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THE PROBLEM

The major decision problem in the design of products and services is the selection of attributes. The assumption is that purchasers will choose the product which is expected to provide the greatest utility from among those with which they are familiar. Utility, in turn, is a function of the product's perceived attributes. There is a distinction here between the objective attributes designed into a product and the attributes affecting the purchase. Purchasers actively evaluate those objective attributes they consider relevant and usually perceive additional attributes of a subjective nature. Attributes defining a product's image or status may have been deliberately contemplated by the product's designer or may emerge only in the minds of users in the marketplace. It is important for the marketer to be aware of both types of attributes.

One issue in data analysis is whether it is productive to include all attributes of interest or whether redundancy should be reduced by examining dependency among attribute dimensions. This requires an analysis of the degree to which attributes relate to one another. The purpose of this paper is to present a methodology for measuring the strength of relationship (covariation) among product attributes.

The problem of measuring attribute covariation is particularly pressing when there are a large number of attribute dimensions, such as is commonly found in marketing and competitive analyses. In a study of the cigarette market, for example, one of the authors worked with 183 product attributes. In a multivariate situation involving metric variables, it is common to seek a parsimonious set of relatively uncorrelated variables by techniques like factor analysis. However, while some of the product attributes in the cigarette study were interval or ratio scaled, most were categorical, so factor analysis was inappropriate. A different measurement methodology was needed.

SIGNIFICANCE OF ATTRIBUTE ANALYSIS

Besides product design decisions, it is important to understand perceived attributes and their relationships for market partitioning, product positioning, and advertising decisions. Market partitioning, which has been popular since the early 1970s, seeks to define a market's competitive structure in terms of hierarchical structures of attributes [4,5,6,8]. Closely competing products will have important attributes in common. For example, tea and coffee share attributes such as "hot drink," whereas two brands of instant coffee share the attributes "hot drink," "coffee flavor," and "instant." Tea and coffee will be perceived by most purchasers as less substitutable than the former and therefore less competitive.

Advertising decisions also require a knowledge of product attributes. For example, Gardner examines the effects of advertising on the product attributes recalled about brands and attributes used to evaluate brands [2]. For commodity products like cigarettes, advertising can communicate objective characteristics like "filter" and "container." In addition it can effectively create perceived attributes which can become a dominant force in choice by the purchaser. If the important attributes for different types of purchasers are known, advertising can be tailored to the interests of those segments.

DATA COLLECTION

A common way of trying to measure attribute perceptions is to ask actual or potential purchasers to respond to questions about attributes and their relationships. This may well uncover some new attribute dimensions but is unlikely to provide valid relationship data. Not only is it costly to obtain such data but they are often at variance with perceptions which underlie actual behavior. In a hypothetical situation, respondents may indicate how they see the relationships among attributes, yet these judgments are often at variance with relationships implied by actual choice behavior. The extent to which attribute values are substitutable is best measured by an individual's sequence of purchases.

The use of purchase sequences to infer substitutability is only valid if the product use context is held constant. For example, if a person is observed to eat at a coffee shop, then a fast-food restaurant, followed by a steakhouse, it would be fallacious to conclude that the three types of institution are closely competitive, or substitutable. Ideally then, context should be controlled for when collecting data on substitutability patterns.

The methodology proposed in this paper is designed for frequently purchased products. It is not suitable for expensive, infrequently purchased products, since the purchaser's utility function probably changes from one purchase occasion to the next. The methodology utilizes diary panel data which are commercially available for a wide range of consumer products.

DATA PREPARATION

The first step in the analysis is to define each product alternative as a vector of attribute states. The step begins with the identification of objective and subjective product attribute dimensions relevant to the designer or marketer. Some examples of attribute dimensions for soft drink products are: brand name, container material, flavor, type of sweetener, and weight of contents. Each product alternative is then defined by a state on each attribute dimension. Thus, soft drink product alternative 1 may be represented by the vector: (Pepsi, can, cola, sugar, 12 ounce).

The next preparatory step is to compute a product alternative switching matrix. To do this, a sequential record of purchases is read for a panel of respondents. Each time a purchaser buys a unit of the product of interest, the product alternative vector is compared to the one last purchased and the corresponding cell in the switching matrix is augmented. Multiple purchases of the same item are treated as a string of repeat purchases, since the purchaser elects to forgo interim switches.

Consider the following example:

Attribute dimension Brand (brand a, brand b, brand c)

Attribute dimension Color (color x, color y)

Attribute dimension Size (size r, size s, size t, size u)

Product alternative A is (brand a, color x, size r)

Product alternative B is (brand b, color y, size s)

Product alternative C is (brand c, color x, size t)

Product alternative D is (brand c, color x, size r)

Product alternative E is (brand a, color y, size u)

Assume that the following transition matrix has been computed.

		TO				
		PA1 (a,x,r)	PA2 (b,y,s)	PA3 (c,x,t)	PA4 (c,x,r)	PA5 (a,y,u)
FROM	PA1 (a,x,r)	10	1	0	1	5
	PA2 (b,y,s)	2	11	1	2	1
	PA3 (c,x,t)	1	0	9	5	2
	PA4 (c,x,r)	1	1	6	13	0
	PA5 (a,y,u)	5	1	2	1	19

ENTROPY AS A SWITCHING INDEX

In the physical sciences, entropy has been viewed as a measure of the disorder, uncertainty or randomness of a probabilistic system. At equilibrium, the assumption is that all systems will be at the maximum entropy consistent with the constraints on the system. Shannon defined the entropy of a probability distribution $\{p_i\}$ as:

$$H\{p_i\} = -k \sum_{i=1}^{n} p_i \log p_i,$$

where k is an arbitrary constant [7]. Assume that k=1. Then the random variable has n values. this function has its maximum value of log n when the p_i are equal, indicating maximum uncertainty in a choice situation. At the other extreme, the value of H is 0 if there is no uncertainty.

One of the strengths of the entropy measure is that it can be used to study dependencies in sequences of outcomes [1,3]. First, second, and third order dependencies can be analyzed quite readily. The present research assumes a first order process; the last item purchased depends only on the one purchased the time before last. In the transition matrix, rows represent product alternative vectors purchased the time before last and columns represent vectors purchased last. By dividing switching frequencies in a row by row total frequency, row-conditional relative frequencies can be computed. These will be treated as empirically determined conditional probabilities. Likewise, the row totals may be divided by total transitions to give a measure of market share.

From these conditional and share probabilities, a measure of switching entropy can be computed:

Switching entropy H_s = $- \sum_i p_i \sum_j p_{i|j} \log p_{i|j}$

where p_i = share probability for product alternative i,

$p_{i|j}$ = conditional probability of choosing product alternative j, given that i was chosen last time.

H_s measures the propensity purchasers have for switching from one alternative to others. It is a share-weighted average of the switching entropy for each product alternative (row). If purchasers are very loyal and seldom switch product alternatives, the row-conditional probability distributions will be close to {0,0,...1,..0}, where the high probability is on the main diagonal of the switching matrix. Low loyalty to a product alternative will produce a more uniform distribution, and a higher switching entropy for that row. If all conditional distributions were uniform, indicating complete randomness of choice from each starting point, H_s would equal log n.

ATTRIBUTE COVARIATION

For measuring covariation of quantitative variables, covariance or Pearsonian correlation are commonly employed. Since most attribute data are categorical, a different measure is needed. In the present context, covariation means the extent to which switching on one attribute accounts for switching on another in purchase decisions. For example, if a switch from brand A to brand B necessarily means a switch from one flavor to another, the attribute "brand name" fully accounts for the attribute "flavor." But if there are several flavors within a brand, the attribute "flavor" may not fully account for "brand name." So, unlike Pearsonian correlation, the relationship is asymmetrical.

Attribute covariation is the extent to which switching across values of one attribute dimension accounts for the switching on another attribute dimension. To measure switching on each attribute dimension, a switching matrix must be computed for each dimension. Switching matrices for brand, color, and size can be computed from the product alternative switching matrix, as follows:

<u>Brand</u>

TO				
FROM		Brand a	Brand b	Brand c
	Brand a	39	2	4
	Brand b	3	11	3
	Brand c	4	1	33

<u>Color</u>

TO			
FROM		Color x	Color y
	Color x	46	9
	Color y	13	32

<u>Size</u>

TO					
FROM		Size r	Size s	Size t	Size u
	Size r	25	2	6	5
	Size s	4	11	1	1
	Size t	6	0	9	2
	Size u	6	1	2	19

To illustrate the computation of switching entropy, H_s is computed for the color dimension. The $p_{i|j}$ and p_i are as follows:

TO				
		Color x	Color y	p_i
	Color x	.836	.164	.550
FROM	Color y	.289	.711	.450

$$H_s = -\{0.550[0.836\log 0.836 + 0.164\log 0.164] + 0.450[0.289\log 0.289 + 0.711\log 0.711]\}$$
$$= 0.516$$

This represents a high degree of switching relative to the maximum value of log 2, or 0.693.

To see how attribute dimensions brand and color co-vary, all transitions involving a brand switch are suppressed, then residual color switching is measured. Holding brand 'a' constant, the 39 transitions on the color dimension are as follows:

		TO	
		Color x	Color y
	Color x	10	5
FROM	Color y	5	19

This gives a conditional H_s value of 0.560. Holding brand 'b' constant, there is no color switching, and likewise when brand 'c' is held constant. In both cases the conditional $H_s = 0$. Residual switching on the color dimension is the brand-share- weighted average of the above conditional H_s values, namely:

$$\text{Residual } H_s = 0.45*0.560 + 0.17*0 + 0.38*0$$

$$= 0.252$$

ENTROPY COVARIATION INDEX

The covariation of the brand attribute and the color attribute when color is the dependent variable is measured by the percentage of color's initial switching entropy accounted for by extracting brand switching. If there were no residual switching on the dependent attribute, then the covariation would be 100%. The interpretation is similar to that for R^2 in regression.

$$C = \frac{H_s - \text{Residual } H_s}{H_s} *100$$

Applying the formula to the brand - color covariation gives:

$$C = \frac{0.516 - 0.252}{0.516} * 100$$

$$= 51.2\%$$

This means that 51.2% of color switching is accounted for by brand switching. The color attribute dimension is certainly not independent of the brand dimension.

The covariation index values for all pairwise attribute relationships are shown in the following table:

Causes the following percentage reduction in switching entropy on dimension				
		Brand	Color	Size
Suppressing switching on dimension	Brand	100	51.2	48.2
	Color	56.3	100	57.3
	Size	80.4	100	100

INTERPRETATION

It is interesting to note that brand and color have a fairly symmetrical relationship. When brand is considered primary, it accounts for 51.2% of switching on the color dimension. When the roles are reversed, color accounts for 56.3% of brand switching. By contrast, brand switching accounts for only 48.2% of size switching, whereas size switching accounts for 80.4% of brand switching. Also, when size switching is suppressed, there is no residual switching of colors. This means that all color switching is accounted for by size switching, since there is only one color associated with each size. But when the color dimension is primary, there is some switching among sizes within color; for example, product alternatives 1, 3 and 4 have color x and there is some size switching observed between 1 and 3 and between 3 and 4. This results in 57.3% of size switching being accounted for by switching on the color dimension.

SUMMARY

The analysis of product attributes provides an important tool for making marketing decisions in product design, product positioning, and advertising. It is particularly useful in market positioning where a market is defined by a hierarchical structure of attributes. Reducing the number of attributes under consideration to those that are most influential in consumer purchase decisions, simplifies this analysis.

In order to find the most significant attributes, a measure of the relative strength of the relationship among product attributes must be found. Because of the categorical nature of many attribute dimensions, traditional statistical measures such as factor analysis are not appropriate. This paper introduces the concept of using entropy to measure covariation.

Brand switching data collected from consumer panels can be tabulated after product attributes have been identified. These data are used to calculate the total and residual switching entropies. The switching entropy for an attribute dimension measures the propensity purchasers have for switching from one attribute to others. The residual switching entropy for an attribute dimension measures the propensity to switch among the attributes in one product dimension while holding the second product attribute constant. The entropy covariation index between two attribute dimensions measures the percentage of switching entropy on one dimension accounted for by switching on the second dimension.

Entropy covariation can be interpreted in the same manner as the traditional coefficient of determination. Values close to 100 percent indicate a strong relationship between two attribute dimensions. Therefore, utilizing both attribute dimensions, in this situation, would add little information to an analysis using only one dimension.

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