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В книге представлены материалы международной научной конференции «Двенадцатые Смирновские чтения по логике», посвященной памяти В. А. Смирнова (1931 – 1996) и Е.Д. Смирновой (1929 – 2017), выдающихся российских ученых, профессоров кафедры логики философского факультета МГУ, блестящих педагогов, оставивших после себя большое количество учеников. В. А. и Е. Д. Смирновы обладали крупнейшим научным авторитетом как в нашей стране, так и за ее пределами, являясь, в то же время, талантливыми организаторами науки. Во многом благодаря их многолетней самоотверженной деятельности сложилась отечественная научная школа, объединяющая в настоящее время специалистов из самых разных областей логики и философии.

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The Subject-Matter of Intensional Conditionals

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Abstract: We consider the matter of how intensional conditional connectives contribute to the subject-matter of sentences in which they appear. We review Kit Fine's model theory for William Parry's logic of analytic implication and some preliminary suggestions made by Fine to address the influence of intensional connectives on subject-matter. After outlining some thoughts on an appropriate formalization, we modify Fine's model theory to give a more natural account and show how several semantic conditions are characterized by individual axioms.

Keywords: *subject-matter, analytic implication, intensional conditionals*

The Content of Conditional Sentences

As discussed in [2], the hallmark feature of William Parry's propositional logic of *analytic implication* AI is the *Proscriptive Principle* that

> no formula with analytic implication as main relation holds universally if it has a free variable occurring in the consequent but not the antecedent.[5, P. 151]

AI was given a model theory by Kit Fine in [3]. Fine's models essentially equip each world w of an S4 Kripke model with a lattice of *concepts* $\langle I_w, \cup_w \rangle$.

Definition 1. An AI model is a tuple $\langle W, R, I, \cup, v, \gamma \rangle$ where:

— $\langle W, R \rangle$ is an S4 Kripke frame
— for each $w \in W$ there is a semilattice $\langle I_w, \cup_w \rangle \in I$
— v is a valuation from atomic formulae to W
— for each $w \in W$ there is an assignment γ_w from atomic formulae to I_w

Fine extends γ_w to cover the entire language as follows:

— $\gamma_w(\neg\varphi) = \gamma_w(\varphi)$
— $\gamma_w(\varphi \circ \psi) = \gamma_w(\varphi) \cup_w \gamma_w(\psi)$ for binary connectives $\circ$

This induces the relation of truth at a world:

— $w \Vdash p$ if $w \in v(p)$
— $w \Vdash \neg\varphi$ if $w \nVdash \varphi$
— $w \Vdash \varphi \wedge \psi$ if $w \Vdash \varphi$ and $w \Vdash \psi$
— $w \Vdash \varphi \to \psi$ if $\begin{cases} \text{for all } w' \text{ such that } wRw', \text{ if } w' \Vdash \varphi \text{ then } w' \Vdash \psi \\ \gamma_w(\psi) \leq_w \gamma_w(\varphi) \end{cases}$

Note the two components of Fine's truth conditions for $\varphi \to \psi$, which we might think of as the *alethic* and *content-theoretic* elements.

The definition of γ_w does not distinguish between *extensional* and *intensional* connectives; neither provides any content beyond that of subformulae. As Fine points out in his [3], Parry's [4] offers the Proscriptive Principle as a thesis about the inclusion of *concepts* (*Begriffe*) while simultaneously describing analytic implication itself as a concept (*Begriff*).

> Another change, suggested by Parry..., arises from treating analytic implication as a concept. No proposition not containing this concept could then analytically imply a proposition containing that concept.[3, P. 177]

Fine offers a preliminary method of incorporating this suggestion into the model theory. The contribution of the intensional connective $\to$ is represented by a concept $\gamma_w(\to)$ in each I_w. The definition of γ_w is then revised as follows:

— $\gamma_w(\varphi \circ \psi) = \gamma_w(\varphi) \cup_w \gamma_w(\psi)$ for *extensional connectives* $\circ$
— $\gamma_w(\varphi \to \psi) = \gamma_w(\varphi) \cup_w \gamma_w(\psi) \cup_w \gamma_w(\to)$

If γ_w is understood as an assignment of subject-matter, there are reasons to believe the preliminary suggestion to be too coarse. Our guiding thesis is this:

Remark 1. Intensional connectives are *transformative*; the subject-matter of a conditional *overlaps* the subject-matter of its subformulae but the two are *incommensurable*. The structure and nesting of conditionals influences their subject-matter as well. In contrast, extensional connectives, acting as mere punctuation marks, are inert and add no content.

To illustrate the proposed *incommensurability*, consider the following statements, intended to give readings to formulae $\varphi \to \psi$ and $\varphi \supset \psi$, respectively:

1. The concept *bachelor* analytically contains the concept of *being unmarried*.
2. Every member of the class of bachelors is unmarried.

Let us investigate the subject-matters by asking what the two are *about*. Intuitively, $\varphi \to \psi$ is *about* the concepts *bachelor* and *being unmarried*—and a purported relationship between them; it is not *about* ground facts about bachelors. $\varphi \supset \psi$, on the other hand, is *about* particular instances of these concepts; it is an *assertoric* gesture categorizing the individuals populating the world.

In this sense, despite a degree of *a priori* overlap between the subject-matters of $\varphi \to \psi$ and $\varphi \supset \psi$, each is about strictly distinct topics. Consequently, two characteristic axioms of [4] are clearly too strong and should fail in an appropriate modification to AI:

11. $(\varphi \to \psi) \to (\varphi \supset \psi)$
13. $f(\varphi) \to (\varphi \to \varphi)$, $f(\varphi)$ any formula in which φ appears

As far as the influence of structure and nesting, we assert that the subject-matters of $\varphi \to (\psi \to \xi)$ and $(\varphi \to \psi) \to \xi$ seem distinct. The former describes a relationship between φ (on the one hand) and a further relationship that holds between ψ and ξ (on the other); the latter is about an entirely different relationship. If the content of a conditional is determined in part by the depth and ordering of its subconditionals, an adequate refinement to Parry's AI is likely to enjoy a property like Brady's notion of *depth relevance* (see *e.g.* [1]).

Apart from these modest suggestions, our position is to remain largely *agnostic* about the subject-matter of a conditional.

Positive Remarks

Despite our agnosticism, there are some positive steps that can be taken. First, we have noted that important distinctions are obliterated by Fine's preliminary modification to his model theory. In response, we first introduce a slightly revised model theory AI$\rightarrow$ that respects the theses of our Remark 1:

Definition 2. An AI$\rightarrow$ model modifies Definition 1 by adding:

— for each $w \in W$ there is function $\rightarrow_w : I_w \times I_w \longrightarrow I_w$

For these models, we retain the truth conditions but modify γ_w as follows:

— $\gamma_w(\varphi \rightarrow \psi) = \gamma_w(\varphi) \rightarrow_w \gamma_w(\psi)$

The modest assumptions made of $\rightarrow_w$ respect our doctrine of agnosticism.

Given the presence of the $\rightarrow_w$ functions, we are able to characterize arguably plausible conditions on $\rightarrow_w$. First, consider the following axiom from [4]:

8. $(\varphi \rightarrow \psi \wedge \xi) \rightarrow (\varphi \rightarrow \psi)$

Axiom 8 seems like the most plausible thesis about inclusion relationships between subject-matters of conditionals among Parry's axioms. There is a sense in which the axiom communicates a true *analytic decomposition* of the antecedent's subject-matter. We can ensure its validity against a particular class of AI$\rightarrow$ models enforcing a semantic condition:

Proposition 3. *Axiom 8 is valid in those models where each $\rightarrow_w$ satisfies:*

$$a \rightarrow_w b \leq_w a \rightarrow_w (b \cup_w c)$$

Proof. The above semantic condition immediately entails that $\gamma_w(\varphi \rightarrow \psi) \leq_w \gamma_w(\varphi \rightarrow \psi \wedge \xi)$, so we focus on the veridical component of the truth conditions.

Suppose that there exists an accessible w' such that $w' \Vdash \varphi \rightarrow \psi \wedge \xi$ but $w' \nVdash \varphi \rightarrow \psi$. There are two explanations for the failure of $\varphi \rightarrow \psi$. If there is an accessible w'' such that $w'' \Vdash \varphi$, then because $w' \Vdash \varphi \rightarrow \psi \wedge \xi$, also $w'' \Vdash \psi \wedge \xi$, whence $w'' \Vdash \psi$. So it must hold that $\gamma_{w'}(\psi) \nleq_{w'} \gamma_{w'}(\varphi)$. But this is impossible; between $\gamma_{w'}(\psi) \leq_{w'} \gamma_{w'}(\psi \wedge \xi)$ (by conditions on $\gamma_{w'}$) and $\gamma_{w'}(\psi \wedge \xi) \leq_{w'} \gamma_{w'}(\varphi)$ (because $w' \Vdash \varphi \rightarrow \psi \wedge \xi$), we know that $\gamma_{w'}(\psi) \leq_{w'} \gamma_{w'}(\varphi)$. $\square$

The semantic condition characteristic of Axiom 8 is modest and plausible, which reinforces the intuition that the axiom is justified. We proceed to consider several less obvious—yet reasonable—theses, including the following axiom:

7. $(\varphi \to \psi) \wedge (\psi \to \xi) \to (\varphi \to \xi)$

While less obviously correct than Axiom 8, there is clear intuitive appeal to Axiom 7. It admits an intuitive reading that the *analyticity* of hypothetical syllogism means that it adds no content beyond its premises.

As in the case of Axiom 8, we describe a condition on $\to_w$ that validates the thesis.

Proposition 4. *Axiom 7 is valid in models in which each $\to_w$ satisfies:*

$$a \to_w c \leq_w (a \to_w b) \cup_w (b \to_w c)$$

Proof. The condition immediately establishes that $\gamma_w(\varphi \to \xi) \leq_w \gamma_w((\varphi \to \psi) \wedge (\psi \to \xi))$, satisfying the content inclusion half of the truth conditions.

Suppose that there is an accessible world w' at which $w' \Vdash \varphi \to \psi$, $w' \Vdash \psi \to \xi$, but $w' \nVdash \varphi \to \xi$. The failure must be either due to the content or the alethic component of the truth conditions. We know that $\gamma_{w'}(\xi) \leq_{w'} \gamma_{w'}(\psi) \leq_{w'} \gamma_{w'}(\varphi)$, whence $\gamma_{w'}(\xi) \leq_{w'} \gamma_{w'}(\varphi)$. So the failure requires the existence of an accessible w'' such that $w'' \Vdash \varphi$ but $w'' \nVdash \xi$. But $w' \Vdash \varphi \to \psi$ entails that $w'' \Vdash \psi$, which jointly with $w' \Vdash \psi \to \xi$ entails that $w'' \Vdash \xi$. □

The difficulty with Axiom 7 is that it is a thesis about the inclusion of a single conditional's subject-matter within the *fusion* of two distinct conditionals. To conclude, we consider Axioms 9 and 10 of [4], which have a similar structure:

9. $((\varphi \to \xi) \wedge (\psi \to \zeta)) \to (\varphi \wedge \psi \to \xi \wedge \zeta)$
10. $((\varphi \to \xi) \wedge (\psi \to \zeta)) \to (\varphi \vee \psi \to \xi \vee \zeta)$

We don't have overwhelming intuitions about Axioms 9 and 10, other than to say that their plausibility is on a par with that of Axiom 7. Nevertheless, it is a worthwhile exercise to establish a corresponding semantic condition:

Proposition 5. *Axioms 9 and 10 are valid in models satisfying:*

$$(a \cup_w b) \to_w (c \cup_w d) \leq_w (a \to_w c) \cup_w (b \to_w d)$$

Proof. By the semantic condition on $\to_w$, $(\gamma_w(\varphi) \cup_w \gamma_w(\psi) \to_w \gamma_w(\xi) \cup_w \gamma_w(\zeta)) = \gamma_w(\varphi \wedge \psi \to \xi \wedge \zeta)$ is included in $\gamma_w(\varphi \to \xi) \wedge (\psi \to \zeta)$.

Now, consider an arbitrary w' such that wRw' and let $w' \Vdash (\varphi \to \xi) \wedge (\psi \to \zeta)$. We show that $w' \Vdash \varphi \wedge \psi \to \xi \wedge \zeta$. First, we treat the alethic component. At any accessible w'' at which $\varphi \wedge \psi$ is true, φ and ψ are true individually. The truth of $\varphi \to \xi$ and $\psi \to \zeta$ then ensure that ξ and ζ, too, are true at w'', whence $w'' \Vdash \xi \wedge \zeta$. Also, the truth of $\varphi \to \xi$ and $\psi \to \zeta$ at w' entails that $\gamma_{w'}(\xi) \leq_{w'} \gamma_{w'}(\varphi)$ and $\gamma_{w'}(\zeta) \leq_{w'} \gamma_{w'}(\psi)$. But the lattice-theoretic properties of $\langle I_{w'}, \cup_{w'} \rangle$ entail that $\gamma_{w'}(\xi) \cup_{w'} \gamma_{w'}(\zeta) \leq_{w'} \gamma_{w'}(\varphi) \cup_{w'} \gamma_{w'}(\psi)$, satisfying the content-theoretic component as well. $\square$

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