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Upper Nilradical In Terms Of A Left Ideal In χ The Ring Of Real Matrix

Azir JUSUFI[1], Kristaq FILIPI[2], Hazbie Vata[3]

[1]Universiteti i Tetovës
azir.jusufi@unite.edu.mk
[2]Universiteti Politeknik i Tiranës
f_kristaq@hotmail.com
[3]Universiteti i Tetoves
hazbievata99@gmail.com

Abstract – Based on the idea of Andrunakievic and Rjabuhin for linking Jacobson's radical to a right-wing ideal [3], we interfered with the theory of the upper nilradical, examining them concerning a unilateral ideal of an associative ring. We'll be here briefly as soon as possible.

Keywords – hi matrices, ideal, ring, nil radical.

1. Upper Nilradical in terms of an ideal of LEFT

Let it be the ideal left P of an *R* ring.

Definition 1.1. Ring *A* is called A-ring in terms of the left ideal *P*, if $\forall a \in A,\ \exists n \in \square, .\ a^n \in P$

The ideal (whether left or right) $Is \triangleleft R$ called the nilideal (left or right) in terms of the left *ideal P*, if *I* is Nilenaese in terms of the left ideal *P*.

Definition 1.2. The upper nilradical in terms of the left ideal *P* in the *Ring R* we call the nilideal (left or right) greater in terms of the left ideal *P* in it. We mark *K* (*R, P*).

These theorems are proven to have room:

Theoremat 1.1. [11]). In an *R* ring, containing the left ideal *P*, the sum of each end community of nilideals (left) related to the left ideal *P* is nillideal in terms of the left ideal *P*.

Theoremat 1.2. [11]). In an *R* ring, containing the left ideal *P*, there is the upper nilradical in terms of the left *ideal P* and it matches the sum of all the nilideals (left) of *R* related to its left ideal *P*.

Theoremat 1.3. [13]). If *I* is ideal for the *R* ring, which contains its left *P* ideal, then. $K(I,P) = I \cap K(R,P)$

Definition 1.3. ([7]). *Ring R* is called K-radical in terms of

ideal left *P*, if *R*=*K* (*R, P)*.

Theoremat 1.4. [7]). Ring *R* is K-radical in terms of the left ideal *P*, then and only then, when it is Niluan in terms of the ideal left *P*.

If *R* is K-radical in terms of the ideal left *P* and *I* ideal containing the ideal left *P* in *R*, then. We take this as well.
$$K(I,P) = I \cap K(R,P) = I \cap R = I$$

Theoremat 1.5. [7]). Every I ideal in the K-radical ring related to the left p ideal is a K-radical ring related to the left ideal *P*.

2. Ring of χ k-MATRIGIVE. UPPER NILRADICAL

Related to a LEFT IDEAL in it

Definition 2.1. χ k-matrix we call the square matrix of the order 2 k+1, *k*=2, 3, ..., which has the following shape

$$\begin{bmatrix}
\alpha_{1,1} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \alpha_{1,2k+1} \\
0 & \alpha_{2,2} & \alpha_{2,3} & \cdots & \alpha_{2,k} & \alpha_{2,k+1} & \alpha_{2,k+2} & \cdots & \alpha_{2,2k-1} & \alpha_{2,2k} & 0 \\
0 & 0 & \alpha_{3,3} & \cdots & \alpha_{3,k} & \alpha_{3,k+1} & \alpha_{3,k+2} & \cdots & \alpha_{3,2k-1} & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \alpha_{k,k} & \alpha_{k,k+1} & \alpha_{k,k+2} & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & \alpha_{k+2,k} & \alpha_{k+2,k+1} & \alpha_{k+2,k+2} & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & \alpha_{2k-1,3} & \cdots & \alpha_{2k-1,k} & \alpha_{2k-1,k+1} & \alpha_{2k-1,k+2} & \cdots & \alpha_{2k-1,2k-1} & 0 & 0 \\
0 & \alpha_{2k,2} & \alpha_{2k,3} & \cdots & \alpha_{2k,k} & \alpha_{2k,k+1} & \alpha_{2k,k+2} & \cdots & \alpha_{2k,2k-1} & \alpha_{2k,2k} & 0 \\
\alpha_{2k+1,1} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \alpha_{2k+1,2k+1}
\end{bmatrix}, \quad (1)$$

Where the (*k*+1) row is zero.

Let it be A_{2k+1} community of χ k-matrixes on *the field* □ real numbers, where *k* is a number fixed among natural numbers 2. 3*, ...*.

We note that $\forall$ *A, B* $\in$ A_{2k+1}, *A*+*B* $\in$ A_{2k+1} and $A \cdot B \in A_{2k+1}$. The first is obvious, and for the second we have

$$(2)\qquad A\cdot B=\begin{bmatrix}
\alpha_{1,1} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \alpha_{1,2k+1}\\
0 & \alpha_{2,2} & \cdots & \alpha_{2,k} & \alpha_{2,k+1} & \alpha_{2,k+2} & \cdots & \alpha_{2,2k} & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \unicode{x22F0} & \vdots & \vdots\\
0 & 0 & \cdots & \alpha_{k,k} & \alpha_{k,k+1} & \alpha_{k,k+2} & \cdots & 0 & 0\\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0\\
0 & 0 & \cdots & \alpha_{k+2,k} & \alpha_{k+2,k+1} & \alpha_{k+2,k+2} & \cdots & 0 & 0\\
\vdots & \vdots & \unicode{x22F0} & \vdots & \vdots & \vdots & \unicode{x22F0} & \vdots & \vdots\\
0 & \alpha_{2k,2} & \cdots & \alpha_{2k,k} & \alpha_{2k,k+1} & \alpha_{2k,k+2} & \cdots & \alpha_{2k,2k} & 0\\
\alpha_{2k+1,1} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \alpha_{2k+1,2k+1}
\end{bmatrix}\cdot$$

$$\begin{bmatrix}
\beta_{1,1} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \beta_{1,2k+1}\\
0 & \beta_{2,2} & \cdots & \beta_{2,k} & \beta_{2,k+1} & \beta_{2,k+2} & \cdots & \beta_{2,2k} & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \unicode{x22F0} & \vdots & \vdots\\
0 & 0 & \cdots & \beta_{k,k} & \beta_{k,k+1} & \beta_{k,k+2} & \cdots & 0 & 0\\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0\\
0 & 0 & \cdots & \beta_{k+2,k} & \beta_{k+2,k+1} & \beta_{k+2,k+2} & \cdots & 0 & 0\\
\vdots & \vdots & \unicode{x22F0} & \vdots & \vdots & \vdots & \unicode{x22F0} & \vdots & \vdots\\
0 & \beta_{2k,2} & \cdots & \beta_{2k,k} & \beta_{2k,k+1} & \beta_{2k,k+2} & \cdots & \beta_{2k,2k} & 0\\
\beta_{2k+1,1} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \beta_{2k+1,2k+1}
\end{bmatrix}=$$

$$\begin{bmatrix}
\sum\limits_{j=1,2k+1}\alpha_{1,j}\beta_{j,1} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \sum\limits_{j=1,2k+1}\alpha_{1,j}\beta_{j,2k+1}\\
0 & \sum\limits_{j=2,2k}\alpha_{2,j}\beta_{j,2} & \cdots & \sum\limits_{j=2}^{2k}\alpha_{2,j}\beta_{j,k} & \sum\limits_{j=2}^{2k}\alpha_{2,j}\beta_{j,k+1} & \sum\limits_{j=2}^{2k}\alpha_{2,j}\beta_{j,k+2} & \cdots & \sum\limits_{j=2,2k}\alpha_{2,j}\beta_{j,2k} & 0\\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \unicode{x22F0} & \vdots & \vdots\\
0 & 0 & \cdots & \sum\limits_{j=k,k+2}\alpha_{k,j}\beta_{j,k} & \sum\limits_{j=k,k+2}\alpha_{k,j}\beta_{j,k+1} & \sum\limits_{j=k,k+2}\alpha_{k,j}\beta_{j,k+2} & \cdots & 0 & 0\\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0\\
0 & 0 & \cdots & \sum\limits_{j=k,k+2}\alpha_{k+2,j}\beta_{j,k} & \sum\limits_{j=k,k+2}\alpha_{k+2,j}\beta_{j,k+1} & \sum\limits_{j=k,k+2}\alpha_{k+2,j}\beta_{j,k+2} & \cdots & 0 & 0\\
\vdots & \vdots & \unicode{x22F0} & \vdots & \vdots & \vdots & \unicode{x22F0} & \vdots & \vdots\\
0 & \sum\limits_{j=2,2k}\alpha_{2k,j}\beta_{j,2} & \cdots & \sum\limits_{j=2}^{2k}\alpha_{2k,j}\beta_{j,k} & \sum\limits_{j=2}^{2k}\alpha_{2k,j}\beta_{j,k+1} & \sum\limits_{j=2}^{2k}\alpha_{2k,j}\beta_{j,k+2} & \cdots & \sum\limits_{j=2,2k}\alpha_{2k,j}\beta_{j,2k} & 0\\
\sum\limits_{j=1,2k+1}\alpha_{2k+1,j}\beta_{j,1} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \sum\limits_{j=1,2k+1}\alpha_{2k+1,j}\beta_{j,2k+1}
\end{bmatrix}$$

From there it appears that (A_{2k+1}, +, ·) is the subnumeration of the M_{2k+1} ring () of the matrix of the order $P2k+1$ over the field of real numbers$P$.

We distinguish in ring A_{2k+1} the community of $\chi_{its\ k}$ –matrixes, which have zeroed all elements except perhaps the first and last element of the first pillar and we mark this community P_{2k+1}:

$$\mathscr{P}_{2k+1}=\left\{\begin{bmatrix} p & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ q & 0 & \cdots & 0 & 0 \end{bmatrix}_{(2k+1)\times(2k+1)} \middle| p,q\in \mathrm{R}\right\}.$$

Statement 2.1. The P_{2k+1} community is the left ideal of the A_{2k+1} ring.

This is because it seems that $\forall$ P *1,* $P_2 \in \mathrm{P}_{2k+1}$, P_1 - $P_2 \in \mathrm{P}_{2k+1}$. In addition, $\forall$ P $P\in$ $_{2k+1}$ and $\forall$ A A $\in$ $_{2k+1}$ we have

$A{\cdot}P=$

$$\begin{bmatrix} \alpha_{1,1} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \alpha_{1,2k+1} \\ 0 & \alpha_{2,2} & \cdots & \alpha_{2,k} & \alpha_{2,k+1} & \alpha_{2,k+2} & \cdots & \alpha_{2,2k} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots \\ 0 & 0 & \cdots & \alpha_{k,k} & \alpha_{k,k+1} & \alpha_{k,k+2} & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & \alpha_{k+2,k} & \alpha_{k+2,k+1} & \alpha_{k+2,k+2} & \cdots & 0 & 0 \\ \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots \\ 0 & \alpha_{2k,2} & \cdots & \alpha_{2k,k} & \alpha_{2k,k+1} & \alpha_{2k,k+2} & \cdots & \alpha_{2k,2k} & 0 \\ \alpha_{2k+1,1} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \alpha_{2k+1,2k+1} \end{bmatrix} \cdot \begin{bmatrix} p & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ q & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} \alpha_{1,1}p+\alpha_{1,2k+1}q & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \alpha_{2k+1,1}p+\alpha_{2k+1,2k=1}q & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathscr{P}_{2k+1} \ .$$

We would already easily state that;

$$P{\cdot}A=\begin{bmatrix} pa_{11} & 0 & 0 & \dots & 0 & 0 & pa_{1,2k+1} \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ qa_{11} & 0 & 0 & \dots & 0 & 0 & qa_{1,2k+1} \end{bmatrix} \notin \mathscr{P}_{2k+1},$$

If A is such that it indicates that $a_{1,2k+1} \neq 0$, P_{2k+1} is only ideal left of ring $A_{2k+1.}$

Let *it* be L a χ_k–matrix of whatever, d.m.th. $L \in A_{2k+1}$ and has the trajectory (1). We divide the A_{2k+1} community initially into two K-grades $_0$, K_1, where K_0 is the class of L matrixes, in which are zero elements of the marginal $_{1.2k+1}$, $a_{2k+1,2k+1}$ of the last pillar and K_1 is the class of matrixes L, in which at least one of these elements is nonzero.

It is understood that a χ_k –matrix of class K_1 has the shape (1), where $a_{1.2k+1} \neq 0 \vee a_{2k+1,2k+1} \neq 0$, while a χ_k –matrix of class K_0 has the shape

$$\begin{bmatrix}
\alpha_{1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & \alpha_{2,2} & \alpha_{2,3} & \alpha_{2,4} & \cdots & \alpha_{2,k} & \alpha_{2,k+1} & \alpha_{2,k+2} & \cdots & \alpha_{2,2k-2} & \alpha_{2,2k-1} & \alpha_{2,2k} & 0 \\
0 & 0 & \alpha_{3,3} & \alpha_{3,4} & \cdots & \alpha_{3,k} & \alpha_{3,k+1} & \alpha_{3,k+2} & \cdots & \alpha_{3,2k-2} & \alpha_{3,2k-1} & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & \alpha_{k-1,k} & \alpha_{k-1,k+1} & \alpha_{k-1,k+2} & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & \alpha_{k,k} & \alpha_{k,k+1} & \alpha_{k,k+2} & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & \alpha_{k+2,k} & \alpha_{k+2,k+1} & \alpha_{k+2,k+2} & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & \alpha_{k+3,k} & \alpha_{k+3,k+1} & \alpha_{k+3,k+2} & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \alpha_{2k-1,3} & \alpha_{2k-1,4} & \cdots & \alpha_{2k-1,k} & \alpha_{2k-1,k+1} & \alpha_{2k-1,k+2} & \cdots & \alpha_{2k-1,2k-2} & \alpha_{2k-1,2k-1} & 0 & 0 \\
0 & \alpha_{2k,2} & \alpha_{2k,3} & \alpha_{2k,4} & \cdots & \alpha_{2k,k} & \alpha_{2k,k+1} & \alpha_{2k,k+2} & \cdots & \alpha_{2k,2k-2} & \alpha_{2k,2k-1} & \alpha_{2k,2k} & 0 \\
\alpha_{2k+1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0
\end{bmatrix} \quad (3)$$

We examine the elements of χ_{the} k-matrix (3), which are located in the two areas

Triangledrawn as above.

We mark T_1 community of elements of the upper triangle area, excluding elements of its "side ribs"; we mark T_2 community of elements of the lower triangle area, excluding elements of its "side ribs". We also mark D the community of elements located in the diagonals of χ_{the} k-matrix (3), except for the peak elements of $_{1.1}$ and $_{2k+1.1}$. So,

D= {and 2,2; and 3, 3; ...; *and* k+$_{1,\ k+1}$; ...*;* and 2k,2k; and $_{2k,2;\ \text{and}\ 2k-1,3}$; ...*;* a k+2, k;a $_{k,k+2}$;...,$a_{2,\ 2k}$}

In terms of zero value of elements of *communities* D and $T = T_1 \cup T_2$ are possible these four cases:

1) All elements of the D and T communities are zero;

2) All *Elements of D* are zero and at least one element of T is nonzero;

3) At least one element of D is nonzero and all Elements of T are zero;

4) At least one element of D is nonzero and at least one element of T is also nonzero.

We share class K_0 according to them respectively in grades I_0, K_{01}, K_{10}, K_{11}.

Grade I L matrixes have the shape

$$L = \begin{bmatrix} a_{11} & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ a_{2k+1,1} & 0 & 0 & \dots & 0 \end{bmatrix} \in \mathscr{P}_{2k+1.}$$

So I_0 = P_{2k+1}

Class K_{01} *L* matrixes have form

$$L=\begin{bmatrix}
\alpha_{1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & \alpha_{2,3} & \alpha_{2,4} & \cdots & \alpha_{2,k} & \alpha_{2,k+1} & \alpha_{2,k+2} & \cdots & \alpha_{2,2k-2} & \alpha_{2,2k-1} & 0 & 0 \\
0 & 0 & 0 & \alpha_{3,4} & \cdots & \alpha_{3,k} & \alpha_{3,k+1} & \alpha_{3,k+2} & \cdots & \alpha_{3,2k-2} & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & \alpha_{k-1,k} & \alpha_{k-1,k+1} & \alpha_{k-1,k+2} & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \alpha_{k,k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & \alpha_{k+2,k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & \alpha_{k+3,k} & \alpha_{k+3,k+1} & \alpha_{k+3,k+2} & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \alpha_{2k-1,4} & \cdots & \alpha_{2k-1,k} & \alpha_{2k-1,k+1} & \alpha_{2k-1,k+2} & \cdots & \alpha_{2k-1,2k-2} & 0 & 0 & 0 \\
0 & 0 & \alpha_{2k,3} & \alpha_{2k,4} & \cdots & \alpha_{2k,k} & \alpha_{2k,k+1} & \alpha_{2k,k+2} & \cdots & \alpha_{2k,2k-2} & \alpha_{2k,2k-1} & 0 & 0 \\
\alpha_{2k+1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0
\end{bmatrix},$$

where at least one of the elements, $a_{ij} \neq 0$ *i=2, 3, ..., 2k* and *j=3, 4, ..., (2k-1).*

The Grade K_{10} And K_{11} matrixes have the form (3), where at least one element of the *D* community is nonzero. Thus, the community A_{2k+1} is divided into classes I_0, K_{01}, K_{10}, K_{11}, K_1.

We mark *N* (A_{2k+1}, P_{2k+1}) community of $LA \in {}_{2k+1}$ matrixes such as $L^m \in P_{2k+1}$ for any natural number $mk \geq$, d.m.th.

$N (A\ 2\ k+1, P\ 2\ k+1) = \{L\ A\ 2\ k+1, mk\ ,\ L^m\ P\ {}_2 \in {}_{k+} \big| \forall m \in N \geq \in {}_1\ \}$ (4)

It is clear that $P_{2k+1} \subseteq$ N (A_{2k+1}, P_{2k+1}), because, being ideally left, P_{2k+1} is ring, therefore along with the matrix of whatever *L* from P_{2k+1} and $L^m \in P_{2k+1}$, for every natural number *m,* so also for *mk.* $\geq$

Let's focus at χ_{the} L-matrix, which appears as;

$$L=\begin{bmatrix} \alpha_{11} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \alpha_{23} & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \alpha_{2k+1,1} & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathscr{A}_{2k+1}, \ \alpha_{23} \neq 0,$$

So $L \notin$ P_{2k+1}. For m=2,

$L^2 =$

$$\begin{bmatrix} \alpha_{11} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \alpha_{23} & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \alpha_{2k+1,1} & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} \alpha_{11}^2 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \alpha_{2k+1,1}\alpha_{11} & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \mathscr{P}_{2k+1};$$

According to claim 2.1, even $L^m \in P_{2k+1}$, for every natural m, $mk.$ $\geq$

That's how we got this.

Statement 2.2. Tek community N (A_{2k+1}, P_{2k+1}) bp $_{2k+1}$ is rigorously included.

Statement 2.3. Community N (A_{2k+1}, P_{2k+1}) includes grades I_0, K_{01} and does not include the other three k $_{10}$, K_{11}, K_1 classes.

Proof. Since $I_0 = P_{2k+1}$ and, according to statement 2.2, we have

$P_{2k+1} \subset N(A_{2k+1}, P_{2k+1})$, remains to prove that $K_{01} \subset N(A_{2k+1}, P_{2k+1})$,

$K_{10} \not\subset N$ (A 2 k+1, P 2 k+1), N (A 2 k+1, P 2 k+1) and N (A 2 k+1 $K_{11} \not\subset$, $P_2 K_1 \not\subset$ $_{k+1}$).

Let's note that for every one, $L \in K_{01}$

$$L^2=\begin{bmatrix}
\alpha_{1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & \alpha_{2,3} & \alpha_{2,4} & \cdots & \alpha_{2,k} & \alpha_{2,k+1} & \alpha_{2,k+2} & \cdots & \alpha_{2,2k-2} & \alpha_{2,2k-1} & 0 & 0\\
0 & 0 & 0 & \alpha_{3,4} & \cdots & \alpha_{3,k} & \alpha_{3,k+1} & \alpha_{3,k+2} & \cdots & \alpha_{3,2k-2} & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \vdots & \cdot^{\cdot^{\cdot}} & \vdots & \vdots & \vdots\\
0 & 0 & 0 & 0 & \cdots & \alpha_{k-1,k} & \alpha_{k-1,k+1} & \alpha_{k-1,k+2} & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & \alpha_{k,k+1} & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & \alpha_{k+2,k+1} & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & \alpha_{k+3,k} & \alpha_{k+3,k+1} & \alpha_{k+3,k+2} & \cdots & 0 & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & \cdot^{\cdot^{\cdot}} & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & 0 & \alpha_{2k-1,4} & \cdots & \alpha_{2k-1,k} & \alpha_{2k-1,k+1} & \alpha_{2k-1,k+2} & \cdots & & 0 & 0 & 0\\
0 & 0 & \alpha_{2k,3} & \alpha_{2k,4} & \cdots & \alpha_{2k,k} & \alpha_{2k,k+1} & \alpha_{2k,k+2} & \cdots & & \alpha_{2k,2k-1} & 0 & 0\\
\alpha_{2k+1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & & 0 & 0 & 0
\end{bmatrix}^2 =$$

$$=\begin{bmatrix}
\alpha_{1,1}^2 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \alpha^{(1)}_{2,4} & \cdots & \alpha^{(1)}_{2,k} & \alpha^{(1)}_{2,k+1} & \alpha^{(1)}_{2,k+2} & \cdots & \alpha^{(1)}_{2,2k-2} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & \alpha^{(1)}_{3,k} & \alpha^{(1)}_{3,k+1} & \alpha^{(1)}_{3,k+2} & \cdots & 0 & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \vdots & \cdot^{\cdot^{\cdot}} & \vdots & \vdots & \vdots\\
0 & 0 & 0 & 0 & \cdots & 0 & \alpha^{(1)}_{k-1,k+1} & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & \alpha^{(1)}_{k+3,k+1} & 0 & \cdots & 0 & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & \cdot^{\cdot^{\cdot}} & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\
0 & 0 & 0 & 0 & \cdots & \alpha^{(1)}_{2k-1,k} & \alpha^{(1)}_{2k-1,k+1} & \alpha^{(1)}_{2k-1,k+2} & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \alpha^{(1)}_{2k,4} & \cdots & \alpha^{(1)}_{2k,k} & \alpha^{(1)}_{2k,k+1} & \alpha^{(1)}_{2k,k+2} & \cdots & \alpha^{(1)}_{2k,2k-2} & 0 & 0 & 0\\
\alpha^{(1)}_{2k+1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0
\end{bmatrix}$$

It is noted that in L^2 the number of elements perhaps nonzero in the "heights" of triangle areas decreases by at least one element; in L they have k-1 elements each (precisely $\alpha_{2,\,k+1}$, $\alpha_{3,\,k+1}$, ..., $\alpha_{k-1,\,k+1}$, $\alpha_{k,\,k+1}$ in the upper), while in L^2 they have from

k-2 elements such (precisely, ..., in the upper). $\alpha^{(1)}_{2,k+1}$ $\alpha^{(1)}_{3,k+1}$ $\alpha^{(1)}_{k-1,k+1}$

With further increase in power indicator, after k-1 steps we take

$$L^k=\begin{bmatrix}
\alpha_{1,1}^k & 0 & 0 & \dots & 0\\
0 & 0 & 0 & \dots & 0\\
0 & 0 & 0 & \dots & 0\\
\dots & \dots & \dots & \dots & \dots\\
\alpha^{(k-1)}_{2k+1,1} & 0 & 0 & \dots & 0
\end{bmatrix}\in \mathscr{P}_{2k+1}.$$

Therefore $L^m \in P_{2k+1}$, for every natural m, mk, $\geq$ d.m.th. $K_{01} \subset N$ (A_{2k-1}, P_{2k-1}).

It is further easily shown that $K_{10} \not\subset N$ (A_{2k+1}, P_{2k+1}), $K_{11} \not\subset N$ (A_{2k+1}, P_{2k+1}) and $K_1 \not\subset N$ (A_{2k+1}, P_{2k+1}). For example, the first relation serves the implication

$$L=\begin{bmatrix} \alpha_{1,1} & 0 & 0 & \ldots & 0 \\ 0 & \alpha_{2,2} & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \alpha_{2k+1,1} & 0 & 0 & \ldots & 0 \end{bmatrix}, \alpha_{2,2} \neq 0 \Rightarrow L^k = \begin{bmatrix} \alpha_{1,1}^k & 0 & 0 & \ldots & 0 \\ 0 & \alpha_{2,2}^k & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \alpha_{2k+1,1}\alpha_{1,1}^{k-1} & 0 & 0 & \ldots & 0 \end{bmatrix} \notin \mathscr{P}_{2k+1}$$

for every $k \in N$.

We mark $A_{2k+1}^{(0)}$ subcommunity $I_0 \cup K_{01}$ of A_{2k+1}. We examine the following subcommunities of $A_{2k+1}^{(0)}$:

$$I_0 = \left\{ \begin{bmatrix} a_{11} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ a_{2k+1,1} & 0 & \cdots & 0 & 0 \end{bmatrix} \middle| a_{ij} \in \mathrm{R} \right\},$$

$$I_{23} = \left\{ \begin{bmatrix} \alpha_{1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \alpha_{2,3} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \alpha_{2k,3} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\ \alpha_{2k+1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix} \middle| a_{ij} \in \mathrm{R} \right\},$$

$$I_{34}=\left\{\begin{bmatrix} \alpha_{1,1} & 0 & 0 & 0 & 0 & ...0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \alpha_{2,3} & \alpha_{2,4} & 0 & ...0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_{3,4} & 0 & ...0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & ...0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & ...0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & ...0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \alpha_{2k-1,4} & 0 & ...0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \alpha_{2k,3} & \alpha_{2k,4} & 0 & ...0 & 0 & \cdots & 0 & 0 & 0 \\ \alpha_{2k+1,1} & 0 & 0 & 0 & 0 & ...0 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix} \middle| a_{ij}\in \mathrm{R}\right\},$$

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$$I_{k-1,k}=\left\{\begin{bmatrix} \alpha_{1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_{2,3} & \alpha_{2,4} & \cdots & \alpha_{2,k} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_{3,4} & \cdots & \alpha_{3,k} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \alpha_{k-1,k} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & \alpha_{k+3,k} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \alpha_{2k-1,4} & \cdots & \alpha_{2k-1,k} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_{2k,3} & \alpha_{2k,4} & \cdots & \alpha_{2k,k} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ \alpha_{2k+1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \end{bmatrix} \middle| a_{ij}\in \mathrm{R}\right\},$$

$$I_{k,k+1}=\left\{\begin{bmatrix} \alpha_{1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_{2,3} & \alpha_{2,4} & \cdots & \alpha_{2,k} & \alpha_{2,k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_{3,4} & \cdots & \alpha_{3,k} & \alpha_{3,k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \alpha_{k-1,k} & \alpha_{k-1,k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & \alpha_{k,k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & \alpha_{k+2,k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & \alpha_{k+3,k} & \alpha_{k+3,k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \iddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \alpha_{2k-1,4} & \cdots & \alpha_{2k-1,k} & \alpha_{2k-1,k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_{2k,3} & \alpha_{2k,4} & \cdots & \alpha_{2k,k} & \alpha_{2k,k+1} & 0 & \cdots & 0 & 0 & 0 & 0 \\ \alpha_{2k+1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \end{bmatrix} \middle| a_{ij}\in \mathrm{R}\right\},$$

$$I_{k-1,k+2}=\left\{\begin{bmatrix}
\alpha_{1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & \alpha_{2,3} & \alpha_{2,4} & \cdots & \alpha_{2,k} & \alpha_{2,k+1} & \alpha_{2,k+2} & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \alpha_{3,4} & \cdots & \alpha_{3,k} & \alpha_{3,k+1} & \alpha_{3,k+2} & \cdots & 0 & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & 0 & 0 & \cdots & \alpha_{k-1,k} & \alpha_{k-1,k+1} & \alpha_{k-1,k+2} & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & \alpha_{k,k+1} & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & \alpha_{k+2,k+1} & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & \alpha_{k+3,k} & \alpha_{k+3,k+1} & \alpha_{k+3,k+2} & \cdots & 0 & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & 0 & \alpha_{2k-1,4} & \cdots & \alpha_{2k-1,k} & \alpha_{2k-1,k+1} & \alpha_{2k-1,k+2} & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & \alpha_{2k,3} & \alpha_{2k,4} & \cdots & \alpha_{2k,k} & \alpha_{2k,k+1} & \alpha_{2k,k+2} & \cdots & 0 & 0 & 0 & 0\\
\alpha_{2k+1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0
\end{bmatrix}\middle| a_{ij}\in \mathrm{R}\right\},$$

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$$I_{3,2k-2}=\left\{\begin{bmatrix}
\alpha_{1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & \alpha_{2,3} & \alpha_{2,4} & \cdots & \alpha_{2,k} & \alpha_{2,k+1} & \alpha_{2,k+2} & \cdots & \alpha_{2,2k-2} & 0 & 0 & 0\\
0 & 0 & 0 & \alpha_{3,4} & \cdots & \alpha_{3,k} & \alpha_{3,k+1} & \alpha_{3,k+2} & \cdots & \alpha_{3,2k-2} & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & 0 & 0 & \cdots & \alpha_{k-1,k} & \alpha_{k-1,k+1} & \alpha_{k-1,k+2} & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & \alpha_{k,k+1} & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & \alpha_{k+2,k+1} & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & \alpha_{k+3,k} & \alpha_{k+3,k+1} & \alpha_{k+3,k+2} & \cdots & 0 & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & 0 & \alpha_{2k-1,4} & \cdots & \alpha_{2k-1,k} & \alpha_{2k-1,k+1} & \alpha_{2k-1,k+2} & \cdots & \alpha_{2k-1,2k-2} & 0 & 0 & 0\\
0 & 0 & \alpha_{2k,3} & \alpha_{2k,4} & \cdots & \alpha_{2k,k} & \alpha_{2k,k+1} & \alpha_{2k,k+2} & \cdots & \alpha_{2k,2k-2} & 0 & 0 & 0\\
\alpha_{2k+1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0
\end{bmatrix}\middle| a_{ij}\in \mathrm{R}\right\},$$

$$I_{2,2k-1}=\left\{\begin{bmatrix}
\alpha_{1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & \alpha_{2,3} & \alpha_{2,4} & \cdots & \alpha_{2,k} & \alpha_{2,k+1} & \alpha_{2,k+2} & \cdots & \alpha_{2,2k-2} & \alpha_{2,2k-1} & 0 & 0\\
0 & 0 & 0 & \alpha_{3,4} & \cdots & \alpha_{3,k} & \alpha_{3,k+1} & \alpha_{3,k+2} & \cdots & \alpha_{3,2k-2} & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots\\
0 & 0 & 0 & 0 & \cdots & \alpha_{k-1,k} & \alpha_{k-1,k+1} & \alpha_{k-1,k+2} & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & \alpha_{k,k+1} & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & 0 & \alpha_{k+2,k+1} & 0 & \cdots & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \cdots & \alpha_{k+3,k} & \alpha_{k+3,k+1} & \alpha_{k+3,k+2} & \cdots & 0 & 0 & 0 & 0\\
\vdots & \vdots & \vdots & \vdots & ⋰ & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & 0 & \alpha_{2k-1,4} & \cdots & \alpha_{2k-1,k} & \alpha_{2k-1,k+1} & \alpha_{2k-1,k+2} & \cdots & \alpha_{2k-1,2k-2} & 0 & 0 & 0\\
0 & 0 & \alpha_{2k,3} & \alpha_{2k,4} & \cdots & \alpha_{2k,k} & \alpha_{2k,k+1} & \alpha_{2k,k+2} & \cdots & \alpha_{2k,2k-2} & \alpha_{2k,2k-1} & 0 & 0\\
\alpha_{2k+1,1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0
\end{bmatrix}\middle| a_{ij}\in \mathrm{R}\right\}$$

It is easy to see from their construction that these communities form the verse

$$I_0\subset I_{2,3}\subset I_{3,4}\subset \ldots\subset I_{k-1,k}\subset I_{k,k+1}\subset I_{k-1,k+2}\subset \ldots\subset I_{3,2k-2}\subset I_{2,2k-1}=\mathscr{A}^{(0)}_{2k+1} \qquad (5)$$

It's obvious that $\forall L_1, L_2 \in I_{pq} \Rightarrow L_1 - L_2 \in I_{pq}$, për çdo q=3, 4, ..., 2k-1 and for every

$$p=\begin{cases} q-1, & q=3,4,...,k+1 \\ 2k+1-q, & q=k+2, k=3,...,2k-1. \end{cases}$$

It's not hard to obey as well that

$$\forall L \in I_{pq}, \forall X \in \mathscr{A}_{2k+1}, X \cdot L \in I_{pq}.$$

So, we obtain this.

Statement 2.4. The communities of the verse (*5*) are left-wing niles in terms of left ideal $\mathscr{P}_{2k+1}$.

Since $\mathscr{A}_{2k+1}^{(0)}$ results the left nile in terms of the left ideal $\mathscr{P}_{2k+1}$, Then from the theorem 1.4 it is a K-radical ring, therefore from definition 1.3 we obtain the draw

$$\mathscr{A}_{2k+1}^{(0)} = K(\mathscr{A}_{2k+1}^{(0)}, \mathscr{P}_{2k+1}). \quad (6)$$

In this way we have confirmed the statement:

Statement 2.5. The upper nilradical in terms of the left ideal $\mathscr{P}_{2k+1}$ of a ring $\mathscr{A}_{2k+1}^{(0)}$ It's the ring itself. $\mathscr{A}_{2k+1}^{(0)}$.

We stress that this conclusion we can also take from verse (5), where it seems that $\mathscr{A}_{2k+1}^{(0)}$ $\,_{2k+1}^{(0)}$ it's the greatest nile in terms of the left ideal $\mathscr{P}_{2k+1}$ ring $\mathscr{A}_{2k+1}^{(0)}$. But is this the greatest nile in terms of the left ideal P_{2k+1} of ring A_{2k+1} ?

This is why we examine the range of communities

$$\mathscr{A}_{2k+1}^{(0)} \subset I^{(2,2)} \subset I_{(2k,2)}^{(2,2)} \subset I^{(3,3)} \subset I_{(2k-1,3)}^{(3,3)} \subset \ldots \subset I_{(k+2,k)}^{(k,k)} \subset I^{(k+1,k+1)} \subset I_{(k+2,k+2)} \subset$$

$$I_{(k+2,k+2)}^{(k,k+2)} \subset \ldots \subset I_{(2k+1,2k+1)} \subset I_{(2k+1,2k+1)}^{(1,2k+1)} = \mathscr{A}_{2k+1}, \quad (7)$$

where with $I^{(p,p)}, p=2,3,...,k+1$ dhe me $I_{(p,p)}, p=k+2,k+3,...2k+1$ we have marked the community that includes $\mathscr{A}_{2k+1}^{(0)}$ and has zero as the elements of the main diagonal from the end to the element $a_{p,p}$ and secondary diagonal elements, excluding the element $a_{2k+1,1}$, and with $I_{(2k-p+2,p)}^{(p,p)}, p=2,3,...,k$ dhe me $I_{(p,p)}^{(2k-p+2,p)}, p=k+2,k+3,...2k+1$, we have marked the community that includes $\mathscr{A}_{2k+1}^{(0)}$ and has zero as the elements of the main diagonal from the end to the element $a_{p,p}$ both secondary diagonal elements from the element $a_{1,2k+1}$ Up to the element $a_{2k-p+2,p}$.

In each of the terms $I^{(p,p)}, p=2,3,...,k+1$ *dhe* $I_{(p,p)}, p=k+2,k+3,...2k+1$ the range (7) has matrixes that have at least one element in the main diagonal (excluded first) not zero. If that's the case $a_{p,p}$, then for the matrix

$$L=\begin{bmatrix} \alpha_{1,1} & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \alpha_{p,p} & \cdots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \alpha_{2k+1,1} & 0 & 0 & \ldots & 0 \end{bmatrix}, \alpha_{p,p} \neq 0, \text{ kemi } L^k = \begin{bmatrix} \alpha_{1,1}^k & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \alpha_{p,p}^k & \cdots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \alpha_{2k+1,1}\alpha 1,1 & 0 & 0 & \ldots & 0 \end{bmatrix} \notin \mathscr{B}_{2k+1} \text{ for every } k \in N.$$

Also, in each of the other terms $I_{(2k-p+2,p)}^{(p,p)}, p = 2,3,...,k$ and

$I_{(p,p)}^{(2k-p+2,p)}, p = k+2, k+3, ...2k+1$ range (7) has matrixes that do not meet the property of nilpotence in terms of ideal $\mathscr{B}_{2k+1}$. It is easily shown that such are matrixes that have not zero at least one element in the main diagonal (excluded first) along with its symmetric versus the k+1 row located in the secondary diagonal.

In this way we have proven this

Statement 2.6. $\mathscr{A}_{2k+1}^{(0)}$ It's the upper nilradical in terms of the left ideal $\mathscr{B}_{2k+1}$ even for the ring $\mathscr{A}_{2k+1}$.

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