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Abstract

The literature on the wage curve provides considerable evidence in favor of a negative relationship between unemployment and wages. It is thus often seen as a refutation of the Harris-Todaro model, who point to a positive relationship. This paper shows that both strands of literature are special cases of a more general approach by combining a New Economic Geography model with monocentric cities and efficiency wages. Whether the relationship is positive or negative depends on the transportation costs between the cities and commuting costs within them. The model helps explain whether and under which conditions the agglomeration of economic activity is associated with higher unemployment and why controls for agglomeration should be included in wage curve regressions.

JEL Classification: R12, R14, R23

Keywords: New Economic Geography, Urban Economics, Efficiency Wages, Unemployment, Disparities, Regional Migration

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1   Introduction

The wage curve, i.e. the negative relationship between unemployment and wages, is a well-observed fact. More recently, models introducing the labor market into the New Economic Geography (NEG) have been proposed to explain how a wage curve might arise endogenously. In the strand of literature following Harris and Todaro (1970), it is often argued that migration is based on expected income and that in the migration equilibrium there should exist a positive relationship between unemployment and wages. The empirical evidence in favor of the wage curve is therefore often seen as a refutation of the Harris-Todaro model (Freeman; 2009).

This paper presents a model to show that both strands of literature, the wage curve and the Harris-Todaro model, are special cases of a more general approach. It encompasses an NEG model with monocentric cities and efficiency wages to explain how a wage curve can arise endogenously, while the degree of agglomeration shifts the wage curve due to commuting costs. The wage is higher in the agglomeration because of the centripetal forces of the NEG part of the model. However, the unemployment rate is only lower in the agglomeration when the wage surplus over the periphery is large enough to compensate for the higher commuting costs in the agglomeration. The size of the wage surplus depends on the centripetal forces and hence on the transport costs. Whether the relationship between unemployment and wages is positive or negative therefore depends on transportation and commuting costs.

The wage curve is empirically well understood. Blanchflower and Oswald (1994) first presented empirical results for this relationship and numerous studies followed, providing empirical evidence in favor of the wage curve relationship.[1] They further point to several theoretical motivations for the wage curve. Following their analysis, several authors have presented theoretical models to explain the wage curve relationship, such as Campbell and Orszag (1998) using efficiency wages or Sato (2000) using search theory. Blien (2001) presents a survey of such approaches. Nevertheless, these models are usually unable to explain how regional labor market disparities arise endogenously but rather rely on exogenous disparities in productivity, sectoral structure or labor immobility.

In recent years, several authors have introduced the labor market into the NEG. In these models, there typically are two regions, an agglomeration and a periphery, where the wage (unemployment rate) is generally higher (lower) in the agglomeration compared with the periphery. These models explain how the agglomeration of economic activity and disparities in wages and unemployment rates (and thus how the wage curve) arise endogenously.

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[1] See Blanchflower and Oswald (2005) for an overview of existing studies and Nijkamp and Poot (2005) for a meta-analysis.

Monthly Compensation per Employee, 2009

(a) Monthly Compensation per Employee
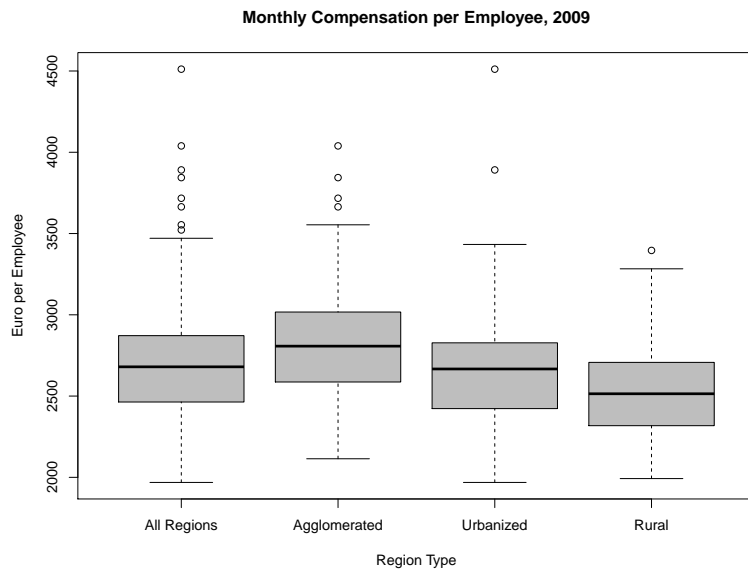
Wage Curves and Region Types
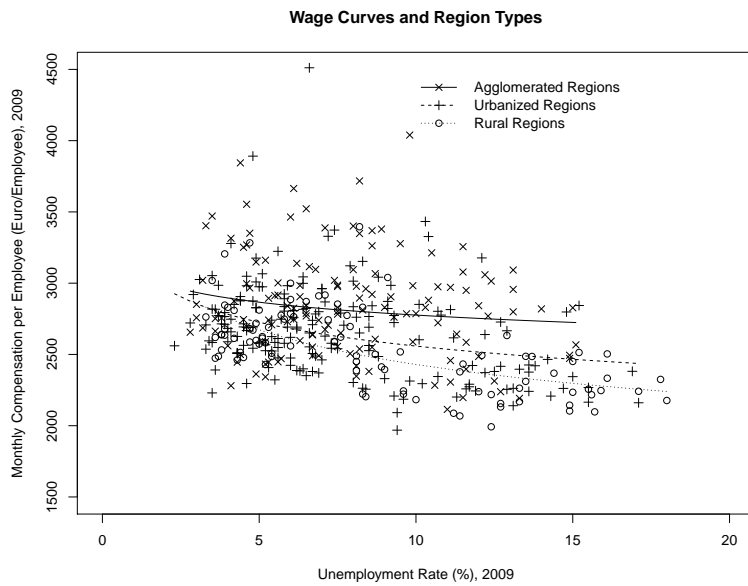
(b) Wage Curves and Region Types

Source: BBSR and BBR (2011), authors own illustration.

Figure 1: Regional Labor Market Disparities in German NUTS 3 Regions

There exists a broad empirical strand of literature showing that wages are indeed typically higher in agglomerated regions compared with peripheral regions.[2] To illustrate this, Figure 1(a) presents box plots of the monthly compensation per employee by region type for German Kreise in 2009.

The relationship between agglomeration and unemployment is, however, less clear. As Elhorst (2003) points out in his survey, empirical evidence of the influence of density on the unemployment rate is mixed at the regional level. On one hand, the wage curve relationship suggests that higher productivity and higher wages in more agglomerated regions are linked to lower unemployment there. On the other hand, the strand of literature following Harris and Todaro (1970) suggests that when people base their migration decisions on expected incomes, in the migration equilibrium higher wages (due to higher productivity) in the agglomeration must be compensated for by higher unemployment or other factors.[3]

Figure 1(b) contains the wage curves for different levels of agglomeration (measured by population density), visualizing both arguments. It becomes obvious that the degree of agglomeration works as a shift parameter, shifting the wage curve outwards. This shift might represent the arguments of Harris and Todaro (1970): a situation of higher wages in the agglomeration compared with the periphery is only stable (i.e. no adjustment through migration) when the wage surplus of the agglomeration is compensated for by higher unemployment or other factors. This is supported by recent empirical studies on the wage curve. For example, Eckey et al. (2008) find that population density is a shift parameter of the wage curve, whereas Baltagi et al. (2010) show that the absolute wage elasticity is higher in rural regions compared with agglomerated regions.

The present paper therefore builds a theoretical model to show that the wage curve and Harris-Todaro model are both special cases of a more general approach rather than contradictions. It is based on an NEG model including unemployment, where each region contains a single monocentric city with intra-city commuting and inter-city migration. Commuting is costly and serves as a centrifugal force and as a shift parameter for the wage curve. The model shows that it depends on the level of commuting costs within the cities and on the level of trade costs between the cities whether the unemployment rate is higher or lower in the agglomeration compared with the periphery. The sign of the relationship between unemployment and wages thus depends on transportation and commuting costs.

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[2]This strand of literature follows the seminal paper by Glaeser and Mare (2001). Heuermann et al. (2010) present an overview of this strand of literature. However, this strand of literature (on the urban wage premium) usually focuses on the technological external effects of agglomeration as the underlying economic mechanism, whereas the present model focuses on pecuniary effects.

[3]Partridge and Rickman (1997) argue that more research is necessary to discuss whether the wage curve strand of literature really refutes the Harris-Todaro model.
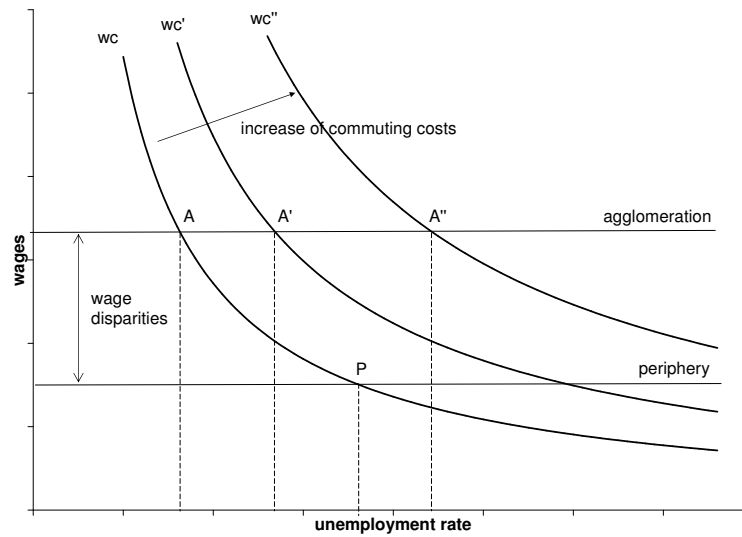
Figure 2: Visualization of the Core Arguments

The basic arguments of this paper are illustrated in Figure 2. Owing to efficiency wages, there is a negative relationship between unemployment and wages in each region. Further wages in the agglomeration are higher compared with the periphery due to centripetal forces. The wage difference between the agglomeration and periphery depends on centripetal forces, which are driven by transport costs. When there are no commuting costs, the incentive to shirk solely depends on wages and unemployment so that the wage curve is identical in both regions. Higher wages in the agglomeration then are accompanied by lower unemployment compared with the periphery. However, when commuting costs are non-negligible, the wage curve of the agglomeration shifts outwards compared with the periphery since there is an additional incentive to shirk, which is more prevalent in the agglomeration. Therefore, the relative unemployment rate between the agglomeration and periphery depends on both the wage differential and the shift of the wage curve. Since wages are always higher in the agglomeration, the relationship between unemployment and wages also depends on the wage differential and shift of the wage curve, where the former is driven by transport costs and the latter by commuting costs. Figure 2 is discussed in more detail in Section 6.2 to illustrate the results of this paper.

The remainder of the paper is organized as follows. Section 2 briefly reviews the related literature and the basic model is presented in Section 3. Sections 4 and 5 contain the discussion of the equilibria and their stability. In Section 6, the implications of the model for regional labor market disparities are presented. The last Section draws the conclusions.

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2   Related Literature

The model presented here is linked to two strands of literature: (1) models that combine Urban Economic Theory with the NEG and (2) models that introduce the labor market into the NEG. It is further linked to Zenou and Smith (1995), who introduce efficiency wages into a two-city model with intra-city commuting and inter-city migration but without endogenous agglomeration.[4]

Regarding the first strand of literature, Tabuchi (1998) perhaps offered the first approach combining the NEG, as presented by Krugman (1991), with an urban economic model in the spirit of Alonso (1964). In his model, Tabuchi (1998) compares the influence of inter-city transportation costs on agglomeration advantages in the form of market size with the influence of intra-city commuting costs on the corresponding agglomeration disadvantages. His main result is that economic activity will be dispersed for any set of parameters if inter-city transportation costs become negligible. Murata and Thisse (2005) use a similar approach but derive more results analytically. They show that, contrary to Krugman (1991), agglomeration takes place for high but not for low inter-city transportation costs. They further show that agglomeration is always stable for sufficiently low commuting costs and that there are sets of parameters for which dispersion is always stable irrespective of inter-city transportation costs. Further contributions are delivered by, among others, Tabuchi and Thisse (2006) who focus on the interaction of the preference for variety and increasing returns of production with urban costs or by Cavailhes et al. (2007) who discuss polycentric cities in the interplay of commuting, communication and transportation costs.

The second strand of literature has emerged in recent years. The assumption of full employment has often been regarded as a drawback of the NEG and hence several authors have developed models that introduce imperfect labor markets into the NEG. Many of these models focus on the international level, because they assume substantial differences between the institutional settings of labor markets[5] or because they neglect migration.[6] The present model, however, focuses on the regional level where the labor force migrates between regions and there are no differences between the institutional settings of regional labor markets. Most models of this kind introduce frictions in job matching into the NEG, such as those of Epifani and Gancia (2005), Francis (2009) and vom Berge (2011b,a). Models introducing efficiency wages are presented by Francis (2007) and Zierahn (2011).[7] Egger and Seidel (2008) use a fair wage approach. These

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[4]Their model is presented in more detail by Zenou (2009).

[5]Examples are Peeters and Garretsen (2004) and Pflüger (2004).

[6]Examples are Chen and Zhao (2009), Helpman and Itskhoki (2010), Helpman et al. (2011), Méjean and Patureau (2010), Monford and Ottaviano (2002), Picard and Toulemonde (2006) and Strauss-Kahn (2005).

[7]Südekum (2005) presents an analytically solvable agglomeration model with efficiency wages that, however, does not contain centrifugal forces.

models can explain how the agglomeration of economic activity arises endogenously and why this leads to higher real wages in the agglomeration compared with the periphery. Further they usually find lower unemployment in the agglomeration compared with the periphery.

Exceptions are presented by vom Berge (2011b,a). In vom Berge (2011b), the unemployment rate might be higher or lower in the agglomeration compared with the periphery, because nominal wages can be lower in the agglomeration so that the replacement ratio is higher there, which causes higher unemployment. Only when transportation costs are very low are centripetal forces strong enough to enable higher nominal wages and lower unemployment in the agglomeration compared with the periphery. However, as only symmetry and full agglomeration are possible outcomes, no unemployed are left in the periphery. Further, higher unemployment in the core is caused by the replacement ratio only and thus this crucially depends on lower nominal wages in the agglomeration, which is counterfactual.[8] In vom Berge (2011a), the unemployment rate of low-skilled workers is always higher in the agglomeration because these jobs are more valuable there and the higher unemployment rate compensates for the higher wages, similar to the arguments of the Harris-Todaro model. Nevertheless, only low-skilled workers can become unemployed and they are further immobile between regions so that a major incentive to migrate – escaping unemployment – is neglected.

In the present model, the unemployed are instead allowed to escape their situation by searching for employment in another city and reasons other than the replacement ratio explain why the unemployment rate might be higher in more density populated areas.

As mentioned above, this model is closely linked to Zenou and Smith (1995). They combine efficiency wages with a monocentric city model and extend this to the case of two cities. However, in their model the number of firms in cities and their productivity are assumed to be fixed. Disparities between cities' labor markets emerge because their levels of productivity are different. The authors discuss unemployment disparities between the cities, namely the unemployment rate might be higher or lower in the larger city than it is in the smaller. The present model, by contrast, does not rely on exogenous differences in the productivity levels between cities, but instead relies on an endogenous formation of industry location. From this perspective, the present model therefore might also be viewed as an extension of the Zenou and Smith (1995) framework by introducing the endogenous location of industries.

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[8]The author is well aware of this point and argues that the unemployment rate is higher in the agglomeration as long as the replacement ratio is higher there, which might also be caused by higher nominal benefits in the core instead of lower nominal wages.

3 Basic Model

The model consists of two regions and two sectors, agriculture and manufacturing. Each region contains a single monocentric city with a central business district (CBD). Workers are immobile between sectors, but manufacturing workers are mobile between regions. Agriculture is located outside cities, whereas manufacturing is located in the city centers. Manufacturing workers thus have to commute to the city centers to work or to search for work if they are unemployed.

3.1 Households

Households j receive utility $\mathfrak{U}$ from the consumption of manufactured goods C_M and agricultural goods C_A. They receive the disutility of work effort e. Their utility function is

$$\mathfrak{U}_j = C_{Mj}^{\mu} C_{Aj}^{1-\mu} - e. \tag{1}$$

Households maximize their utility subject to the budget constraint,

$$p_M C_{Mj} + p_A C_{Aj} = Y_j, \tag{2}$$

where p_M and p_A are the prices of manufactured and agricultural goods, and Y_j represents household income. Utility maximization yields

$$p_M C_{Mj} = \mu Y_j, \tag{3}$$

$$p_A C_{Aj} = (1 - \mu)Y_j. \tag{4}$$

$p_A = 1$ is the numeraire, while C_M is a constant elasticity of substitution (CES) bundle of manufactured goods,

$$C_M = \left[\int_0^n c_i^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{5}$$

where θ is the elasticity of substitution between the varieties c_i of the manufactured goods. Shephard's (1953) Lemma,

$$min \int_0^n p_i c_i di \ s.t. \ \left[\int_0^n c_i^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} = C_M, \tag{6}$$

delivers the price index for region r (and s accordingly). p_i represents the prices of varieties of manufactured goods. In the case of two regions where firms are identical and there exist iceberg

transport costs τ, the manufacturing price index P_r in region r is

$$P_r = \left[n_r p_r^{1-\theta} + n_s (\tau p_s)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{7}$$

where n_r and n_s are the number of manufacturing firms, and p_r and p_s are the prices of the varieties of manufactured goods in region r and region s. Household j's demand for variety i of the manufactured good is

$$c_{ij} = \left(\frac{p_i}{p_M} \right)^{-\theta} C_{Mj} = \left(\frac{p_i}{p_M} \right)^{-\theta} \mu \frac{Y_j}{p_M}. \tag{8}$$

Indirect utility is calculated using the income shares,

$$\mathfrak{U}_j = K P_r^{-\mu} Y_j - e, \tag{9}$$

where $K = \mu^\mu (1-\mu)^{1-\mu}$ and where p_M is replaced by P_r as the model focuses on the two-region case.

3.2 Cities

To build two monocentric cities with efficiency wages this model relies on Zenou and Smith (1995). However, the commuting costs differ from their framework in the sense that here commuting costs are measured in units of time lost. The proportion of time spent commuting is not available for effective labor supply, and thus commuting costs have the nature of an iceberg (Murata and Thisse; 2005), similar to iceberg transport costs. The monocentric city framework is now presented for region r (the analogous framework applies to region s).

Manufacturing is located in the CBDs of the two cities and does not consume any space. Workers locate around the CBD (for simplicity, each city is a line). Assume that the cities are so far away from each other that there is no inter-city commuting. Workers choose their places of residence depending on their status, land rents and commuting costs. Owing to the efficiency wage framework, workers can be employed or unemployed. When they are employed, they can decide whether they shirk or not.

Each employee is equipped with one unit of time. Owing to commuting, he or she can only use a proportion $(1 - \chi d)$ of that time for working, where $0 \le \chi \le 1$ are the commuting costs per distance d.[9] Employees earn wages w_r lowered by the tax rate t and have to pay land rents $R_r(d)$

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[9]Later, the maximum distance will be set to the world labor supply (which is one) so that the theoretical maximum distance of commuting is one, implying that a worker at the urban fringe, when there is full agglomer-

depending on their locations (i.e. distance from the CBD). Similar to employees, the unemployed have to commute into the CBD to search for jobs. However, they commute less by the factor α. Their commuting costs are measured in terms of lost time/income as well, while they receive unemployment benefit w_0. The income of non-shirking employees NS, shirking employees S and unemployed 0 then is given by

$$Y_j^{NS} = (1 - \chi d_j)(1 - t)w_r - R_r(d_j), \tag{10}$$

$$Y_j^{S} = (1 - \chi d_j)(1 - t)w_r - R_r(d_j), \tag{11}$$

$$Y_j^{0} = (1 - \alpha\chi d_j)w_0 - R_r(d_j). \tag{12}$$

Making use of the respective income, the indirect utility of non-shirking employees, shirking employees and unemployed is

$$\mathfrak{U}_r^{NS} = KP_r^{-\mu}\left((1 - \chi d)(1 - t)w_r - R_r(d)\right) - e, \tag{13}$$

$$\mathfrak{U}_r^{S} = \mathfrak{U}_r^{NS} + e, \tag{14}$$

$$\mathfrak{U}_r^{0} = KP_r^{-\mu}\left((1 - \alpha\chi d)w_0 - R_r(d)\right). \tag{15}$$

Given these utility levels, the location problem is solved by making use of standard arguments (Fujita; 1989). The bid rents for non-shirking employees and the unemployed (in the labor market equilibrium, there are no shirking employees) are given by

$$R_r^{NS}(d, \mathfrak{U}_r^{NS}) = max\left[(1 - \chi d)(1 - t)w_r - (e + \mathfrak{U}_r^{NS})P_r^{\mu}/K, 0\right], \tag{16}$$

$$R_r^{0}(d, \mathfrak{U}_r^{0}) = max\left[(1 - \alpha\chi d)w_0 - \mathfrak{U}_r^{0}P_r^{\mu}/K, 0\right]. \tag{17}$$

Every household consumes one unit of space, so that the distance to the urban fringe $\bar{d}_r$ is given by the manufacturing labor force N_r in that city (region): $\bar{d}_r = N_r$. Further, since $0 < \alpha < 1$ and $w_0 < (1 - t)w_r$, the slopes of the bid rent curves for the unemployed are lower than those for employees, so that the unemployed can only live at the peripheral zones of the cities. As there exists an opportunity rent $\bar{R}$ for land-owners, $R_r^{0}(\bar{d}_r, \mathfrak{U}_r^{0}) = \bar{R}$ must hold. In the land use equilibrium, the unique utility level of the unemployed then is

$$\mathfrak{U}_r^{0} = KP_r^{\mu}\left[(1 - \alpha\chi N_r)w_0 - \bar{R}\right]. \tag{18}$$

The border between the unemployment and employment districts is located at the distance

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ation and $\chi = 1$, uses all of her time to commute. Hence here the maximum value of χ is one to ensure that the proportion of time spent commuting is never larger than 100 %.

$\bar{b}_r = L_r$, where L_r is the number of manufacturing employees in the corresponding region/city. In the land use equilibrium, it must hold that $R_r^0(\bar{b}_r, \mathfrak{U}_r^0) = R_r^{NS}(\bar{b}_r, \mathfrak{U}_r^{NS})$, which implies

$$\mathfrak{U}_r^{NS} = K P_r^{-\mu} \left[(1 - \chi L_r)(1 - t)w_r + (1 - \alpha)\chi(N_r - L_r)w_0 - \bar{R} \right] - e. \tag{19}$$

3.3 Labor Market

The labor market is based on the Shapiro and Stiglitz (1984) efficiency wage framework. That is, workers can decide whether they shirk or not. Workers who shirk face a probability $1 - \gamma$ of getting caught shirking and getting fired. There is an exogenous job destruction rate ψ and an endogenous job generation rate δ_r. ρ is the discount rate of utility. The Bellman equations for the expected life-time utilities of non-shirking employees, shirking employees and the unemployed are

$$\rho V_r^{NS} = \mathfrak{U}_r^{NS} - \psi(V_r^{NS} - V_r^0), \tag{20}$$

$$\rho V_r^{S} = \mathfrak{U}_r^{NS} + e - (\psi + 1 - \gamma)(V_r^{S} - V_r^0), \tag{21}$$

$$\rho V_r^0 = \mathfrak{U}_r^0 + \delta_r(V_r^{NS} - V_r^0). \tag{22}$$

The efficiency wage framework is now expressed for region r (the analogous framework applies to region s).

Employers want to prevent shirking and thus they pay wages that are sufficient to prevent shirking at the margin where $V_r^{NS} = V_r^S$. Using this and plugging the above equations into each other delivers

$$\mathfrak{U}_r^{NS} = \mathfrak{U}_r^0 + e\frac{\rho + \psi + \delta_r}{1 - \gamma}. \tag{23}$$

Inserting the utility levels from the location decision delivers the wage curve,

$$w_r = \left[\frac{P_r^\mu}{K} e \frac{\rho + \psi + \delta_r + 1 - \gamma}{1 - \gamma} + (1 - \alpha)\chi L_r w_0 \right] \frac{1}{(1 - \chi L_r)(1 - t)}, \tag{24}$$

where δ_r is the endogenous rate of job generation. In equilibrium, the outflow of unemployment $\delta_r(N_r - L_r)$ must equal the inflow to unemployment ψL_r, which defines $\delta_r = \psi L_r/(N_r - L_r)$. This further defines the relationship between the endogenous job generation rate δ_r and unemployment rate $U_r = \psi/(\delta_r + \psi)$.

Manufacturing workers can decide to migrate between the two cities. They base their migration decision on the expected life-time utilities in both regions. Assume that migrants have to search for a new job at their destinations, namely they are unemployed at first. Then, it

is sufficient to compare the expected life-time utilities of the unemployed to ascertain whether migration occurs (Zenou and Smith; 1995). The expected life-time utility of the unemployed in region r is

$$
\begin{aligned}
\rho V_r^0 =& \frac{(\rho + \psi)\mathfrak{U}_r^0 + \delta_r \mathfrak{U}_r^{NS}}{\rho + \psi + \delta_r}, \\
=& K P_r^{-\mu} \left((1-\alpha)\chi N_r w_0 - \bar{R} \right) \\
&+ \frac{\delta_r}{\rho + \psi + \delta_r} \left[K P_r^{-\mu} \left[(1 - \chi L_r)(1-t)w_r - (1-\alpha)\chi L_r w_0 \right] - e \right].
\end{aligned}
\tag{25}
$$

The comparison of expected life-time utilities to derive migration behavior in the NEG literature has been criticized. However, Baldwin (2001) shows analytically that the global stability properties of the underlying core-periphery model do not change when forward-looking expectations rather than static expectations (as implied by the simple comparison of expected life-time utilities) are used. Since the key results do not change, the present model rests on static expectations to keep the analysis as traceable as possible.

3.4 Goods Market

The goods market is separated into the agricultural and manufacturing markets. The labor force is immobile between sectors but the manufacturing labor force is mobile between regions, whereas the agricultural labor force is not. It is assumed that the productivity and marginal productivity of each agricultural worker is one. It is further assumed that there is perfect competition on the agricultural labor and goods markets so that agricultural wages and prices are normalized to one. These wages and prices serve as a reference for the manufacturing sector (Fujita et al.; 1999).

In manufacturing, the effective labor supply $s(d_j)$ of an individual worker is lowered by commuting time depending on distance d_j (Murata and Thisse; 2005),

$$
s(d_j) = 1 - \chi d_j \text{ where } 0 < d_j < L_r.
\tag{26}
$$

Total effective labor supply S_r then is

$$
S_r = \int_0^{L_r} s(d_j) dd_j = L_r (1 - \frac{1}{2}\chi L_r).
\tag{27}
$$

Employers pay wages per unit of effective labor supply. Except for the distinction between labor supply and effective labor supply, the manufacturing sector is constructed analogous to

Fujita et al. (1999). The manufacturing sector is characterized by increasing returns to scale and monopolistic competition. The trade in manufactured goods takes place between the two regions with iceberg transport costs τ. The production function in manufacturing is

$$S_i = \beta + \phi q_i + \mathfrak{s}_i, \tag{28}$$

where S_i is the effective labor input of firm i, β is the fixed labor input per firm, ϕ is the variable labor input per unit of production q_i and $\mathfrak{s}_i$ is the labor input needed due to shirking. However, in the labor market equilibrium there is no shirking, so $\mathfrak{s}_i = 0$. Firms maximize their yields,

$$\pi_i = p_i q_i - w_r(\beta + \phi q_i), \tag{29}$$

with respect to prices p_i,

$$\frac{\partial \pi_i}{\partial p_i} = 0 \rightarrow p_i = \frac{\theta}{\theta - 1} w_r \phi = p_r. \tag{30}$$

Hence, the prices of firms within a region do not differ. New firms enter the market until profits decrease to zero,

$$q_i = \frac{\beta(\theta - 1)}{\phi}, \tag{31}$$

$$S_i = \beta\theta. \tag{32}$$

This means that all firms share the same size irrespective of their region of residence. Then, the number of firms per region is

$$n_r = \frac{S_r}{S_i} = \frac{S_r}{\beta\theta}. \tag{33}$$

The usual normalizations $\phi = (\theta - 1)/\theta$ and $\beta = \mu/\theta$ lead to

$$p_r = w_r, \tag{34}$$

$$q_i = \theta\beta = S_i = \mu, \tag{35}$$

$$n_r = S_r/\mu. \tag{36}$$

These and the demand equations for the varieties of manufactured goods are used to calculate the wage rate at which the zero-profit condition holds, i.e. the wage rate up to which new firms enter the market. This is known as the wage equation in the NEG literature,

$$w_r = \left[Y_r P_r^{\theta-1} + Y_s P_s^{\theta-1} \tau^{1-\theta} \right]^{\frac{1}{\theta}}. \tag{37}$$

Owing to the direct link between the number of firms and employment, the zero-profit condition, or wage equation, implicitly reflects labor demand. When market access increases (i.e. firms can serve a larger market, by increasing income or decreasing transport costs), the break-even point is shifted to a higher wage. New firms thus enter the market, employing more workers.

The manufacturing price index is

$$P_r = \left[\frac{1}{\mu} \left(S_r w_r^{1-\theta} + S_s (\tau w_s)^{1-\theta} \right) \right]^{\frac{1}{1-\theta}}. \tag{38}$$

Here, Y_r and Y_s stand for the regional income disposable for consumption (i.e. total wage income lowered by space costs). For simplicity, assume $\bar{R} = 0$, so that the total wage income in region r is $(1-t)w_r S_r$. There are $N_r - L_r$ unemployed in region r. Their total income is, analogously to the employed, given by $w_0(N_r - L_r) - \frac{1}{2}w_0\alpha\chi(N_r^2 - L_r^2)$, where the second term represents space costs.

Further, assume that the total population is one, of which a proportion μ are manufacturing workers and a proportion $1-\mu$ are agricultural workers. Agricultural workers are equally distributed among both regions, whereas a proportion λ of manufacturing workers is located in region r and a proportion $1-\lambda$ is located in region s. The total regional income disposable for consumption then is given by

$$Y_r = (1-t)w_r S_r + w_0(N_r - L_r) - \frac{1}{2}w_0\alpha\chi(N_r^2 - L_r^2) + \frac{1-\mu}{2}. \tag{39}$$

4 Equilibria and Migration

For any given level of $0 \leq \lambda \leq 1$, the equilibrium in the short-term (i.e. without migration) is defined by equations 24, 27, 37, 38 and 39 and by the definition $\delta_r = \psi L_r/(N_r - L_r)$ for region r and the corresponding equations for region s (refer to the appendix for the system of equations). Additionally, the balanced national budget of unemployment insurance is included, which is $t(w_r S_r + w_s S_s) = w_0(N_r + N_s - L_r - L_s)$. No closed form solution exists and the model is solved numerically, as is standard in the NEG.[10]

Whether a short-term equilibrium is also a long-term equilibrium depends on the migration decisions given by the difference in the expected life-time utilities of the unemployed (equation 25), as illustrated in Figure 3.

In Figure 3, the difference in expected life-time utilities (vertical axis) is plotted against the

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[10]All simulations of this paper are based on the parameter constellation A from Table 1 in the appendix, except for the additional simulations in the appendix.
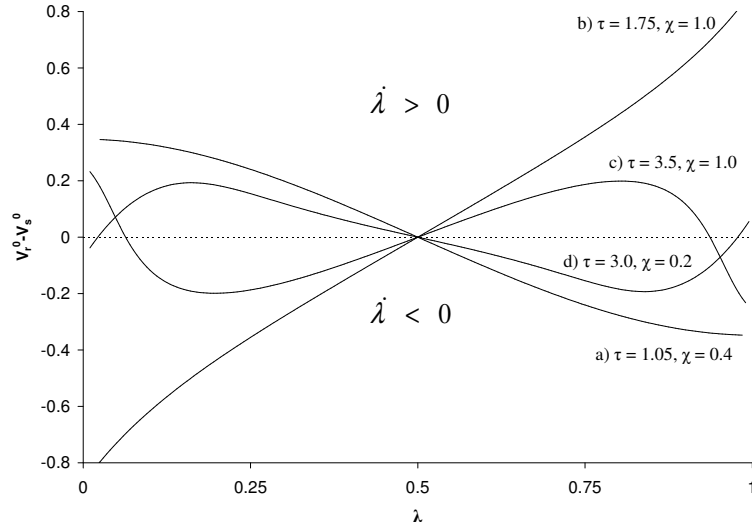
Figure 3: Equilibria and Migration

distribution of manufacturing employees (horizontal axis); the latter represents the degree of agglomeration. When $\lambda = 0.5$, both regions share the same size and there are no differences between them (symmetry). When $\lambda > 0.5$, region r is the agglomeration and region s the periphery (and vice versa for $\lambda < 0.5$). For $\lambda = 1$ and $\lambda = 0$, there is full agglomeration. Qualitatively, there are thus four situations:

- In situation a ($\tau = 1.05$, $\chi = 0.4$), the expected life-time utility is always higher in the periphery, so that people immigrate to the periphery and the system returns to symmetry in the long run (only symmetry is stable).

- In situation b ($\tau = 1.75$, $\chi = 1.0$), the expected life-time utility is always higher in the agglomeration, so that people leave the periphery and the system becomes full agglomeration of either region r or region s (only agglomeration is stable).

- In situation c ($\tau = 3.5$, $\chi = 1.0$), the expected life-time utility is higher in the agglomeration, but only up to a certain degree where the line crosses the no-migration line ($V_r^0 = V_s^0$) at $\lambda \neq 0.5$. Here, partial agglomeration is stable, and both symmetry and full agglomeration are unstable.

- In situation d ($\tau = 3.0$, $\chi = 0.2$), both, full agglomeration and symmetry are stable, but partial agglomeration is unstable. The system thus might move to full agglomeration or symmetry depending on the initial distribution λ.
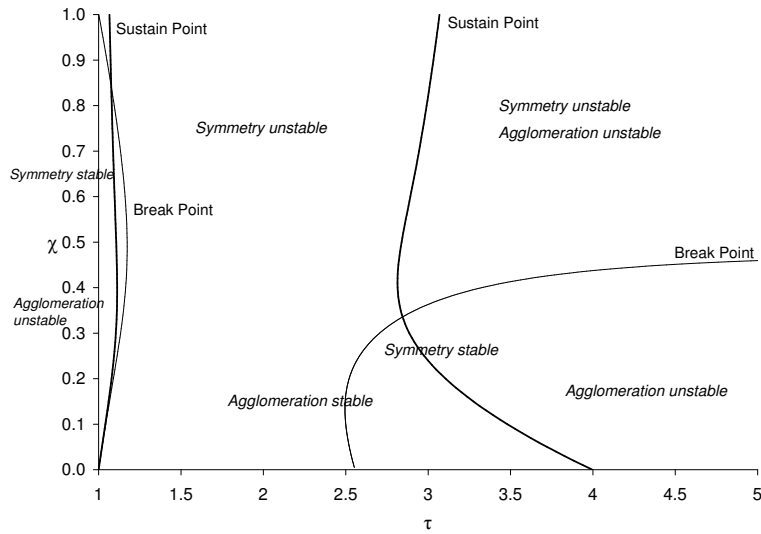
15

Figure 4: Break and Sustain Points

Note that partial agglomeration is only stable when neither full agglomeration nor symmetry is stable. Hence, it is sufficient to discuss the stability of symmetry and full agglomeration in order to describe the dynamics of the model.

5   Break and Sustain Points

The stability of symmetry and agglomeration is discussed with reference to the two key parameters of the model, τ and χ. τ is the parameter for transportation costs between the two regions and represents the degree of economic integration between them. χ is the parameter for commuting costs within the cities and represents the negative congestion externalities of them. Break and sustain points are thus used to describe the dynamics. The break point is the point at which a change in the two key parameters leads to symmetry changing from unstable to stable or vice versa. Analogously, the sustain point is the point at which a change in the two key parameters leads to full agglomeration changing from unstable to stable or vice versa.

To calculate the sustain point all combinations of τ and χ are derived, where in the case of full agglomeration the expected life-time utilities are equal in the agglomeration and in the periphery. The break point is calculated by searching for all combinations of τ and χ, where in the case of symmetry a marginal deviation from symmetry leads to a zero change in expected life-time utilities.

The break and sustain points are illustrated by thin and thick lines in Figure 4, respectively. Hence, left of the left-hand thin line symmetry is stable, whereas it is unstable to the right of

this line and stable again in the lower right corner, defined by the right-hand thin line. Similarly, agglomeration is unstable to the left of the left-hand thick line, stable to its right and unstable to the right of the right-hand thick line. Combining this information, one can distinguish areas where only symmetry, only agglomeration or both are stable. In the upper right-hand corner demarcated by the right-hand thick and thin lines, symmetry and full agglomeration are unstable, so that there is partial agglomeration. The same holds true for the small area in the upper left corner demarcated by the left-hand thick and thin lines.

Note that agglomeration behavior reduces to the model of Fujita et al. (1999) when there are no commuting costs ($\chi = 0$). In this case, there is a sustain point, which lies to the right of the break point, simultaneous break and sustain points at $\tau = 1$ and no partial agglomeration.

The model thus encompasses a multiple bifurcation pattern depending on the parameter constellation. Although most other NEG models include only one bifurcation pattern, there exist examples of other models with multiple patterns, such as that of Pflüger and Südekum (2011).

# 6   Labor Market Disparities

6.1   Numerical Examples

This section focuses on the implications of the model for regional labor market disparities by presenting numerical examples. In Figure 5, τ is set to 3.5 in order to to illustrate the degree of agglomeration λ and unemployment disparities. This highlights that there is full agglomeration or symmetry for low χ, symmetry for medium χ and partial agglomeration for large χ, whereas the size of the partial agglomeration increases as χ increases.

It is interesting to see that in the full agglomeration, the unemployment rate might be higher or lower compared with the periphery depending on χ, whereas in the partial agglomeration the unemployment rate is always higher compared with the periphery. This is true for all combinations of τ and χ, where there is partial agglomeration.

Additionally, Figure 6 illustrates the unemployment disparities for different levels of τ, holding constant χ at 1.0 or 0.2. In Figure 6(a), χ is 1. Starting at $\tau = 1$, there is a symmetric equilibrium with zero unemployment disparities. However, this symmetry is broken once τ is larger than 1 and partial agglomeration arises, growing in size as τ increases. This growth in the partial agglomeration is accompanied by increasing unemployment disparities, whereas the unemployment rate is larger in the agglomeration compared with the periphery. At a particular value of τ (the sustain point), the partial agglomeration becomes a full agglomeration and the disparities in unemployment rates peak. Further increases in τ now first lead to a decrease in the
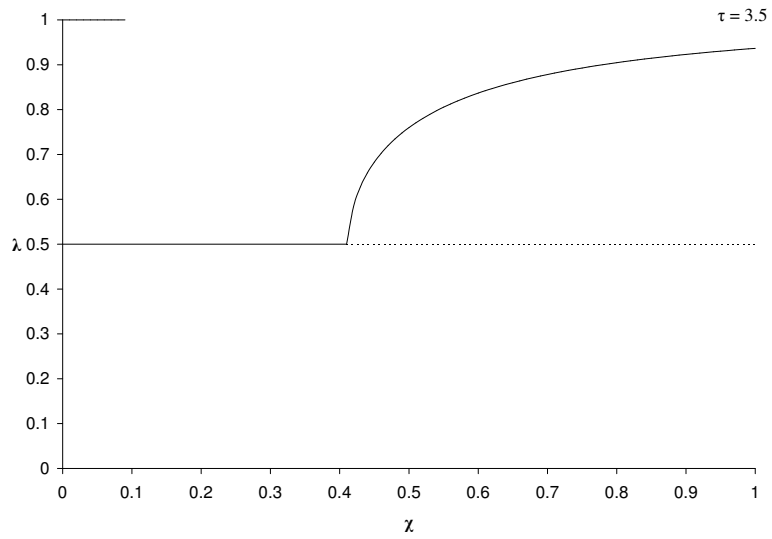
(a) Agglomeration Size
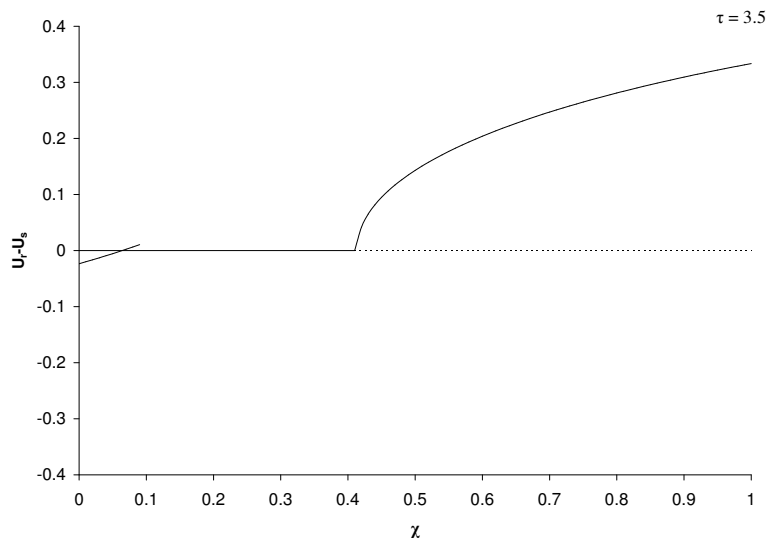
(b) Unemployment Disparities

Figure 5: Partial Agglomeration

unemployment rate of the agglomeration relative to the periphery, before this pattern is reversed once τ is large enough. Therefore, the unemployment rate is lower in the agglomeration compared with the periphery only for a certain range. The unemployment rate of the agglomeration relative to the periphery reaches another peak at which the full agglomeration is broken and the system changes to a partial agglomeration. Increases in τ then lead to a decrease in the agglomeration's unemployment rate relative to the periphery as the agglomeration size shrinks with increasing τ.

A similar pattern holds for $\chi = 0.2$ (Figure 6(b)). However, for $\chi = 0.2$ there is no partial agglomeration. Starting at $\tau = 1$, there is symmetry until this is broken once τ is large enough and the system switches to full agglomeration with the agglomeration's unemployment rate at first being higher than that of the periphery. Again, the agglomeration's relative unemployment rate first decreases, but then increases in τ. When τ is large enough, the agglomeration becomes unstable and the system returns to symmetry.

These examples illustrate that the unemployment rate is higher in the partial agglomeration than it is in the corresponding periphery, but might be lower or higher in the full agglomeration compared with the corresponding periphery. The next section provides an explanation of why this is the case.

6.2   Main Results

To compute for which constellations of the parameters τ and χ the unemployment rate is higher (lower) in the agglomeration compared with the periphery, Figure 7 illustrates all agglomerations where the agglomeration's unemployment rate is equal to the unemployment rate of the corresponding periphery ($U_r = U_s$). These points are arranged in Figure 7 as thick lines, whereas the thin and dashed lines correspond to the break and sustain points as in Figure 4. In the area between the thick lines, the agglomeration's unemployment rate is always lower than it is in the corresponding periphery, whereas the opposite is true outside this field.

To understand why the unemployment rate might be higher or lower in the agglomeration compared with the periphery one has to consider the wage curve from above,

$$w_r = \left[\frac{P_r^\mu}{K} e \frac{\rho + \psi + \delta_r + 1 - \gamma}{1 - \gamma} + (1 - \alpha)\chi L_r w_0 \right] \frac{1}{(1 - \chi L_r)(1 - t)}. \tag{40}$$

Assume that $\chi = 0$. In this case, the wage curve is equal for both regions and commuting costs do not exist. Thus, the difference between the unemployment rates depends only on the wage differential between both regions. The wages in the model depend on the wage equation (equation 37), which expresses up to which wage level new firms enter the market. This wage is higher in regions where access to markets is higher, i.e., in the agglomeration. Thus, the
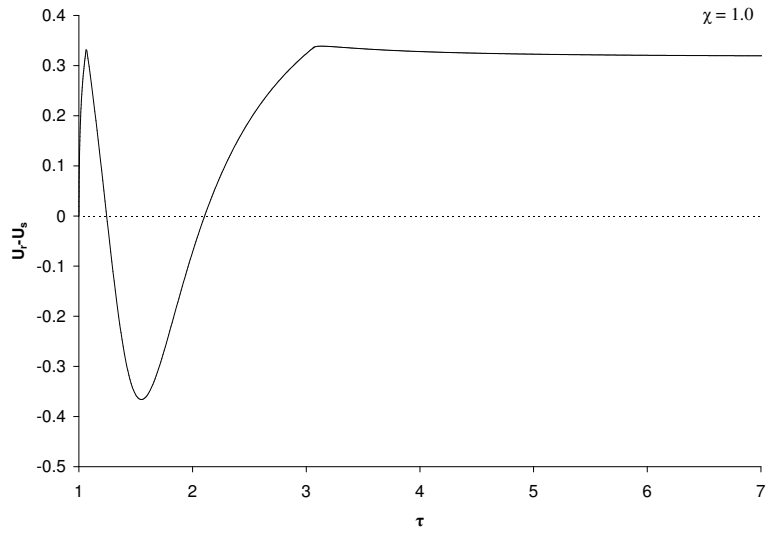
(a) $\chi = 1.0$
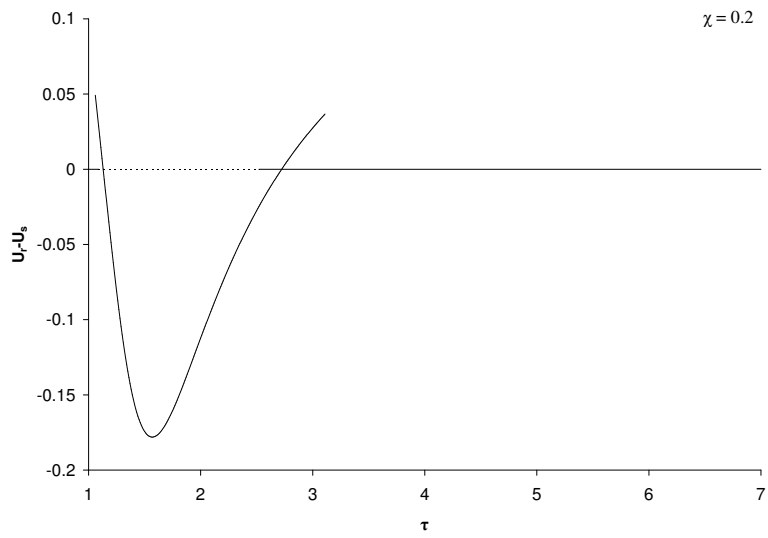
(b) $\chi = 0.2$

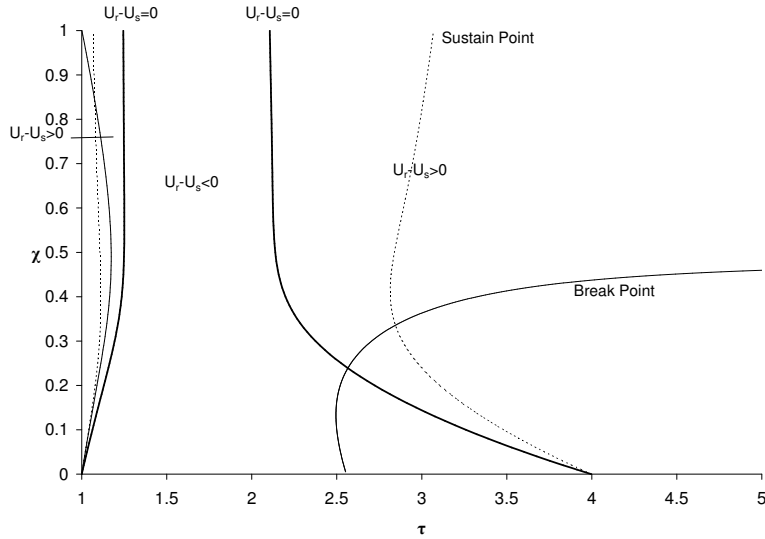Figure 6: Bifurcation Diagram for Unemployment Disparities

Figure 7: Unemployment Disparities

unemployment rate is lower in the agglomeration when there are no commuting costs. This is illustrated in Figure 2, where initially both the agglomeration A and the periphery P are located on the same wage curve. The higher wages in the agglomeration are associated with lower unemployment compared with the periphery.

However, when there are commuting costs ($\chi > 0$), the wage curve of the agglomeration is different from that of the periphery. Since commuting costs are larger in the agglomeration compared with the periphery, individuals in the agglomeration demand additional compensation for these costs. This is represented in the wage curve of the model such that the wage, which is necessary to ensure that employees do not shirk, is higher in the agglomeration compared with the periphery, holding constant the unemployment rate. Likewise, a given wage, resulting from the wage equation, is associated with a higher unemployment rate when commuting costs increase. Hence, the unemployment rate differential between the agglomeration and the periphery depends not only on the wage differential between regions but also on commuting costs.

These arguments are illustrated in Figure 2. Owing to the higher commuting costs, the wage curve of the agglomeration is shifted outwards compared with that of the periphery.[11] The wage curve shifts outwards since higher commuting costs imply an additional incentive to shirk, which is higher in the agglomeration compared with the periphery since commuting is more pronounced in the former. Depending on the size of χ, this shift might be small or large.

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[11]Actually, the wage curve for the periphery also shifts outwards when commuting costs χ increase, although this shift is smaller. In Figure 2, only the shift for the agglomeration is presented to illustrate the relative shift and to keep the illustration simple.

When the shift is small (large), the unemployment rate will remain smaller (become larger) compared with the periphery, holding constant the wage difference. As the wage is always higher in the agglomeration, the aggregate relationship between unemployment and wages depends on the wage difference and commuting costs. Further, this illustrates that commuting costs are positively linked to the unemployment rate.

The wage differential between regions depends on the strength of the centripetal forces in the NEG part of the model. Those centripetal forces are strongest for intermediate levels of transport costs and small for both high and low levels of transport costs.[12] Hence, the relative unemployment rate is likely to be lower in the agglomeration for intermediate levels of transport costs and higher in the agglomeration for low and high levels of transport costs. Regarding commuting costs, the relative unemployment rate in the agglomeration is likely to be higher the larger commuting costs are. For commuting costs of zero, the unemployment rate in the agglomeration is always lower than that in the periphery.

This feature is visible in Figure 7, which highlights the importance of transport and commuting costs on unemployment rates. The unemployment rate is lower in the agglomeration than it is in the periphery when the wage surplus of the agglomeration over the periphery is large (for intermediate levels of transport costs). For low or high transport costs, the centripetal forces are weak and the unemployment rate instead is higher in the agglomeration. The range of transport costs for which the unemployment rate is lower in the agglomeration compared with the periphery increases as commuting costs decrease. When commuting costs are zero, the unemployment rate will always be lower in the agglomeration than that in the periphery.[13]

Why is the unemployment rate only lower in the agglomeration than it is in the periphery when there is full agglomeration? Consider the case of partial agglomeration where the unemployment rate is higher than it is in the periphery. The expected life-time utilities in the partial agglomeration must equal those in the periphery to ensure stability. In principle, one could reach a stable partial agglomeration with relatively lower unemployment, when higher unemployment is compensated for by higher wages or lower commuting costs.[14] However, within a single re-

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[12]This is a basic feature of the core-periphery model. See Fujita et al. (1999) for a discussion of the underlying reasons.

[13]In principle, one could calibrate the model using estimates for the parameters to make predictions for the real world. However, the author decided not to do so since the aim of this model is to provide general conclusions on the influence of centripetal and centrifugal forces on unemployment disparities and on the interrelation between the Harris-Todaro model and the wage curve literature. For this reason, the model is rather stylized to keep the analysis traceable, which also means that features that are relevant in reality, such as restrictions to mobility, have been left aside. The appendix provides a large variety of simulations and the reader might choose a constellation to make predictions, although the simulations do not include all features that are important in reality because of the focus of this model.

[14]Note that commuting costs refer to overall regional commuting costs and not only to the commuting cost

gion there is a negative relationship between unemployment and wages (which is represented by movements on the wage curve). Further, there is a positive link between the unemployment rate and commuting costs, as discussed above (as represented by shifts in the wage curve). Hence, one cannot reach a *stable* partial agglomeration where the unemployment rate is lower compared with the periphery. Of course, one can reach an unstable partial agglomeration with relatively lower unemployment. However, since this must be associated with relatively higher wages and/or relatively lower commuting costs, there will be an incentive to immigrate to this partial agglomeration, turning it into a full agglomeration. In fact, the only reason why the unemployment rate can be lower in a *stable* full agglomeration is that there the relative life-time utilities need do not need to be equal across regions, as stability is assured by the lack of potential migrants in the empty periphery.

The results presented above illustrate how the Harris-Todaro model and wage curve literature can be special cases of a more general model. Harris and Todaro argue that there must be a positive relationship between unemployment and wages in the no-migration equilibrium when migration is based on expected life-time utilities. In the present model, commuting costs are included as a disincentive to migrate. Hence, the relationship between unemployment and wages is altered when overall commuting costs differ across regions (due to population size differences). In contrast to Harris and Todaro, the wage curve literature suggests a negative relationship between unemployment and wages, which is introduced in the present model based on efficiency wages. However, this is augmented by commuting costs in the present model, and the relationship between unemployment and wages in the agglomeration and the periphery depends on these commuting costs. Therefore, both the arguments of Harris and Todaro and the wage curve literature are included in the present model. Whether the arguments of Harris and Todaro or those of the wage curve literature dominate depends on the importance of commuting costs relative to transport costs. More generally, this depends on the size of negative agglomeration externalities (represented by commuting costs) and the size of positive agglomeration externalities (represented by transport costs).

7 Conclusions

This paper presents an NEG model with monocentric cities and efficiency wages to show that the sign of the relationship between unemployment and wages depends on transportation and commuting costs. The model encompasses the wage curve and Harris-Todaro model as special cases. Whereas most other models that introduce unemployment into the NEG usually find a

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parameter χ.

lower unemployment rate in the agglomeration compared with the periphery, this model shows under which conditions the unemployment rate might be higher in the agglomeration. Vom Berge (2011b,a) presents two models where the unemployment rate is higher in the agglomeration, too. However, in his models this rests either on lower nominal wages (and thus a higher replacement ratio in the agglomeration) or on the fact that the unemployed are unable to escape their situation through emigration. In the present model, whether the unemployment rate is higher or lower in the agglomeration compared with the periphery depends on commuting and transport costs, while the unemployed are fully mobile between cities.

The results are comparable to Zenou and Smith (1995). Nevertheless, in contrast to their model here the production structure is endogenous. Hence, the labor market disparities in the present model do not depend on an exogenously defined production structure, but rather endogenously emerge simultaneously with the location of industries. The present model shows that the question of whether the unemployment rate is higher or lower in the agglomeration crucially depends on the level of transportation costs, which are a measure of the degree of integration between cities. These define the strength of the centripetal forces. Lower unemployment rates in the agglomeration compared with the periphery only emerge for intermediate levels of transportation costs. When transportation costs are too low or too high, centripetal forces are too weak to generate a wage surplus of the agglomeration that is large enough to compensate for the commuting disadvantages. Then the unemployment rate is higher in the agglomeration. Further, the lower the commuting costs, the larger is the range of transportation costs for which the unemployment rate is lower in the agglomeration compared with the periphery. Therefore commuting costs affect the unemployment rate through their effects on work incentives.

The model presented here therefore shows that the Harris-Todaro model and wage curve are both special cases of a more general model. Whether the relationship between unemployment and wages is positive or negative depends on transportation and commuting costs. The model thus delivers a theoretical foundation for the effects of agglomeration on the wage curve. Such effects have been considered by only few recent empirical wage curve analyses. The results presented here show that the wage curve literature does not refute the Harris-Todaro model and that the empirical applications of the wage curve should include the degree of agglomeration or measures of negative agglomeration externalities, such as commuting costs, to control for shifts in the wage curve.

A Short-term Equilibrium

The equations for the short-term equilibrium are now presented for region r. Analogous expressions hold for region s. In the forthcoming equations, N_r is defined as $N_r = \mu\lambda$ and $N_s = \mu(1-\lambda)$, as described in Section 3.4,

$$P_r = \left[\frac{1}{\mu}\left(S_r w_r^{1-\theta} + S_s(\tau w_s)^{1-\theta}\right)\right]^{\frac{1}{1-\theta}}, \tag{41}$$

$$w_r = \left[Y_r P_r^{\theta-1} + Y_s P_s^{\theta-1}\tau^{1-\theta}\right]^{\frac{1}{\theta}}, \tag{42}$$

$$Y_r = (1-t)w_r S_r + w_0(N_r - L_r) - \frac{1}{2}\alpha\chi w_0(N_r^2 - L_r^2) + \frac{1-\mu}{2}, \tag{43}$$

$$\delta_r = \psi\frac{L_r}{N_r - L_r}, \tag{44}$$

$$S_r = L_r(1 - 0.5\chi L_r), \tag{45}$$

$$w_r = \left(\frac{P_r^\mu}{K}e\frac{\rho + \psi + \delta_r + 1 - \gamma}{1-\gamma} + (1-\alpha)\chi L_r w_0\right)\frac{1}{(1-\chi L_r)(1-t)}, \tag{46}$$

$$\rho V_r^0 = KP_r^{-\mu}(1-\alpha)\chi N_r w_o$$
$$+ \frac{\delta_r}{\rho + \psi + \delta_r}\left[KP_r^{-\mu}\left((1-\chi L_r)(1-t)w_r - (1-\alpha)\chi L_r w_0\right) - e\right], \tag{47}$$

$$t(w_r S_r + w_s S_s) = w_0(N_r + N_s - L_r - L_s). \tag{48}$$

B Sustain Point

Based on the short-term equilibrium above, it is easy to derive the sustain point. The sustain point is the point at which $\lambda = 1$, so that $N_r = \mu$, $N_s = 0$, $L_s = 0$, $S_s = 0$ and $V_r^0 = V_s^0$, as described in Section 5. In the simulations of appendix D, the restriction $V_r^0 = V_s^0$ is not included, but instead $V_r^0 - V_s^0$ is calculated in the (τ, χ)-space.

$$P_r = \left[\frac{1}{\mu}S_r w_r^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{49}$$

$$P_s = \left[\frac{1}{\mu}S_r(\tau w_r)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{50}$$

$$w_r = \left[Y_r P_r^{\theta-1} + Y_s P_s^{\theta-1}\tau^{1-\theta}\right]^{\frac{1}{\theta}}, \tag{51}$$

$$w_s = \left[Y_s P_s^{\theta-1} + Y_r P_r^{\theta-1}\tau^{1-\theta}\right]^{\frac{1}{\theta}}, \tag{52}$$

$$Y_r = (1-t)w_r S_r + w_0(N_r - L_r) - 0.5\alpha\chi w_0(N_r^2 - L_r^2) + \frac{1-\mu}{2}, \tag{53}$$

$$Y_s = \frac{1-\mu}{2}, \tag{54}$$

$$\delta_r = \frac{\psi L_r}{N_r - L_r}, \tag{55}$$

$$S_r = L_r(1 - 0.5\chi L_r), \tag{56}$$

$$w_r = \left[(1-\alpha)\chi L_r w_0 + \frac{P_r^\mu}{K}e\frac{\rho + \psi + \delta_r + 1 - \gamma}{1-\gamma}\right]\frac{1}{(1-\chi L_r)(1-t)}, \tag{57}$$

$$w_s = \left[(1-\alpha)\chi L_s w_0 + \frac{P_s^\mu}{K}e\frac{\rho + \psi + \delta_s + 1 - \gamma}{1-\gamma}\right]\frac{1}{(1-\chi L_s)(1-t)}, \tag{58}$$

$$\rho V_r^0 = KP_r^{-\mu}(1-\alpha)\chi N_r w_0$$
$$+ \frac{\delta_r}{\rho + \psi + \delta_r}\left[KP_r^{-\mu}\left((1-\chi L_r)(1-t)w_r - (1-\alpha)\chi L_r w_0\right) - e\right], \tag{59}$$

$$\rho V_s^0 = KP_s^{-\mu}(1-\alpha)\chi N_s w_0$$
$$+ \frac{\delta_s}{\rho + \psi + \delta_s}\left[KP_s^{-\mu}\left((1-\chi L_s)(1-t)w_s - (1-\alpha)\chi L_s w_0\right) - e\right], \tag{60}$$

$$t(w_r S_r + w_s S_s) = w_0(N_r + N_s - L_r - L_s). \tag{61}$$

C Break Point

The break point is the point at which the symmetry is stable at the margin, i.e. where the regional system is in symmetry ($\lambda = 0.5$) and people neither gain nor lose utility by emigrating. It is thus defined by the situation where the derivative of the difference in expected life-time utilities between the two regions with respect to λ is exactly zero ($dV_r^0 - dV_s^0 = 0$). Since the regional system is in symmetry at the break point, one can exploit this symmetry by dropping the indexes r and s for the regions. This is because the levels of the variables are equal in both regions, and thus the change in the variables share the same absolute value but differ in the sign as long as one considers a marginal deviation from symmetry, such as by calculating the derivative with respect to λ. Subsequently, the indexes are dropped and all variables are expressed in units of region r. Since $N_r = \mu\lambda$, it is obvious that $dN_r = \mu d\lambda$ and since $\partial t/\partial \lambda = 0$, dt is excluded.

$$P = \left[\frac{1}{\mu} S w^{1-\theta} (1 + \tau^{1-\theta}) \right]^{\frac{1}{1-\theta}}, \tag{62}$$

$$w = \left[Y P^{\theta-1} (1 + \tau^{1-\theta}) \right]^{\frac{1}{\theta}}, \tag{63}$$

$$Y = (1-t)wS + w_0(N-L) - 0.5\alpha\chi w_0(N^2 - L^2) + \frac{1-\mu}{2}, \tag{64}$$

$$w = \left[\frac{P^\mu}{K} e \frac{\rho + \psi + \delta + 1 - \gamma}{1 - \gamma} + (1-\alpha)\chi L w_0 \right] \frac{1}{(1-\chi L)(1-t)}, \tag{65}$$

$$\delta = \frac{\psi L}{N-L}, \tag{66}$$

$$\rho V^0 = K P^{-\mu}(1-\alpha)\chi N w_0$$
$$+ \frac{\delta}{\rho + \psi + \delta} \left[K P^{-\mu} ((1-\chi L)(1-t)w - (1-\alpha)\chi L w_0) - e \right], \tag{67}$$

$$S = L(1 - 0.5\chi L), \tag{68}$$

$$t = w_0 \frac{N-L}{wS}, \tag{69}$$

$$dP = \frac{1}{\mu}(1 - \tau^{1-\theta})P^\theta \left(\frac{w^{1-\theta}}{1-\theta}dS + Sw^{-\theta}dw \right), \tag{70}$$

$$dw = \frac{1}{\theta}(1 - \tau^{1-\theta})P^{\theta-1}w^{1-\theta}\left(dY + Y(\theta - 1)\frac{dP}{P} \right), \tag{71}$$

$$dY = (1-t)(Sdw + wdS) + w_0(dN - dL) - w_0\alpha\chi(NdN - LdL), \tag{72}$$

$$dw = \left(w - \frac{(1-\alpha)w_0\chi L}{(1-\chi L)(1-t)} \right)\mu\frac{dP}{P} + \frac{P^\mu}{K}\frac{\frac{e}{1-\gamma}}{(1-\chi L)(1-t)}d\delta$$
$$+ \frac{(1-\alpha)\chi w_0}{(1-\chi L)(1-t)}dL + w\left(\frac{\chi dL}{1-\chi L} - \frac{dt}{1-t} \right), \tag{73}$$

$$d\delta = \frac{\psi}{(N-L)^2}(NdL - LdN), \tag{74}$$

$$\rho dV^0 = KP^{-\mu}(1-\alpha)\chi w_0\left(dN - \mu N\frac{dP}{P} \right) - e\frac{\rho + \psi}{(\rho + \psi + \delta)^2}d\delta$$
$$+ \frac{1}{\rho + \psi + \delta}KP^{-\mu}\left((1-\chi L)(1-t)w - (1-\alpha)\chi Lw_0 \right)\left(\frac{\rho + \psi}{\rho + \psi + \delta}d\delta - \mu\delta\frac{dP}{P} \right)$$
$$+ \frac{\delta}{\rho + \psi + \delta}KP^{-\mu}\left((1-\chi L)(1-t)dw - \chi(1-t)wdL - (1-\alpha)\chi w_0dL \right), \tag{75}$$

$$dS = dL - \chi LdL. \tag{76}$$

D   Additional Simulations

This appendix provides additional simulations of the model for the break points, sustain points and unemployment disparities. The simulations for the break points illustrate the derivative of the difference in expected life-time utilities in both regions with respect to λ in the case of symmetry ($\lambda = 0.5$). When this derivative is larger (smaller) than zero, symmetry is unstable (stable). The points at which this derivative is exactly zero are the break points. In each of the forthcoming figures, this derivative is presented for a total of 16,281 points on the grid ($1 \leq \tau \leq 5$, $0 \leq \chi \leq 1$). Owing to symmetry, it is sufficient to calculate only the derivative of expected life-time utilities in region r, because the derivative of the difference in expected life-time utilities is simply double the former derivative.

The results for the sustain points are provided by calculating the difference in expected life-time utilities in the case of full agglomeration ($\lambda = 1$) on the same grid for τ and χ. When this difference is larger (smaller) than zero, full agglomeration is stable (unstable). All points at which this difference is zero are termed sustain points. For each of the full agglomerations, the difference in unemployment rates is also presented.

The parameter constellations are summarized in Table 1. They have to fulfill the so-called

no-black-hole condition $(\theta - 1)/\theta \geq \mu$. This condition constrains the strength of the centripetal forces in the model and ensures that they are not so strong that they would lead to a collapse of the economy into a single point(see Fujita et al. (1999, p.58-59) for a detailed discussion).

Example A (Figure 8) replicates the simulations from Figures 4 and 7, although the method of illustration is different here, as all equilibria on the grid are calculated, not only the break and sustain points. The forthcoming examples show variations of this baseline specification, first by changing θ and μ, and later by changing the labor market parameters. The parameters θ and μ are at the core of the model since they have a crucial impact on the strength of the centripetal forces. The higher μ and the lower θ, the stronger the centripetal forces are and the larger (smaller) the area is, where full agglomeration (symmetry) is stable. When the centripetal forces are weak, the space in which agglomerations are stable reduces to a small area where χ is small and τ is intermediate. Further, the area in which the unemployment rate is lower in the agglomeration compared with the periphery is always a subset of the area in which the full agglomeration is stable. However, example E (Figure 12) shows that when θ and μ get closer to the no-black-hole condition, the relationship between χ and unemployment disparities becomes non-linear. This is because the effect of commuting on the attractiveness of a region depends not only on the size of the region, but also on the wages in that region, and thus on the strengths of the centripetal forces. When wage disparities are very large, the loss of income due to commuting might be significant, which affects the unemployment rate through the effect of commuting costs on shirking incentives.
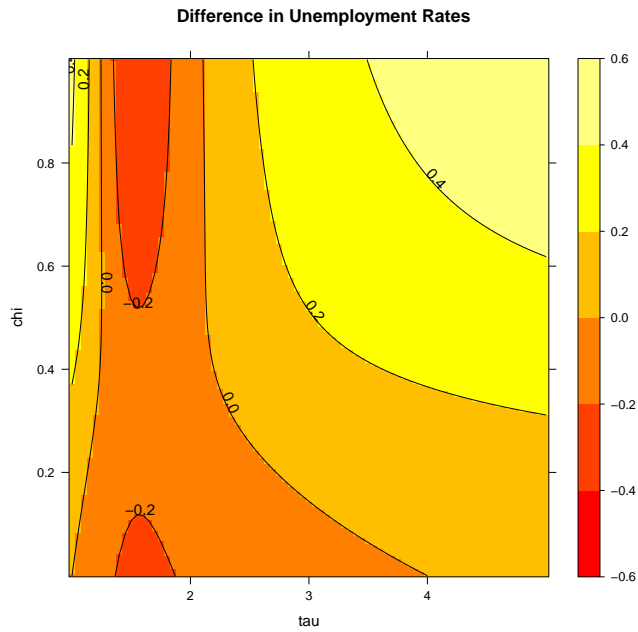
Simulation	μ	θ	ψ	γ	ρ	e	w_0	α
A	0.5	4	0.1	0.1	0.05	0.3	0.6	0.4
B	0.4	4	0.1	0.1	0.05	0.3	0.6	0.4
C	0.4	3	0.1	0.1	0.05	0.3	0.6	0.4
D	0.5	5	0.1	0.1	0.05	0.3	0.6	0.4
E	0.6	6	0.1	0.1	0.05	0.3	0.6	0.4
F	0.4	6	0.1	0.1	0.05	0.3	0.6	0.4
G	0.5	4	0.1	0.1	0.05	0.3	0.6	0.6
H	0.5	4	0.15	0.1	0.05	0.3	0.6	0.4
I	0.5	4	0.1	0.2	0.05	0.3	0.6	0.4
J	0.5	4	0.1	0.1	0.01	0.3	0.6	0.4
K	0.5	4	0.1	0.1	0.05	0.2	0.6	0.4
L	0.5	4	0.1	0.1	0.05	0.3	0.5	0.4

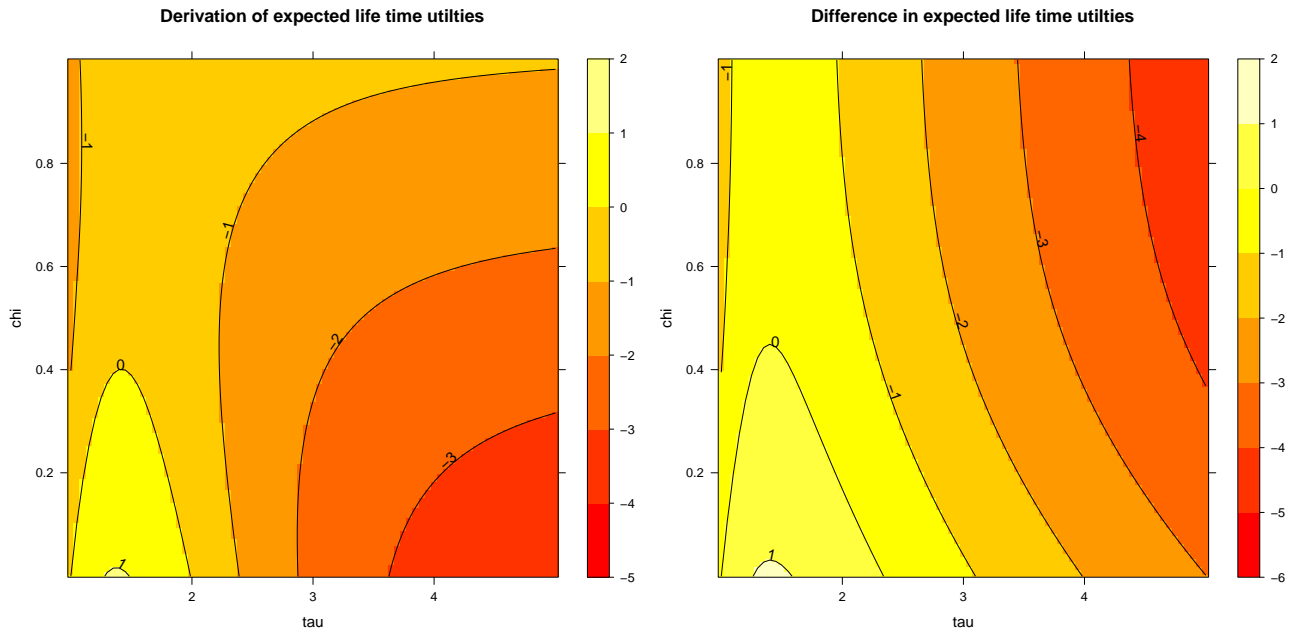Table 1: Parameter Constellations
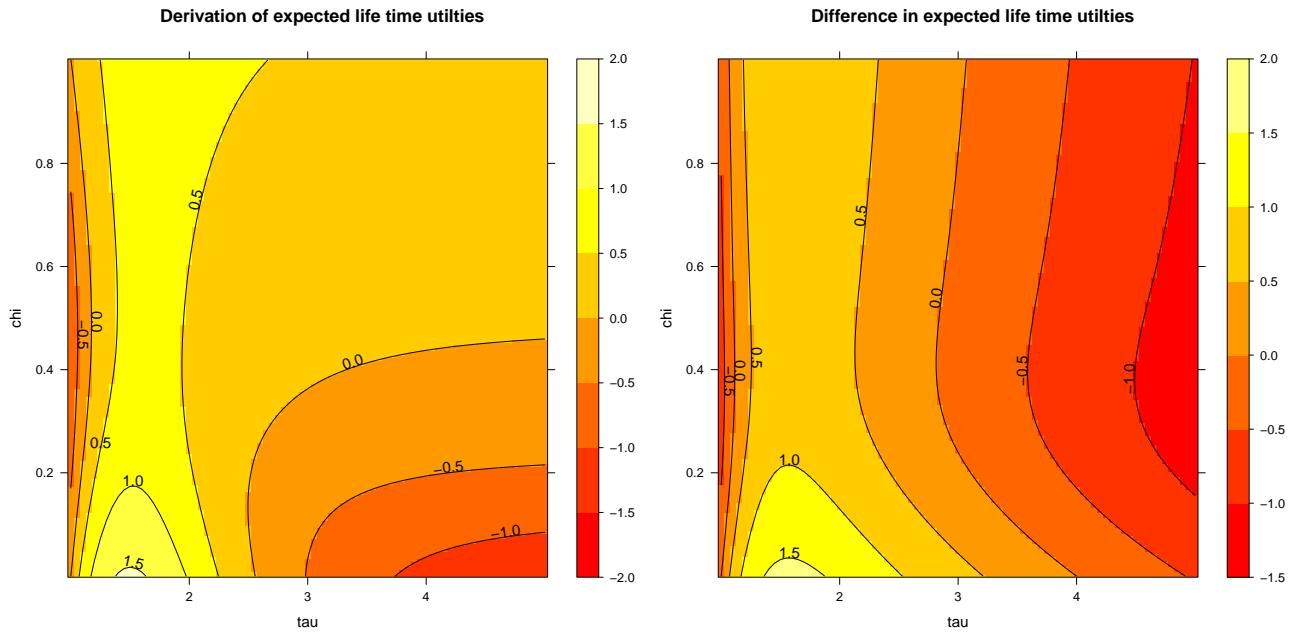
(a) Break Point

(b) Sustain Point
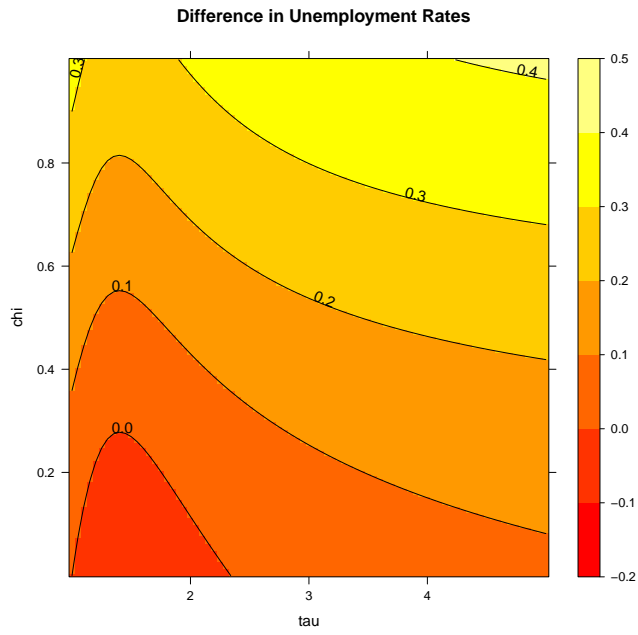

(c) Unemployment Disparities

See Table 1 for the Parameter Constellation.
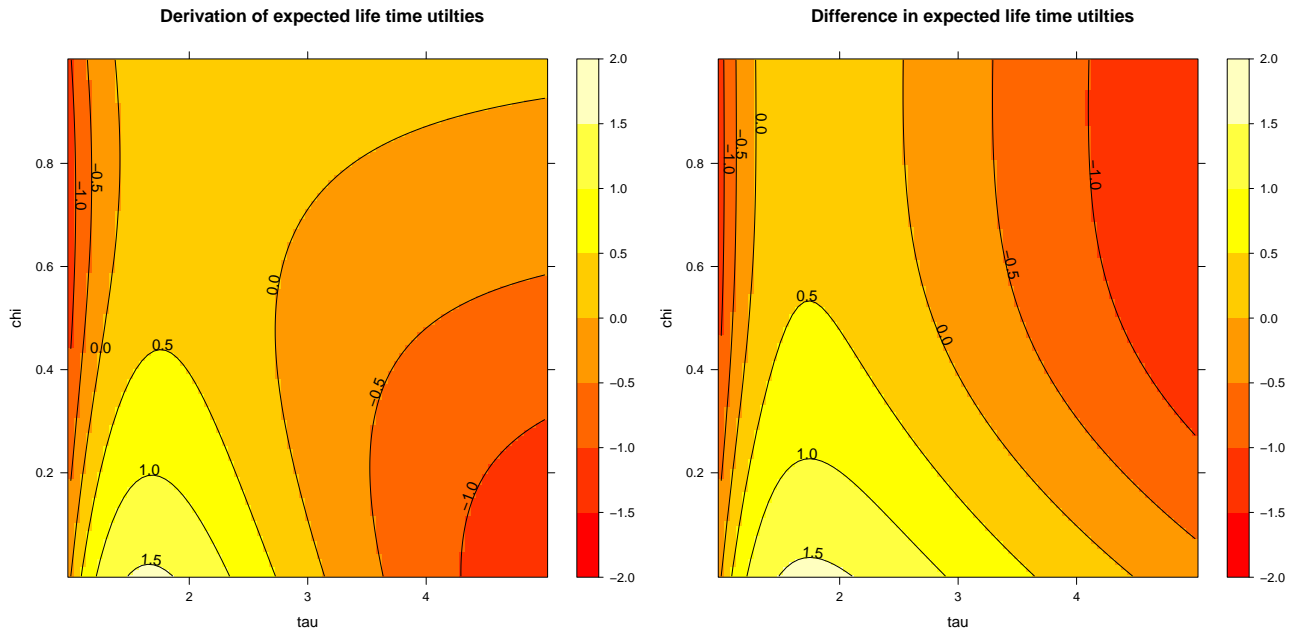
Figure 8: Simulation A

(a) Break Point
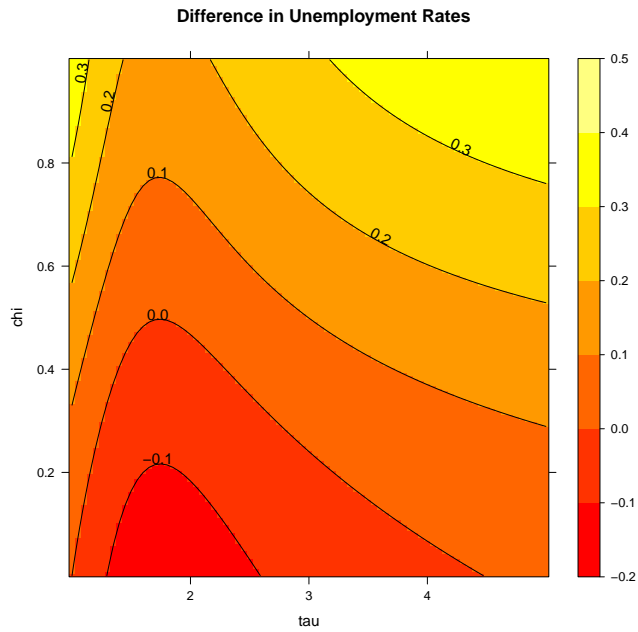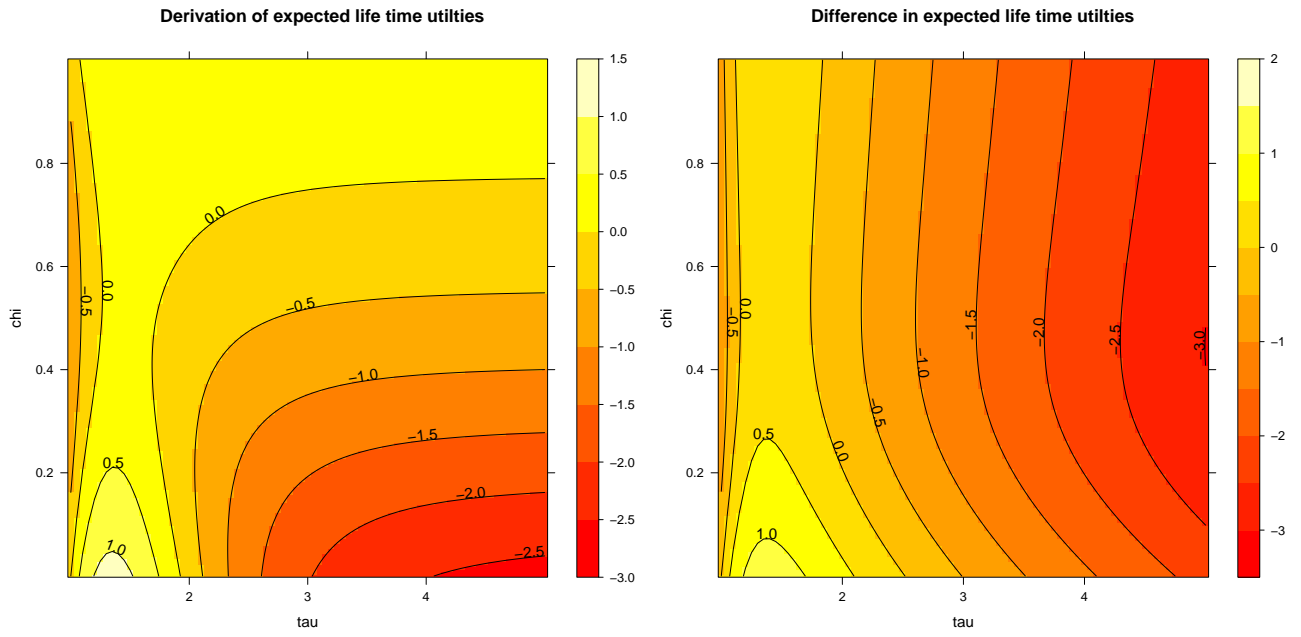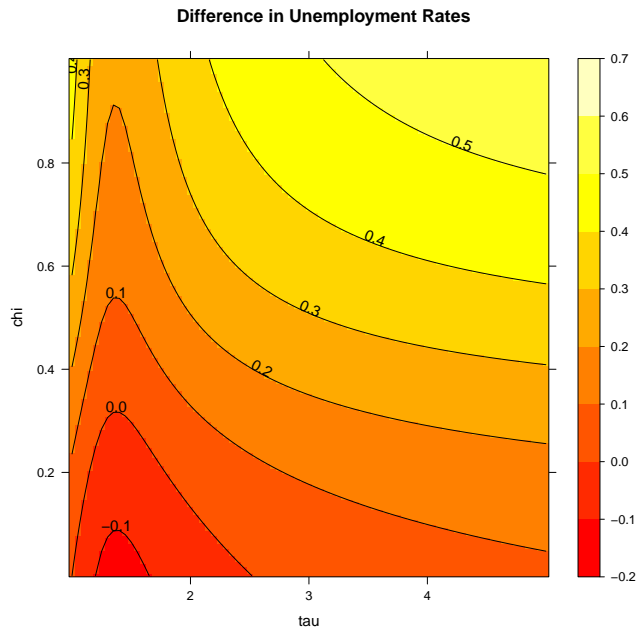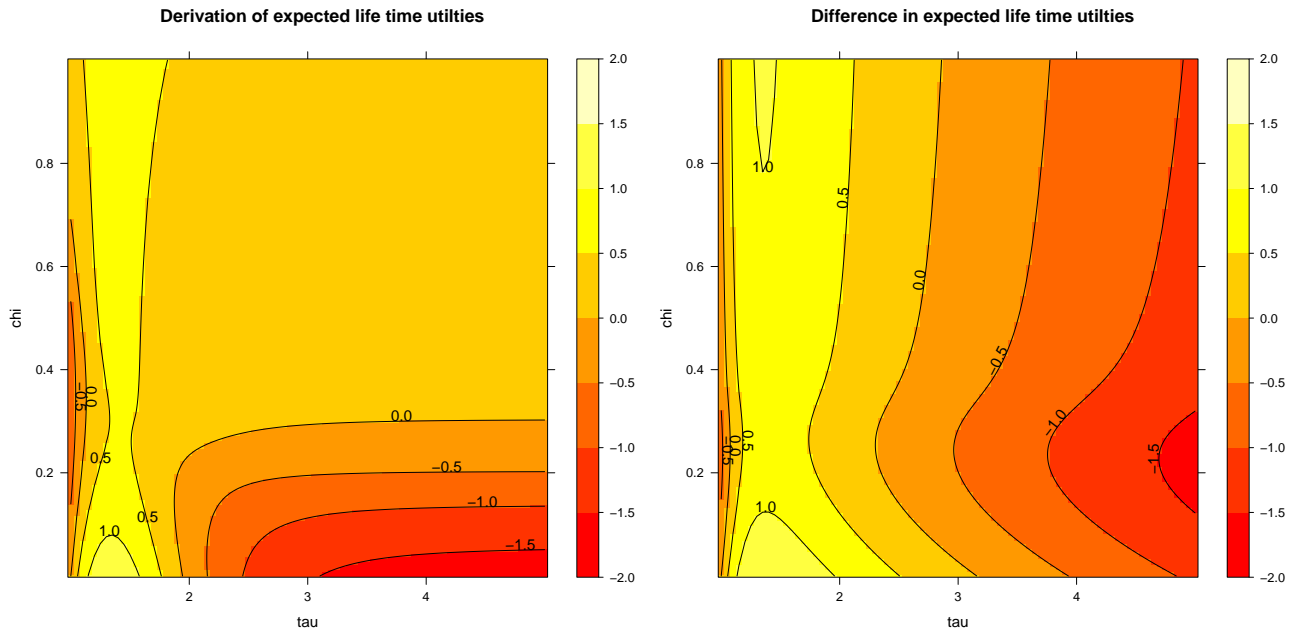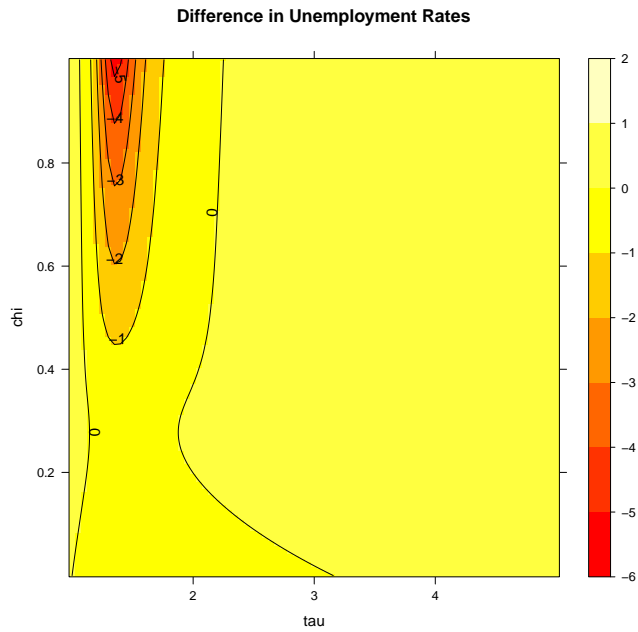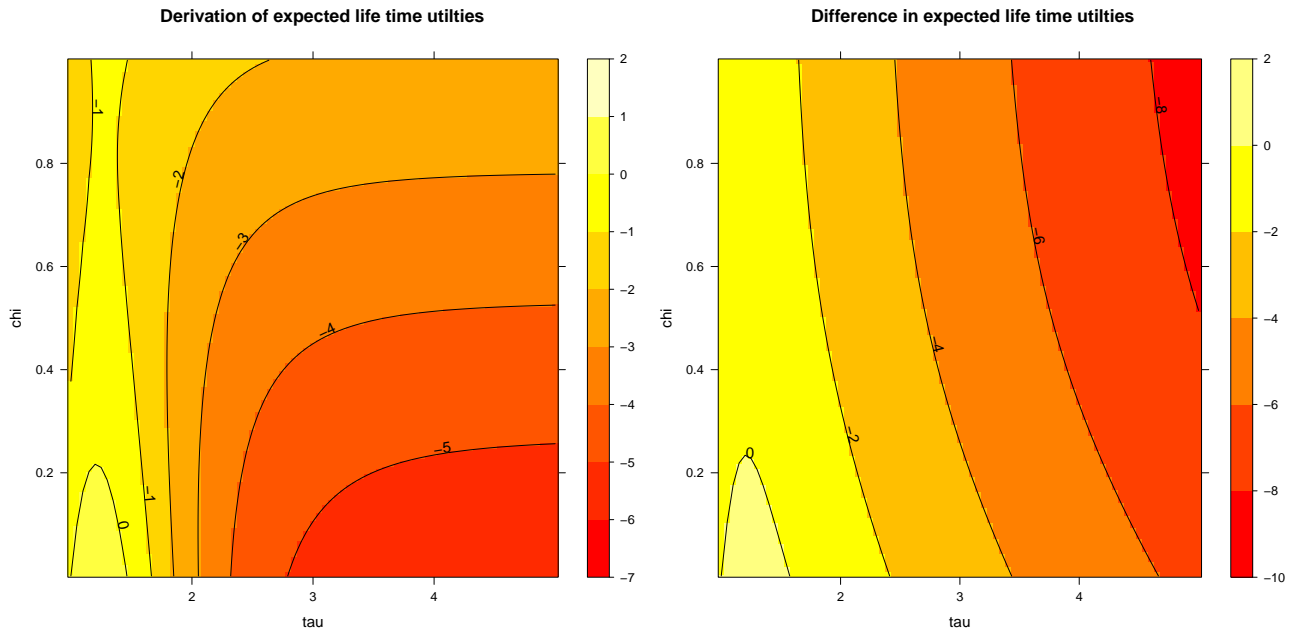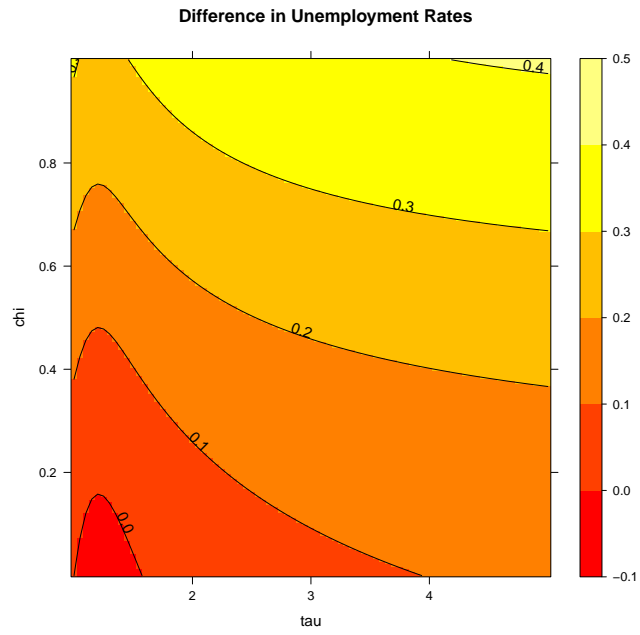
(b) Sustain Point

(c) Unemployment Disparities

See Table 1 for the Parameter Constellation.

Figure 9: Simulation B

Derivation of expected life time utilties

Difference in expected life time utilties

(a) Break Point

(b) Sustain Point

Difference in Unemployment Rates

(c) Unemployment Disparities

See Table 1 for the Parameter Constellation.

Figure 10: Simulation C

Derivation of expected life time utilties

(a) Break Point

Difference in expected life time utilties

(b) Sustain Point

Difference in Unemployment Rates

(c) Unemployment Disparities

See Table 1 for the Parameter Constellation.

Figure 11: Simulation D

Derivation of expected life time utilties

Difference in expected life time utilties

(a) Break Point

(b) Sustain Point

Difference in Unemployment Rates

(c) Unemployment Disparities

See Table 1 for the Parameter Constellation.

Figure 12: Simulation E

Derivation of expected life time utilties

Difference in expected life time utilties

(a) Break Point

(b) Sustain Point

Difference in Unemployment Rates

(c) Unemployment Disparities

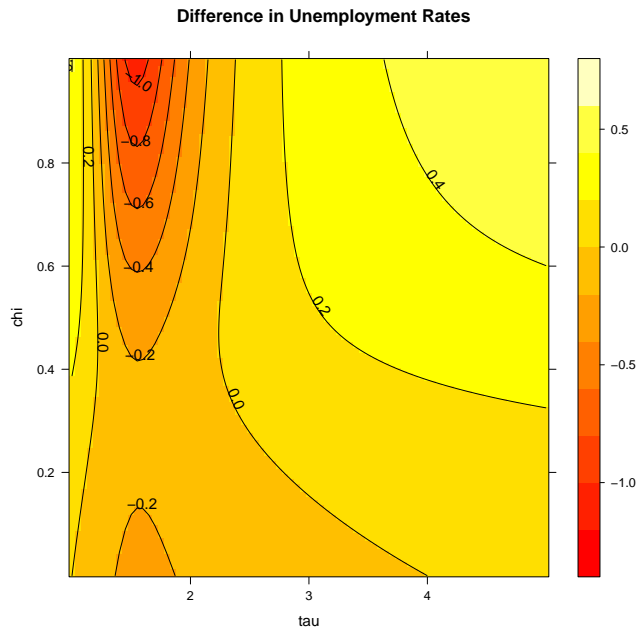See Table 1 for the Parameter Constellation.

Figure 13: Simulation F
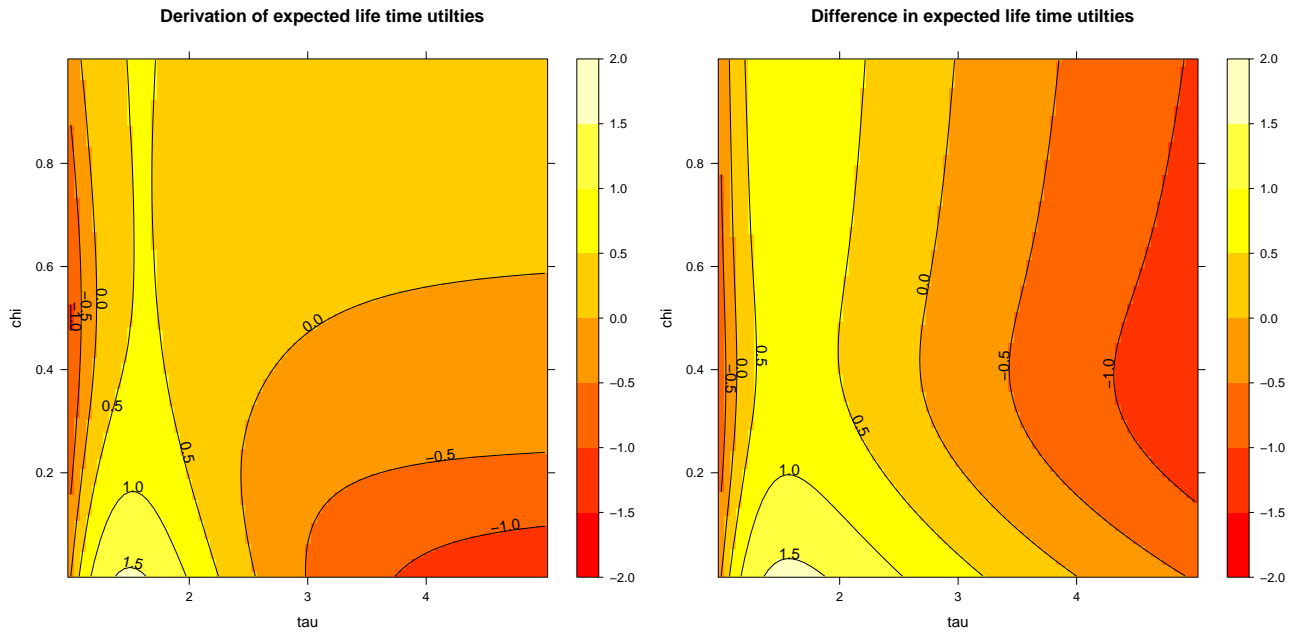
(a) Break Point

(b) Sustain Point

(c) Unemployment Disparities

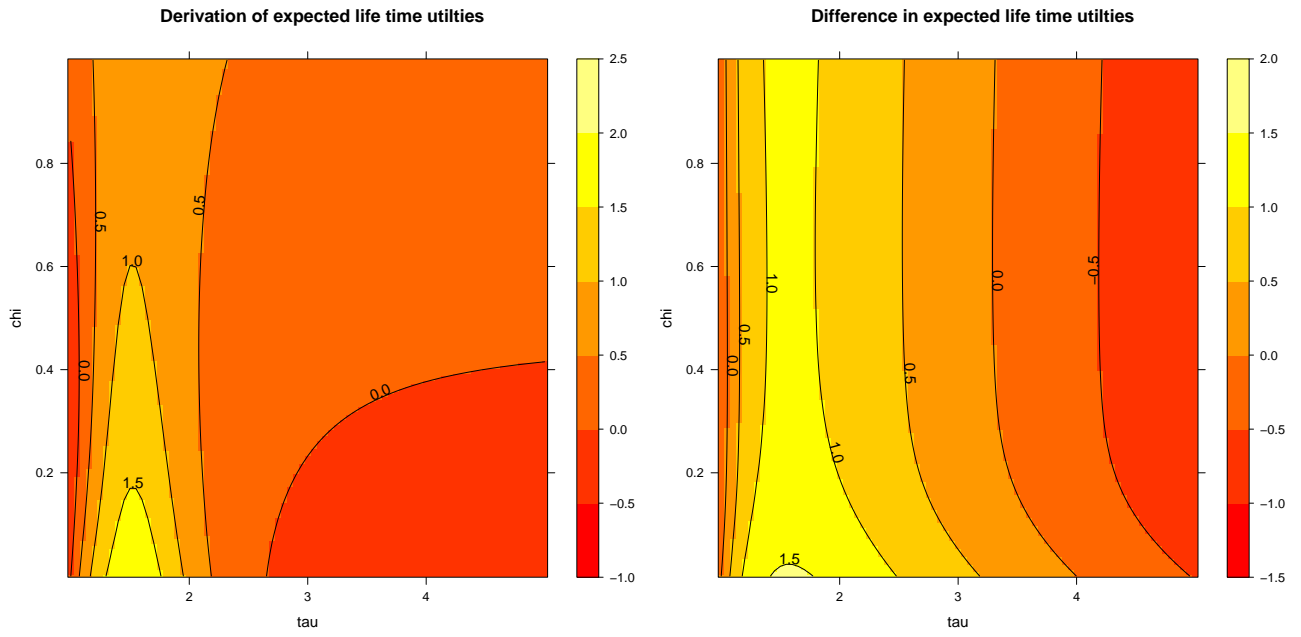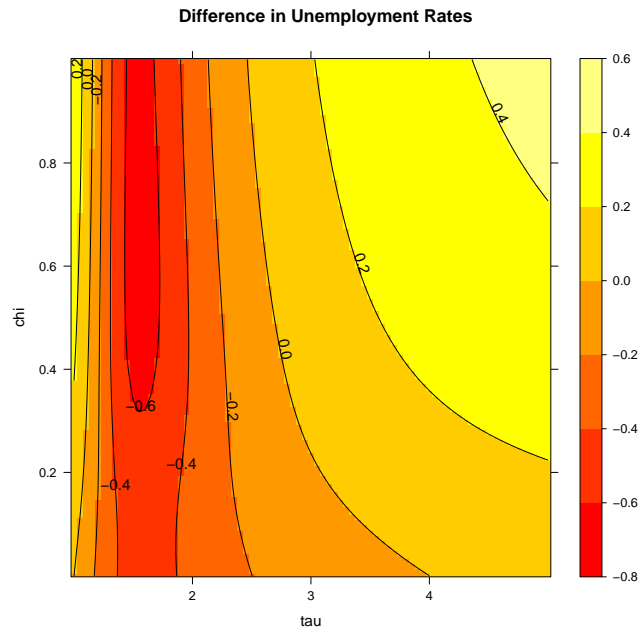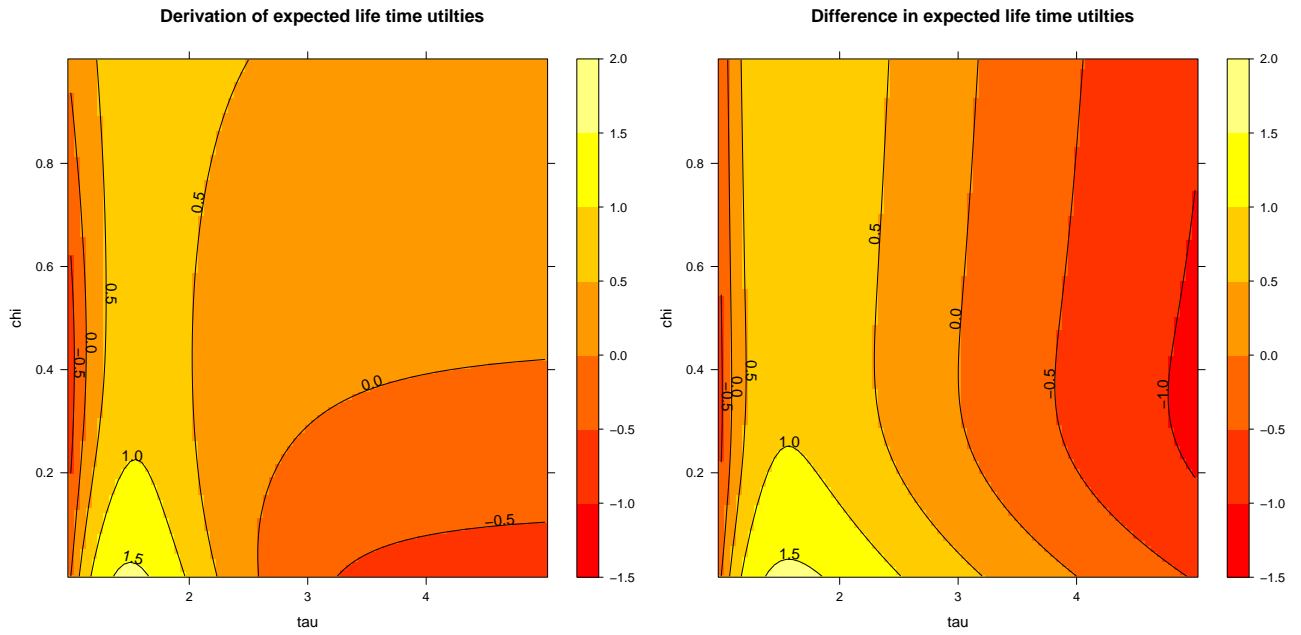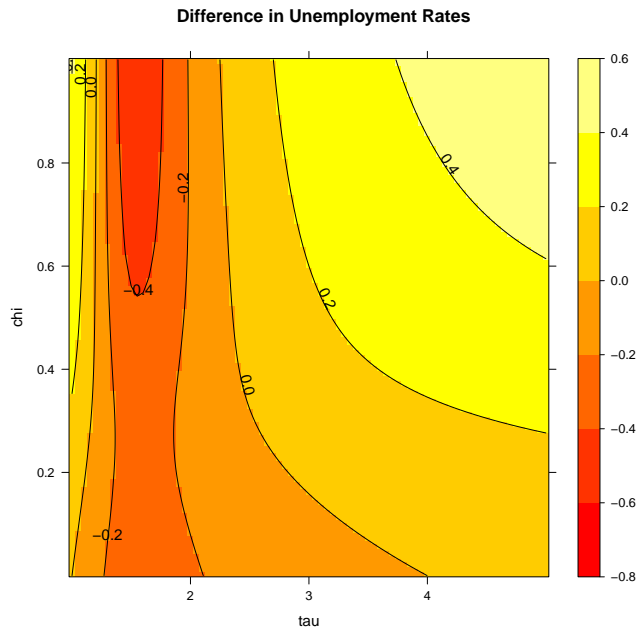See Table 1 for the Parameter Constellation.

Figure 14: Simulation G

Derivation of expected life time utilties

Difference in expected life time utilties

(a) Break Point

(b) Sustain Point

Difference in Unemployment Rates

(c) Unemployment Disparities

See Table 1 for the Parameter Constellation.

Figure 15: Simulation H

Derivation of expected life time utilties

Difference in expected life time utilties

(a) Break Point

(b) Sustain Point

Difference in Unemployment Rates

(c) Unemployment Disparities

See Table 1 for the Parameter Constellation.

Figure 16: Simulation I

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Derivation of expected life time utilties

Difference in expected life time utilties

(a) Break Point

(b) Sustain Point

Difference in Unemployment Rates

(c) Unemployment Disparities

See Table 1 for the Parameter Constellation.

Figure 17: Simulation J

(a) Break Point

(b) Sustain Point

(c) Unemployment Disparities

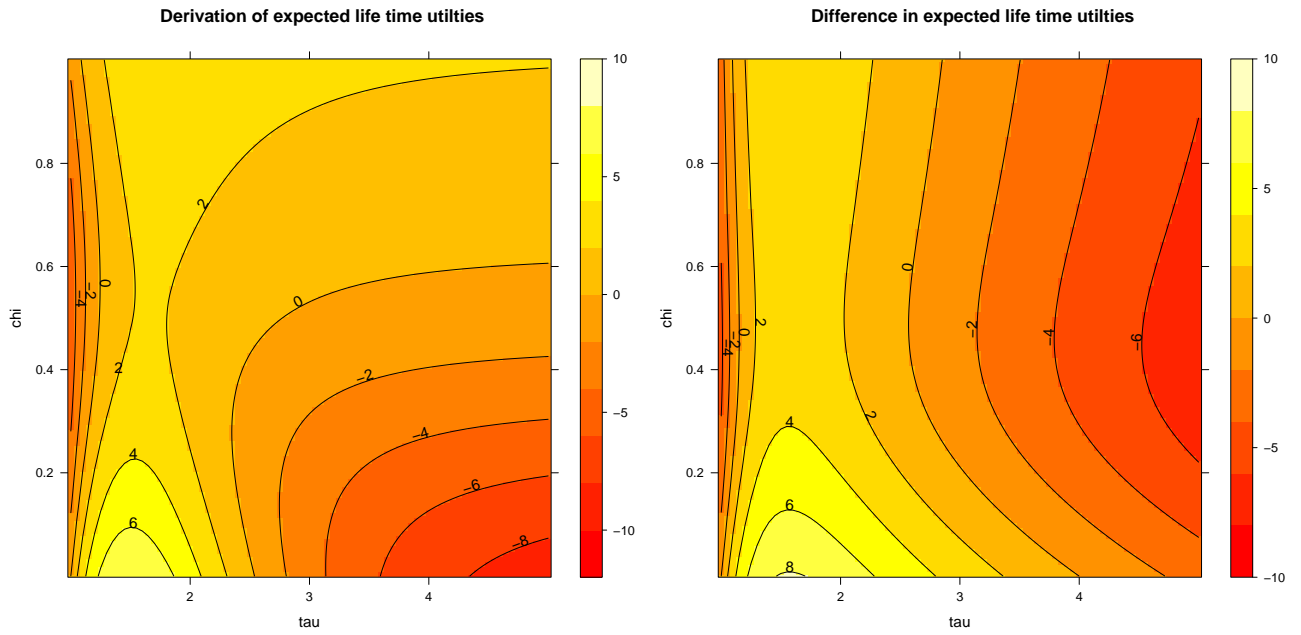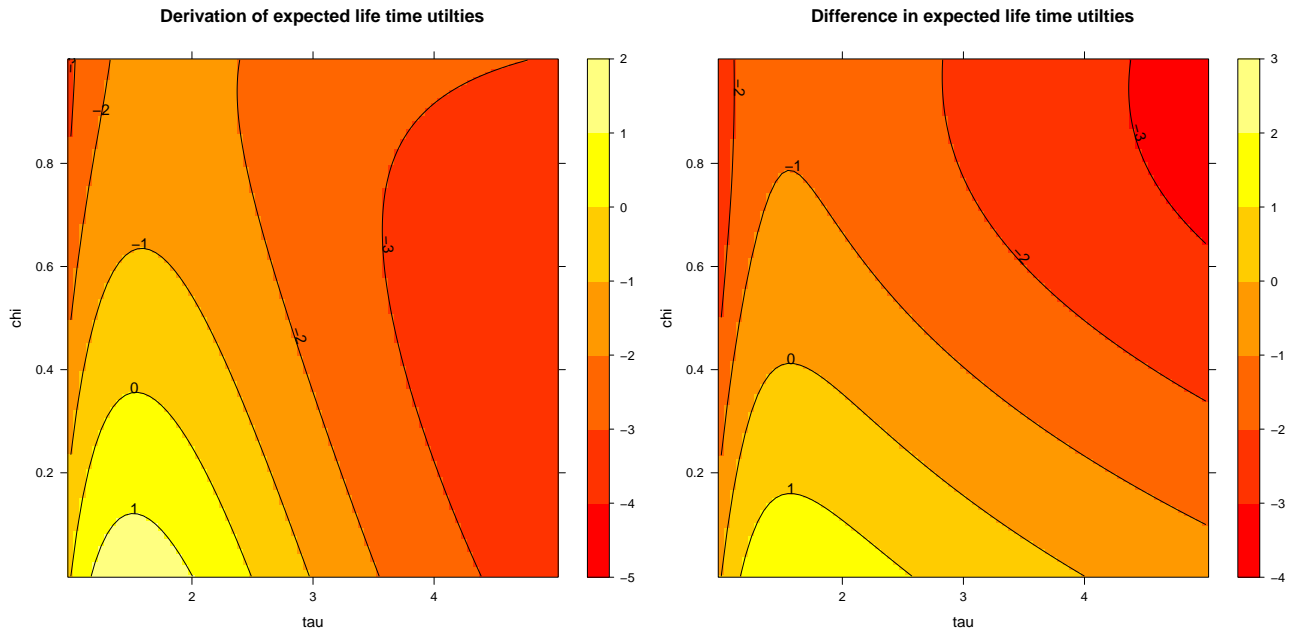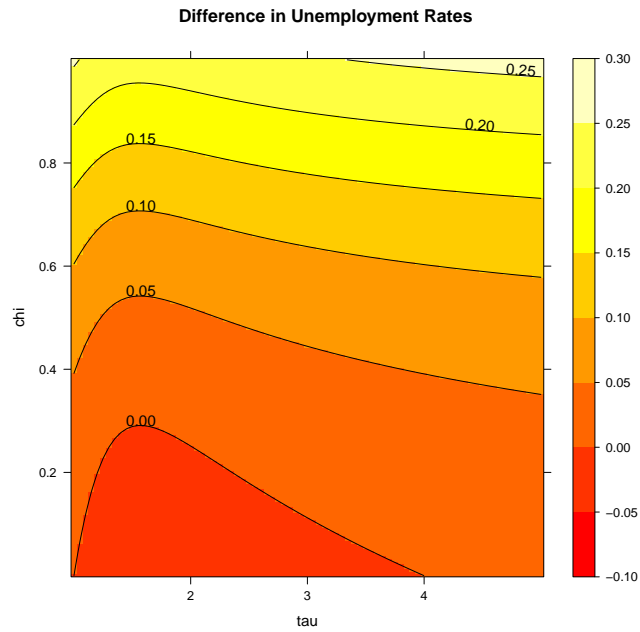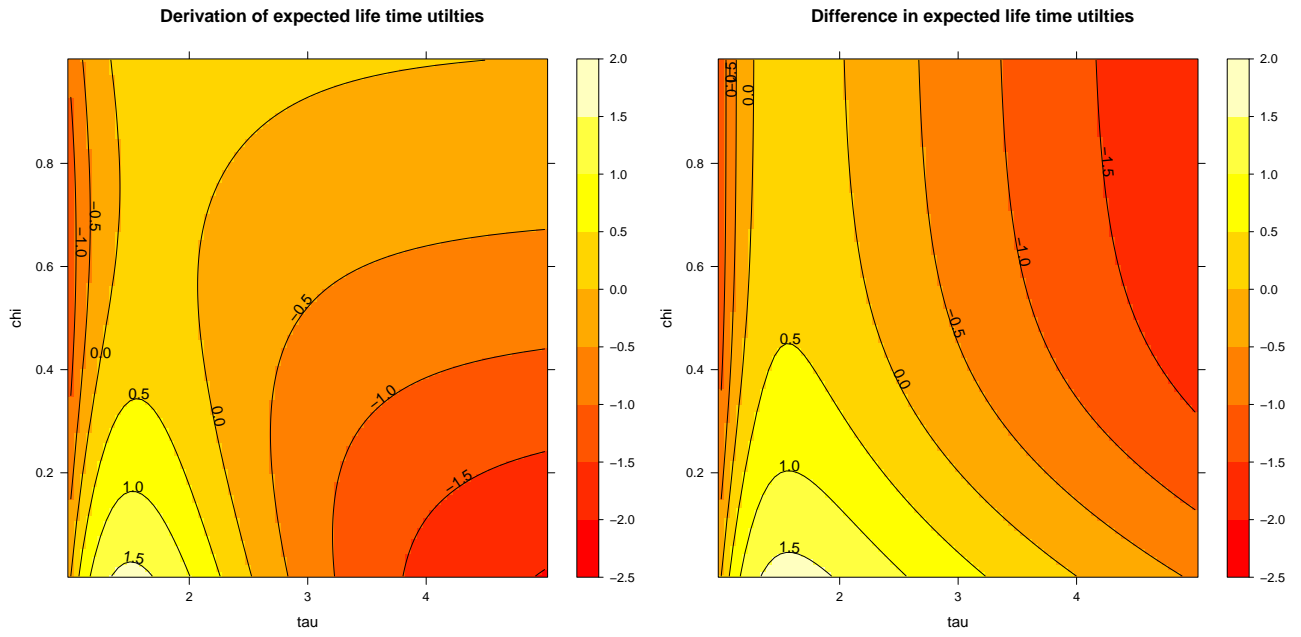See Table 1 for the Parameter Constellation.

Figure 18: Simulation K

Derivation of expected life time utilties

(a) Break Point

Difference in expected life time utilties

(b) Sustain Point

Difference in Unemployment Rates
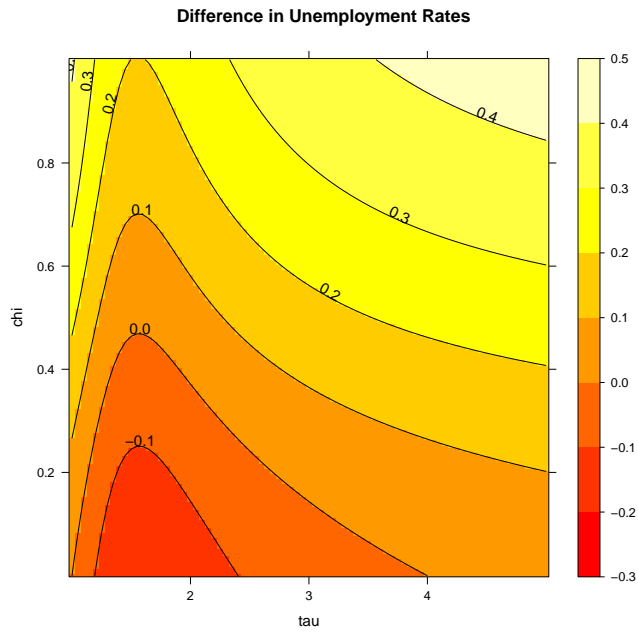
(c) Unemployment Disparities

See Table 1 for the Parameter Constellation.

Figure 19: Simulation L

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