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Hybrid Approach to Solve the Capacitated Vehicle Routing Problem

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Abstract— The Capacitated Vehicle Routing Problem (CVRP) is one of the most studied problems in the field of transport logistics. This given that transport represents up to 10% of the value of a product. CVRP addresses the situation of finding the optimal number of vehicles to meet the demands of a set of customers. The assignment of vehicles to customers must be consolidated in routes of minimum cost, distance or time. Because this problem is of the NP-hard type, there are few instances that can be solved optimally. In this article, a greedy algorithm is presented to obtain approximate solutions to this problem in a fast way with the free license software Octave. For the comparison of results, instances with optimal values of the CVRPLIB database were used. The results showed that this algorithm was able to obtain feasible solutions with errors less than 14.0% for medium to large CVRP instances. In some cases, near-optimal solutions were obtained.

Keywords— vehicle routing problem; heuristic algorithm; combinatorial optimization; software design.

I. INTRODUCTION

The field of Logistics encompasses all activities of transportation, distribution, storage, handling of materials and inventories at all stages of product manufacturing [1]. In particular, the transportation of products represents a significant part of their value (approximately 10%) given the resources (vehicles, fuel, personnel) involved [2]. Therefore, it is necessary to have transportation planning to increase the use of resources and minimize associated costs.

The Capacitated Vehicle Routing Problem (VRP) describes the design of routes which must start and return from a warehouse or distribution center. These routes are covered by vehicles of finite capacity, where each vehicle must cover the demands of a finite set of customers. As restrictions of this problem, we have the following:

a) only one vehicle can visit a customer;

b) the demands of the customers assigned to the route of a vehicle cannot exceed the capacity of the same.

The designed routes must minimize a performance metric such as distance or total transport time. Consequently, this tends to minimize costs for fuel, wages (overtime), and pollutant emissions [3].

The design of routes with these characteristics is constantly being improved given the complexity of the CVRP, which is of the NP-hard type. While there are exact methods to solve the CVRP such as Branch & Bound [4], these methods lose efficiency when the number of customers is greater than 50 clients [5]. Due to this, the design of approximate methods such as heuristics has been considered for CVRP in its different modalities (e.g., periodic CVRP or with time windows) for larger numbers of clients [6]. Among the most efficient heuristic methods, the following can be mentioned: Genetic Algorithms (GA), Ant Colony Optimization (ACO), Particle Swarm, Savings by Clarke & Wright, and Bee Colony [6, 7].

In this work, we develop a hybrid Greedy-Nearest Neighbor (GNN) algorithm to find a good local solution (with an error gap less than 15%) in reasonable time for medium and large instances of the CVRP. The GNN has the following practical features:

a) low computational complexity;

b) use of nearest-neighbor approach and sequential saturation of vehicle capacities to obtain minimum number of routes and route costs;

c) integration of diversification operators (based on GA mutation operators) to improve solutions under exploratory and exploitative schemes;

d) stable performance for different numbers of customers (particularly for route design for more than 50 customers);

The details of this method and the analysis of the results obtained are presented as follows: Section II presents the technical details of the random method, while Section III shows the results of the method on symmetric instances of the library. TSPLIB 95. Finally, Section IV presents the conclusions of this work and future plans for it.

II. Development

2.1. The Capacitated Vehicle Routing Problem

The Capacitated Vehicle Routing Problem (CVRP) is described as follows [8]: consider the complete graph G={V, A}, where V={0,1,2,…, n} is the set of vertices (at vertex 0 is the warehouse or distribution center, and vertices 1 to n represent the customers to visit), and A={(i, j) | i, j∈V, i ≠ j} is the set of arcs between the vertices. C_{ij} denotes the non-negative cost of traveling from vertex i to vertex j. It is assumed that there are K identical vehicles with capacity Q. It is considered that S ⊆ V{0} is the set of vertices – customers and r(S) the minimum number of vehicles required to serve all customers in S. The variable of decision is x_{ij} and its value is 1 if the arc (i, j) belongs to the optimal path and 0 otherwise. In this way the basic model of the CVRP is given by:

$$Minimize\ \sum_{i\in V}\sum_{j\in V} C_{ij}x_{ij} \quad (1)$$

Subject to:

$$\sum_{i\in V} x_{ij} = 1, \forall j \in V\{0\} \quad (2)$$

$$\sum_{j\in V} x_{ij} = 1, \forall i \in V\{0\} \quad (3)$$

$$\sum_{i\in V} x_{i0} = K \quad (4)$$

$$\sum_{j\in V} x_{0j} = K \quad (5)$$

$$\sum_{i\notin S}\sum_{j\notin S} x_{ij} \geq r(S), \forall S \subseteq V\{0\}, S \neq 0 \quad (6)$$

$$x_{ij} \in \{0,1\}, \forall i,j \in V \quad (7)$$

The objective function (1) minimizes the total travel distance while the constraints defined by (2) and (3) indicate that only one arc enters and leaves each node. The constraints defined by (4) and (5) indicate that a number K of vehicles leave and return to vertex 0 (distribution center). Finally, constraint (6) ensures that customer demand does not exceed the capacity of the assigned vehicle and that there is connectivity between assigned customers.

2.2. The GNN Algorithm

Fig. 1 shows the block diagram of the GNN algorithm proposed for the CVRP. The search for a possible solution begins with a single route constructed by the Nearest Neighbor algorithm. This route represents a feasible solution to the Traveling Salesman Problem (TSP), where no restrictions of demands or capacities exist and it is only required to find the route that connects all customers in the problem. This route is improved by a set of inversion and exchange operations which are randomly applied (see Figure 2). This enables the creation of variations of the TSP solution.

Then, a segmentation algorithm cuts this TSP solution into K sub-routes. The cutting criterion for these sub-routes consider the demands of the customers and the capacity restriction given by Q. Also, each sub-route is improved by the same set of inversion and exchange operations used for the TSP solution.

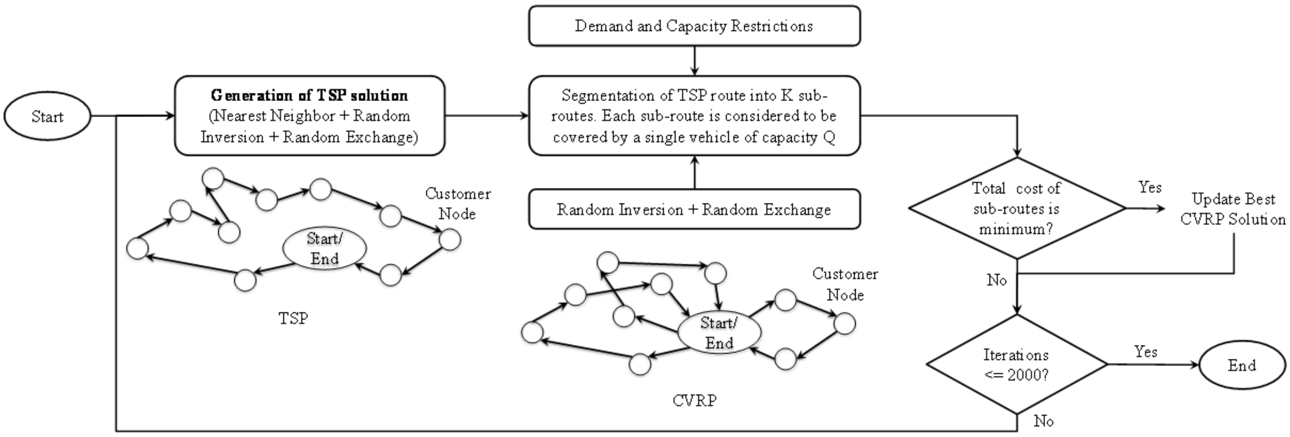

Fig. 1. Stages of the proposed greedy local search algorithm for the CVRP.

1) Two random positions are selected

Route → 1-2-3-4-5-6-7-8-9-10-11-12-1

2) Random Inversion:
The order of the nodes within the selected positions is reversed.

Route Inversion → 1-2-3-4-11-10-9-8-7-6-5-12-1 **Explorative Changes**

3) Random Exchange:
The nodes in the selected positions are exchanged between them.

Route Exchange → 1-2-3-4-11-6-7-8-9-10-5-12-1 **Exploitative Changes**

Fig. 2. Inversion and exchange operators to improve TSP/CVRP solutions.

Each time a segmented TSP solution with K sub-routes is obtained, the total distance of these sub-routes is computed as this is the metric to assess the CVRP solution. As presented in Figure 1, the process of generating TSP routes and cutting them into K sub-routes is iteratively performed 2000 times. After these iterations are performed, we keep the segmentation with the least number of K sub-routes and minimum total CVRP distance.

III. Results

The proposed algorithm was programmed in Octave v7.1.0 and it was executed in a LANIX Laptop with Intel Core i5 CPU at 1.0 GHz and 8GB RAM. For CVRP testing, we used the CVRPLIB database and selected 14 instances with medium to large number of customers. As this database provides the best-known solutions for each instance, we were able to compute the error gap of the proposed algorithm. The results are presented in Table 1. Because the algorithm is constituted by two algorithms: a

Greedy algorithm which performs exchange and inversion to improve solutions, and the Nearest Neighbor algorithm to provide suitable initial solutions, there are two sets of reported solutions: (1) NN (just the Nearest Neighbor algorithm is performed) and (2) GNN (the hybrid algorithm is performed).

It is observed that the Greedy algorithm significantly improves the solutions obtained with just the NN algorithm. The hybrid GNN leads to errors less than 15.0% for all CVRP instances. Particularly, 5 instances achieved an error less than 10.0% with 3 large instances (X-n376-k94, X-n502-k39, X-n655-k131) achieving near-optimal results. It is interesting to observe that small instances (less than 350 customers) and very large instances (more than 680 customers) achieved the largest error rates (12.0%). However, even the largest error rate of 14.57% (X-n685-k75) is within the acceptable level of 15.0%.

TABLE I. ASSESSMENT OF CVRP SOLUTIONS OBTAINED WITH THE GNN ALGORITHM

Instance	***n***	***K***	***NN Error Gap (%)***	***GNN Error Gap (%)***
X-n101-k25	101	25	17.42	11.25
X-n190-k8	190	8	16.31	12.35
X-n266-k58	266	58	18.50	11.03
X-n289-k60	289	60	15.54	12.29
X-n336-k84	336	84	15.79	14.57
X-n359-k29	359	29	15.22	11.48
X-n376-k94	376	94	4.77	1.98
X-n439-k37	439	37	16.13	12.13
X-n480-k70	480	70	12.57	9.59
X-n502-k39	502	39	8.79	3.28
X-n548-k50	548	50	9.55	5.57
X-n655-k131	655	131	5.81	2.77
X-n685-k75	685	75	16.78	13.85
X-n701-k44	701	44	18.80	11.31

IV. CONCLUSIONS

The hybrid algorithm can achieve suitable solutions for large instances of the CVRP. In some cases, the GNN algorithm can lead to near-optimal solutions. Although there are methods which can lead to more competitive results with error gaps less than 5.0%, these require more complex operators and structures (i.e., populations of solutions, order/cycle crossover operators, tabu lists, etc.).

The proposed GNN is faster to implement and can provide very suitable solutions within reasonable time. These are two main features frequently sought in real logistics scenarios. Also, it can be implemented in free-license software, which is an advantage for some enterprises and managers.

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