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Stability of Smoker Population Model

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Abstract— **In this article, a model for the smoking habits distribution is studied. The distribution model of smoking habits is divided into three compartments, namely potential smokers (P), active smokers (S), and permanent non-smokers (R). Based on the model analysis, two equilibrium points are obtained, they are the smoke-free equilibrium point and the endemic equilibrium point. Analytical study is carried out by analyzing the stability of the model around the equilibrium point based on the eigen values of the Jacobian matrix. The stability of the model is also associated with the basic reproduction number ($\mathfrak{R}_0$) or the parameter that determines the population is free from active smokers or the prevalence of smoking habits occurs. The numerical simulations performed show asymptotically stable smoking population around the smoker-free equilibrium point with $\mathfrak{R}_0$ <1 and asymptotically stable endemic equilibrium point with $\mathfrak{R}_0$ >1.**

Keywords—PSR model, Stability, Basic Reproduction number

I. INTRODUCTION

Cigarettes are one of the processed tobacco products (nicotiana) which are made for the purpose of being burned, smoked or inhaled. Cigarettes are one of the chemicals that have a negative impact on health and the environment [4]. Chemicals found in cigarettes can cause heart disease, hypertension, cancer and several other diseases [3]. One effort to minimize the problem of the smoking population is to formulate it into a mathematical model. Verma in [12] proposed a mathematical model of the dynamics of the smoking population in the form of an optimal control problem which is divided into three compartments, namely, the population of potential smokers (P), the population of active smokers (S), and the population of permanent non-smokers (R).

In this article, the Verma model in [8] is reviewed by looking at the behavior of the solution from the equilibrium point in order to analyze the stability of the equilibrium point. Determination of the basic reproduction number is sought using $S^* > 0$. The purpose of this study is to analyze the stability of the equilibrium point of the dynamics model of the distribution of the smoking population.

II. MATHEMATICS MODEL

The smoking population model is expressed in the following compartment diagram [8..

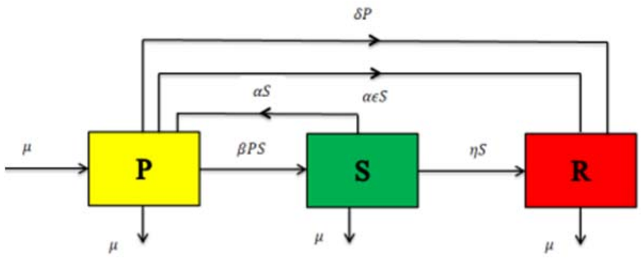

Fig. 1 Compartment Distribution of Smoker Population

From the schematic Fig. 1 above, the distribution of the PSR smoking population can be modeled in the form of a non-linear system of differential equations as follows

$$\frac{dP}{dt} = \Lambda - \beta PS + \alpha(1-\varepsilon)S - \mu P - \delta P$$

$$\frac{dS}{dt} = \beta PS - \mu S - \alpha S - \eta S \quad (1)$$

$$\frac{dR}{dt} = \alpha\varepsilon S - \mu R + \delta P + \eta S$$

The definition of the parameters used can be seen in the following table

Table 1: Definition of the Parameters

Simbol	Deskripsi
P	Potential smoking population rate
S	Active smoking population rate
R	Permanent smokers population rate
Λ	The recruitment rate
β	Distribution rate
α	The cure rate
δ	The awareness level of potential smokers
ε	The resolve to recover
μ	The mortality rate
η	Media implementation rate

III. Results And Discussion

3.1. Equilibrium Point

The equilibrium point is free equilibrium point (E^0) and endemic equilibrium point (E^*) is

$$E^0 = (P^0, S^0, R^0) = \left(\frac{\Lambda}{\mu+\delta}, 0, \frac{\delta\Lambda}{\mu(\mu+\delta)}\right)$$

$$E^* = (P^*, S^*, R^*)$$

with

$$P^* = \frac{\alpha + \eta + \mu}{\beta}$$

$$S^* = \frac{\Lambda\beta - \mu(\alpha + \eta) - (\mu + \alpha)(\mu + \delta)}{\beta(\alpha\varepsilon + \eta + \mu)}$$

$$R^* = \frac{\Lambda\alpha\varepsilon\beta - \alpha^2\varepsilon\mu - \alpha\varepsilon\eta\mu - \alpha\varepsilon\mu^2 + \Lambda\beta\eta + \alpha\delta\mu - \alpha\eta\mu + \delta\eta\mu + \delta\mu^2 - \eta^2\mu - \eta\mu^2}{\beta\mu(\alpha\varepsilon + \eta + \mu)}$$

The basic reproduction number $\mathfrak{R}_0$ is a parameter to determine whether a population is free from active smokers or there is a spread of smoking habits obtained by assuming $S^* > 0$

$$S^* = \frac{\Lambda\beta - \big(\mu(\alpha + \eta) + (\mu + \alpha)(\mu + \delta)\big)}{\beta(\alpha\varepsilon + \eta + \mu)} > 0$$

So that it is obtained

$$\mathfrak{R}_0 = \frac{\Lambda\beta}{\big(\mu(\alpha + \eta) + (\mu + \alpha)(\mu + \delta)\big)}$$

Definition 1 [1] *If* $\mathfrak{R}_0 > 1$ *then the smoker-free equilibrium point is asymptotically stable so that smoking habits will disappear, and If* $\mathfrak{R}_0 > 1$ *then the endemic equilibrium point is asymptotically stable so that smoking habits will become epidemic.*

3.2. Equilibrium Point Stability Analysis

Model (1) is a nonlinear system of differential equations. Therefore, to determine the stability of the system (1) at the smoke-free equilibrium point and the endemic equilibrium point, the system (1) needs to be linearized using the Jacobian matrix as follows [4].

$$\boldsymbol{J} = \begin{pmatrix} -\beta S - \mu - \delta & -\beta P + \alpha(1 - \varepsilon) & 0 \\ \beta S & \beta P - \mu - \alpha - \eta & 0 \\ \delta & \alpha\varepsilon + \eta & -\mu \end{pmatrix} \qquad (2)$$

At the smoke-free equilibrium point E^0, the Jacobian J_{E^0} is obtained by substituting the smoke-free equilibrium point into (2). The eigenvalues are obtained by solving the equation $\det(J_{E^0} - \lambda I) = 0$ so that the characteristic equation is obtained as follows:

$$(-\mu - \lambda)\left(\frac{\beta\Lambda}{\mu + \delta} - \mu - \alpha - \eta - \lambda\right)(-\mu - \lambda) = 0$$

The eigen value of the matrix J_{E^0} is

$$\lambda_1 = -\mu, \qquad \lambda_2 = -\mu - \delta, \quad \lambda_3 = \frac{\beta\Lambda}{\mu + \delta} - \mu - \alpha - \eta$$

Based on the stability criteria, the equilibrium point is asymptotically stable if $\lambda_3 < 0$.

$$\frac{\beta\Lambda}{\mu + \delta} - \mu - \alpha - \eta < 0$$

$$\frac{\beta\Lambda}{(\mu + \alpha + \eta)(\mu + \delta)} < 1$$

$$\Re_0 < 1$$

Hence, the smoker-free equilibrium point E^0 is asymptotically stable.

At the endemic equilibrium point E^*, the Jacobian J_{E^*} is obtained by substituting the endemic equilibrium point into (2). The eigen value is obtained by solving the equation $\det(J_{E^*} - \lambda I) = 0$ so that the characteristic equation is obtained as follows:

$$(\lambda + \mu)(\lambda^2 + (\beta P - \beta S + \alpha + \delta + \eta + 2\mu)\lambda + \beta S\,(\mu + \alpha + \eta) + (\mu + \alpha)(\mu + \delta) + \eta\mu + \eta\delta - \beta P\mu - \beta P\delta - \beta S\alpha - \beta S\alpha\varepsilon) = 0 \quad (3)$$

From equation (3), the eigen values of matrix J_{E^*} are obtained $\lambda_1 = -\mu$, λ_2, λ_3 the roots of the polynomial

$$(\lambda^2 + (\beta P - \beta S + \alpha + \delta + \eta + 2\mu)\lambda + \beta S(\mu + \alpha + \eta) + (\mu + \alpha)(\mu + \delta) + \eta\mu + \eta\delta - \beta P\mu - \beta P\delta - \beta S\alpha - \beta S\alpha\varepsilon)$$

Based on the stability criteria, the equilibrium point E^* will be asymptotically stable if the eigenvalues are negative. This condition is caused by the result of the conditions that meet the eigenvalues whose real part is negative. Therefore, it is obtained:

$$\frac{\beta\Lambda}{(\mu + \delta)(\mu + \alpha + \eta)} > 1$$

$$\Re_0 > 1$$

Thus E^* will be asymptotically stable because $\Re_0 > 1$.

3.3. Numerical Simulation

In this section, a numerical simulation is carried out with the help of MATLAB software which is simulated using the Runge Kutta method of order 4 to see the population changes for cases $\Re_0 < 1$ and $\Re_0 > 1$.

a) The Numerical Simulation for $\Re_0 < 1$

The initial value used is $P(0) = 0.4$, $S(0) = 0.4$, $R(0) = 0.2$. All parameter values are presented in table 2 below. In the simulation, the initial time $t_0 = 0$ day and the end time $t_n = 60$ were used.

Table 2 : Parameters value

Parameter	Λ	β	α	δ	ε	μ	η
Nilai	0.2	0.5	0.1	0.1	0.02	0.2	0.17

The graph of the smokers population is obtained as follows

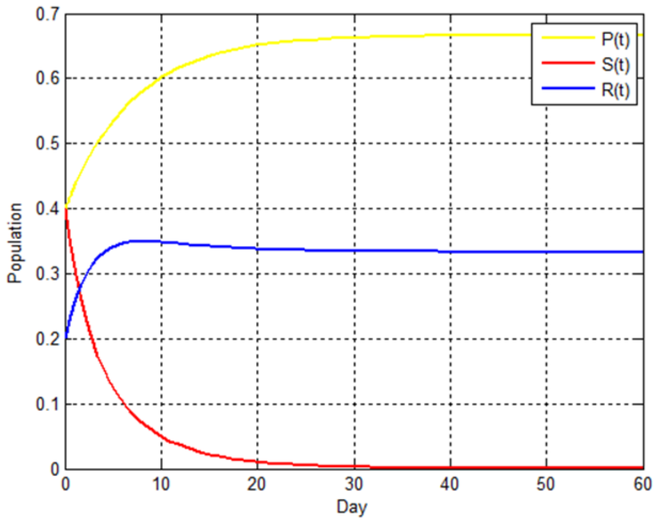

Fig. 2 Distribution Dynamics of Smoker Population$\Re_0 < 1$

The smoker-free equilibrium point is $E^0 = (P^0, S^0, R^0) = (0.67, 0, 0.33)$ with a value of $\Re_0 = 0.7 < 1$. From Fig 2, it can be seen that the population is stable to a smoke-free equilibrium point, so that the smoke-free equilibrium point with $\Re_0 < 1$ condition is asymptotically stable. It means that there is no spread or distribution of smoking habits.

b) The Numerical Simulation for $\Re_0 > 1$

The initial value used is $P(0) = 0.5$, $S(0) = 0.2$, $R(0) = 0.3$. All parameter values are presented in table 3 below. In the simulation, the initial time $t_0 = 0$ day and the end time $t_n = 60$ were used.

Table 3 : Parameters value

Parameter	Λ	β	α	δ	ε	μ	η
Nilai	0.2	0.5	0.1	0.1	0.02	0.2	0.03

The graph of the smokers population is obtained as follows

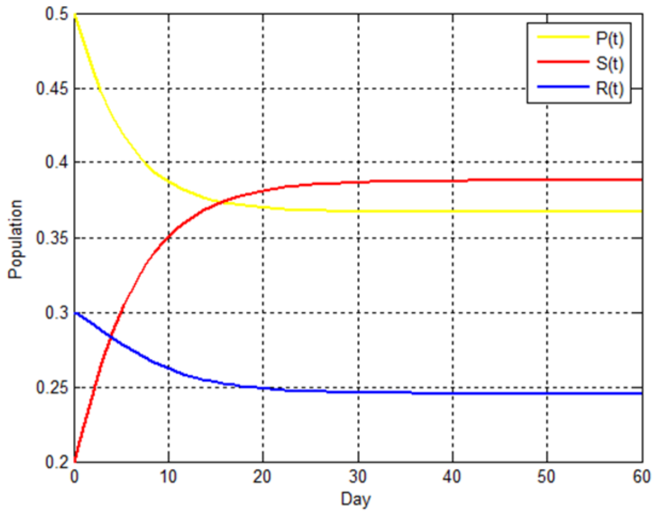

Fig. 3 Distribution Dynamics of Smoker Population $\Re_0 > 1$

The smoker-free equilibrium point is $E^0 = (P^0, S^0, R^0) = (0.67, 0, 0.33)$ and endemic equilibrium point is $E^* = (P^*, S^*, R^*) = (0.37, 0.39, 0.24)$ with a value of $\Re_0 = 1.81 > 1$. From Fig 2, it can be seen that the population is stable to a smoke-free equilibrium point, so that the smoke-free equilibrium point with $\Re_0 > 1$ condition is asymptotically stable. It means that there is distribution of smoking habits.

IV. Conclusion

The distribution model of the smoking habit population produces asymptotically stable smokers-free equilibrium point when $\Re_0 < 1$ and asymptotically stable endemic equilibrium point when $\Re_0 > 1$.

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