International Journal of Progressive Sciences and Technologies (IJPSAT)
ISSN: 2509-0119.

Vol. 37 No. 1 February 2023, pp. 636-645

On Ramsey Minimal Graphs For (P_4,P_n), For $n \geq 5$

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Abstract – For given two graphs G and H, the notation $F \rightarrow (G, H)$ means that any red-blue coloring of all the edges of F contains a red copy of G as a subgraph or a blue copy of H as a subgraph. A graph F is Ramsey (G,H)-minimal if $F \rightarrow (G, H)$ and for any edge e in F then $F - e \nrightarrow (G, H)$. The class of all (G,H)-minimal graph, is denoted by $\mathcal{R}(G, H)$. In this paper, some graph in $\mathcal{R}(P_4, P_5)$ are obtained. Then, a graph in $\mathcal{R}(P_4, P_n)$ for even n, $n \geq 6$ and a graph in $\mathcal{R}(P_4, P_n)$ for odd n, $n \geq 7$ is also obtained.

Keywords – Ramsey minimal graph; path graph; cycle graph; complete graph

I. Introduction

All graphs considered in this paper are simple, finite, and undirected. Let G and H be two graphs. We write $F \rightarrow (G, H)$ if any red-blue coloring of the edges of F implies that either F contains a red subgraph G or a blue subgraph H. Graph F is Ramsey (G,H)-minimal if $F \rightarrow (G, H)$ but $F^* \nrightarrow (G, H)$ for any proper subgraph $F^* \subset F$. The class of all minimal graph is denoted by $\mathcal{R}(G, H)$ [6].

There are some previous results for Ramsey (G,H)-minimal graphs, for some G and H. Baskoro and Wijaya [1] determined some graphs in $\mathcal{R}(2K_2, C_4)$. Muhsi and Baskoro [10] determined the graph in $\mathcal{R}(2K_2, P_3)$.Baskoro and Yulianti [2] gave some characterization of graphs in $\mathcal{R}(2K_2, P_n)$ for $n \geq 2$, where P_n is a path graph on n vertices. Wijaya et al. [16] determined subdivision of graph in $\mathcal{R}(mK_2, P_4)$. Next, Wijaya et al. [15], [17] gave complete list of graphs in $\mathcal{R}(2K_2, K_4)$, $\mathcal{R}(2K_2, C_4)$. Mengersen and Oeckermann [9] discussed about Ramsey set for matching.

In [8] the graphs belonging to $\mathcal{R}(2K_2, K_{1,n})$ for $n \geq 3$ were characterized. Borowiecki et al. [5] determined the graphs in $\mathcal{R}(K_{1,2}, C_3)$. Then, Borowiecki et al. [4] gave some characterization of all graphs in $\mathcal{R}(K_{1,2}, C_4)$. Tatanto and Baskoro [13] determined the graphs belonging to $\mathcal{R}(2K_2, 2P_n)$, for $n \geq 2$. Baskoro et al. [3] gave an infinite family belonging to $\mathcal{R}(K_{1,2}, C_4)$.

Vetrik et. al. [14] determined some class of graphs belonging to $\mathcal{R}(K_{1,2}, C_4)$, where $K_{1,2}$ is a star graph on 3 vertices and C_4 is a cycle graph with 4 vertices. Then, Yulianti et. al. [18] determined some graphs in $\mathcal{R}(K_{1,2}, P_4)$, where P_4 is a path graph on 4 vertices. Haluszczak [7] studied the graphs belonging to $\mathcal{R}(K_{1,2}, K_n)$, where K_n is a complete graph on n vertices. Rahmadani et. al. [11] determined some graphs in $\mathcal{R}(P_3, P_6)$. Then, Rahmadani and Nusantara [12] determined some graphs in $\mathcal{R}(P_4, P_4)$.

A path P_n is a connected graph with n vertices and $n - 1$ edges, where its end vertices have one degree and the others have two degree. In this paper, we will determine some graphs in the class of Ramsey minimal for $\mathcal{R}(P_4, P_n)$, for $n \geq 5$.

II. Main Result

In Theorem 1 we determine some graphs that belongs to $\mathcal{R}(P_4, P_5)$

Theorem 1. Let P_4 and P_5 be two paths on 4 and 5 vertices. Let F_1, F_2, F_3 and F_4 be the graphs in Figure 1., then $\{F_1, F_2, F_3, F_4\} \subseteq$

$\mathcal{R}(P_4, P_5)$.

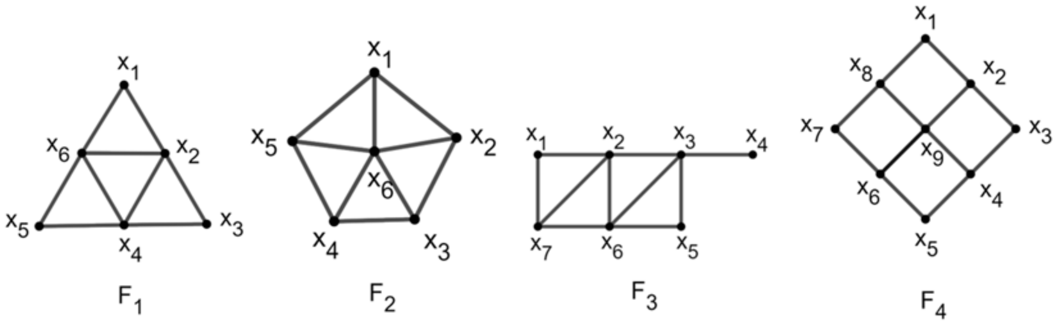


Figure 1. F_1, F_2, F_3, F_4

Proof. Let P_4 and P_5 be two given graphs. We will show that (1). $F_1 \rightarrow (P_4, P_5)$, (2). $F_1 \nrightarrow (P_4, P_5)$. The proof for F_2, F_3, F_4 as similar to F_1. Consider the following cases.

Case 1. First, we prove that $F_1 \rightarrow (P_4, P_5)$. Consider any red-blue coloring of all edges of F_1 containing no red P_4. If F_1 does not contain red P_4, then the red subgraph will be in the form of $K_{1,4}, C_3 \cup P_3, C_3, 3P_2$. Consider Figure 2. for all possibilities of coloring against F_1, the remaining edges will contain a blue P_5 as in Figure 2. Thus, $F_1 \rightarrow (P_4, P_5)$.

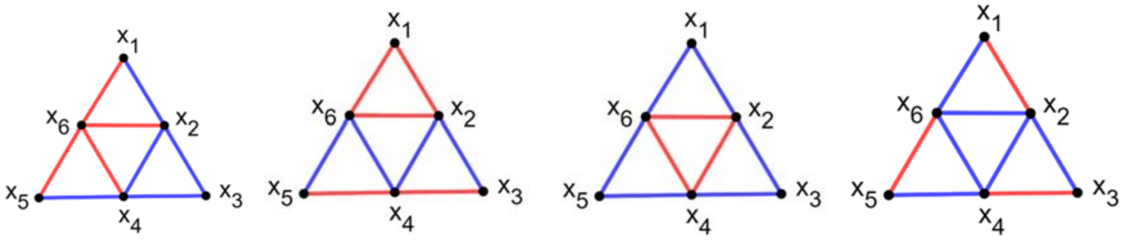


Figure 2. $F_1 \rightarrow (P_4, P_5)$

Second, we prove that $F_1 \backslash e \nrightarrow (P_4, P_5)$, for any edge e in F_1. Consider that if $e = x_1x_6, x_1x_2, x_2x_3, x_3x_4, x_4x_5$, or x_5x_6, then give coloring as in Figure 3 (i). If $e = x_2x_6, x_2x_4$ or x_4x_6, then give coloring as in Figure 3 (ii). Obviously, no blue P_5 as a subgraph. Therefore, $F_1 \backslash e \nrightarrow (P_4, P_5)$, for any edge e.

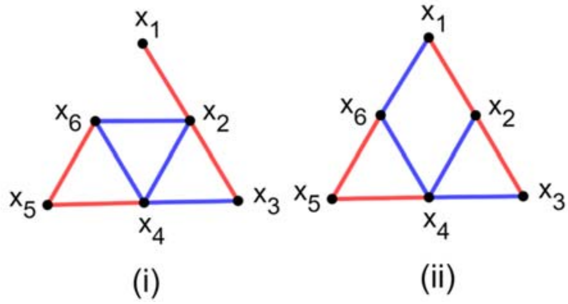


Figure 3. $F_1 \backslash e \nrightarrow (P_4, P_5)$

Case 2. First, we show that $F_2 \rightarrow (P_4, P_5)$. Consider any red-blue coloring of all edges of F_2 containing no red P_4. If F_2 does not contain red P_4, then the red subgraph will be in the form of $C_3 \cup P_3, K_{1,5}, K_{1,3} \cup P_2, 3P_2$. Consider Figure 4. for all possibilities coloring against F_2, the remaining edges will contain a blue P_5 as in Figure 4. Hence, $F_1 \rightarrow (P_4, P_5)$.

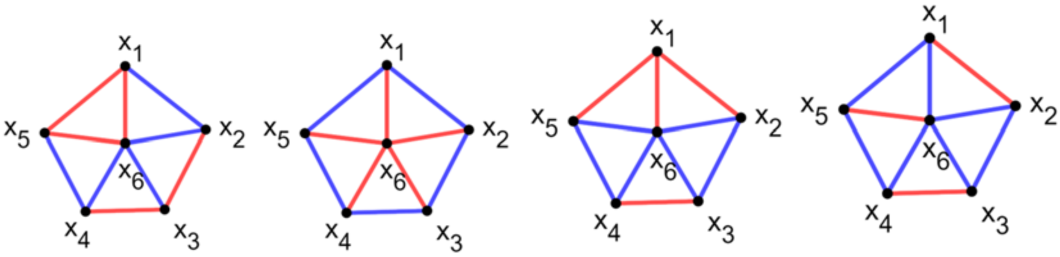


Figure 4. $F_2 \rightarrow (\mathrm{P}_4, \mathrm{P}_5)$

Next, we show that $F_2 \backslash \mathrm{e} \nrightarrow (\mathrm{P}_4, \mathrm{P}_5)$, for any edge e. Consider that if $\mathrm{e} = \mathrm{x}_1\mathrm{x}_5, \mathrm{x}_1\mathrm{x}_2, \mathrm{x}_2\mathrm{x}_3, \mathrm{x}_3\mathrm{x}_4$, or $\mathrm{x}_4\mathrm{x}_5$, then give coloring as in Figure 5(i). If $\mathrm{e} = \mathrm{x}_5\mathrm{x}_6, \mathrm{x}_1\mathrm{x}_6, \mathrm{x}_2\mathrm{x}_6, \mathrm{x}_3\mathrm{x}_6$ or $\mathrm{x}_4\mathrm{x}_6$, then give the coloring as in Figure 5(ii). Consequently, neither red P_4 nor blue P_5 occurs. Therefore, $F_2 \backslash \mathrm{e} \nrightarrow (\mathrm{P}_4, \mathrm{P}_5)$, for any edge e.

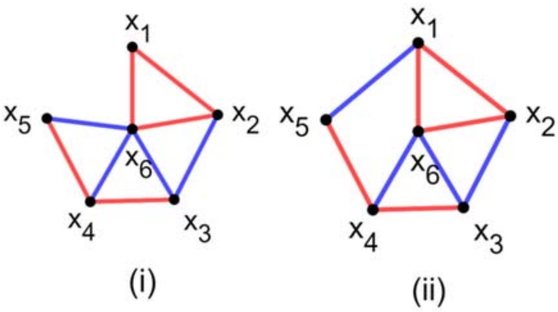


Figure 5. $F_2 \backslash \mathrm{e} \nrightarrow (\mathrm{P}_4, \mathrm{P}_5)$

Case 3. First, we show that $F_3 \rightarrow (\mathrm{P}_4, \mathrm{P}_5)$. Consider any red-blue coloring of all edges of F_3 containing no red P_4. If F_3 does not contain red P_4, then the red subgraph will be in the form of $K_{1,3} \cup P_3, 2C_3, K_{1,4} \cup P_3, 2P_2 \cup P_3, K_{1,4}$. Consider Figure 6. for all possibilities coloring against F_3, the remaining edges will contain a blue P_5 as in Figure 6. Thus, $F_3 \rightarrow (\mathrm{P}_4, \mathrm{P}_5)$.

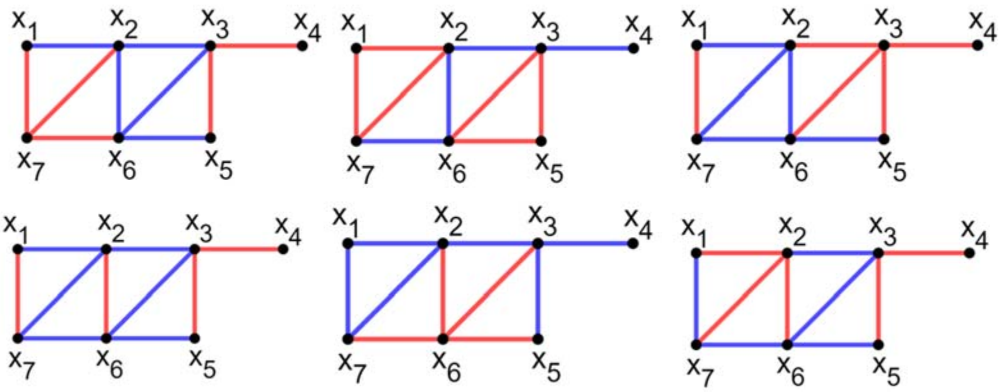


Figure 6. $F_3 \rightarrow (\mathrm{P}_4, \mathrm{P}_5)$

Second, we prove that $F_3 \backslash \mathrm{e} \nrightarrow (\mathrm{P}_4, \mathrm{P}_5)$, for any edge e. Consider that if we remove one edge for any edge e of graph F_3, then do the coloring as in Figure 7. This coloring implies that there is no red P_4 nor blue P_5. Therefore, $F_3 \backslash \mathrm{e} \nrightarrow (\mathrm{P}_4, \mathrm{P}_5)$, for any edge e in F_3.

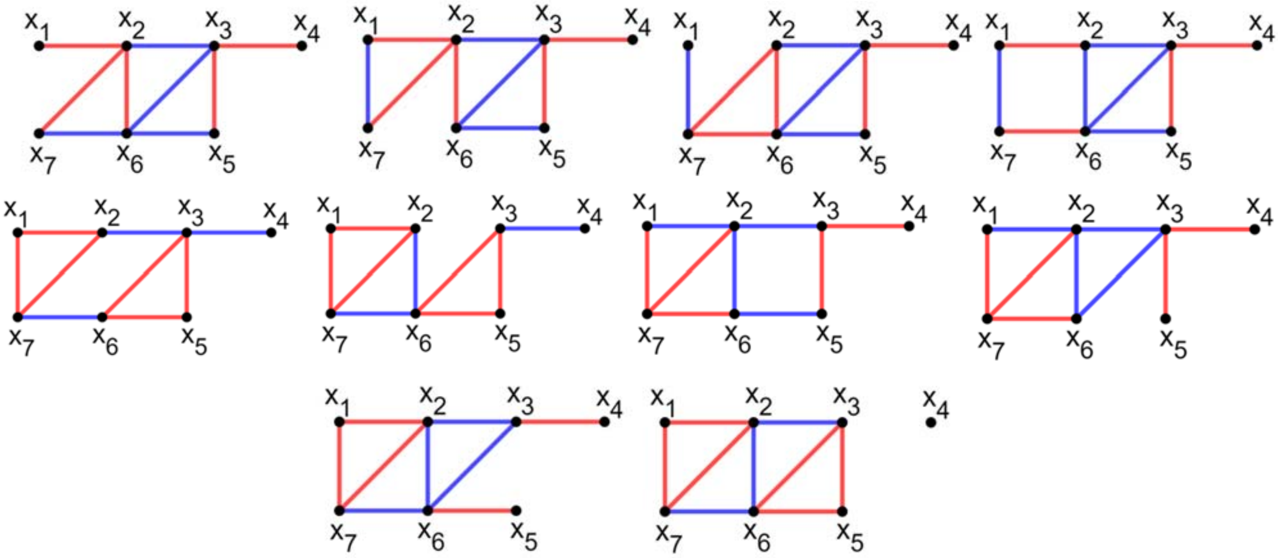


Figure 7. $F_3\backslash e \nrightarrow (P_4, P_5)$

Case 4. First, we show that $F_4 \rightarrow (P_4, P_5)$. Consider any red-blue coloring of all edges of F_4 containing no red P_4. If F_4 does not contain red P_4, then the red subgraph will be in the form of $K_{1,4}$, $K_{1,3} \cup P_3, 3P_3, 4P_2$. Consider Figure 8. for all possibilities coloring againts F_4, the remaining edges will contain a blue P_5 as in Figure 8. Therefore, $F_4 \rightarrow (P_4, P_5)$.

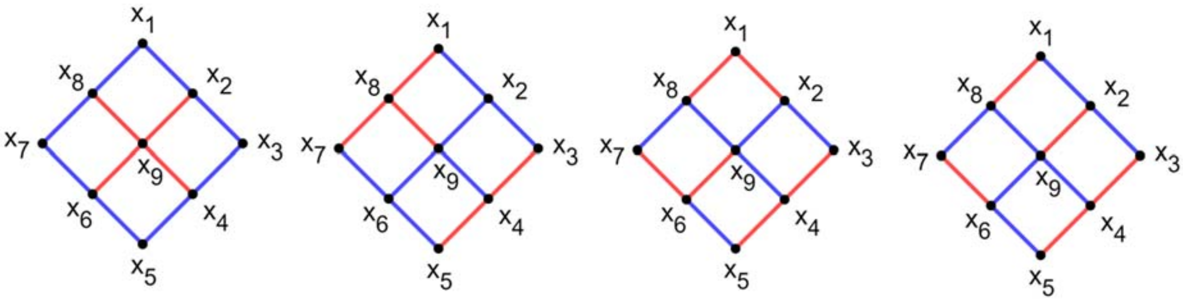


Figure 8. $F_4 \rightarrow (P_4, P_5)$

Second, we prove that $F_4\backslash e \nrightarrow (P_4, P_5)$, for any edge e. Consider that if e is one of x_ix_{i+1} for $1 \leq i \leq 7$ or x_1x_8, then give coloring as in Figure 9(i). If e is one of x_ix_9 for $2 \leq i \leq 8$ and even i, then $F_4\backslash e \nrightarrow (P_4, P_5)$ as in Figure 9(ii). Clearly, no blue P_5 as a subgraph. Therefore $F_4\backslash e \nrightarrow (P_4, P_5)$, for all e in F_4.

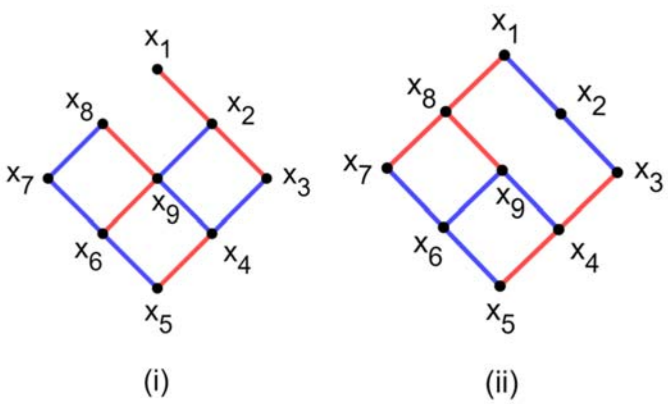


Figure 9. $F_4\backslash e \nrightarrow (P_4, P_5)$

Based on case 1 to case 4, it is proven that $\{F_1, F_2, F_3, F_4\} \subseteq \mathcal{R}(P_4, P_5)$.

In Theorem 2 we determine a graph that belong to $\mathcal{R}(P_4, P_n)$, for even n, n ≥ 6

Theorem 2. Let P_4 and P_n be the path graphs on 4 and n vertices, then A_n in Figure 10. is a Ramsey minimal graph of (P_4, P_n), for even n, n ≥ 6.

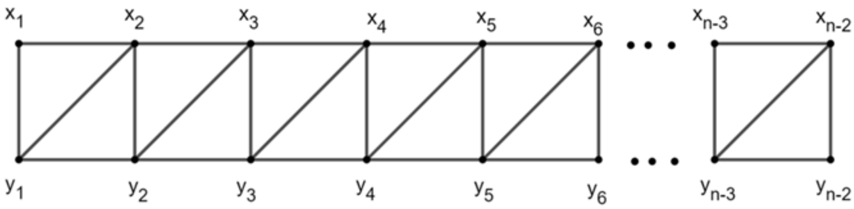
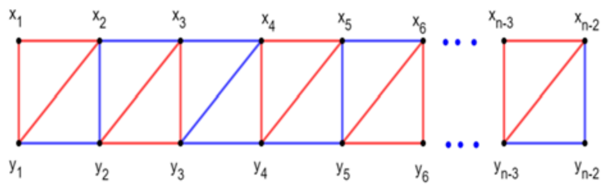
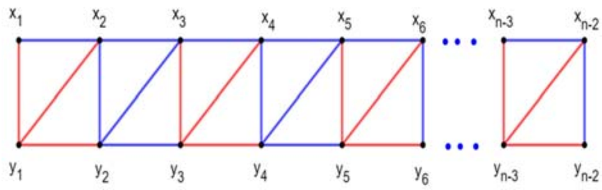
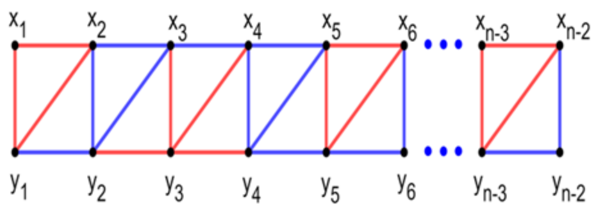
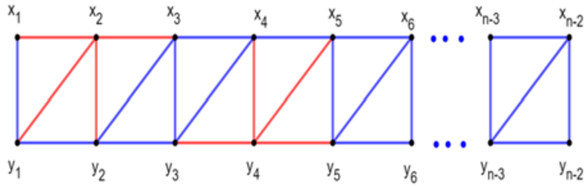


Figure 10. Graph A_n

Proof. Let P_4 and P_n be two given paths. First, we prove that $A_n \rightarrow (P_4, P_n)$. Consider any red-blue coloring of all edges of A_n containing no red P_4. If A_n does not contain red P_4, then the red subgraph will be in the form of $K_{1,3}, C_3, K_{1,4}$. Consider Table 1for all possibilities coloring of A_n that does not contain red P_4 as follows.

Table 1.

Cases	Incident edge	Coloring steps	Illustration
1	x_1	1. Give a red color to each incident edges of x_1, i.e x_1x_2 and x_1y_1 2. Color the incident edges to y_1, i.e x_2y_1 inred, y_1y_2 in blue 3. Give a blue color to the incident edges of x_2 4. Give a red color to the incident edges of y_2 5. Color the incident edges to $x_3, y_3, x_4, y_4, \ldots, x_{n-2}, y_{n-2}$, respectively by maximizing the red edge as long as it doesn't contain red P_4	
2	y_1	1. Give a red color to each incident edges of y_1 2. Give a blue color to the incident edge of x_1, i.e x_1x_2 3. Give a blue color to each incident edge of y_2 4. Color the incident edge to x_2, i.e x_2x_3 with red color. 5. Color the incident edges to $y_3, x_3, y_4, x_4, \ldots, y_{n-2}, x_{n-2}$, respectively by maximizing the red edge as long as it doesn't contain red P_4	
3	y_3	1. Give a red color to each incident edges of y_3 2. Give a blue color to the incident edges of x_3, i.e x_2x_3, x_3y_2, x_3x_4. 3. Color the incident edges to x_3, i.e x_1x_2 and x_2y_1 in red, x_2y_2 in blue 4. Color the incident edges to y_2, i.e x_1x_2 in blue 5. Color the incident edges to $x_1, y_1, x_4, y_4, x_5, y_5, \ldots, x_{n-2}, y_{n-2}$,	

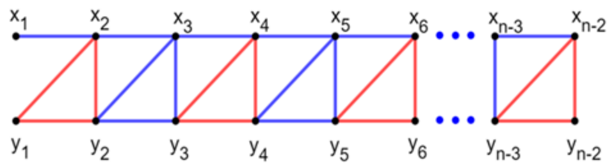
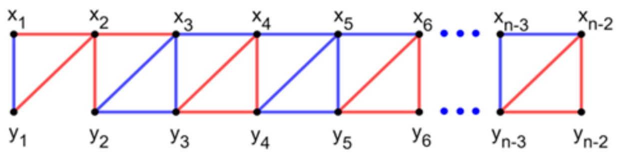
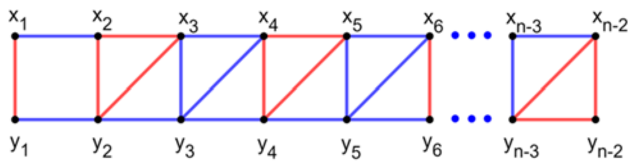
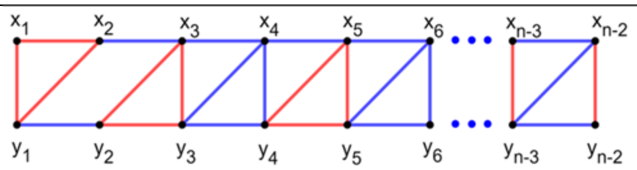
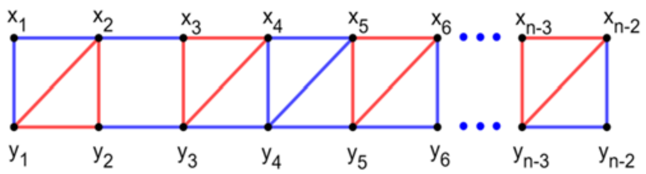
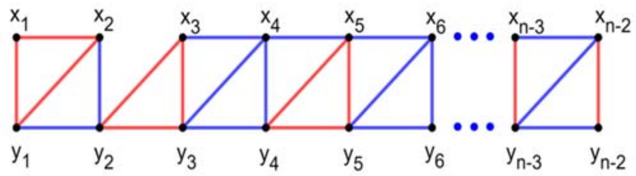
		respectively by maximizing the red edge as long as it doesn't contain red P_4	
4	y_4	1. Give a red color to each incident edges of y_4 2. Give a red color to each incident edges of x_2 3. Give a red color to each incident edges of $x_6, y_8, x_{19}, y_{12}, \ldots$	

As shown in Table 1 which consists of 4 cases. Color the incident edges, consider the coloring steps and see the illustration.

In any case where A_n does not contain a red P_4, the remaining edges will contain a blue P_n. Hence $A_n \rightarrow (P_4, P_n)$, for even n, $n \geq 6$.

Second, we prove that $A_n \backslash e \nrightarrow (P_4, P_n)$, for any edge e. Consider the form of coloring in Table 2 for each edges removed.

Table 2.

Remove edge (e)	The form of red subgraph	Illustration	The longest path
$x_1y_1, x_{n-2}y_{n-2}$, x_1x_2, or $y_{n-3}y_{n-2}$	$\frac{n-2}{2}C_3$		$y_2y_3x_3x_4x_5x_6\ldots x_{n-4}$ $y_{n-3}y_{n-4}$
y_1y_2 or $x_{n-3}x_{n-2}$	$K_{1,4} \cup \frac{n-4}{2}C_3$		$y_2y_3x_3x_4x_5x_6\ldots x_{n-3}$ $y_{n-3}y_{n-2}$
x_2y_1 or $x_{n-2}y_{n-3}$	$\frac{n-2}{2}C_3 \cup P_2$		$y_1y_2y_3y_4y_5y_6\ldots y_{n-4}$ $y_{n-3}x_{n-3}x_{n-2}$
x_iy_i for $2 \leq i \leq n-3$	$\frac{n-2}{2}C_3 \cup P_2$		$x_2x_3x_4x_5x_6\ldots x_{n-3}$ $x_{n-2}y_{n-3}y_{n-2}$
x_iy_{i-1} for $3 \leq i \leq n-3$	$\frac{n}{2}C_3 \cup P_2$		$y_2y_3y_4y_5y_6\ldots y_{n-3}$ $y_{n-2}x_{n-2}$
x_ix_{i+1} for $2 \leq i \leq n-3$, or y_ny_{n+1} for $2 \leq n \leq n-3$	$\frac{n-2}{2}C_3 \cup P_2$		$y_3y_4x_4x_5x_6\ldots x_{n-3}$ $x_{n-2}y_{n-3}y_{n-2}$

If $A_n \backslash e$, for any edge e, then give the coloring with the form of red subgraph and see the illustration as in Table 2. Obviously, in the longest path there is no blue P_n as a subgraph. Therefore, $A_n \backslash e \nrightarrow (\mathrm{P_4}, \mathrm{P_n})$, for even n, n ≥ 6 for any edge e.

In Theorem 3 we determine a graph that belong to $\mathcal{R}(\mathrm{P_4}, \mathrm{P_n})$, for odd n, n ≥ 7

Theorem 3. Let P_4 and P_n be the path graphs on 4 and n vertices, then B_n in Figure 11. is a Ramsey minimal graph of $(\mathrm{P_4}, \mathrm{P_n})$, for odd n, n ≥ 7.

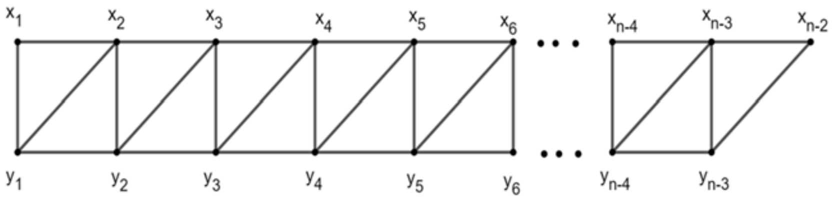
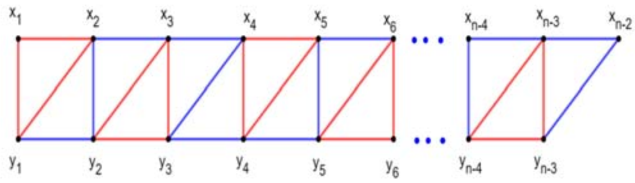
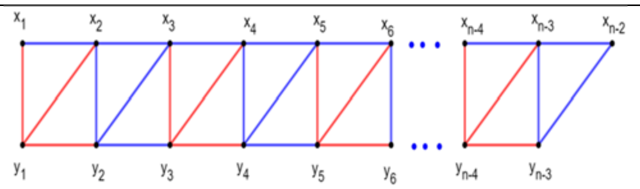
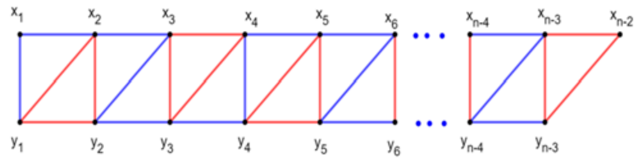
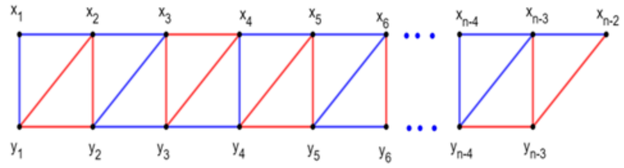
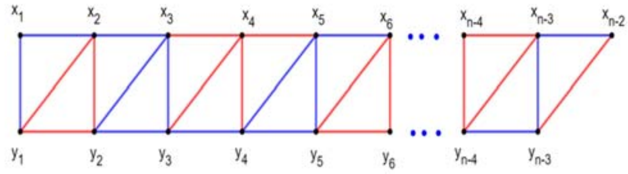
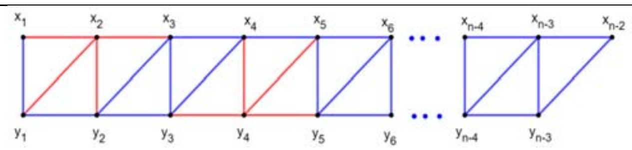


Figure 10. Graph B_n

Proof. First, we show that $B_n \rightarrow (\mathrm{P_4}, \mathrm{P_n})$. Consider any red-blue coloring of all edges of B_n containing no red $\mathrm{P_4}$. If B_n does not contain red $\mathrm{P_4}$, then the red subgraph will be in the form of $K_{1,3}, C_3, K_{1,4}$. Consider Table 3 for all possibilities coloring of B_n that does not contain red $\mathrm{P_4}$ as follows.

Table 3.

Cases	Incident edge	Coloring steps	Illustration
1	x_1	1. Give a red color to each incident edges of x_1, i.e x_1x_2 and x_1y_1 2. Color the incident edges to y_1, i.e x_2y_1 in red, y_1y_2 in blue 3. Give a blue color to the incident edges of x_2 4. Give a red color to the incident edges of y_2 5. Color the incident edges to $x_3, y_3, x_4, y_4, \ldots, x_{n-3}, y_{n-3}, x_{n-2}$, respectively by maximizing the red edge as long as it doesn't contain red $\mathrm{P_4}$	
2	y_1	1. Give a red color to each incident edges of y_1 2. Give a blue color to the incident edge of x_1, i.e x_1x_2 3. Give a blue color to each incident edge of y_2 4. Color the incident edge to x_2, i.e x_2x_3 with blue color. 5. Color the incident edges to $y_3, x_3, y_4, x_4, \ldots, y_{n-2}, x_{n-2}$, respectively by maximizing the red edge as long as it doesn't contain red $\mathrm{P_4}$	

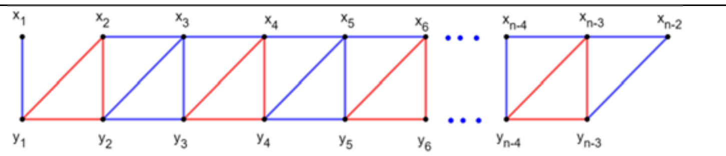
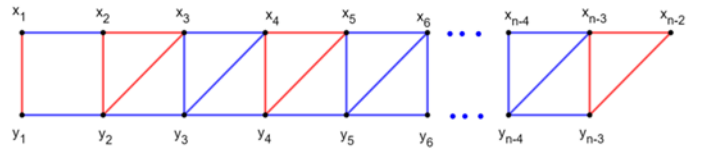
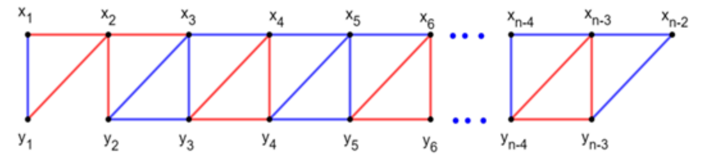
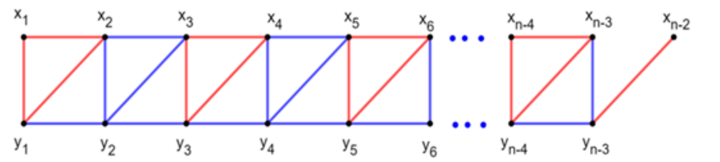
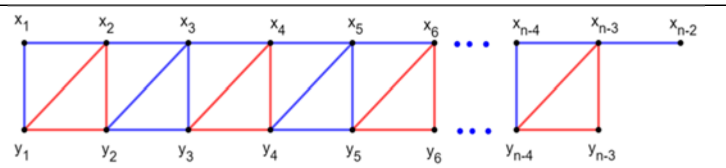
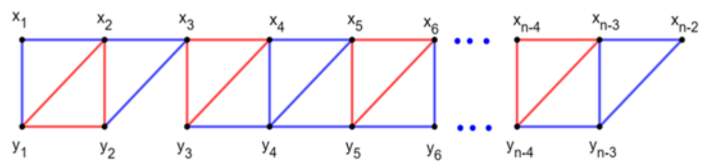
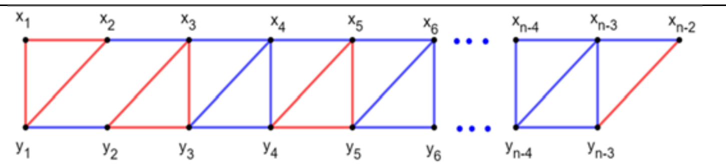
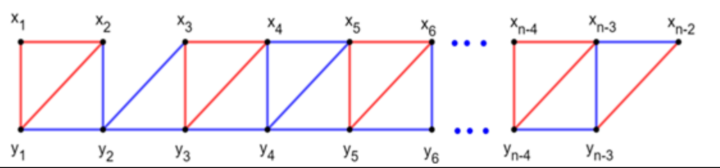
3	x_{n-2}	1. Give a red color to each incident edges of x_{n-2} 2. Color the incident edges to x_{n-3}, i.e $x_{n-3}y_{n-3}$ in red, othewise, give a blue color 3. Color the incident edges to y_{n-3}, i.e $y_{n-4}y_{n-3}$ in blue 4. Give a blue color to the incident edges of y_{n-4}, i.e $x_{n-4}y_{n-4}$ and $y_{n-3}y_{n-4}$ 5. Color the incident edges to $x_{n-4}, x_{n-3}, y_{n-3}, y_{n-2}, \ldots, y_1, x_1$, respectively by maximizing the red edge as long as it doesn't contain red P_4	
4	y_{n-3}	1. Give a red color to each incident edges of y_{n-3} 2. Give a blue color to the incident edges of x_{n-3}, i.e $x_{n-4}x_{n-3}$, $x_{n-3}y_{n-4}$, $x_{n-3}x_{n-2}$ 3. Give a blue color to the incident edges of y_{n-4} 4. Color the incident edges to $x_{n-4}, y_{n-5}, x_{n-5}, \ldots, y_1, x_1$, respectively by maximizing the red edge as long as it doesn't contain red P_4	
5	x_4	1. Give a red color to each incident edges of x_4 2. Give a blue color to each incident edges of x_3 3. Color the incident edges to x_2, i.e x_2y_1 and x_2y_2 in red, otherwise, give a blue color	
6	x_2	1. Give a red color to each incident edges of x_2 2. Give a red color to each incident edges of y_4 3. Give a red color to each incident edges of $x_6, y_8, x_{19}, y_{12}, \ldots$	

As shown in Table 3 which consists of 6 cases. Color the incident edges, consider the coloring steps and see the illustration.

In any case where B_n does not contain a red P_4, the remaining edges will contain a blue P_n. Thus, $B_n \rightarrow (P_4, P_n)$, for odd n, n ≥ 7.

Second, we show that $B_n \backslash e \nrightarrow (P_4, P_n)$, for any edge e. Consider the form of coloring in Table 4 for each removed edges.

Table 4.

Remove edge (e)	The form of red subgraph	Illustration	The longest path
x_1y_1, or x_1x_2	$\frac{n-3}{2}C_3$		$y_2y_3x_3x_4x_5x_6\ldots x_{n-3}$ $x_{n-2}y_{n-3}$
x_2y_1	$\frac{n-3}{2}C_3$ $\cup P_2$		$y_1y_2y_3y_4y_5y_6\ldots y_{n-4}$ $x_{n-4}x_{n-3}$
y_1y_2	$K_{1,4}$ $\cup \frac{n-5}{2}C_3$		$y_2y_3x_3x_4x_5x_6\ldots x_{n-3}$ $x_{n-2}y_{n-3}$
$x_{n-3}x_{n-2}$ or $x_{n-3}y_{n-3}$	$\frac{n-3}{2}C_3$ $\cup P_2$		$x_3x_2y_2y_3y_4y_5\ldots y_{n-4}$ $y_{n-3}x_{n-3}$
$x_{n-2}y_{n-3}$	$\frac{n-3}{2}C_3$		$y_1x_1x_2x_3x_4x_5\ldots x_{n-3}$ x_{n-2}
y_iy_{i+1} for $3 \le i \le n-4$	$\frac{n-3}{2}C_3$		$x_5x_4y_4y_5y_6\ldots y_{n-4}$ $y_{n-3}x_{n-2}$
x_iy_i for $2 \le i \le n-4$	$\frac{n-3}{2}C_3$ $\cup P_2$		$x_2x_3x_4x_5x_6\ldots x_{n-4}$ $x_{n-3}y_{n-3}y_{n-4}$
x_ix_{i+1} for $2 \le i \le n-4$	$\frac{n-3}{2}C_3$ $\cup P_2$		$y_1y_2y_3y_4y_5y_6\ldots y_{n-4}$ $y_{n-3}x_{n-3}x_{n-2}$

If $B_n \backslash e$, for any edge e, then give the coloring with the form of red subgraph and see the illustration as in Table 4. Consequently, neither red P_4 nor blue P_n occurs. Therefore, $B_n \backslash e \nrightarrow (P_4, P_n)$, for odd n, $n \geq 7$ for any edge e.

III. CONCLUSIONS

In this paper, we have obtained some graphs that belongs to $\mathcal{R}(P_4, P_5)$. Then, we have obtained a graph in $\mathcal{R}(P_4, P_n)$ for even n, $n \geq 6$ and a graph in $\mathcal{R}(P_4, P_n)$ for odd n, $n \geq 7$ is also obtained.

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