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Stability Analysis Of The Rumor Spreading Model

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Abstract— The process of spreading rumors is the same as the process of spreading infectious diseases. In this case, a rumor-spreading model is discussed in this study by considering the credibility of rumors, the correlation between rumors and lives, and the classification of groups based on personality. Furthermore, the stability of the model is analyzed for the rumor-free equilibrium point and the rumor-spreading endemic equilibrium point.

Keywords—Stability; Jacobian Matrix; Routh's Criteria; Eigenvalue; Rumor Spreading Model

I. Introduction

Mathematical modeling gets into a developmental phase occurring in the structure of society called the information society – people who use information technology facilities with high intensity in their daily lives, including economic, political, and cultural activities [9]. The spread of rumors is one of the phenomena or problems that occur in the structure of the information society, so mathematical modeling is needed to overcome and provide solutions to these problems. The spread of rumors is one of the phenomena or problems that frequently occur in the structure of society or the information society, so mathematical modeling is needed to overcome and provide solutions to these problems. Spread of information and the spread of infectious diseases are analogous phenomena, In 1964, Daley and Kendal first introduced the rumor model, known as the DK model. The DK model is formulated based on the SIR epidemic model (susceptible, infected, removed). In the DK model, the population is divided into three class, namely class of individuals has not heard rumor (analogous to class of individuals susceptible to disease),class of individuals activly spreading rumor (analogous to class of individuals infectious case), and class of individuals no longest spreading rumor (analogous to class of individuals dead, isolated, or immune) [4]. Various researches related to the spread of rumors have been conducted by Huo et al. (2017) [6] comparing the model of spreading rumors with the model of the spread of infectious diseases and adding groups of people who are hesitant to the spread of rumors. Then, Xia et al (2017) [10] conducted a previous study concerning the rumor spreading model by considering the group of people who are uncertain about the rumors and adding the ambiguity of the rumored content as a model parameter. Therefore, the rumor spreading model is discussed in this study by considering the credibility of rumors, the correlation between rumors and lives, and the classification of groups based on personality [3]. Furthermore the stability of the model for the rumor-free equilibrium point and the rumor-spreading endemic equilibrium point is analyzed.

II. Equilibrium Point, Jacobian Matrix, Routh's Stability Criteria

A. Equilibrium Point

The aim of analyzing the stability of the equilibrium point in the rumor-spreading model is to figure out whether rumors are spreading or disappearing from a population.

Definition 2.1 [8] *It is provided a system of differential equations* $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$. The point $\hat{x} \in R^n$ *is called the equilibrium point* $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ *if it satisfies* $f(\hat{x}) = 0$.

A balance point is needed in the process of analyzing the spread of rumors in the rumor spreading model. The equilibrium point of the rumor spread model can be classified into two parts, i.e. [3]

1. The rumor-free equilibrium point means that the entire population cannot spread rumors or equilibrium point when the rumor-spreading class is zero.
2. The endemic equilibrium point of spreading rumor, which means that the equilibrium point is at the non-zero class of rumor spreaders or when rumors spread in the population

B. **Jacobian Matrix**

Definition 2.2. [7] *Let* $f: R^n \Longrightarrow R^n$ *is a function that is differentiable and continuous on the set* $D \subset R^n$ *and* $\hat{x} \in R^n$. *The Jacobian matrix of* f *around* $\hat{x}$, *written as* $J_f(\hat{x})$, *is defined as*

$$J_f(\hat{x}) = \begin{bmatrix} \frac{\partial f_{1(\hat{x})}}{\partial x_1} & \frac{\partial f_{1(\hat{x})}}{\partial x_2} & \cdots & \frac{\partial f_{1(\hat{x})}}{\partial x_n} \\ \frac{\partial f_{2(\hat{x})}}{\partial x_1} & \frac{\partial f_{2(\hat{x})}}{\partial x_2} & \cdots & \frac{\partial f_{2(\hat{x})}}{\partial x_n} \\ \vdots & \vdots & \vdots & \ddots \\ \frac{\partial f_{n(\hat{x})}}{\partial x_1} & \frac{\partial f_{n(\hat{x})}}{\partial x_2} & \cdots & \frac{\partial f_{n(\hat{x})}}{\partial x_n} \end{bmatrix}$$

Furthermore, stability around the equilibrium point is obtained by considering the eigenvalues of the Jacobian matrix for each equilibrium point.

Theorem 2.3. [1] *Let matrix* $n \times n$. $M\boldsymbol{x} = 0$ *only has a non-zero solution if and only if* $\det(M) \neq 0$.

Definition 2.4. [1] *If A is a matrix* $n \times n$, *then a nonzero vector* $\boldsymbol{x}$ *on* R^n *is called* ***an eigenvector*** *of A if Ax is a scalar multiple of* $\boldsymbol{x}$, *i.e.*

$$A\boldsymbol{x} = \lambda\boldsymbol{x} \quad (1)$$

The scalar λ *is called the* ***eigenvalue*** *A.*

According to definition 2.4, the equation (1) becomes

$$J\boldsymbol{z} = \boldsymbol{\lambda z}$$

Note that the equation (1) is equivalent to

$$(J - \lambda I)\boldsymbol{z} = \boldsymbol{0} \quad (2)$$

where I is the identity matrix. According to the Theorem 2.3 , the equation (2) has a non-zero solution if and only if the determinant of the matrix $(J - \lambda I)$ is non-zero. Therefore, the necessary and sufficient conditions for being the eigenvalues of the J matrix are

$$det(J - \lambda I) = 0$$

Theorem 2.5. [7] *Let the system (2.2.17),* J *the Jacobian matrix of size* $n \times n$ *has* k *eigenvalues* $\lambda_1, \lambda_2, \dots, \lambda_k$, *with* $k \leq n$.

1) *If $Re(\lambda_i < 0)$, for $i = 1,2,\dots,n$ then the equilibrium point $\hat{x}$ is asymptotically stable.*
2) *If $Re(\lambda_i \le 0)$, for $i = 1,2,\dots,n$ then the equilibrium point of $\hat{x}$ is stable and for the eigenvalues of $Re(\lambda_i = 0)$ corresponds to an eigenvector that is linearly independent as much as the multiplicity of λ_i.*
3) *If there is an eigenvalue of the Jacobian matrix that has a positive real part, then the equilibrium point $\hat{x}$ is unstable.*

C. Routh's Stability Criteria

Look at the following characteristics of polynomial equations.

$$P(\lambda) = \lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_0 \qquad (3)$$

According to Brannan et. al. [2], to determine whether all the roots (3) have negative real parts, Routh's criteria can be used based on array formulation, where the characteristic coefficients (3) are presented in array form. Routh's criteria are presented in the following table.

Table II.1 Routh's Criteria Table

λ^n	1	a_{n-2}	a_{n-4}	$\dots$
λ^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	$\dots$
λ^{n-2}	b_1	b_2	b_3	$\dots$
λ^{n-3}	c_1	c_2	c_3	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	

where $(1, a_{n-1}, a_{n-2}, \dots, a_0)$ is the coefficient of (3) and $(b_1, b_2, \dots, c_1, c_2, \dots)$ is defined as follows:

$$b_1 = -\frac{\begin{vmatrix} 1 & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}}{a_{n-1}}, b_2 = -\frac{\begin{vmatrix} 1 & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}}{a_{n-1}}, \dots$$

$$c_1 = -\frac{\begin{vmatrix} 1 & a_{n-3} \\ b_1 & b_2 \end{vmatrix}}{b_1}, c_2 = -\frac{\begin{vmatrix} a_{n-1} & a_{n-5} \\ b_1 & b_3 \end{vmatrix}}{a_{n-1}}, \dots$$

Theorem 2.6. [2] *All roots of the polynomial equation* (3) *have a negative real part if and only if the elements of the first column in Routh's criteria table have the same sign. Meanwhile, the number of roots with positive real parts is equal to the number of changes in sign.*

III. Result and Discussion

Assume the rumors spreading model, which considers rumor credibility, correlation between rumors and people's lives and crowd calssification based on personality as follows [3].

- Individuals devide into 5 classes :
  - Steady ignorant (G^P). This class includes people who do not know the rumor; if they hear the rumor, they prefer to contemplate it and seek confirmation before making decisions.

- Radical Ignorant (G^H). This class includes people who do not know the rumor, if they hear the rumor, they are mosy likely to believe it without contemplating it or seeking iformation.
- Exposed (G^E). This class includes people who know the rumor but hestitate to believe it and do not spread it.
- Spreader (G^S). This class includes people who spread the rumor
- Stifler (G^R). The class includes people who know the rumor but never spread it ora stop spreading it.

- If a rumor is more credible and more relevant to people's lives, people who belong to G^H are more likely to switch to class S when they hear the rumor. Thus, when an indvidual in class G^H comes to into contact with individual in class$G^S$and get to know the rumor, he or she will switch to class G^S with probability $\gamma\alpha\beta$.
- When an individual in class G^P comes into contact with an individual in class G^S, he or she will switch to class G^S or class G^E with different probabilities. The probability that individuals in class G^P will switch to class G^S can be defined as $\gamma\alpha\beta\mu$ and the probability that individuals in class G^P will switch to class G^E is expressed as $\gamma(1-\gamma)\alpha\beta$, where the relationship is that the proportion of people who hestitate is positively correlated with $\gamma(1-\gamma)$.
- Individuals in class G^E can switch to class G^S or G^R depending on the class which the individuals they are in contact with belong to. Inviduals in class G^E will decide wheter to spread rumors when they are influenced by others. Therefore, individuals in class G^E switch to class S with the probability θ when they are in contact with an individual in class G^S and switch to class R with the probability ϕ when they are in contact with an individual in class G^R.
- When individual in class G^S comes into contact with an individual in class G^S, G^E, or G^R, the former would switch to class G^R with probability η_1 because a spreader will realize that the rumor is not new when he or she meets another person who knows the rumor. Additionally, individuals in class G^S switch to class G^R at the rate η_2 due to the forgetting mechanism, which means that some individuals may forget the rumor in the spreading process.

Based on the assume, construct a flow diagram of the rumor spreading process show in Fig.1.

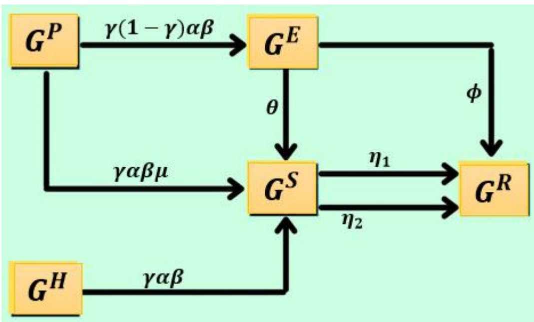

FIGURE 1. Flow diagram of rumor spreading process.

The meanings of the parameters of the spreading model are summarized in Table 1.

TABLE 1. Parameter of Spreading Model

Parameter	Description	Condition
α	The correlation coefficient between rumor and peoples's lives	$0 < \alpha \leq 1$
γ	The credibility of a rumor	$0 \leq \gamma < 1$
β	The probability that an ignorant individual hears a rumor via contact with individual in class spreader	$0 \leq \beta \leq 1$
μ	The spreading desire ratio	$0 < \mu \leq 1$
θ	Thepropbability that individuals in class G^E will switch to class G^S	
ϕ	The probability that individuals class G^E will witch to class G^R	
η_1	The probability that individuals in class G^S will switch to class G^R	
η_2	Forgetting rate	

Based on the assume of the rumor spreading model, given the following system of differential equations.

$$
\begin{aligned}
\frac{dP}{dt} &= -kPS(\gamma\alpha\beta + \gamma(1-\gamma)\alpha\beta) \\
\frac{dH}{dt} &= -kHS\gamma\alpha\beta \\
\frac{dE}{dt} &= kPS\gamma(1-\gamma)\alpha\beta - kSE\theta - kRE\phi \\
\frac{dS}{dt} &= kS(\mu P + H)\gamma\alpha\beta + kSE\theta - kS(R+S+E)\eta_1 - S\eta_2 \\
\frac{dR}{dt} &= kS(R+S+E)\eta_1 + S\eta_2 + kRE\phi
\end{aligned}
\tag{4}
$$

Where k denotes the average degree. Value of avarage degree k on the network the same as determining the value of the average degree k in theory graph.

Theorem 3.1. [5] *In a graph G, the sum of the degrees of the vertices is equal to twice the number of edges. Consequently, the number of vertices with odd degree is even.*

For calculate the average degree by the formula $k = \frac{\Sigma k_i}{n}$, where n the number of vertices and Σk_i the number of of node degrees.

Let $P(t)$, $H(t)$, $E(t)$, $S(t)$ and $R(t)$ be the densities of individuals in class G^P, G^H, G^E, $G^S$and G^R, respectively at the time. These densities satisfy the normalization condition as follows :

$$P(t) + H(t) + E(t) + S(t) + R(t) = 1 \tag{5}$$

The steady state of Eq. (4) is expressed as follows :

$$\frac{dP}{dt} = \frac{dH}{dt} = \frac{dE}{dt} = \frac{dS}{dt} = \frac{dR}{dt} = 0 \qquad (6)$$

Eq. (4) can be simplified to

$$\begin{aligned}
\frac{dP}{dt} &= -PS(A+B) \\
\frac{dH}{dt} &= -HSC \\
\frac{dE}{dt} &= PSB - k_1SE - k_2RE \\
\frac{dS}{dt} &= PSA + HSC + k_1SE - k_3S(R+S+E) - S\eta_2 \\
\frac{dR}{dt} &= k_3S(R+S+E) + S\eta_2 + k_2RE
\end{aligned} \qquad (7)$$

Where , $A = k\gamma\alpha\beta\mu$, $B = k\alpha\beta\gamma(1-\gamma)$, $C = k\gamma\alpha\beta$, $k_1 = k\theta$, $k_2 = k\phi$, dan $k_3 = k\eta_1$.

Eq. (4) has a rumor free equilibrium point $E_0 = \big(x; n; 0; 0; (1-x-n)\big)$ where x mean the densities of individuals in class Steady Ignirant and n mean the densities of individuals in class Radical Ignoarant. The density of individuals in class Stifler are represented as $(1-x-n)$ according to Eq. (5).

Eq. (4) has an endemic equilibrium point for spreading rumors is $E^* = (P_1, H_1, E_1, S_1, R_1) = \left(0; 0; -\frac{\theta\eta_1 q}{\phi(\theta-\eta_1)} + \frac{\eta_1 q}{(\theta-\eta_1)} + \frac{\eta_2}{k(\theta-\eta_1)}; q; -\frac{\theta q}{\phi}\right)$

where,

$$q = \frac{-b}{a}$$

$$a = -\frac{k\eta_1\theta}{\phi} + k\eta_1 - \frac{k{\eta_1}^2\theta}{\phi(\theta-\eta_1)} + \frac{k{\eta_1}^2}{\theta-\eta_1} + \frac{k\theta^2\eta_1}{\phi(\theta-\eta_1)} - \frac{k\theta\eta_1}{\theta-\eta_1}$$

$$b = \frac{\eta_1\eta_2}{\theta-\eta_1} + \eta_2 - \frac{\theta\eta_2}{\theta-\eta_1}$$

Employe the Jacobian matrix method to discuss stability of Eq. (7). The Jacobian matrix of Eq. (7) is expressed as follows:

$$J = \begin{bmatrix} -S(A+B) & 0 & 0 & -P(A+B) & 0 \\ 0 & -SC & 0 & -HC & 0 \\ SB & 0 & -k_1S - k_2R & PB - k_1E & -k_2E \\ SA & SC & k_1S - k_3S & PA + HC + k_1E - k_3(R+2S+E) - \eta_2 & -k_3S \\ 0 & 0 & k_3S + k_2R & k_3(R+2S+E) + \eta_2 & k_3S + k_2E \end{bmatrix}$$

Theorem 3.2. *Let a system of differential equations (7) , system of differential equations (7) has eigenvalues at free equilibrium point. Stability of the sytem (7) at free equilibrium point depends on* $xA + nC - k_3(1-x-n) - \eta_2$.

(i) *If* $xA + nC - k_3(1-x-n) - \eta_2 < 0$ *then free equilibrium point is stable*

(ii) *If* $xA + nC - k_3(1-x-n) - \eta_2 > 0$ *then free equilibrium point is not stable*

Proof.

The Jacobian matrix at free equilibrium point is a following:

$$J(E_0) = \begin{bmatrix} 0 & 0 & 0 & -x(A+B) & 0 \\ 0 & 0 & 0 & -nC & 0 \\ 0 & 0 & -k_2(1-x-n) & PB - k_1E & 0 \\ 0 & 0 & 0 & xA + nC - k_3(1-x-n) - \eta_2 & 0 \\ 0 & 0 & k_2(1-x-n) & k_3(1-x-n) + \eta_2 & 0 \end{bmatrix}$$

The characteristic equation of $J(E_0)$ can be obtained as follows :

$$\lambda^3(k_2(1-x-n)+\lambda)(-xA-nC+k_3(1-x-n)+\eta_2+\lambda) = 0 \quad (8)$$

The eigenvalues of the Jacobian matrix $J(E_0)$ by solving Eq. (8) are $\lambda_1 = \lambda_2 = \lambda_3 = 0$ and $\lambda_4 = -k_2(1-x-n) < 0$ or the eigenvalues is non positive, It can be conclude that r free equilibrium point is stable depends on the $xA + nC - k_3(1-x-n) - \eta_2$. *If* $xA + nC - k_3(1-x-n) - \eta_2 < 0$ then free equilibrium point is stable and if $xA + nC - k_3(1-x-n) - \eta_2 > 0$ then free equilibrium point is not stable.

Theorem 3.3. *Let a system of differential equations (7) , system of differential equations (7) has eigenvalues at endemic equilibrium point for spreading rumors. Stability (7) at endemic equilibrium point for spreading rumors depends on. If* $m_{11} \leq 0$ *and* $m_{22} \leq 0$ *then endemic equilibrium point for spreading rumors is stable.*

Proof.

The Jacobian matrix at endemic equilibrium point for spreading rumors as follows :

$$J(E_*) = \begin{bmatrix} -S_1(A+B) & 0 & 0 & 0 & 0 \\ 0 & -S_1C & 0 & 0 & 0 \\ S_1B & 0 & -k_1S_1 - k_2R_1 & -k_1E_1 & -k_2E_1 \\ S_1A & S_1C & k_1S_1 - k_3S_1 & k_1E_1 - k_3(R_1 + 2S_1 + E_1) - \eta_2 & -k_3S_1 \\ 0 & 0 & k_3S_1 + k_2R_1 & k_3(R_1 + 2S_1 + E_1) + \eta_2 & k_3S_1 + k_2E_1 \end{bmatrix}$$

The charecteristic equation of $J(E^*)$ can be obtained as follows :

$$(\lambda - m_{11})(\lambda\lambda - m_{22})(\lambda^3 - a_2\lambda^2 - a_1\lambda - a_0) = 0 \quad (9)$$

Where,

$m_{11} = -S_1(A+B)$
$m_{22} = -S_1C$
$m_{31} = S_1B$
$m_{33} = -k_1S_1 - k_2R_1$
$m_{34} = -k_1E_1$
$m_{35} = -k_2E_1$
$m_{41} = S_1A$
$m_{42} = S_1C$
$m_{43} = k_1S_1 - k_3S_1$

$m_{44} = k_1E_1 - k_3(R_1 + 2S_1 + E_1) - \eta_2$
$m_{45} = -k_3S_1$
$m_{53} = k_3S_1 + k_2R_1$
$m_{54} = k_3(R_1 + 2S_1 + E_1) + \eta_2$
$m_{55} = k_3S_1 + k_2E_1$

and

$a_0 = m_{33}m_{44}m_{55} - m_{33}m_{45}m_{54} + m_{43}m_{54}m_{35} - m_{43}m_{34}m_{55} + m_{53}m_{34}m_{45} - m_{53}m_{35}m_{44}$
$a_{11} = m_{53}m_{55} - m_{33}m_{44} - m_{33}m_{55} - m_{44}m_{55} + m_{45}m_{54} + m_{43}m_{34}$
$a_2 = m_{33} + m_{44} + m_{55}$

The eigenvalues of the Jacobian matrix $J(E^*)$ by solving Eq. (9) and obtain the eigenvalues

$$\lambda_1 = m_{11}$$
$$\lambda_2 = m_{22}$$

Based on Theorem 2.5 if $m_{11} \leq 0$ and $m_{22} \leq 0$ so endemic equilibrium point for spreading rumors is stable.

Look at the following equation

$$\lambda^3 - a_2\lambda^2 - a_1\lambda - a_0 = 0. \quad (10)$$

Theorem 3.4. *All roots of the polynomial equation (10) have a negative real part if and only if* $\frac{a_0+a_1a_2}{-a_2} > 0, a_0 < 0$ *and* $a_2 < 0$ *then endemic equilibrium points stable.*

Proof.

All roots of the polynomial equation (10) have a negative real part if and only if the elements of the first column in Routh's criteria table have the same sign. Meanwhile, the number of roots with positive real parts is equal to the number of changes in sign.

Table 2. Tabel Kriteria Routh

$\boldsymbol{\lambda^3}$	1		$\boldsymbol{-a_1}$
$\boldsymbol{\lambda^2}$	$-a_2$		$-a_0$
$\boldsymbol{\lambda^1}$	b_1	0	
$\boldsymbol{\lambda^0}$	c_1	0	

where,

$$b_1 = \frac{a_0 + a_1a_2}{-a_2}$$

$$c_1 = -a_0$$

Endemic equilibrium points stable asymptotic if he elements of the first column of the Routh table have positive, so it must be $\frac{a_0+a_1a_2}{-a_2} > 0, a_0 < 0$ and $a_2 < 0$.

IV. Conclusion

According to the results of the discussion, it can be concluded that the rumor-free equilibrium point is $E_0 = \big(x, n, 0{,}0, (1-x-n)\big)$ with the eigenvalues as follows $\lambda_2 = 0$, $\lambda_3 = 0$, $\lambda_4 = -k_2(1-x-n)$ and $\lambda_5 = xA + nC - k_3(1-x-n) - \eta_2$ because $\lambda_1 = 0$ $\lambda_2 = 0$, $\lambda_3 = 0$ and $\lambda_4 < 0$ then the stability around the rumor-free equilibrium point depends on λ_5. Furthermore, the equilibrium point of endemic rumor spreading is $E_* = \left(0{,}0, -\frac{\theta\eta_1 q}{\phi(\theta-\eta_1)} + \frac{\eta_1 q}{(\theta-\eta_1)} + \frac{\eta_2}{k(\theta-\eta_1)}, q, -\frac{\theta q}{\phi}\right)$ with the eigenvalues as follows $\lambda_1 = m_{11}$ and $\lambda_2 = m_{22}$, the stability around the endemic equilibrium point is stable if $\lambda_1 \leq 0$ dan $\lambda_2 \leq 0$. Furthermore, the stability around the rumor endemic equilibrium point also depends on all the roots of the equation $\lambda^3 - a_2\lambda^2 - a_1\lambda - a_0 = 0$. So, to determine whether all the roots of the equation have a negative real part, it must be fulfilled $\frac{a_0 + a_1 a_2}{-a_2} > 0$, $a_0 < 0$ and $a_2 < 0$.

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