International Journal of Progressive Sciences and Technologies (IJPSAT)
ISSN: 2509-0119.

Vol. 31 No. 2 March 2022, pp. 54-60

Students' Epistemological Obstacles on Geometric-algebraic Relations of Transformation Geometry

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Abstract—Learning obstacles are naturally experienced by students, including the topic of transformation geometry. This study aims to describe the epistemological obstacles experienced by students on geometric-algebraic relations of transformation geometry. A qualitative approach is used with a case study as a research model. The subjects were 9 students of 11th grade obtained through snowball sampling. The data were collected through paper tests and think aloud method then analyzed through data reduction, data presentation, and concluding. The results showed that there are epistemological obstacles found in students' understanding of the geometric-algebraic relations of transformation geometry. Students know the geometric meaning of a transformation process they have done, indicated by the absence of errors that occur in the tests they take or in their articulation of thinking through think aloud. However, there are some limitations of knowledge experienced by students including the limited basic knowledge about matrix algebra operations, limitations in realizing the geometric-algebraic connections of the cases they encounter, and do not know the matrix of transformation that should be used. These limitations then become the epistemological obstacles for students in understanding and problem solving the transformation problem in the form of algebraic (analytically). These epistemological obstacles cause students to be unable to understand the algebraic meaning of transformation geometry. Thus, geometric-algebraic relationships are not formed completely and result in hindering students in understanding transformation geometry.

Keywords— learning obstacle, epistemology, transformation geometry

I. INTRODUCTION

Transformation Geometry (TG) is a “stretch along the royal road to geometry”, that is, a modern approach based on bringing together geometry and algebra [1]. The transformations that are fundamental to geometry consist of translations, reflections, and rotations and compositions of these which preserve distance and angles, together with dilations, which expand or contract [2]. Furthermore, TG also provide a bridge between algebra and geometry that makes it possible for geometric problems to be solved algebraically (or analytically), and to solve algebraic problems geometrically [3].

TG is studied in school mathematics. Indonesia’s curriculum of education (The Kurikulum 2013) classifies TG into two levels that are studied in junior and senior high school. At junior high school level, the knowledge that must be possessed by students is related to explaining and solving TG in contextual problems. Moreover, matrices are involved in solving analytical problems at the high school level [4].

In applying the understanding process, TG is important in terms of developing artistic and aesthetic feelings with the awareness of the beauties in nature, so the contextual approach can support the understanding of TG visually and geometrically [5]. The analytical approach seems attractive to students because all the formulas are already available. Using routine procedures such as formulas to solve analytical problems requires a basic understanding of algebra, in particular. For TG analytically to be fully understood, students must first understand the rules in algebra and use them, analytic strategies in TG are dependent on the use of algebraic rules, learners' misapplication of rules can be largely attributed to poor algebraic skills [6]. Students who experience limitations in algebra will have difficulty learning TG due to their background [7]. A study explains that high school students may be more suitable if presented with a combination of the two because they are exposed to approaches both contextually and analytically [6]. However, operating algebra based on TG is not as simple as learners perceive it to be [8]–[10], at some point, students naturally experience errors.

Errors that arise due to erratic and unpredictable basic understanding are called obstacles [11]. These obstacles occur during the learning process by students, which are consequently then understood as learning obstacles [12]. Furthermore, the causes of learning obstacles are varied, one of which is the limited knowledge possessed by students at certain points and contexts called epistemological obstacles [13]. Students with epistemological obstacles will have difficulty in developing the knowledge they already have because of the limitations of their previous knowledge [14]. The obstacles become important to study considering that in TG students' understanding is expected to develop from geometric to algebraic understanding. This study describes how students' limitations in understanding the concept of transformation geometry in a contextual approach can be a barrier to student learning when studying and using matrices (algebra) to express and solve problems related to TG. The epistemological obstacles are essentially related to the limited knowledge of students in certain contexts, different contexts will hinder students in learning [15]. Learning obstacles cannot be ignored because they are an important part of the learning process. Another important part is how teachers and students themselves can find out so that the optimization of meaningful learning can be achieved.

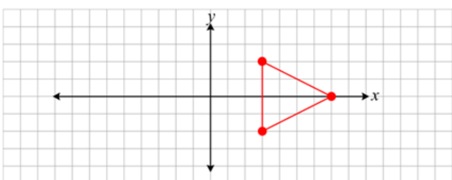
II. RESEARCH METHODOLOGY

This study uses a qualitative approach with a case study as the research model. The purpose of qualitative research is to understand the phenomena that occur in the subject, and case studies emphasize the depth of understanding of the problems that occur [16]. Researchers as the main instrument use a paper test that had previously been validated by experts. The test given represents the TG problem in a geometric approach as well as other problems that require analytical solutions.

Respondents in this study were students of 11th grade in one of the private schools in Surakarta, Indonesia. A total of 9 students were obtained through snowball sampling from the rolling process. Data were collected through a two-question test. The exam consists of geometric questions and analytic (algebraic) questions of transformation geometry. Researchers use the think aloud method as a technique to find out students' thinking processes that describe students' understanding of the questions being worked on. Both questions are based on reflection transformations. The first problem begins by illustrating a triangle with three coordinate points made by the student and then reflects on it, and the second problem continues to solve the reflection of the given coordinates using a matrix. In detail, the test papers given to students are shown in Fig 1.

The goal of these questions is to have students demonstrate their ability to complete transformations both geometrically and algebraically. The two problems given became a bridge to connect geometric transformations to be solved with identical problems but with different methods. Students are also not given the same starting point but form their structure.

Question 1.
Using the grid book, create Cartesian plane, then draw three coordinate points and connect them to form a triangle. Use the image below as an example. Then, determine the coordinates according to your creativity with different coordinates from the examples given. After that, determine the image of the triangle after reflecting on the y-axis!



Question 2.
Give the coordinates of previous triangle with P, Q, and R notations of each coordinates respectively. Then, determine the image coordinates P', Q', R' of P, Q, R after reflection on the y-axis, use the matrix to reflect each coordinate!

Fig. 1. Paper test questions

The data obtained from the students' test results were then documented and the words from the students during the thinking process in solving the given questions were recorded objectively before data reduction was carried out. The data obtained is then analyzed based on the students' ability to complete and explain their understanding of the questions and then describe them as part of the analysis technique. To maintain the reliability of the results, researchers used data triangulation by comparing the results of the analysis between subjects and then compared with the data from the respondents' analysis to see the interpretation of the data about students' epistemological obstacles.

III. FINDINGS

Overall, there are several epistemological obstacles experienced by 9 subjects in understanding geometric transformation in geometric-algebraic relationships. These obstacles occur based on the limitations experienced by students in algebraic operations, limitations in realizing the geometric-algebraic connections of the cases they encounter, and do not know the matrix of transformation that should be used. A total of 2 answers are described as a representation of the epistemological obstacles experienced by students. Both will be analyzed per student to describe the relation between geometric-algebraic knowledge and the obstacles possessed by students.

Two students with the names Vias and Moh represent the epistemological obstacles experienced by all subjects. Both suffer from all of the aforementioned limitations. The limitations experienced by the rest of the students were identical. Researchers describe these findings based on Vias and Moh's answers and followed by discussions as follow:

Starting to answer question number 1, Vias drew 3 dots all of which were in quadrant 1. Then, he connected the three lines into a triangle as can be seen in Fig 2. He then realized that the triangle he drew was an isosceles triangle as stated, "…, now if we look at the base of 4 squares unit, the height is only 2 squares unit located in the middle -middle, so isosceles…"

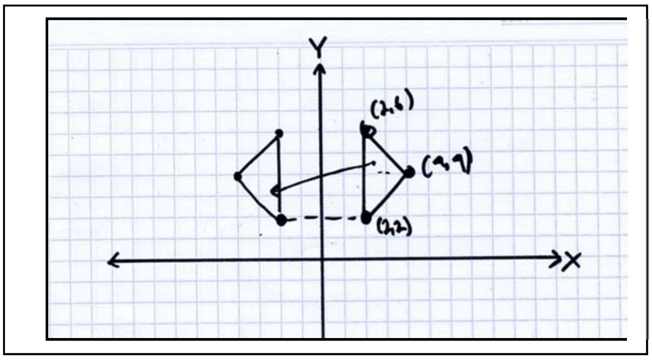


Fig. 2. Vias' answer to answer to question number 1

After Vias saw the next command which was to determine the shadow, he then paused before finally measuring the number of squares between the points he made and the y-axis. "One... four, so the image is here [points to four squares unit on the left]" then he marked out the images' points. Next, he saw a third point on the right which is the vertex of an isosceles triangle. The hand gesture showed that Vias was calculating the distance between the top of the triangle and the base, a few moments later he said "the distance is 2, so it is supposed to be here", he again showed the shadow points, and after that, he reconnected the second triangle. At this time, the triangle is a reflection of the first triangle, "this is the image ..." as he said.

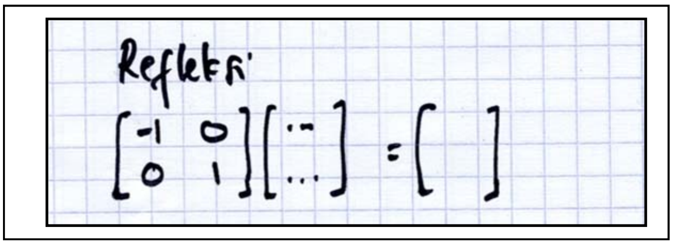


Fig. 3. Vias' answer to answer to question number 2.

Continuing on question number 2, here Vias immediately gives the notation P, Q, and R at the point he made earlier. After that, vias realized that in this problem the steps he had to take were different, as he said “using a matrix… formula… reflection?” with the pen in the hand supporting his head. Then, he wrote a small note at the bottom of his worksheet, where it was the matrix form used in the transformation, precisely the reflection on the y-axis as can be seen in Fig 3. However, Vias kept quiet and looked back at the answer to the first question. He then drew a line from the first and second triangles, saying "if this is a reflection, this is also a reflection, it means..." after a moment of silence. Vias then gave the coordinates of the points on the triangle he made by writing (2,2), (2,6), and (4,4). However, then he got confused again by asking "how to multiply by a matrix if it's a triangle? I mean... usually in the questions that are reflected it is “a” and “b” [expressing the coordinates A and B]". He made another gesture of looking at the matrix notes he had made, back at the triangle, and so on. Then he went back to his notes and tried to write another one but it did not work. Vias then simply wrote its reflection matrix multiplied by another matrix with empty members (Fig 3). He left the transformed coordinates blank and did not write the answer. He went on to say, "OK, I need help here, I can't go on". In the end, he scribbled back on the notes he had previously made as can be seen in Fig 4.

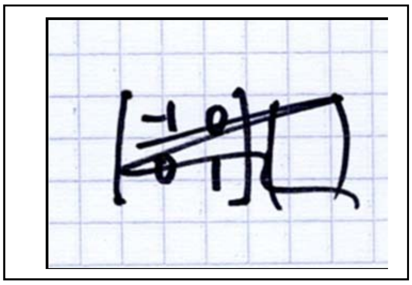


Fig. 4. Vias’s note on transformation reflection matrix

Continuing to the second student, Moh started working in a slightly different way whereafter he made three dots, he immediately gave notations to all three, but not as given in the problem. Moh gave the notation A, B, and C at each point. After he made the triangle, he immediately read the question’s instruction to reflect the triangle about the y-axis as can be seen in Figure 4. Then he said, “… the y-axis, means the triangle is inverted” he came up with the idea that reflection or “mirror” or reversing the position of an object, then Moh drew the image of the triangle he made earlier with the exact result.

Moh came back for reading question number 2, he realized that the notation should have been P, Q, R and not A, B, C. So he immediately replaced the notation and marked out the images with P’, Q’, and R’. After that, he was silent for a moment after reading the questions, and then he looked up. “Matrix… reflection” is what he said after reading question number 2. Then, he wrote down the reflection matrix which he wrote did not work. Moh wrote the reflection matrix about the x-axis. Next, Moh wrote the reflection transformation matrix about the x-axis and multiplies it by the coordinates of the triangle point (1,0). Next, he paused again, continued by saying, "I doubt whether this is an x-axis or y-axis reflection matrix", then crossed out the value "-1" and added a "-" notation to the number 1. So, the current transformation matrix he has is a reflection transformation matrix about the y-axis. Then, he went on to say, "if this is reflected, it means it is the same, just using a matrix".

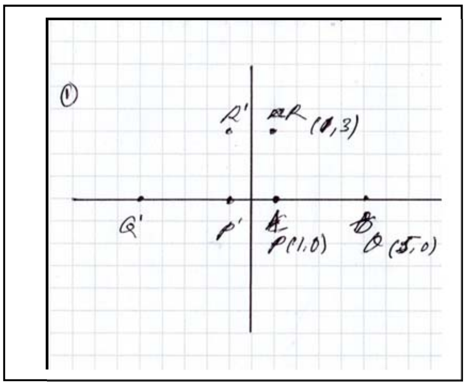


Fig. 5. Moh' answer to question number 1

Then, he wrote down the reflection matrix and multiplied it by each coordinate as he said "…, this reflection matrix formula, multiplied by the coordinates". He then proceeded to do the calculations and got the coordinates of the image.

However, Moh then stopped for a moment after doing the calculations. He looked at the results of the coordinates one by one and then looked at the reflection results that he determined earlier. He pointed to the result with a pen and said, "Wait..., this should not be different. If the image is the same it should be the same as in the picture". Then he looked for possible mistakes and realized nothing.

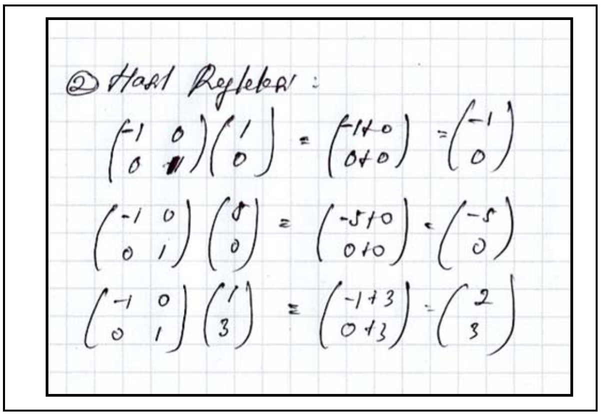


Fig. 6. Moh' answer to question number 2

At the end of his work, Moh was unable to make any changes and left the answer exactly as it can be seen in Figure 5. He wrote the answers to questions number 1 and number 2 with coordinates mismatch. He said "I don't know what's going on here, but the result is different, maybe I got an error but I don't know where because I think I have used the correct matrices", then submits the answer.

IV. DISCUSSIONS

Based on the process that Vias went through in the first problem, no significant obstacles were found in working on the geometric TG problem where Vias managed to determine the image of the triangle that he formed with the correct result, he also gave reasons for the steps he took. However, Vias has difficulties in determining the relationship between the transformations he performed on geometric objects and the transformations in analytical work. This is shown by how he crossed out from the first triangle to the second triangle and what he conveyed was that he knew that the reflection images in questions number 1 and 2 were the same. However, he could not continue his work because of his limited ability to understand the relationship between the two things, so this limitation prevented him from working on question number 2 because question number 2 had the same concept and even the answer but in a different context. This is then under what was previously explained that students can experience difficulties in developing knowledge, especially analytical TG problem solving due to limited knowledge in certain contexts, which in this case is the use of matrices and the relationship between geometric shapes and the coordinates [15]. In this situation, Vias has not yet arrived at an algebraic operation because the matrix he provided has not yet been operated on.

Next, on Moh's answers, the epistemological obstacle experienced where due to limited knowledge of reflection matrices, he has difficulty in working on the next stage. What should be noted is how Moh has realized how to solve this reflection problem analytically. He no longer thinks visually or geometrically and imagined how the reflection is or about mirroring, but he focused on how the coordinates can be algebraically transformed so that the image is found. However, in contrast to Vias, Moh seems to be aware that the coordinates of the triangle are (1,0), (5,0), and (1,3) is the one he has to transform. He realized that what he was transforming from the triangle were the coordinates that made up the triangle. Therefore, what he did was first write down the coordinates next to the notation P, Q, and R.

Moh also experienced an error in the operation of algebra or multiplication in the matrix. Moh did the wrong procedure in multiplying the reflection matrix by the third coordinate, (1,3). He found that the coordinates he transformed with the resulting matrix were different from what he observed in the triangular image. From here, Moh actually tried to relate geometric-algebraic relations in the transformation process he did geometrically on triangles and algebraically on their coordinates. However, he got results that did not match between the two. The result occurred because of the matrix calculation error that he did at the third coordinate. Instead of multiplying like the procedure in the previous coordinates, he wrote down the matrix multiplication steps $\begin{pmatrix}-1+3\\0+3\end{pmatrix}$ where the matrix multiplication process has an error. At this moment, it was known that in the end, Moh's final answer is not what he expected. He thought that the overall coordinates of the shadow would be the same as in the triangle he drew geometrically, but in reality, it wasn't, one of the coordinates was different.

This will be different when multiplication occurs in other matrices. So in this case, Moh not only experienced obstacles in the form of his limited knowledge of transformation matrices but also encountered obstacles in the algebraic operations he carried out. As a result, in the context of solving this problem, according to how it is related to algebra [7], students who experience limitations in algebra will have difficulty learning TG due to their background.

V. Conclusion

There are epistemological learning obstacles experienced by students in geometric-algebraic relations in transformation geometry. These obstacles are limitations in matrix algebraic operations, limitations in realizing the geometric-algebraic connections of the cases they encounter and do not know the matrix of transformation that should be used. All students are able to solve transformation geometry problems contextually or geometrically, but as a result of these three obstacles, students have difficulty solving analytic geometry transformation problems.

Curriculum designers as well as teachers who teach geometric transformations at both junior and high school levels can emphasize geometric-algebraic relations as important points in learning geometric transformations. This is because analytic geometry requires algebraic and geometric skills which are both needed by students so that geometric problems can be solved analytically and vice versa.

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