4(n+1)^2\theta^e}\\ \frac{3\underline{q}(\Delta^e)^D-2\tau+\sqrt{-9\underline{q}^2+6\underline{q}(\Delta^e)^D\left((\Delta^e)^D-\theta^e\right)+6(1-q)(\Delta^e)^D\theta^e-(n+1)^2(\theta^e)^2+\tau^2}}{3n\underline{q}}\end{pmatrix}$$

, with $\tau\equiv n\theta^g-\lambda\theta^e$

We will approximate the upper and lower boundaries by linear functions intersecting point B and D, respectively. Our observation above, that the slop $\Delta^{e,Min}(\Delta^g)$ is decreasing in q, gives us a lower bound for the slop by $\frac{\theta^e}{\theta^g}$. Figure 3 illustrates our approximation procedure. Note that for our approach MES is not empty if and only if the area spanned by $X^g(1)>0$ and the two approximating linear function is non-empty.

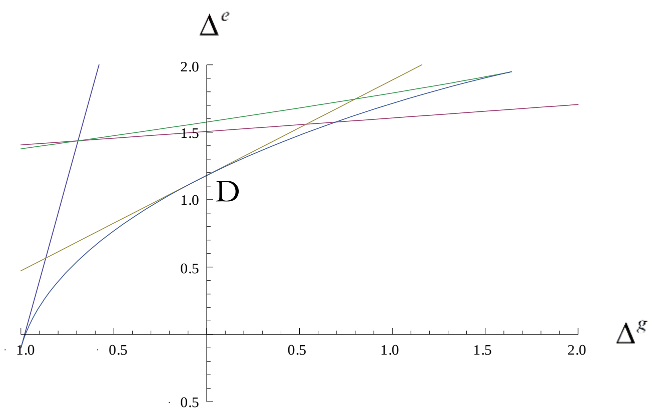

Figure 3: Approximation of MES: red line: approximation of $\Delta^{e,Min}\left(\Delta^g\right)$; yellow line: approximation of $\Delta^{e,Max}\left(\Delta^g\right)$.
$\theta^e = 0.1,\ \theta^g = 1,\ \sigma_h/\sigma_a = 1,\ n = 4,\ \alpha = 0,\ \lambda/\kappa = 4/5$

Lemma 6: If $\alpha = 0$ and $0 < \theta^e < \dfrac{n}{\lambda}\theta^g$, $\dot{\tilde{q}} = 0$ has three solutions if (sufficient condition):

$$\left(\Delta^e,\Delta^g\right) \in \left\{ \begin{array}{c} \left(\Delta^e,\Delta^g\right) \middle| \Delta^e < -\theta^e + \dfrac{n}{\lambda}\left(\Delta^g + \theta^g\right), \Delta^e < \dfrac{\theta^e}{\theta^g}\Delta^g + \dfrac{\left(n\theta^g - \lambda\theta^e\right)^3}{4n\left(n+1\right)^2\theta^g\theta^e} \\ \Delta^e > \dfrac{\left(\Delta^e\right)^D \underline{q} - \theta^e}{\left(\Delta^g\right)^D \underline{q} - \theta^g}\Delta^g + \left(\Delta^e\right)^D - \dfrac{\left(\Delta^e\right)^D \underline{q} - \theta^e}{\left(\Delta^g\right)^D \underline{q} - \theta^g}\left(\Delta^g\right)^D \end{array} \right\}$$

The effects of market parameter variations on the location of $\dot{\tilde{q}}(q)$ and on the number of equilibria are best understood by observing that they only enter via $\tilde{X}^e$ and $\tilde{X}^g$ into equation (7). Since $\dfrac{d\dot{\tilde{q}}(q)}{d\tilde{X}^e} > 0 > \dfrac{d\dot{\tilde{q}}(q)}{d\tilde{X}^g}$, the derivatives are all straight forward and mention of their signs may be left to the discussion.

Before studying the effect of the cognitive bias and the transition rates $\sigma_{a,h}$, it is worth mentioning that these parameters are not subject to policy measures. They reflect the dynamics of norm adoption before the innovation takes place. In particular the cognitive dissonance from having adopted a norm to which one cannot comply is beyond the reach of political measures. Discussing these parameters is thus only relevant for understanding the circumstances within which any policy has to act. Since the transition rates $\sigma_{a,h}$ occur in each and every term of the right-hand side of equation (7), it is only their ratio which is relevant. If σ_a/σ_h is small, there will be only few norm adopters in equilibrium before the innovation takes place, in particular because too much cognitive dissonance is implied by having the norm. After the innovation, small values of σ_a/σ_h imply that the range of Δ^e for which multiple equilibria occur shifts upwards and stretches along the Δ^e-axis.

If the cognitive bias *CB* is large, that is, if the innovation removes a lot of cognitive dissonance from norm adopters, then the innovation tends to have a particularly positive effect on norm adoption. Starting from the pre-innovation equilibrium value of the rate of norm adoption, $q^o = \sigma_a/(\sigma_h + \sigma_a)$, exemplifies the effect of the size of *CB* and its interplay with the conformity bias on which most of our hitherto discussion was concentrated. The following Lemma states necessary and sufficient condition for a positive growth rate in norm adoption at the pre-innovation level.

Lemma 7: $$\dot{q}(q^o) > 0 \Leftrightarrow 2CB\frac{\alpha}{1-\alpha} + \left.\frac{\tilde{X}^e}{\tilde{X}^g}\right|_{q=q^o} - \left.\frac{\tilde{X}^g}{\tilde{X}^e}\right|_{q=q^o} > 0 \tag{19}$$

Which may be transformed to $$\Delta^e > \frac{\mu}{\mu+1}\frac{n}{n+1}\left(\frac{\theta^g}{q^o} + \Delta^g\right) + \frac{1}{\mu+1}\left(1 - \mu\frac{\lambda/\kappa}{n+1}\right)\frac{\theta^e}{q^o}, \tag{20}$$

$$\text{where } \mu = \left(-CB\frac{\alpha}{1-\alpha} + \sqrt{CB^2\frac{\alpha^2}{(1-\alpha)^2} + 1}\right)\frac{\lambda/\kappa}{n+1}.$$

Equation (20) describes a straight and increasing line, above which $\dot{q}(q^o)$ is positive so that the innovation induces a growth of norm adoption, while below this line, norm adoption will decline when the innovation occurs. The straight line moves upward, if *CB* or α increase.

If $\dot{q}(q^o) < 0$, then this implies that the positive cognitive bias is offset by a negative conformity bias with a sufficiently large weight α. Obviously, the conformity bias is negative only if at q^o the market-equilibrium quantity of the norm-compliant variant of the good is less than the corresponding quantity of the norm-violating variant.

If the quantities of the two variants of the good are hardly affected by the number of norm adopters or the quantity of the norm-compliant variant grows only slightly as compared to the quantity of the norm-violating variant, i.e. if the effects of norm adoption on individual demand are small or not too much diverging, then $\dot{q} < 0$ may hold true for all $q \geq q^o$. However, if the effects of norm adoption are strong and induce a quick growth of $\frac{\tilde{X}^e}{\tilde{X}^g} - \frac{\tilde{X}^g}{\tilde{X}^e}$ in *q* (see (19)), then $\dot{q}$ may turn positive for some $q \in (q^o, 1)$ so that a (second) stable equilibrium with a large level of norm adoption is generated by the conformity bias. In the next section we will enlighten the effects that the discontinuity of the number of firms adds to our discussion of the cognitive and conformity bias.

3.2.2 Discontinuity of Firm Number

We now drop the simplifying assumption of continuity of the equilibrium number of firms producing the norm-compliant variant of the product. We first study the effect of the discreteness of this number of firms on the pace at which norm adoption changes and then infer consequences for the number and location of equilibria with reference to the structure of the market of the innovative good.

A helpful first insight is the following:

Lemma 8: 1. Except for the discontinuities, where $\dot{q}(q)=\dot{\tilde{q}}(q)$ holds true, we have:

1. $\dot{q}(q)<\dot{\tilde{q}}(q)$ and $\frac{d\dot{q}(q)}{dq} \lessgtr \frac{d\dot{\tilde{q}}(q)}{dq} \Leftrightarrow \Delta^e \gtrless 0$ for all q.

2. Let q_1 and q_2 be two instances of discontinuity of $\dot{q}$ with $q_2>q_1$. Then:

   a. $q_2-q_1=\eta\frac{\sqrt{k\kappa}}{\Delta^e}$ where $\eta\in\{1,2,\ldots\}$

   b. $\dot{q}(q_1)-\lim_{q\uparrow q_1}(\dot{q}(q))>\dot{q}(q_2)-\lim_{q\uparrow q_2}(\dot{q}(q))>0$ if $\Delta^e>0$ and
   $\dot{q}(q_1)-\lim_{q\downarrow q_1}(\dot{q}(q))<\dot{q}(q_2)-\lim_{q\downarrow q_2}(\dot{q}(q))<0$ if $\Delta^e<0$.

Figure 4 visualizes the relationship between $\Delta^e>0$ and $\dot{\tilde{q}}(q)$ reported in the lemma.

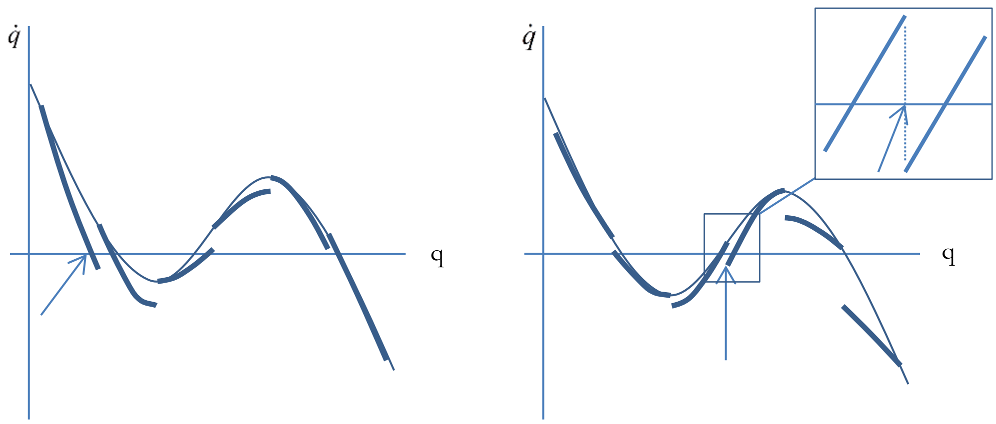

Figure 4: Effects of discontinuity on $\dot{q}(q)$. Left: $\Delta^e>0$, right $\Delta^e<0$: Additional stable equilibria marked by an arrow.

The discontinuities described in Lemma 8 may increase the number of instances, at which the sign of $\dot{q}(q)$ changes from positive to negative as q increases, i.e. the number of stable equilibria. It does not reduce this number. The additional stable equilibria may not occur over the entire range of q, but only in those intervals, in which the “jumps” and the slope in the neighborhood of the discontinuities are in opposite directions. Only then the discontinuities may result in additional sign changes. We state the argument more precisely in the following:

Corollary 9: Additional stable equilibria due to the discontinuities of $\dot{q}(q)$ occur if and only if the discontinuities entail additional sign changes of $\dot{q}(q)$. If $\Delta^e>0$, every additional stable equilibrium is in one of the intervals in which $\dot{q}(q)$ is continuous and which has its lower bound in one of the decreasing branches of $\dot{\tilde{q}}(q)$. If $\Delta^e < 0$, almost all[12] additional stable equilibria occur at discontinuities which form the lower bound of a continuity interval of $\dot{q}(q)$ which is at least partly in the increasing branch of $\dot{\tilde{q}}(q)$.

We note that this corollary implies that with negative Δ^e and a monotonously decreasing function $\dot{\tilde{q}}(q)$ the discontinuity will never induce additional equilibria. The relevance of this insight becomes obvious if one remembers that with negative Δ^e the existence of an increasing branch of $\dot{\tilde{q}}(q)$ is only possible if Δ^g is sufficiently smaller than Δ^e.

With more stable equilibria, temporary policies are more likely to induce a permanent shift in market structures or market outcomes, but as the larger number of stable equilibria become less distant, such permanent effects of temporary policies tend to be smaller. Much of the discussion in the following section on policy implications is based on this insight.

4. Policy implications

Policy implications of our model depend to some degree on the exact definition of policy goals. Within the realm of environmental policy in general and traffic-emissions policy in particular, policy goals may run the gamut from the dissemination of environment-friendly products over a reduction of particularly polluting products to straight emission reductions. Very often, improvement of the environment and emission reductions may be the final goal, but political activism concentrates on preliminary targets such as electrically driven cars replacing gasoline-driven cars. General adoption of environmental norms, such as the sustainable-transportation norm we have been using as a running example in our model, may also serve as one of the more immediate goals.

All these goals may be affected by the occurrence of an innovation such as electrically driven cars with similar consumption properties as conventional cars have today. If the innovation is unrelated to a norm, or if adoption and abandonment of the norm do not depend on the relative frequency of the consumption of the new, norm-compliant product variant, then there would be few arguments for government support of the new technology, except for the internalization of external effects. However, if the dissemination of the innovation is linked to a norm in the two ways we have described in our model, namely both higher valuation of the new product by norm bearers and the feedback of norm-compliant consumption on the dissemination of the norm, then the introduction of a norm-compliant innovation ceases to have unambiguous effects.

We have discussed the case that the conformity bias may be so strong that it hinders the dissemination of the innovation. In fact, as the innovation allows for the observable choice

[12] The only case in which an additional equilibrium may be in a continuity interval of $\dot{q}(q)$ occurs if $\dot{\tilde{q}}(q)$ has a minimum, this minimum is positive, and a continuity interval of $\dot{q}(q)$ embraces this minimum, has an interior minimum which is negative and has positive limits at both bounds.

between norm-compliant and norm-violating behavior, the innovation may reduce the number of norm adopters, if it enters the market only in small numbers at the beginning, and thereby hinders its own further dissemination into the market. It is particularly in these cases were political interference with market forces (and norm formation!) is appropriate. However, policy measures should be carefully chosen. It would be detrimental, if policy aimed at (and succeeded in) increasing the influence of the normative sphere on the market by strengthening the conformity bias in society. Such policy measures would only reinforce the innovation-curbing effects of the conformity bias. However, policy should be willing to strongly support the innovation in an early stage by improving the market parameters in order to shift the market-norm system into the region of attraction of the high level of norm adoption. Only in the long run, such policies should be replaced by supporting the conformity bias in order to further shift the "good" equilibrium towards more norm adoption. The reverse order of these measures may have detrimental effects: the system may be driven to the bad equilibrium if it exists, and this may make later successful market interference extremely expensive.

Among the market parameters to be influenced politically, choices should be made according to the dissemination of the norm in society. Political measures which alter the effect that the norm imposes on demand should only be taken when norm adoption is wide already. If it is not, the effect is not only diminished by the small number of individuals who may react to the policy measure, but also by a possible reintroduction of at least some cognitive dissonances from having the norm but not complying with it, which in our model would be tantamount to reducing *CB*. The effect would be less norm adoption and thus even less effectiveness of the political instruments. Policies which affect the valuation of both norm adopters and hedonists in the same way (such as a subsidy for consumption of norm-compliant behavior) or operate on the supply side (such as cost reductions) will of course have the desired effects too, but cannot be tailored to the level of norm adoption.

If the norm compels individuals to use electric mobility rather than to avoid gasoline-driven cars, i.e. if the effect of norm adoption on individual demand for electric cars (Δ^e in our model) is positive, then discontinuity of the number of firms may have to be considered in making decisions on political action to support the innovation of electric cars. In particular, if the number of suppliers is small due to an initially low demand for such cars, discontinuity effects tend to be large. If they are, temporary policy measures supporting the innovation are more likely to have permanent effects. In addition, the permanence of the effects is triggered faster than if multiplicity of equilibria only stems from positive feedback loops in norm formation (in our model working via the market). However, this permanence cuts both ways. Not only the return to an initial equilibrium with lower consumption of the innovation is avoided, but also further increases in consumption may be blocked. If additional stable equilibria occur on the way from an equilibrium of little consumption to an equilibrium of much consumption, then their regions of attraction may trap the system before it can evolve to the region of attraction of the "best" equilibrium. Hence, if policy suspects the existence of multiple equilibria due to positive feedback loops in the norm formation process and the market structure on the new market is a small oligopoly or even a monopoly, then policies aiming at overcoming equilibria of little norm adoption have to be particularly strong and patient.

5. Conclusions

In this paper, we have shown that norms may not only be self-stabilizing if norm adoption is a frequency dependent opinion formation process with direct positive feedback loops, but also if norm adoption depends on observed market behavior, in particular on the proportion of norm compliant consumption. The positive feedback loop necessary for self-stabilizing norms is then mediated through the market effects of norms. We also have discussed a second source of norm stabilization, the market structure, which allows easier norm adoption and compliance if more suppliers offer a norm-compliant good at lower prices. It turned out that this feedback loop may reinforce already existing positive frequency dependency as source of multiplicity of equilibria, but will rarely induce multiple equilibria on its own.

We have shown that norm stabilization via the market is only possible, if adoption of the norm strongly alters preferences. Only when the norm-compliant variant of a good is hardly consumed at all by "hedonists", i.e. individuals who have not adopted the norm, but replaces the consumption of the norm-violating variant to a very large extent, when an individual adopts the norm, may we get multiple equilibria in the norm-market dynamics. Even when this is the case, equilibria tend to be multiple unless the effect of the norm on the consumption levels is extreme. With multiple equilibria, there is a good reason for policy to support the dissemination of the norm-compliant variant of the product and thereby the dissemination of the supporting norm.

For the case of electric mobility, one may well have severe doubts whether a norm favoring this form of sustainable transportation may affect consumption decisions strongly enough to allow for self-stabilization of the norm. As a consequence, political measures to replace conventional by electric individual transportation should probably not rely too much on social norms and the hope that they influence markets in a way which stabilizes the norms again.

If, however, this pessimism is not fully justified, then a narrow supply side on the market for electric mobility may have to be considered as well. If at least initially, only a small number of oligopolistic producers supply on the market, the discontinuity of the number of firms may entail additional equilibria of the norm-market system. If it does, permanent effects of temporary parameter changes are more likely, but are smaller than without the discontinuity.

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Appendix

Proof of Proposition 1: The demand system (12) in vector notion is given by $\begin{pmatrix} p^e \\ p^g \end{pmatrix} = A\begin{pmatrix} X^e \\ X^g \end{pmatrix} + b$. According to Okuguchi and Szidarovszky (1990, p.34) given the linear structure of the model negative definiteness of $A+A^T$ is sufficient for uniqueness of the Cournot equilibrium. Eigenvalues of $A+A^T$ are given by

$-\frac{1}{\kappa-\lambda}\left(1 \pm \sqrt{1-\frac{4\lambda(\kappa-\lambda)}{(\kappa+\lambda)^2}}\right)$ and negative by inspection. QED

Proof of Lemma 2: $\frac{\partial m^*}{\partial q} \gtreqless 0 \Leftrightarrow \frac{\chi_a^e - \chi_h^e}{\sqrt{k\kappa}} \gtreqless 0$. QED

Equilibrium prices of system (13) are given by:

$$p^e = \frac{1}{\kappa^2-\lambda^2}\left(\left(q\chi_a^e + (1-q)\chi_h^e\right)\left(\frac{\kappa^2-\lambda^2}{(1+m)\kappa} + \frac{\lambda^2}{(n+1)\kappa}\right) + \frac{\lambda}{n+1}\left(q\chi_a^g + (1-q)\chi_h^g\right)\right) + c^e\frac{m}{m+1} + c^g\frac{(n-m)\lambda}{(n+1)(m+1)\kappa}$$

$$p^g = \frac{1}{n+1}\frac{\left(q\chi_a^g + (1-q)\chi_h^g\right)\kappa + \left(q\chi_a^e + (1-q)\chi_h^e\right)\lambda}{\kappa^2-\lambda^2} + \frac{n}{n+1}c^g$$

Proof of Lemma 3: is given in the paper QED

Derivation of vertices of MES:

The lower left vertex (point A) is given by $\begin{pmatrix} \Delta^e \\ \Delta^g \end{pmatrix}^A = \begin{pmatrix} -\theta^e + \frac{(1-CB)\alpha\delta_h}{(1+n)(1-\alpha)\delta_a}\left(n\theta^g - \lambda\theta^e\right) \\ -\theta^g + \frac{(1-CB)\alpha\delta_h\lambda}{n(1+n)(1-\alpha)\delta_a}\left(n\theta^g - \lambda\theta^e\right) \end{pmatrix}$, because $q^{Max}\left(\left(\Delta^e,\Delta^g\right)^A\right) = 1$ and $\dot{q}\left(q^{Max} = 1\right) = -\frac{X^g}{X^e}(1)$. Hence $\left(\Delta^e,\Delta^g\right)^A$ $\left(\Delta \mathrm{e}^A, \Delta g^A\right)$ is a solution to $X^g(1) = 0$ and $\dot{q}\left(q^{Max} = 1\right) = 0$.

The upper left vertex (point B) is derived by similar conditions, $\dot{q}\left(q^B\right) = 0$, $\dot{q}'\left(q^B\right) = 0$ and $\tilde{X}^g(1) = 0$. The first two condition reduce to: $\frac{\alpha}{1-\alpha}(lq+k)Z + (1-q)Z^2 = q,\ \frac{\alpha}{1-\alpha}l - Z + (1-q)Z' - \frac{1}{Z} + q\frac{Z'}{Z^2} = 0$, where $lq + k = (1+CB)\sigma_a - q\left((1+CB)\sigma_a + (1-CB)\sigma_h\right)$. After some algebra it turns out that

$$q^B = \frac{\theta^e - \frac{\alpha}{1-\alpha}\frac{(n+1)\left(l\theta^e - k\Delta^e\right)}{n\theta^g - \lambda\theta^e}\theta^e}{\Delta^e + 2\theta^e - \frac{\alpha}{1-\alpha}\frac{(n+1)\left(l\theta^e - k\Delta^e\right)}{n\theta^g - \lambda\theta^e}\Delta^e}$$

. Again, to obtain a relation between Δ^e and Δ^g we plug this value into $\dot{q}\left(q^B\right) = 0$. This gives us the third vertex:

$$\begin{pmatrix}\Delta^e \\ \Delta^g\end{pmatrix}^{\mathrm{B}} = \begin{pmatrix} \dfrac{1}{(1+CB)^2(1+n)^2\alpha^2\delta_a^{\,2}(-2+\delta_h)} \begin{pmatrix} -(1+CB)(1+n)^2\alpha^2\delta_a\delta_h(-1+\delta_a+CB(1+\delta_a-\delta_h)+\delta_h)\theta^e \\ +(1+n)\delta_a((1+n)(-2+\alpha(4+\alpha(-2+2(1+CB)^2\delta_a+\delta_h-CB^2\delta_h)))\theta^e \\ +(1+CB)n(-1+\alpha)\alpha\theta^g-(1+CB)(-1+\alpha)\alpha\lambda\theta^e)+\sqrt{\Upsilon} \end{pmatrix} \\ \dfrac{\lambda}{n}\left(\Delta^e\right)^{\mathrm{B}}+\dfrac{\lambda}{n}\theta^e \end{pmatrix}$$

$$\begin{aligned}\Upsilon \equiv{} & ((1+n)^2\delta_a^{\,2}(((1+n)(2+\alpha(-4+\alpha(2+(-1+CB^2)\delta_h)))\theta^e-(1+CB)n(-1+\alpha)\alpha\theta^g+(1+CB)(-1+\alpha)\alpha\theta^e\lambda)^2 \\ & +(1+CB)^2\alpha^2\delta_h^{\,2}(-n\theta^g+\lambda\theta^e+\alpha((-1+CB)(1+n)\theta^e+n\theta^g-\lambda\theta^e))^2- \\ & 2(1+CB)\alpha^2\delta_h((1+CB)n^2(-1+\alpha)^2\left(\theta^g\right)^2+(1+CB)n(-1+\alpha)\theta^e\theta^g(-(-1+CB)(1+n)\alpha(-3+\delta_h)+2\lambda-2\alpha\lambda) \\ & +\left(\theta^e\right)^2((-1+CB)(1+n)^2(2(-1+\delta_h)+\alpha(4-4\delta_h+\alpha(-2+\delta_h+CB^2\delta_h))) \\ & +(-1+CB^2)(1+n)(-1+\alpha)\alpha(-3+\delta_h)\lambda+(1+CB)(-1+\alpha)^2\lambda^2))))\end{aligned}$$

We can solve for the upper right vertex (point C) only for $\alpha=0$. It is given by the intersection of $\Delta^{e,Max}\left(\Delta g\right)$ and $\Delta^{e,Min}\left(\Delta g\right)$, characterized by solutions to $\dot{q}\left(q^{Max}\right)=0$ and $\dot{q}\left(q^{Min}\right)=0$. It follows that such a $\left(\Delta^e,\Delta^g\right)$-pair is given by three condition $\dot{q}\left(q^C\right)=0$, $\dot{q}'\left(q^C\right)=0$ and $\dot{q}''\left(q^C\right)=0$. We can solve for q^{IP} explicitly:

$q^C=\dfrac{n\theta^g-\lambda\theta^e}{3\left(n\Delta^g-\Delta^e\lambda\right)+4\left(n\theta^g-\lambda\theta^e\right)}$. However $\left(\Delta^e,\Delta^g\right)^C$ cannot be solved analytically.

q^C is derived by rewriting $\dot{q}\left(q^C\right)=0$, $\dot{q}'\left(q^C\right)=0$ and $\dot{q}''\left(q^C\right)=0$ as $Z^2\left(q\right)=\dfrac{q}{1-q}$,

$\left(1-q\right)Z'-Z-\dfrac{1}{Z}+q\dfrac{Z'}{Z^2}=0$ and $-2Z'+2\dfrac{Z'}{Z}+\left(1-q\right)Z''+q\dfrac{Z''}{Z}-2q\dfrac{\left(Z'\right)^2}{Z^3}=0$, where $Z=\dfrac{\tilde{X}^e}{\tilde{X}^g}$. The definition of Z implies the following relation between Z and its second derivative: $Z''=-2Z'\dfrac{\tilde{X}'^g}{\tilde{X}^g}$.

<u>Proof of Lemma 5:</u>

The situation where MES is empty corresponds to the case point A, B and C are equal, i.e. where $q^{Max}=q^{Min}\equiv q^{IP}=1$, $\dot{q}\left(1\right)=0$ and $X^g\left(1\right)=0$. The latter two condition give a solution for Δ^g as a function of α: $\Delta^g\left(\alpha\right)=\dfrac{\alpha\lambda\delta_h\left(1-CB\right)\left(n\theta^g-\lambda\theta^e\right)}{n(n+1)(1-\alpha)\delta_a}-\theta^g$. The first condition amounts to a condition for α as a function of Δ^g:

$$\Delta\alpha\left(\Delta^g\right)=$$

$$\frac{2n(n+1)^2\delta_a(\Delta^g+\theta^g)\left(n\Delta^g+\tau\right)+\delta_h\lambda^2\tau^2}{2n(n+1)^2\delta_a(\Delta^g+\theta^g)\left(n\Delta^g+\tau\right)+\delta_h\lambda^2\tau^2+(1+n)\lambda\tau\left(n\left((1+CB)\delta_a+2(1-CB)\delta_h\right)(\Delta^g+\theta^g)-(1-CB)\delta_h\theta^e\lambda\right)}$$

, where $\tau=\left(n\theta^g-\lambda\theta^e\right)$. Solving these two equations for α yields the critical value stated in the Lemma. QED

<u>Proof of Lemma 6:</u>

For $\alpha=0$ the approximation strategy is described in the paper. We therefore present here only the general solution for the tangent point D:

$$\left(\Delta^e\right)^D = \frac{1}{4(1+n)^2(1+\alpha(-2+CB^2\alpha))\delta_a}\left(-\Omega + \sqrt{\frac{8(1+n)^2(1+\alpha(-2+CB^2\alpha))\delta_a\theta^e}{q^3(1-\alpha)\delta_h}\Psi + \Omega^2}\right)$$

$$\left(\Delta^g\right)^D = \frac{1}{2nq^2(1-\alpha)\delta_h}\left(\begin{array}{l} -q^2\left(\Delta^e\right)^D\left(\begin{array}{l}(1+n)\alpha\left(\left(1+CB\right)\delta_a + \left(1-CB\right)\delta_h\right) \\ -2(1-\alpha)\delta_h\lambda\end{array}\right) \\ +q\left(\begin{array}{l} -2\delta_h\tau + \alpha((1+CB)(1+n)\delta_a(\left(\Delta^e\right)^D - \theta^e) \\ +\delta_h((-1+CB)(1+n)\theta^e + 2\tau)) + (1+n)\left(\Delta^e\right)^D\Sigma \end{array}\right) \\ +(1+n)\theta^e\left((1+CB)\alpha\delta_a + \Sigma\right) \end{array}\right)$$

, where

$\tau \equiv \left(n\theta^g - \lambda\theta^e\right)$

$\Sigma \equiv \sqrt{(1+CB)^2(-1+q)^2\alpha^2\delta a^2 - 2(-1+q)q(2+\alpha(-4+\alpha+CB^2\alpha))\delta a\delta h + (-1+CB)^2q^2\alpha^2\delta h^2}$

$$\begin{aligned}\Psi \equiv\ & (1+n)(2q\alpha\delta a\delta h(-3(1+n)(-3+2q)\theta e + 2(1+CB)n(-1+q)\theta g - 2(1+CB)(-1+q)\theta e\lambda) + \\ & \alpha^3(-2q\delta a\delta h((1+n)(-1+CB^2(-2+q)+q)\theta e - (1+CB)(1+CB^2)n(-1+q)\theta g + (1+CB)(1+CB^2)(-1+q)\theta e\lambda) + \\ & (1+CB)^2(-1+q)^2\delta a^2(-(1+CB)n\theta g + \theta e(1+n+\lambda+CB\lambda)) + (-1+CB)^2q^2\delta h^2(-(1+CB)n\theta g + \theta e(1+n+\lambda+CB\lambda))) + \\ & \alpha(-(1+CB)(1+n)(-1+q)\delta a\theta e + q\delta h(-(-1+CB)(1+n)\theta e - 4n\theta g + 4\theta e\lambda))\Sigma + 2q\delta h((1+n)(-3+2q)\delta a\theta e + (n\theta g - \theta e\lambda)\Sigma) - \\ & \alpha^2((1+CB)^2(1+n)(-1+q)^2\delta a^2\theta e + \delta a(2q\delta h(-(1+n)(-7+CB^2(-2+q)+5q)\theta e + 4(1+CB)n(-1+q)\theta g - 4(1+CB)(-1+q)\theta e\lambda) - \\ & (1+CB)(-1+q)(-(1+CB)n\theta g + \theta e(1+n+\lambda+CB\lambda))\Sigma) + q\delta h((-1+CB)^2(1+n)q\delta h\theta e + (-(1+CB^2)n\theta g + \theta e(1+n+\lambda+CB(-1-n+CB\lambda)))\Sigma)))\end{aligned}$$

$\Omega \equiv \frac{1}{q^2(-1+\alpha)\delta_h}\left(\Psi + 2(1+n)^2q(-1+\alpha)(1+\alpha(-2+CB^2\alpha))\delta_a\delta_h\theta^e\right)$

To find such a point we follow the following approach: First we express two of the conditions for the inflection point C $\dot{q}(q)=0, \dot{q}'(q)=0$ in terms of $\Delta g(q)$. With these two conditions we can solve for Δe. However, we still have to find a q that will be greater than q^{IP} and independent of Δe and Δg.

For the general case $\alpha \neq 0$ we again choose a q such that we can be sure that it will correspond to a point on the graph of $\Delta^{e,Max}(\Delta g)$. This can be achieved by choosing $(\Delta g)^B$ as a lower bound for Δg and $-\theta e$ as a lower bound for Δe.

$$\underline{q} = q^{IP}\left(\Delta g = \Delta g^{\mathrm{B}}, \Delta e = -\theta e\right)$$

$$= \frac{\begin{array}{l}\delta_h\left[(1-CB)\alpha(1+n)n\left(\Delta g\theta e + \theta g\Delta e\right) + 2n(1-\alpha)\left(n\Delta g - \lambda\Delta e\right)\left(n\theta g - \theta e\lambda\right) - 2\lambda\left(\left((1-CB)(1+n)\alpha\Delta e\right)\theta e\right)\right] \\ +(1+n)\delta_a\left((1+CB)n\alpha\left(\Delta g\theta e + \theta g\Delta e\right) + \Delta e^2((1+n)(-1+\alpha) + (1+CB)\alpha\lambda) - \Delta e\left(n(-2\theta e + \alpha((1+CB)\Delta g + 2\theta e)) + 2\theta e(-1+\alpha+(1+CB)\alpha\lambda)\right)\right)\end{array}}{3((1+n)\delta a\Delta e((1+n)(-1+\alpha)\Delta e - (1+CB)n\alpha\Delta g + (1+CB)\alpha\Delta e\lambda) + \delta h(n\Delta g - \Delta e\lambda)(-n\Delta g + \Delta e\lambda + \alpha((-1+CB)(1+n)\Delta e + n\Delta g - \Delta e\lambda)))}$$

This gives us a lower bound for the maxima that correspond to $\Delta^{e,Max}(\Delta g)$ independent of Δe and Δg. We can then calculate the slope at point D: $\frac{d\Delta^e}{d\Delta^e} = \frac{\left(\Delta^e\right)^{\mathrm{D}}\underline{q} + \theta^e}{\left(\Delta^g\right)^{\mathrm{D}}\underline{q} + \theta^g}$.

<u>Proof of Lemma 7:</u> Inserting equilibrium values in (14) and reformulation yields the result. QED

<u>Proof of Lemma 8:</u>

(1.) At the discontinuities we have $m^* = m^{eq}$ and thus $\dot{q}(q) = \dot{\hat{q}}(q)$. Otherwise, $m^* > m^{eq}$ implies $\tilde{X}^e > \hat{X}^e$ and $\tilde{X}^g < \hat{X}^g$ due to $\frac{dX^{e*}}{dm} > 0$ and $\frac{dX^{g*}}{dm} < 0$. Hence $\dot{q}(q) < \dot{\hat{q}}(q)$ for all q in the intervals of continuity. For the second part of the claim note that we can write $\dot{q} = \dot{q}\left(\frac{X^e}{X^g}(m(q),q), q\right)$ and thus $\frac{d\dot{q}}{dq} = \frac{\partial\dot{q}}{\partial\frac{X^e}{X^g}}\left(\frac{\partial\frac{X^e}{X^g}}{\partial m}\frac{dm}{dq} + \frac{\partial\frac{X^e}{X^g}}{\partial q}\right) + \frac{\partial\dot{q}}{\partial q}$. Since

$\frac{dm^*}{dq}=\frac{\Delta^e}{\sqrt{k\kappa}}$ and $\frac{dm^{eq}}{dq}=0$ for all q in the intervals of continuity, and the other terms in $\frac{d\dot{q}}{dq}$ are the same for the discontinuous version of $\dot{q}$ and its continuous approximation $\dot{\tilde{q}}$, the observation

$\frac{\partial \dot{q}}{\partial \frac{X^e}{X^g}}=(1-\alpha)\left((1-q)\sigma_a+\frac{q\sigma_h}{\left(\frac{X^e}{X^g}\right)^2}\right)>0$ implies the second claim of the lemma.

(2.) The distance between two discontinuities is a natural multiple of $\sqrt{k\kappa}/\Delta^e$ because $\frac{dm^*}{dq}=\frac{\Delta^e}{\sqrt{k\kappa}}$ and thus m^* reaches the next integer at this frequency. Finally for $\Delta^e>0$, the lower limit at the discontinuities is obviously smaller than the upper limit and size of the "jumps" of $\hat{X}^e$ at a discontinuities q_i is given by

$\left(\frac{\hat{m}}{\hat{m}+1}-\frac{\hat{m}-1}{\hat{m}}\right)\left(\chi_h^e+\Delta^e q_i-\kappa c^e+\lambda c^g\right)=\frac{\sqrt{k\kappa}}{\hat{m}}$, where $\hat{m}=m^*(q_i)$. Since m^* grows in q, the size of the "jumps" declines in q. For $\Delta^e<0$, exactly the opposite is true. QED

<u>Derivation of the partial effects on the critical value $\alpha^{crit.}$:</u>

$$\alpha^{crit.}=\frac{(1-CB)(n+1)\theta e+2(n\theta g-\lambda\theta e)-\sqrt{\left((1-CB)(n+1)\theta e\right)^2+4(n\theta g-\lambda\theta e)^2(1-CB^2)}}{2\left(CB^2(n\theta g-\lambda\theta e)+(n+1)(1-CB)\theta e\right)}$$

$$\underset{y=2(n\theta g-\lambda\theta e)}{\overset{x=(1-CB)(n+1)\theta e}{=}}\frac{x+y-\sqrt{x^2+y^2(1-CB^2)}}{CB^2y+2x}$$

$$\Rightarrow\frac{\partial\alpha^{crit.}}{\partial\theta g}=\frac{\left(y'-\frac{2yy'(1-CB^2)}{2\sqrt{x^2+y^2(1-CB^2)}}\right)(CB^2y+2x)-\left(x+y-\sqrt{x^2+y^2(1-CB^2)}\right)(CB^2y')}{(CB^2y+2x)^2}$$

$$=\frac{y'(CB^2y+2x)-\frac{2yy'(1-CB^2)(CB^2y+2x)}{2\sqrt{x^2+y^2(1-CB^2)}}-xCB^2y'-yCB^2y'+CB^2y'\sqrt{x^2+y^2(1-CB^2)}}{(CB^2y+2x)^2}$$

$$=\frac{(2-CB^2)xy'-\frac{2yy'(1-CB^2)(CB^2y+2x)}{2\sqrt{x^2+y^2(1-CB^2)}}+CB^2y'\sqrt{x^2+y^2(1-CB^2)}}{(CB^2y+2x)^2}>0\Leftrightarrow$$

$$(2-CB^2)xy'2\sqrt{x^2+y^2(1-CB^2)}-2yy'(1-CB^2)(CB^2y+2x)+CB^2y'2\left(x^2+y^2(1-CB^2)\right)>0\overset{y'>0}{\Leftrightarrow}$$

$$(2-CB^2)x\sqrt{x^2+y^2(1-CB^2)}-y(1-CB^2)(CB^2y+2x)+CB^2\left(x^2+y^2(1-CB^2)\right)>0\overset{x>0}{\Leftrightarrow}$$

$$(2-CB^2)\sqrt{x^2+y^2(1-CB^2)}-2y(1-CB^2)+xCB^2>0\Leftrightarrow$$

$$(2-CB^2)\sqrt{x^2+y^2(1-CB^2)}>2y(1-CB^2)-xCB^2\Leftarrow$$

$$(2-CB^2)^2\left(x^2+y^2(1-CB^2)\right)>\left(2y(1-CB^2)-xCB^2\right)^2\Leftrightarrow$$

$$(2-CB^2)^2\left(x^2+y^2(1-CB^2)\right)-\left(2y(1-CB^2)-xCB^2\right)^2=(1-CB^2)(2x+yCB^2)^2>0$$

$$\frac{\partial\alpha^{crit.}}{\partial CB}=\frac{\left(x'-\frac{2xx'-2CBy^2}{2\sqrt{x^2+y^2\left(1-CB^2\right)}}\right)\left(CB^2y+2x\right)-\left(x+y-\sqrt{x^2+y^2\left(1-CB^2\right)}\right)\left(2CBy+2x'\right)}{\left(CB^2y+2x\right)^2}$$

$$=\frac{x'\left(CB^2y+2x\right)-\frac{2xx'-2CBy^2}{2\sqrt{x^2+y^2\left(1-CB^2\right)}}\left(CB^2y+2x\right)-\left(x+y-\sqrt{x^2+y^2\left(1-CB^2\right)}\right)\left(2CBy+2x'\right)}{\left(CB^2y+2x\right)^2}>0\Leftrightarrow$$

$$x'\left(CB^2y+2x\right)-\frac{2xx'-2CBy^2}{2\sqrt{x^2+y^2\left(1-CB^2\right)}}\left(CB^2y+2x\right)-\left(2CBy+2x'\right)\left(x+y\right)+\left(2CBy+2x'\right)\sqrt{x^2+y^2\left(1-CB^2\right)}>0\Leftrightarrow$$

$$-\frac{2xx'-2CBy^2}{2\sqrt{x^2+y^2\left(1-CB^2\right)}}\left(CB^2y+2x\right)-\left(2CBy\right)\left(x+y\right)-\left(2-CB^2\right)x'y+\left(2CBy+2x'\right)\sqrt{x^2+y^2\left(1-CB^2\right)}>0\Leftrightarrow$$

$$-\left(2xx'-2CBy^2\right)\left(CB^2y+2x\right)+\left(2CBy+2x'\right)2\left(x^2+y^2\left(1-CB^2\right)\right)-\left(2CBy\left(x+y\right)+\left(2-CB^2\right)x'y\right)2\sqrt{x^2+y^2\left(1-CB^2\right)}>0$$

$$\overset{y>0}{\Leftrightarrow}\underbrace{-xx'CB^2}_{>0}+CBy^2\underbrace{\left(2-CB^2\right)}_{>0}+2xy\underbrace{\left(1+CB-CB^2\right)}_{>0}+2x^2CB-\left(2CB\left(x+y\right)+\left(2-CB^2\right)x'\right)\sqrt{x^2+y^2\left(1-CB^2\right)}>0$$

$$-xx'CB^2+CBy^2\left(2-CB^2\right)-2xy+2x^2CB-\left(2CB\left(x+y\right)+\left(2-CB^2\right)x'\right)\sqrt{x^2+y^2\left(1-CB^2\right)}\overset{x'=-\frac{x}{1-CB}}{=}$$

$$x^2\frac{CB^2}{1-CB}+CBy^2\left(2-CB^2\right)-2xy+2x^2CB-\left(2CB\left(x+y\right)+\left(2-CB^2\right)x'\right)\sqrt{x^2+y^2\left(1-CB^2\right)}>$$

$$x^2\frac{CB^2}{1-CB}+CBy^2\left(2-CB^2\right)-2xy+2x^2CB-\left(2CB\left(x+y\right)+\left(2-CB^2\right)x'\right)\left(x+y\sqrt{\left(1-CB^2\right)}\right)=$$

$$x^2\left(\frac{CB^2}{1-CB}+\frac{2-CB^2}{1-CB}\right)+CBy^2\left(2-CB^2-2\sqrt{1-CB^2}\right)+2xy\left(-1-CB\sqrt{1-CB^2}+\left(2-CB^2\right)\frac{\sqrt{1-CB^2}}{2\left(1-CB\right)}\right)=$$

$$x^2\left(\frac{2}{1-CB}\right)+CBy^2\underbrace{\left(2-CB^2-2\sqrt{1-CB^2}\right)}_{\geq 0}+2xy\left(-1+\frac{\sqrt{1-CB^2}}{2\left(1-CB\right)}\left(1+\left(1-CB\right)^2\right)\right)=$$

$$x^2\left(\frac{2}{1-CB}\right)+CBy^2\left(2-CB^2-2\sqrt{1-CB^2}\right)+2xy\left(-1+\frac{\sqrt{1-CB^2}}{2\left(1-CB\right)}\left(1+\left(1-CB\right)^2\right)\right)=$$

$$1:\left(-1+\frac{\sqrt{1-CB^2}}{2\left(1-CB\right)}\left(1+\left(1-CB\right)^2\right)\right)\geq 0\Rightarrow\text{claim}$$

$$2:\left(-1+\frac{\sqrt{1-CB^2}}{2\left(1-CB\right)}\left(1+\left(1-CB\right)^2\right)\right)<0$$

$$\left(x-y\sqrt{\left(1-\frac{\sqrt{1-CB^2}}{2\left(1-CB\right)}\left(1+\left(1-CB\right)^2\right)\right)}\right)+y^2\underbrace{\left(CB\left(2-CB^2-2\sqrt{1-CB^2}\right)-\left(1-\frac{\sqrt{1-CB^2}}{2\left(1-CB\right)}\left(1+\left(1-CB\right)^2\right)\right)\right)}_{\equiv f(CB)}\geq 0$$

because: $f\left(0\right)=0$ $\quad f'\left(CB\right)=-2CB+\frac{2CB}{\sqrt{1-CB^2}}+\left(1+\left(1-CB\right)^2\right)\frac{\sqrt{1-CB^2}-2CB\sqrt{\frac{1-CB}{1+CB}}}{4\left(1-CB\right)^2}$

$$=\underbrace{-2CB+\frac{2CB}{\sqrt{1-CB^2}}}_{\geq 0}+\frac{\left(1+\left(1-CB\right)^2\right)}{4}\underbrace{\left(\sqrt{\frac{1+CB}{\left(1-CB\right)^3}}-2CB\sqrt{\frac{1}{\left(1+CB\right)\left(1-CB\right)^3}}\right)}_{\geq 0}\geq 0$$
