Physical vs Virtual Corporate Power Purchase Agreements: Meeting Renewable Targets Amid Demand and Price Uncertainty

Seyed Danial Mohseni Taheri[a], Selvaprabu Nadarajah[a,*], Alessio Trivella[b]

[a]College of Business Administration, University of Illinois at Chicago, Chicago, IL 60607, USA. E-mail: {smohse3@uic.edu, selvan@uic.edu}

[b]Industrial Engineering and Business Information Systems, University of Twente, 7522 NH Enschede, Netherlands. E-mail: a.trivella@utwente.nl

This version: July 5, 2023

Power purchase agreements (PPAs) have become an important corporate procurement vehicle for renewable power, especially among companies that have committed to targets requiring a certain fraction of their power demand be met by renewables. PPAs are long-term contracts that provide renewable energy certificates (RECs) to the corporate buyer and take two main forms: Physical vs Virtual. Physical PPAs deliver power in addition to RECs, while virtual PPAs are financial contracts that hedge (at least partially) power price uncertainty. We compare procurement portfolios that sign physical PPAs with ones that sign virtual PPAs, focusing on fixed-volume contracts and emphasizing uncertainties in power demand and the prices of power and RECs. In particular, we first analyze a two-stage stochastic model to understand the behavior of procurement quantities and costs when using physical and virtual PPAs as well as variants that limit risk. We subsequently formulate a Markov decision process (MDP) that optimizes the multi-stage procurement of power to reach and sustain a renewable procurement target. By leveraging state-of-the-art reoptimization techniques, we solve this MDP on realistic instances to near optimality, and highlight the relative benefits of using PPA types to meet a renewable target.

Keywords: Decision analysis; renewable energy targets; procurement; power purchase agreements; Markov decision processes.

1. Introduction

Power purchase agreements (PPAs) are long-term contracts that facilitate the procurement of renewable electricity by corporations directly from generators to meet load. PPAs are used by corporations to protect against power price volatility, reduce their environmental footprint, and enhance brand image. They have been signed at an increasing pace over the last decade; for instance, the renewable capacity contracted via these contracts has increased from 0.9 GW in 2013 to 5.8 GW in 2017, and to 32.2 GW in 2021 (IEA 2021). Corporate PPAs are either physical or virtual in nature. Physical PPAs, which we refer to as physical contracts (PCs), involve the delivery of power from the producer to the consumer. In contrast, virtual PPAs (also known as "synthetic" contracts and henceforth abbreviated SCs) are financial agreements where the producer

*Corresponding author.

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sells power to the grid, the firm buys power from the grid, and payments for differences relative to a reference strike price are made to ensure a price hedge. Although SCs account for the majority of PPA contracts signed by corporations, there is also significant demand for direct delivery, especially when the producer is located near the offtaker (Baker McKenzie 2018).

In addition to the classification into physical and virtual, different PPAs pricing structures exist. For instance, PPAs typically have a fixed strike price, which helps stabilize procurement costs but some variants use a inflation indexed strike price or apply caps, floors, and interval structures to this price (WBCSD 2021). Furthermore, the amount of supply guaranteed in the contract can take different terms in practice. Among them, fixed-volume PPAs operate on a pre-agreed quantity of electricity, regardless of generation intermittency. They are favorable to the offtaker as they transfer supply risk to the generator. At the other extreme, are as-generated PPAs that consider only the electricity that is generated. These PPAs thus transfer supply risk to the buyer and are favorable to the generator (Greenmatch 2020).

We focus on fixed-volume PPAs for firms with uncertain energy demand, which captures technology firms with a growing demand base as more customers move to the cloud, firms that are exhibiting market growth, and firms that are investing in energy efficiency. The firms we consider have also committed to a renewable procurement target (henceforth target). Such targets require validating by a future date that a specified percentage of annual electricity demand is met by renewable electricity and this level of renewable procurement is sustained thereafter. There are currently over 400 companies with a target of 100% (RE100 2023), and over 5500 companies that have set broader goals (Science Based Targets Initiative 2023).

Both PCs and SCs also provide the buyer with renewable energy certificates (RECs), where each REC allows its owner to validate the use of one megawatt hour (MWh) of renewable power. Meeting a target requires procuring RECs equal to the specified percentage of annual electricity demand. Thus, PPAs have become a popular procurement vehicle of choice for companies with such a target. Although RECs can also be procured from a voluntary market, using this practice as the sole source of RECs has been criticized as green washing because voluntary REC prices are very low and do not necessarily support the addition of renewable power capacity (S&P Global 2021). In contrast, PPAs transfer RECs to the offtaker and provide revenue guarantees that can be used to finance new capacity.

In this paper, we provide (i) analytical results to better understand the effectiveness of PCs and SCs, including fixed and interval strike price structures, and the impact of target levels on procurement costs, and (ii) test state-of-the-art decision heuristics to help companies construct dynamic

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PPA policies to meet a target. In addition to the long-term purchase option presented by a PPA, we assume that the company also has access to short-term options to buy power from the grid and purchase RECs from a secondary market[1].

Our analysis considers a two-stage setting where the long-term power purchase option involves a single PPA. We characterize how the optimal expected procurement cost varies with the target when using PCs and SCs, and establish that this cost under the latter contract is lower than with the former contract. Nevertheless, a drawback cited in the practitioner literature is that using SCs, as opposed to PCs, leads to increased variability in procurement costs (Green Power Partnership 2016). We find that this drawback is indeed true when the same quantity of power is purchased via a PC and an SC. However, contrary to this sentiment, an optimized portfolio containing an SC reduces the quantity of short-term power and RECs purchased relative to a comparable portfolio with a PC, which in turn can decrease the variability of procurement costs under the former portfolio. In the same spirit, specifying a target as a percentage of known past demand (i.e., 80% of 2020 demand) is easier to track and would seem preferable to the target being a fraction of uncertain future demand. However, we find that the latter stochastic target can reduce procurement costs when demand exhibits a negative drift, which is likely for firms that are investing in energy efficiency initiatives. Finally, some companies have also opted to use a variant of the fixed strike price, known as the interval strike price, that allows the strike price to vary within a predefined interval similar to a collar option (WBCSD 2021). We show that interval strike prices employed in industry can reduce the procurement cost relative to a fixed strike price only in a market where power prices are skewed. Hence, interval strike prices must be used with caution since the behavior of power prices in several markets change over time.

To compute dynamic procurement decisions in a multi-stage setting, we formulate a Markov decision process (MDP) that minimizes the expected procurement cost to reach and sustain a target. Specifically, the target does not need to be met in the first part of the planning horizon (i.e., the reach period, typically a few years), while it must be satisfied in all the subsequent stages (i.e., the sustain period). The firm decides whether to enter into new PPAs of different length and size at each stage of the MDP. The strike prices of PPAs are specified by a model that factors the effects of the expected power price and the return on investment required by the generator in a manner that is consistent with publicly available software from the National Renewable Energy Laboratory (NREL; NREL 2017). The set of available PPAs depends on the contracts offered by

[1]A short-term power purchase is akin to a spot purchase. The actual nature of a short-term purchase could vary by region. For instance, in the United States, such a purchase could represent power from an index-based pricing program (DirectEnergy 2018).

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generators, which is unpredictable over time. Approximate methods to solve our MDP are limited because it has a non-convex action set and its state space is high dimensional. We thus leverage both a forecast based primal reoptimization technique (Chand et al. 2002) and a state-of-the-art dual reoptimization technique (Trivella et al. 2023) to obtain procurement policies.

We perform a numerical study on realistic instances based on a planning horizon of 40 years and where PPAs with length spanning from 5 to 25 years can be signed each year. We use market data to calibrate stochastic processes for the evolution of uncertain quantities. We compare the dual reoptimization policy against the one based on primal reoptimization and two additional problem-specific benchmarks. The first heuristic relies on short-term purchases of power and RECs alone, that is, it does not consider PPAs. The second uses a single PPA and renews this contract each time it expires, also allowing for short-term power purchases. The procurement policies computed via dual reoptimization are near-optimal and result in lower procurement costs compared to the remaining benchmarks. In particular, the average dual and primal reoptimization optimality gaps are 2.6% and 5.6%, respectively, for PCs, and 4.1% and 6.5% for SCs. The remaining benchmarks have average optimal gaps of 7.8–16.7% for PCs and 11.2–21.5% for SCs. Thus, dual reoptimization outperforms traditional rolling-planning methods (akin to primal reoptimization) used in practice and its near-optimal performance further strengthens existing results that rolling-planning approaches can be extended to effectively compute dynamic procurement portfolios.

Our numerical experiments also provide procurement insights. Portfolios that include multiple PPAs can reduce procurement costs by 10–17% compared to short-term purchases alone. These savings are significant and especially relevant for energy-intensive industries where electricity is a major cost component, such as data centers (Koot and Wijnhoven 2021) (which we focus on in our numerical study), metal production assets (He and Wang 2017, Trivella et al. 2021), and water treatment plants (Lam et al. 2017). The cost reduction from optimized PPA portfolios increases with the target and ranges from 10–12% for a 60% target to 14–17% under a 100% target, suggesting that PPAs are particularly relevant for companies with aggressive targets. PPA portfolios also lead to procurement costs that are stable when contract availability and the market dynamics of REC prices change, while other policies are significantly more exposed to these changes. Overall, our results support the role of PPAs as useful corporate procurement instruments that help tie the knot between climate oriented targets and financial performance. Finally, our numerical study highlights that SCs lead to lower procurement costs than PCs and their difference can reach up to 4.1% under a 100% target, which is substantial.

To summarize, the main contributions of this paper are the following:

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- In a two-stage setting, we analytically characterize optimal procurement decisions and costs under PCs and SCs as well as other contract variants to provide insights to the structuring of PPAs. These results suggest that SCs and PCs yield significantly different procurement quantities and costs when included in a procurement portfolio – they are thus not merely physical and financial versions of PPAs. Moreover, these findings provide insights relevant to structuring portfolios with PPAs, in some cases, contrary to the intuition in practice.

- In a multi-stage setting, we formulate an MDP to meet a target at minimum procurement cost using PCs or SCs, and leverage reoptimization heuristics to compute dynamic policies.

- We compare reoptimization-based policies and simpler benchmarks on realistic instances and find that dual reoptimization is an effective approach to compute dynamic PPA based procurement policies. We shed slight on the behavior of these policies to market parameters and on the use of physical vs virtual PPA portfolios.

These contributions add to the limited research on using PPAs for the procurement of renewable power. Focusing on a single PPA, Mendicino et al. (2019) proposes a levelized cost of energy model to determine the contract price and length. In a similar spirit, Parlane and Ryan (2020) characterize the contract that minimizes the cost of procuring a given amount of electricity from risk-averse renewable energy generators. A few papers consider dynamic procurement of power. Pedrini et al. (2020) considers the power procurement of a large consumer that can either invest in a renewable asset or sign bilateral contracts with a duration of 1 to 3 years. Gabrielli et al. (2022) and Arellano and Carrión (2023) consider the dynamic procurement of power using PPAs. The former paper studies the risk-reward trade-off of using multi-location and multi-technology PPA portfolios. The latter paper considers a portfolio of PCs and SCs. None of the papers on dynamic power procurement above consider RECs as they do not model targets and also assume that future PPA prices are given, as opposed to being uncertain, which is a reality that we model. Trivella et al. (2023) studies the corporate PPA-based procurement to meet a target considering uncertainty in REC and PPA prices by solving an MDP formulation. They consider as-generated SCs for a firm with fixed demand, while we study fixed-volume PPAs for a firm with stochastic demand and compare physical and synthetic variants. Our MDP embeds a well-known PPA pricing approach developed by NREL, which to our knowledge has not been used in a dynamic procurement context. Our analytical study of PCs and SCs and the impact of targets and strike price structure is new to the literature.

Most literature on dynamic optimization of energy procurement has adopted approaches based

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multi-stage stochastic programming (Rocha and Kuhn 2012, Gilbert et al. 2015, Pedrini et al. 2020, Gabrielli et al. 2022, Arellano and Carrión 2023). Rocha and Kuhn (2012), an early paper in this literature, studies the value of using a electricity procurement portfolio containing spot purchases, forward contracts, and options over pure spot procurement but does not consider PPAs. As alluded to above, Pedrini et al. (2020) use a small number of periods. They highlight in their procurement context that scaling to more periods and scenarios is challenging without using scenario reduction and decomposition techniques. Gilbert et al. (2015) support their choice of stochastic programming by highlighting that alternatives such as approximate dynamic programming (Bertsekas 2011, Powell 2011) will find it difficult to account for non-anticipativity constraints along with the combinatorial aspects of tracking contracts over time. We adapt the dual reoptimization approach introduced in Trivella et al. (2023) that is based on approximate dynamic programming principles but ultimately leverages deterministic optimization as do efficient stochastic programming approaches. It is capable of handling a large number of stages and high-dimensional state spaces, as well as non-convexities. Dual reoptimization relaxes the non-anticipativity constraints within the information relaxation and duality framework (Andersen and Broadie 2004, Haugh and Kogan 2004, Brown et al. 2010) and solves multiple deterministic optimization models to account for the effect of uncertainty on decisions. We show that the dual reoptimization policy is near optimal in our setting, which facilitates subsequent numerical experimentation to obtain novel procurement insights. We also test a forecast-based primal reoptimization heuristic that replaces uncertain factors by their respective forecasts at each stage (Chand et al. 2002, Bertsekas 2005, Weber et al. 2009). Others have used reoptimization together with stochastic programming (see, e.g., Guigues and Sagastizábal 2012) or to more efficiently handle time structured deterministic problems (see, e.g., Glomb et al. 2022).

The rest of the paper is organized as follows. In §2, we analyze procurement quantities and costs of different PPA structures in a two-stage procurement setting. In §3, we formulate a multi-stage MDP to reach and sustain a target, and present reoptimization methods to obtain policies and lower bounds for this MDP. We conduct an extensive numerical study and discuss our findings in §4. We conclude in §5. All proofs can be found in an online supplement.

2. Corporate power purchase agreement structures

In this section, we analyze different PPA structures using simplified models that capture key tradeoffs. In §2.1, we characterize the behavior of procurement quantities and costs as functions of the target. In §2.2, we examine targets defined as a percentage of known past demand rather than uncertain future demand. In §2.3, we study the impact of moving from a fixed strike price to one

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that varies within an interval.

2.1 Procurement costs and targets

We model a two-stage procurement problem with stages 0 (now) and 1 (future), where a company has committed to satisfy a target at stage 1. Thus, the reach and sustain periods are each one stage. To fulfill the target, the company can (i) enter into a PPA at stage 0 to receive power and RECs from a renewable generator at stage 1, and (ii) procure in stage 1 any unmet power demand and shortfall in the target, after accounting for the stage 0 PPA purchase, using grid power purchases and unbundled RECs, respectively. These procurement decisions depend on the power price, the REC price, and the power demand at stage $i \in \{0, 1\}$, which we denote by P_i (USD/MWh), R_i (USD/MWh), and D_i (MWh), respectively. To simplify notation we define $w_i := (P_i, R_i, D_i)$. Indeed, at stage 0, the vector w_1 is stochastic. We represent the target as a fraction $\alpha \in [0, 1]$ of the stage-1 firm's power demand D_1; given that D_1 is stochastic at stage 0, the target αD_1 is also stochastic. In this setting, the firm determines a stage 0 PPA quantity to minimize the expected cost of procuring long-term and short-term power to satisfy its power demand and the target at stage 1. We formulate models considering physical and synthetic PPA variants.

Signing a (fixed-volume) PC for z MWh results in the physical delivery of this power at stage 1 and a payment of K USD/MWh. Given a target α and a strike price K, the stage-1 procurement cost as a function of z and w_1 is

$$\widetilde{C}_{\mathrm{PC}}(z, w_1; \alpha, K) := K\, z + P_1(D_1 - z)_+ + R_1(\alpha D_1 - z)_+, \tag{1}$$

where the first term represents the stage-1 cost of procuring z MWh of power through a PC signed at stage 0, and the second and third terms represent the expected cost of fulfilling stage-1 shortfalls in meeting total demand and the target, respectively, using the short-term market. The optimal expected procurement cost of a firm using a PC at stage 0 is thus

$$C_{\mathrm{PC}}(\alpha, K) := \min_{z \geqslant 0} \mathbb{E}_0[\widetilde{C}_{\mathrm{PC}}(z, w_1; \alpha, K)], \tag{2}$$

where we use $\mathbb{E}_0[\cdot] \equiv \mathbb{E}[\cdot | w_0]$ and $(\cdot)_+ \equiv \max\{\cdot, 0\}$ for notational convenience[2].

In contrast to a PC, an SC does not require the physical delivery of power. Instead, the generator sells z MWh to the grid and the company purchases the same amount of power from the grid. If the grid price P_1 is greater than the fixed strike price K, the generator pays the company for each MWh the positive difference $P_1 - K$; otherwise, the company pays the generator $K - P_1$. Formally, the firm's stage-1 cost function is

$$\widetilde{C}_{\mathrm{SC}}(z, w_1; \alpha, K) := P_1 D_1 + (K - P_1)z + R_1(\alpha D_1 - z)_+, \tag{3}$$

[2]The short-term procurement cost at stage 0 is excluded because it is a constant and does not affect the choice of z.

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where the first term is the cost of purchasing the stage-1 power demand from the grid, the second is the cash flow resulting from difference payments between the generator and the company on the z MWh contracted via the SC when the grid price deviates from the strike price, and the third is the cost of procuring RECs to meet the target shortfall. The optimal expected procurement cost when using an SC is

$$C_{\mathrm{SC}}(\alpha, K) := \min_{z \geqslant 0} \mathbb{E}_0[\widetilde{C}_{\mathrm{SC}}(z, w_1; \alpha, K)], \tag{4}$$

Next we compare models (2) and (4) under the following assumption.

Assumption 1. *It holds that (i) the strike price K belongs to the interval $\big[\mathbb{E}_0[P_1], \mathbb{E}_0[P_1 + R_1]\big]$; (ii) the power demand D_1 is uniformly distributed in the interval $[a, b]$, where a and b are positive scalars satisfying $b > a$; (iii) the power price P_1 follows a log-normal distribution; (iv) the expected REC price $\mathbb{E}_0[R_1]$ is positive; and (v) the power demand D_1 is independent of the prices P_1 and R_1.*

The domain of the strike price captures practically relevant values for the parameter K. The lower bound of $\mathbb{E}_0[P_1]$ avoids cases where the generator is better off selling its power directly to the grid as opposed to the company via a PPA, while the upper bound of $\mathbb{E}_0[P_1 + R_1]$ removes situations where the company would save money from procuring power and RECs directly from the short-term market instead of using a PPA. The log-normal assumption on the stage-1 power price is consistent with the long-term components of common electricity price models such as one-factor and two-factor mean-reverting stochastic processes used in the literature, which consider the evolution of the logarithm of the power price[3] (Lucia and Schwartz 2002, Cartea and Figueroa 2005). We do not assume any specific disitrbutional form for the REC price but requires its mean to be positive, which is consistent with the behavior of REC prices across markets in the United States. The uniformly distributed power demand can be viewed as adding variability around a long-term demand forecast. Finally, our assumption of independence between the power price and the power demand stems from the power demand of an individual company not being large enough to affect the market price and companies in several sectors (e.g., high-tech and education) having limited flexibility to adjust their power consumption to fluctuations in the power price. The independence of demand and REC prices follows similar justification.

Proposition 1 characterizes the optimal PPA procurement quantity. Let z^*_{PC} and z^*_{SC} denote the optimal solutions of models (2) and (4), respectively. Further, we define a target threshold $\bar{\alpha} := a\mathbb{E}_0[R_1]/(b\mathbb{E}_0[R_1] - (K - \mathbb{E}_0[P_1])(b - a))$.

[3] Unlike short-term power prices, the long-term power prices that we model do not take negative values, which justifies considering the evolution of the logarithm of the power price.

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Proposition 1. *Under Assumption 1, we have*

$$z^*_{PC} = \begin{cases} \alpha\left(b - \frac{K-\mathbb{E}_0[P_1]}{\mathbb{E}_0[R_1]}(b-a)\right), & if\ \alpha \leqslant \bar{\alpha}, \\ \frac{-K(b-a)+(\mathbb{E}_0[R_1]+\mathbb{E}_0[P_1])(b)}{\frac{1}{\alpha}\mathbb{E}_0[R_1]+\mathbb{E}_0[P_1]}, & if\ \alpha > \bar{\alpha}; \end{cases} \quad and \quad z^*_{SC} = \alpha\left(b - \frac{K-\mathbb{E}[P_1]}{\mathbb{E}[R_1]}(b-a)\right).$$

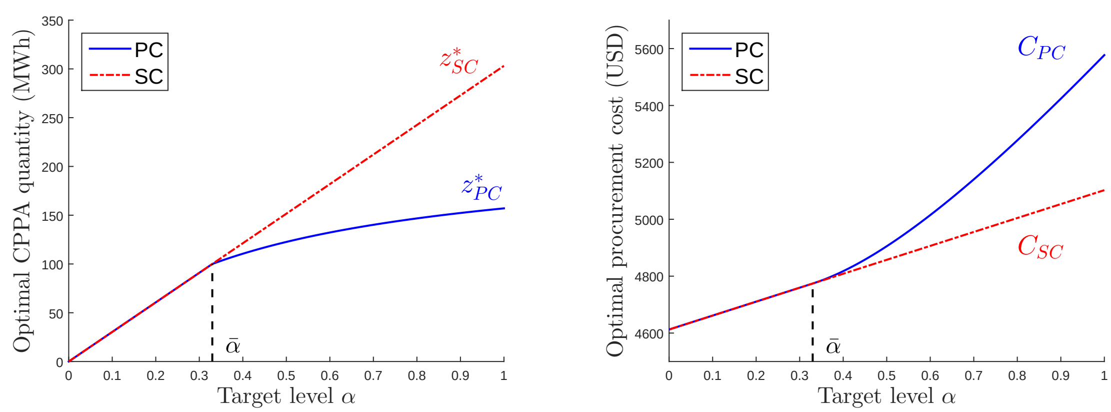
Both the optimal PC and SC procurement quantities are equal and vary linearly with α within the interval $[0, \bar{\alpha}]$. However, for α greater than $\bar{\alpha}$, z^*_{PC} and z^*_{SC} diverge. Specifically, z^*_{SC} continues to vary linearly with α while z^*_{PC} is an increasing concave function of the target. Example 1 and Figure 1(a) illustrate this behavior and show that the optimal PC procurement quantity can be substantially smaller than the SC optimal procurement quantity.

Example 1. *Suppose D_1 is uniformly distributed in the interval $[100, 350]$, P_1 is lognormal with $\mathbb{E}_0[P_1] = 20.5$ USD/MWh, and $\mathbb{E}_0[R_1] = 8$ USD/MWh. Moreover, the strike price K is 22 USD/MWh. In this setting, the target threshold $\bar{\alpha}$ equals 0.32. Figure 1(a) displays the optimal procurement quantities z^*_{PC} and z^*_{SC} as functions of α. For α equal to 1, z^*_{SC} (= 157 MWh) is roughly 93% smaller than z^*_{PC} (= 303 MWh).*

Figure 1: Optimal procurement quantities and costs as a function of the target.

(a) Optimal PPA procurement quantity

(b) Optimal procurement cost



For $\alpha > \bar{\alpha}$, the conservative long-term procurement using a PC can be attributed to over-procurement risk, that is, the event when z is greater than the stage-1 demand. Such over-procurement risk is significant for large α in a PC because one needs to pay for the contracted power even when all of it is not needed[4]. In contrast, this risk is mitigated when using an SC by the fact that the cor-

[4]Results analogous to the assumed Take-and-pay PC also hold when comparing the optimal procurement quantities of an SC and a Take-or-pay PC. However, here one would need to factor in the penalty a company pays for not taking delivery of the contracted power in the Take-or-pay PC when $z > D_1$. We omit this analysis as it does not provide sufficiently new insights but complicates the exposition of the key differences between a PC and an SC.

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poration purchases only the required power from the grid. Over-procurement risk has important practical implications on the procurement cost and its variance, which we study below.

Proposition 2 characterizes optimal expected procurement costs when using a PC and an SC.

Proposition 2. *Suppose Assumption 1 is true. The following hold:*
(a) $C_{PC}(\alpha, K)$ *is linear in* α *for* $\alpha \in [0, \bar{\alpha}]$*, and strictly convex in* α *for* $\alpha \in (\bar{\alpha}, 1]$*;*
(b) $C_{SC}(\alpha, K)$ *is linear in* α *for all* $\alpha \in [0, 1]$*;*
(c) $C_{SC}(\alpha, K) = C_{PC}(\alpha, K)$ *for* $\alpha \in [0, \bar{\alpha}]$*, and* $C_{SC}(\alpha, K) < C_{PC}(\alpha, K)$ *for* $\alpha \in (\bar{\alpha}, 1]$*.*

The behavior of the optimal procurement costs as a function of α are driven by the structure of the optimal procurement quantities in Proposition 1. Specifically, within the interval $[0, \bar{\alpha}]$, the procurement costs when using a PC and an SC are both equal and linear in α. For $\alpha > \bar{\alpha}$, the former cost is increasing convex while the latter remains linear. This difference can be attributed to an interplay between over-procurement (i.e., $z \geqslant D_1$) and under-procurement (i.e., $z < D_1$) risks. To elaborate, a PC procures less long-term power than an SC for large α due to over-procurement risk as already discussed above. The resulting smaller z exposes PC to procuring more power and RECs from the short-term market, an expensive option, when an under-procurement event occurs, which amounts to higher expected costs. This finding suggests that an SC allows a company to manage expected procurement costs more efficiently for high targets than a PC. Example 2 illustrates the preceding discussion by considering under- and over-procurement components of the expected stage-1 cost. Formally, we have

$$\begin{aligned}\mathbb{E}_0[\widetilde{C}_{\mathrm{PC}}(z^*_{\mathrm{PC}}, w_1; \alpha, K)] &= \mathbb{E}_0[\widetilde{C}_{\mathrm{PC}}(z^*_{\mathrm{PC}}, w_1; \alpha, K) | z^*_{\mathrm{PC}} \geqslant D_1] \Pr(z^*_{\mathrm{PC}} \geqslant D_1) \\ &\quad + \mathbb{E}_0[\widetilde{C}_{\mathrm{PC}}(z^*_{PC}, w_1; \alpha, K) | z^*_{\mathrm{PC}} < D_1] \Pr(z^*_{\mathrm{PC}} < D_1),\end{aligned}$$

where we refer to the first and second terms in the right-hand-side of the equality as the over- and under-procurement components of the PC expected cost, respectively. Analogous definitions hold for the SC expected cost.

Example 2. *For the setting considered in Example 1, Figure 1(b) displays the expected procurement costs* $C_{PC}(\alpha, K)$ *and* $C_{SC}(\alpha, K)$ *as functions of* α*. The procurement cost when using a PC (= 5,570 USD) is roughly 9% greater than the analogous cost under an SC (= 5,102 USD) for* α *equals* 1*. The over- and under-procurement components of the PC procurement costs are 800 USD and 4,770 USD, respectively. The analogous cost components corresponding to SC equal 3720 and 1,382 USD, respectively. Therefore,* z^*_{SC} *being greater than* z^*_{PC} *leads to higher expected procurement costs when there is over-procurement but reduces the exposure of SC to the short-term market. This reduced*

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exposure leads to a much smaller expected cost when there is under-procurement, which leads to $C_{SC}(1, K)$ *being strictly smaller than* $C_{PC}(1, K)$.

The expected procurement cost measure discussed above does not consider an important motivation for companies entering into long-term contracts, which is to reduce or eliminate the variability of future costs – stable costs facilitate budgeting. A related sentiment in the practitioner literature is that using an SC could increase the cash flow variability compared to a PC. Proposition 3 shows that this sentiment is in fact true when procuring the same quantity of power using both PPA types. We use $\mathrm{Var}[\cdot]$ to represent the variance of a random variable.

Proposition 3. *Under Assumption 1, we have* $\mathrm{Var}[\widetilde{C}_{SC}(z, w_1; \alpha, K)] = \mathrm{Var}[\widetilde{C}_{PC}(z, w_1; \alpha, K)]$, *if* $\Pr(z > D_1) = 0$, *and* $\mathrm{Var}[\widetilde{C}_{SC}(z, w_1; \alpha, K)] > \mathrm{Var}[\widetilde{C}_{PC}(z, w_1; \alpha, K)]$, *if* $\Pr(z > D_1) > 0$.

To gain some intuition on this result, note that $\widetilde{C}_{\mathrm{PC}}(z, w_1; \alpha, K)$ and $\widetilde{C}_{\mathrm{SC}}(z, w_1; \alpha, K)$ are equal if $z \leqslant D_1$, that is, when there is under-procurement. Therefore, if the over-procurement risk is zero, we have the same variance of cash flows under a PC and an SC. Instead, if the over-procurement risk is positive, the comparison of cash flow variance becomes more involved but we can establish that the variance under an SC is greater than with a PC. Nevertheless, Proposition 3 may not hold for the optimized and unequal power procurement quantities computed by solving (2) and (4) for $\alpha > \bar{\alpha}$. Example 3 provides an instance where the variance of an optimized portfolio with an SC is smaller than an optimized portfolio containing a PC for large α. This example suggests that optimizing PPA purchases is important because a suboptimal SC portfolio may lead to high variance in procurement costs but optimizing this portfolio can mitigate this effect.

Example 3. *Consider an instance with* $\alpha = 0.9$, *uniformly distributed demand* D_1 *in the interval* $[100, 200]$, *and strike price* K *equal to* 22 *USD/MWh. For simplicity, we assume a deterministic power price* P_1 *equal to* 20.5 *USD/MWh and a deterministic REC price* R_1 *of* 3.5 *USD/MWh. Invoking Proposition 1, we find* z^*_{PC} *and* z^*_{SC} *to be* 107 *MWh and* 141 *MWh, respectively. Moreover, we have* $\mathrm{Var}[\widetilde{C}_{PC}(z^*_{PC}, w_1; \alpha, K)]$ *(i.e., optimal PC cost variance),* $\mathrm{Var}[\widetilde{C}_{SC}(z^*_{PC}, w_1; \alpha, K)]$ *(i.e., variance of suboptimal SC cost evaluated at* z^*_{PC}*), and* $\mathrm{Var}[\widetilde{C}_{SC}(z^*_{SC}, w_1; \alpha, K)]$ *(i.e., optimal SC cost variance) equal to 445,840; 455,160; and 394,330, respectively. These values show that the suboptimal SC portfolio that procures long-term power equal to* z^*_{PC} *has higher variance than the optimal PC portfolio, which is consistent with Proposition 3. In contrast, in this example, the variance of an optimized SC portfolio is lower than that of an optimal PC portfolio.*

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2.2 Deterministic renewable target

Thus far, we have assumed a "stochastic" target, that is, a procurement target defined as a percentage of uncertain future demand. Companies also define a "deterministic" target with respect to known past demand as this is easier to track (CDP et al. 2017). Thus, there are two possible choices for reference demand used in practice when specifying a target.

In this section, we analyze procurement costs for a PC and an SC under a deterministic target and compare it with analogous costs under the stochastic target discussed in §2.1. A deterministic target involves satisfying a fraction α of known demand $\bar{D}$, that is, the target is $\alpha\bar{D}$. Choosing a deterministic target only affects the term corresponding to the shortfall in meeting the renewable target in the cost functions of PCs and SCs, i.e. equations (1) and (3), respectively. Specifically, the expression $\mathbb{E}[R(\alpha D - z)_+]$ in both functions is replaced by $\mathbb{E}[R](\alpha\bar{D} - z)_+$.

An analogous result to Proposition 2 holds for the expected procurement cost of PCs and SCs in the presence of a deterministic target (we omit this result for brevity). However, the value of setting a deterministic target instead of a stochastic target is unclear and depends on both future and past power demands. Proposition 4 characterizes a region for $\bar{D}$ in which PCs and SCs with deterministic targets can lead to higher procurement costs than with stochastic targets. In (5a) and (5b) we define the optimal procurement cost, respectively, in PCs and SCs when the target is deterministic.

$$C_{\mathrm{PC,D}}(\alpha, K) := \min_{z \geqslant 0} \left\{ Kz + \mathbb{E}\left[P(D-z)_+\right] + \mathbb{E}[R](\alpha\bar{D} - z)_+ \right\}; \tag{5a}$$

$$C_{\mathrm{SC,D}}(\alpha, K) := \min_{z \geqslant 0} \left\{ \mathbb{E}[PD] + \mathbb{E}[(K-P)z] + \mathbb{E}[R](\alpha\bar{D} - z)_+ \right\}. \tag{5b}$$

Proposition 4. *Suppose Assumption 1 holds. Then, for each $\alpha \in (0,1]$, $C_{SC}(\alpha, K) < C_{SC,D}(\alpha, K)$ if and only if $\bar{D} > \frac{1}{2\alpha} z^*_{SC} + \frac{b}{2}$. Moreover, assuming $\mathbb{E}[R] \leqslant \mathbb{E}[P]$, there exists an $\alpha \in (0,1]$ such that $C_{PC}(\alpha, K) < C_{PC,D}(\alpha, K)$ if $\bar{D} > \mathbb{E}[D] + \frac{(b-a)}{2} \frac{\mathbb{E}[P]}{\mathbb{E}[R]+\mathbb{E}[P]}$.*

This proposition provides support to the fact that deterministic targets are not always cost-beneficial and companies with such targets might incur higher procurement costs compared to a stochastic target. In particular, using a stochastic target can lower expected procurement costs when future power is smaller than past demand, for instance, due to investments in energy efficiency improvements (see, e.g., the supplement of CDP et al. 2017 for more details on companies with both a renewable power target and energy efficiency initiatives). In this case, specifying an $\alpha\%$ deterministic target can lead to procuring more MWh of renewable power than specifying this percentage with respect to uncertain future demand that is unlikely to exceed its historic value.

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Our findings bode well for the use of stochastic targets given recent efforts by companies to reduce their power consumption, in spite of adding uncertainty to their target fulfillment.

2.3 Interval strike price

An interval strike price is defined by a pair (K, δ), where K is the baseline strike price and δ the half-length of the interval $[K - \delta, K + \delta]$ in which the strike price is allowed to fluctuate. In particular, if the power price P exceeds the upper bound $K + \delta$, then the generator pays the company the difference $P - K - \delta$ between the power price and this upper bound. Similarly, if the power price P is less than the lower bound $K - \delta$, then the company has to pay the generator the difference $K - \delta - P$. Instead, when P belongs to the interval $[K - \delta, K + \delta]$ no payment between parties occurs. Therefore, unlike the standard PPA in which the cost of procuring a unit of power is fixed, the interval strike price has the following partially variable cost per unit:

$$K^{\text{INT}}(P; K, \delta) = \begin{cases} K + \delta & \text{if} \quad P > K + \delta; \\ P & \text{if} \quad P \in [K - \delta, K + \delta]\,; \\ K - \delta & \text{if} \quad P < K - \delta. \end{cases} \tag{6}$$

We are only aware of SC with interval strike prices and thus focus on this case here, although our results can be easily adapted to the PC setting. The analogue of the procurement problem (4) when using an interval strike price is

$$C_{\text{SC}}^{\text{INT}}(\alpha, K, \delta) := \min_{z \geqslant 0} \left\{ \mathbb{E}_0[\widetilde{C}_{\text{SC}}(z, w_1; \alpha, K^{\text{INT}}(P_1; K, \delta))] \right\}. \tag{7}$$

When $\delta = 0$, the interval strike price defined in (6) becomes $K^{\text{INT}}(P; K, 0) = K$ and the optimization model (7) satisfies $C_{\text{SC}}^{\text{INT}}(\alpha, K, 0) = C_{\text{SC}}(\alpha, K)$. Proposition 5 compares the procurement cost function of interval versus fixed strike prices in SCs that share the same baseline strike price K, but where the former contract can be optimized by choosing the interval length δ. Consistent with Assumption 1, we assume that the power price is log-normally distributed. The mean and standard deviation of power price are $\mathbb{E}_0[P_1]$ and $\mathbb{E}_0[P_1]\sqrt{\exp(\sigma_P^2) - 1}$, respectively, where σ_P^2 denotes the variance of the natural logarithm of the stochastic power price P_1.

Proposition 5. *There exists a value $\delta > 0$ such that $C_{SC}^{\text{INT}}(\alpha, K, \delta) < C_{SC}(\alpha, K)$ if and only if $K > \mathbb{E}_0[P_1]\exp(-\sigma_P^2/2)$. Moreover, if $K > \mathbb{E}_0[P_1]\exp(-\sigma_P^2/2)$, then $C_{SC}^{\text{INT}}(\alpha, K, \cdot)$ attains its global minimum at δ equal to $\sqrt{K^2 - \mathbb{E}_0[P_1]^2\exp(-\sigma_P^2)}$.*

This proposition shows that an interval strike price can reduce the procurement cost relative to a fixed strike price when K exceeds $\mathbb{E}_0[P_1]\exp(-\sigma_P^2/2)$. This threshold is a decreasing function of the variance of the power price distribution. Thus, as σ_P increases sufficiently, it holds that

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$\exp(-\sigma_P^2/2) < 1$ implying that the interval strike price can reduce the cost even when the strike price is less than the expected power price. This behavior can be attributed to the positive skewness of the log-normal power price distribution, which is $\exp(\sigma_P^2)\sqrt{\exp(\sigma_P^2) - 1}$. As σ_P increases, the distribution becomes right-skewed, that is, lower power prices become more probable and an interval strike price contract with an appropriately defined half-length δ can benefit from it. Proposition 5 also characterizes the optimal interval length that minimizes the procurement cost.

Overall, our analysis unveils a potential advantage of interval strike price in SCs, but suggests some caution as this benefit is tied to the skewness of power prices, which can change over time due to changes in mean-reversion among other factors. Empirical evidence from the literature also suggests that the skewness of power prices could be both positive and negative (Lucia and Schwartz 2002, Geman and Roncoroni 2006, Cartea et al. 2009).

3. Dynamic procurement model

In this section, we discuss a dynamic procurement model to assist a firm in meeting a renewable power target in a multi-period setting. In §3.1, we describe the PPA strike price structure. In §3.2, we formulate an MDP that defines an optimal dynamic procurement policy. Since computing this policy is intractable, in §3.3 we describe procurement heuristics that approximate our MDP.

3.1 PPA strike price

A renewable power generator typically sets a PPA strike price to recoup its project investment and maintenance costs as well as a return on investment (NREL 2017). In addition, historical data and models from NREL show that the PPA strike price is affected by several factors including the average quantity of power produced as a fraction of installed capacity (i.e., capacity factor), tax credits, improvements in technology, the contract duration, and the expected power price over the tenor of the contract (NREL 2010, DOE 2016, Wiser and Bolinger 2017). We describe below a model that accounts for these factors and determines a PPA strike price for a given generator and contract. This model provides the strike prices of PPAs used as input to the procurement model that we describe in §3.2.

Consider a renewable power generator that begins production at year i, where i belongs to a discrete set $\mathcal{I} := \{0, \ldots, I-1\}$ containing the years in our planning horizon. The generator has an expected lifetime of L^{P} years, a capacity factor equal to $\theta_i \in (0, 1]$, and incurs a cost of C_i^{INV} capturing the one-time installation and estimated maintenance costs associated with a MW of production capacity as well as any applicable investment tax credit[5]. We assume that there is a

[5]An investment tax credit represents a one-time federal tax deduction equal to a pre-specified percentage of the

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production tax credit[6] of T_i USD per MWh for the next L_i^{T} years and that future cash flows are discounted at rate $r \in (0, 1]$, which can be chosen to also account for the generator's target return on investment. We begin by computing the fixed strike price $\hat{K}_i$ of a PPA that spans the lifetime of the generator using the net present value (NPV) of the contract's cash flows. This approach is consistent with the System Advisor Model[7] (SAM; NREL 2017). The NPV of 1 MW of installed capacity contracted via such a PPA is

$$\mathrm{NPV}_i = \sum_{l=1}^{L^{\mathrm{P}}} r^l \theta_i \hat{K}_i + \sum_{l=1}^{L_i^{\mathrm{T}}} r^l \theta_i T_i - C_i^{\mathrm{INV}}.$$

Setting NPV_i to zero, we obtain the following strike price formula:

$$\hat{K}_i = \frac{1}{\sum_{l=1}^{L^{\mathrm{P}}} r^l} \left[\frac{C_i^{\mathrm{INV}}}{\theta_i} - \sum_{l=1}^{L_i^{\mathrm{T}}} r^l \mathrm{T}_i \right]. \tag{8}$$

Expression (8) captures the dependence of the strike price on generator vintage (i.e., the year that production begins) by treating the production tax credit T_i, investment cost C_i^{INV}, and capacity factor θ_i as time-dependent quantities. Currently, renewable power generators in the U.S. that start construction before 2025 are eligible for productions tax credits for 10 years from the date the facility starts production (EPA 2023) but this status-quo is likely to change with government regulation. Investment costs and capacity factors typically decrease and increase, respectively, over time due to improvements in technology. The capacity factor, in addition, exhibits significant inter-region variation. For instance, in the case of wind power, capacity factors in the "internal" regions of the United States are significantly higher than those of coastal regions (Wiser and Bolinger 2017).

Next, we describe how the strike price $\hat{K}_i$ in (8) can be modified to account for shorter contract lengths and the expected power price over the tenure of the contract. Consider a PPA with a duration of m years that is less than the lifetime L^P of the generator. Shorter contracts result in additional cash flow risk over the period of the generator's life time for which they do not generate revenue (ACORE 2016). We thus define a risk-adjusted strike price $\hat{K}_{i,m} := \hat{K}_i \cdot K_m^+$, where $K_m^+ \geqslant 1$ is a risk factor that inflates the strike price if $m < L^{\mathrm{P}}$ and equals 1 otherwise, that is, $\hat{K}_{i,m} = \hat{K}_i$ when the contract spans the life of the generator. The PPA strike price is not solely determined by NPV but is also tied to the long-term expected power price because higher expected (future) power prices give the generator leverage to increase the PPA price since the company's outside option is expensive (Wiser and Bolinger 2017). To account for this effect, we

installation cost of a renewable power project.

[6] A production tax credit provides a per-kilowatt-hour tax credit for power generation for a fixed number of future years from the installation of a renewable power project.

[7] SAM is an open source performance and financial tool designed by NREL to access the feasibility of renewable energy projects (e.g., wind, solar, or biomass).

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lower bound the PPA strike price by the average power price over the tenure of the contract, which is $\underline{K}_{i,m} := \left(\sum_{l=1}^{m} \gamma^l \, \mathbb{E}\left[P_{i+l} \middle| P_i\right]\right) / \left(\sum_{l=1}^{m} \gamma^l\right)$, where P_i (USD/MWh) is the power price in year i and $\gamma \in (0, 1]$ a yearly discount factor. Our final strike price expression for a contract delivering power for m years starting in year i is

$$K_{i,m} := \max\left\{\hat{K}_{i,m},\, \underline{K}_{i,m}\right\}. \tag{9}$$

3.2 Markov decision process

We assume the firm can enter into PPAs at each year in the planning horizon represented by $\mathcal{I}$ and/or purchase power and RECs from the short-term market. A stochastic target $\alpha \in (0, 1]$ is enforced from year I^{R}. This means that the target does not need to be fulfilled in the first years $\mathcal{I}^{\mathrm{R}} := \{0, \ldots, I^{\mathrm{R}-1}\}$ but must be met in the rest of the planning horizon $i \in \mathcal{I}^{\mathrm{S}} := \{I^{\mathrm{R}}, \ldots, I-1\}$. We refer to sets $\mathcal{I}^{\mathrm{R}}$ and $\mathcal{I}^{\mathrm{S}}$, respectively, as the reach and sustain periods, with $\mathcal{I} = \mathcal{I}^{\mathrm{R}} \cup \mathcal{I}^{\mathrm{S}}$.

We call $\mathcal{M}$ the set of potentially available PPAs, where $m \in \mathcal{M}$ identifies the duration in years of contract m. Signing a contract m at stage i implies a power delivery from stage $i+1$ to $i+m$ at strike price $K_{i,m}$ given by (9)[8]. PPA availability is modeled with binary vectors $a_i := (a_{i,m} \in \{0, 1\}, m \in \mathcal{M})$, where $a_{i,m} = 0$ and $a_{i,m} = 1$ mean that contract m is unavailable and available, respectively, at stage i. Based on availability, the firm determines a power procurement quantity $z_{i,m}$ (MWh)[9]. Specifically, if $a_i = 1$, then $z_{i,m} = 0$, and if $a_i = 0$, then $z_{i,m} \in \{0\} \cup [z_m^{\min}, z_m^{\max}]$, where $z_m^{\min}$ and $z_m^{\max}$ are minimum and maximum procurement limits. Thus, a decision is a continuous-valued vector $\mathcal{Z}_i(a_i) \subseteq \mathbb{R}_+^{|\mathcal{M}|}$ that specifies procurement quantities for each PPA type $m \in \mathcal{M}$.

The MDP state at a stage i is composed by an endogenous and an exogenous state component $(x_i, w_i) \in \mathcal{X}_i \times \mathcal{W}_i$. The former component $x_i := (x_{i,l}, l \in \{0, \ldots, M-1\})$ tracks the on-hold PPA inventory, where $x_{i,l}$ is the power in MWh delivered in year $i+l$ by all PPAs, and evolves according to the vector transition function $f_i(x_i, z_i)$ when a procurement decision $z_i \in \mathcal{Z}_i(a_i)$ is made

$$x_{i+1,l} = f_i(x_i, z_i)_l = \begin{cases} x_{i,l+1} + \displaystyle\sum_{m \in \mathcal{M}: m > l} z_{i,m}, & \text{if } l \in \{0, \ldots, M-2\}; \\ z_{i,M}, & \text{if } l = M-1. \end{cases}$$

The latter component $w_i := (w_{i,k}, k \in \mathcal{K})$ contains the stochastic factors, indexed by $\mathcal{K}$, that drive the dynamics of power price P_i (USD/MWh), REC price R_i (USD/MWh), and electricity demand D_i (MWh/year). This component evolves according to a Markovian process independently of z_i.

Signing a PPA of length m has an associated procurement cost $\sum_{l=1}^{L_{i,m}} \gamma^l K_{i,m}\, z_{i,m}$ over the tenure

[8] Our MDP formulation and solution methodology are flexible to handle other strike price definitions.

[9] The MWh quantity is the product of the contracted capacity in MW, the duration of a period in hours, and the capacity factor of the generator. This is reasonable for long-term procurement planning.

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of the PPA, where $L_{i,m} := \min\{m,\, I - i\}$ equals the number of periods of power delivery within the planning horizon and $\gamma \in [0, 1)$ is the discount factor. Demand not met by power from PPAs at stage i, that is $u_i := \max\{D_i - x_{i,0}, 0\}$, is procured from the short-term market in both the reach and sustain periods at a price P_i USD per MWh. In addition, any shortfall $v_i := \max\{\alpha D_i - x_{i,0}, 0\}$ in meeting the target during the sustain period requires additional REC purchases at R_i USD per MWh. For each stage $i \in \mathcal{I}$, the cost accrued when entering into PCs is shown below.

$$\textbf{PCs:} \quad c_i(x_i, w_i, z_i) = \sum_{m \in \mathcal{M}} \sum_{l=1}^{L_{i,m}} \gamma^l K_{i,m}\, z_{i,m} + \begin{cases} P_i\, u_i, & \text{if } i \in \mathcal{I}^{\mathrm{R}}; \\ P_i\, u_i + R_i\, v_i, & \text{if } i \in \mathcal{I}^{\mathrm{S}}. \end{cases} \tag{10}$$

When using SCs, the cost incurred at stage i is instead:

$$\textbf{SCs:} \quad c_i(x_i, w_i, z_i) = \sum_{m \in \mathcal{M}} \sum_{l=1}^{L_{i,m}} \gamma^l K_{i,m}\, z_{i,m} + \begin{cases} P_i\,(D_i - x_{i,0}), & \text{if } i \in \mathcal{I}^{\mathrm{R}}; \\ P_i\,(D_i - x_{i,0}) + R_i\, v_i, & \text{if } i \in \mathcal{I}^{\mathrm{S}}. \end{cases} \tag{11}$$

Comparing (10) and (11) shows that the procurement cost at stage i is the same for PCs and SCs if the on-hand contracts delivering power at i do not exceed demand, i.e. $x_{i,0} \leqslant D_i$. On the other hand, if $x_{i,0} > D_i$, SCs allow the firm to purchase from the grid only the power that is needed to meet demand. In this case, the term $(D_i - x_{i,0})$ is negative. We assume that the terminal costs when employing PCs and SCs are $c_I(x_I, w_I) := P_I\, u_I + R_I\, v_I$ and $c_I(x_I, w_I) := P_I(D_I - x_{I,0}) + R_I\, v_I$, respectively. In other words, only short-term procurement of power and RECs is possible.

A stage i procurement policy π_i is a collection of stage-dependent functions $\{Z_j^{\pi_i}, j \in \mathcal{I}_i\}$ mapping states to actions, with $\mathcal{I}_i := \{i, \ldots, I-1\}$. Policy π_i is feasible if any state $(x_j, w_j) \in \mathcal{X}_j \times \mathcal{W}_j$ is associated with a feasible action $z_j(x_j, w_j) \in \mathcal{Z}_j(a_j)$ for $j \geqslant i$. An optimal policy then solves

$$V_i(x_i, w_i) := \min_{\pi_i \in \Pi_i} \mathbb{E}\left[\sum_{j \in \mathcal{I}_i} \gamma^{j-i} c_j(x_j^{\pi_i}, w_j, Z_j^{\pi_i}(x_j^{\pi_i}, w_j)) + \gamma^{I-i} c_I(x_I^{\pi_i}, w_I) \,\middle|\, x_i, w_i \right], \tag{12}$$

where Π_i the set of feasible policies, $V_i(x_i, w_i)$ is the MDP value function at stage i and state (x_i, w_i), $\mathbb{E}$ is the expectation with respect to the future exogenous states, and $x_j^{\pi_i}$ is the endogenous state reached in stage j when following the policy π_i starting from the initial state (x_i, w_i).

MDP (12) is challenging to solve due to well-known curses of dimensionality (Bertsekas 2011, Powell 2011). Specifically, the endogenous state x_i and decision z_i are M- and $|\mathcal{M}|$-dimensional continuous vectors, respectively, and the exogenous state w_i may also be high dimensional when using a multi-factor stochastic model for the evolution of uncertainty. Moreover, it can be easily shown that the value function of (12) is non-convex in general. Thus, using ADP methods that rely on convexity to handle high-dimensional endogenous states is not viable. Instead, we employ reoptimization approaches.

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3.3 Procurement heuristics

We consider four procurement heuristics of increasing complexity from Trivella et al. (2023). The simplest heuristic involves only short-term procurement, that is, the entire power demand D_i is purchased on a short-term basis in each stage $i \in \mathcal{I} \cup \{I\}$. A portion αD_i of unbundled RECs is also procured in the sustain period $\mathcal{I}^{\mathrm{S}} \cup \{I\}$ to meet the target. This policy has no demand risk but is fully exposed to volatile power and REC prices. Since stages correspond to years in our numerical setting, hereafter we refer to *short-term* power purchase as a yearly average purchase[10], as opposed to the long-term (i.e. multi-year) power delivery from PPAs.

The second policy, denoted *forecast-based block heuristic* (FBH_m), uses a single PPA of length m and renews it every m years, that is, each time a contract expires a new one of the same length is signed. To elaborate, the first contract is entered at the last year of the reach period, $I^{\mathrm{R}-1}$, and delivers renewable power during the first m years of the sustain period. The second contract is ordered one year before the first contract expires to ensure the continuous delivery of power from PPAs. This process is repeated until the end of the planning horizon. The quantity $z_{i,m} \in \mathcal{Z}_i(a_i)$ associated with a new contract signed at stage i is obtained by solving a deterministic model that minimizes the procurement cost given forecasts of demand, PPA availability, and power and REC prices over the delivery period of m years[11]. Any shortfall in meeting demand or the target using the incumbent PPA is addressed via purchases of short-term power and/or unbundled RECs.

The third policy, called *forecast-based reoptimization heuristic* (FRH), can sign PPAs at any period, thus allowing for the use of a portfolio of PPAs of different lengths. At stage i and state (x_i, w_i), FRH computes procurement decisions as an optimal solution of a math program obtained by replacing random quantities in MDP (12) by their respective forecasts.

$$\min \sum_{j \in \mathcal{I}_i} \gamma^{j-i} c_j\big(y_j, \mathbb{E}[w_j|w_i], z_j\big) + \gamma^{I-i} c_I\big(y_I, \mathbb{E}[w_I|w_i]\big) \tag{13a}$$

$$\text{s.t.: } y_i = x_i, \tag{13b}$$

$$y_{j+1} = f_j(y_j, z_j), \quad \forall\, j \in \mathcal{I}_i, \tag{13c}$$

$$y_j \in \mathcal{X}_j, \quad \forall\, j \in \mathcal{I}_i \cup \{I\}, \tag{13d}$$

$$z_j \in \mathcal{Z}_j(\bar{a}_{i,j}), \quad \forall\, j \in \mathcal{I}_i. \tag{13e}$$

The objective function (13a) is the sum of discounted procurement costs when using forecasts of random quantities. Constraint (13b) initializes the current state. Constraints (13c), (13d),

[10]Our MDP/methods can handle multiple settlements in a year, e.g. monthly, but will require higher simulation time to estimate costs.

[11]Forecasts are conditional expectations of random prices and demand. For contract availability, a stage-j forecast for contract $m \in \mathcal{M}$ made at stage i, with $j \geqslant i$, is defined as $\bar{a}_{i,j,m} = 1$, if $\mathbb{E}[a_{j,m}|w_i] > 0.5$, and $\bar{a}_{i,j,m} = 0$ otherwise.

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and (13e) ensure the feasibility of state transitions, endogenous states, and actions, respectively. Although (13) computes procurement decisions z_j for stages j from i to I, we only implement z_i^* corresponding to the current stage. Then, an analogue of (13) starting from the updated endogenous state $(x_{i+1} = f_i(x_i, z_i^*), w_{i+1})$ and revised forecasts is formulated, and the procedure is repeated until the end of the planning horizon.

Finally, we consider the *information-relaxation based reoptimization heuristic* (IRH), which employs a rolling horizon framework as FRH but relies on the information relaxation and duality theory (Andersen and Broadie 2004, Haugh and Kogan 2004, Brown et al. 2010) to compute procurement decisions. Specifically, a stage i and state (x_i, w_i), the IRH decision is based on the following steps:
(i) Generate H Monte Carlo sample paths of uncertainty $\{W_i^h = (w_i, w_{i+1}^h, \dots, w_I^h),\ h = 1, \dots, H\}$.
(ii) For each sample path $h = 1, \dots, H$, solve the deterministic math program (14)

$$\min \sum_{j \in \mathcal{I}_i} \gamma^{j-i} \Big[c_j\big(y_j, w_j^h, z_j\big) - q_j\big(y_j, z_j, W_j^h\big) \Big] + \gamma^{I-i} c_I(y_I, w_I^h) \tag{14a}$$

$$\text{s.t.:}\ y_i = x_i, \tag{14b}$$

$$y_{j+1} = f_j(y_j, z_j), \ \ \forall\, j \in \mathcal{I}_i, \tag{14c}$$

$$y_j \in \mathcal{X}_j, \ \ \forall\, j \in \mathcal{I}_i \cup \{I\}, \tag{14d}$$

$$z_j \in \mathcal{Z}_j(a_j), \ \ \forall\, j \in \mathcal{I}_i, \tag{14e}$$

which is similar in structure to (13) but uses perfect information over sample path h and a dual penalty $q_i\big(x_i, z_i, W_i\big)$ to correct the costs for this knowledge, where $\mathbb{E}[q_i\big(x_i, z_i, W_i\big) | w_i] \geqslant 0$.
(iii) Consider the stage i optimal sample-dependent decisions $\{z_{i,h}^*, h = 1, \dots, h\}$ and extract a single non-anticipative decision from this distribution, e.g., the mean or the component-wise median. See Trivella et al. (2023) for related theory and details.

Since dual penalties based on MDP value function approximations are difficult to compute in our setting, we consider a dual penalty that is linear in the procurement decision, which is

$$q_i(x_i, z_i, W_i) := \sum_{m \in \mathcal{M}} z_{i,m} \sum_{l=1}^{m} \sum_{k \in \mathcal{K}} \gamma^l \theta \left(w_{i+l,k} - \mathbb{E}[w_{i+l,k} | w_i] \right). \tag{15}$$

We implement IRH using both zero and linear dual penalties in math program (14). These two variants, dubbed IRH_0 and IRH_+, deliver policies but also dual bounds. We consider hereafter the mean decision in step (iii), as we found its performance to be similar to the median decision.

Table 1 summarizes the policies considered in our numerical study. Estimating the value of any of these policies provides an upper bound on the optimal policy cost. This estimation involves Monte Carlo simulation of this policy and averaging the resulting sum of discounted costs across sample paths.

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Table 1: Summary of procurement policies.

	Policy	Description
(Simple heuristics)	Short-term	Short-term purchase of power and RECs
	FBH_m	Forecast-based block heuristic with single PPA m
(Reoptimization heuristics)	FRH	Forecast-based reoptimization heuristic
	IRH_0	Information-relaxation reoptimization with zero dual penalty
	IRH_+	Information-relaxation reoptimization with linear dual penalty

4. Numerical study

In this section, we assess the cost of meeting a target when using PCs and SCs on realistic instances. We describe our instance sets and computational setup in §4.1. In §4.2, we discuss the performance of PCs and SCs to meet a target under different procurement strategies and market conditions. In §4.3, we present the procurement insights resulting from the numerical study.

4.1 Instances and computational setup

We consider a planning horizon of 40 years (I) and a stochastic target of 90% (α) to be attained by year 5 (I^{R}). We use a contract set $\mathcal{M} = \{5, 10, 15, 20, 25\}$ with minimum procurement quantities $z_m^{\min} = 20\,\theta_i$ MWh for all $m \in \mathcal{M}$ based on the portfolio of Google (Google 2016b) following Trivella et al. (2023), and set a very loose upper bound on these quantities ($z_m^{\max} = 1000\,\theta_i$ MWh). We choose an annual discount factor (γ) of 0.97 so that the corresponding risk-free rate is equal to the 10-year United States treasury rate in May 2018 (Bloomberg 2018).

Table 2: Parameters defining the PPA strike price.

Name	Value	Unit	Name	Value	Unit
C_0^{INV}	1.7×10^6	USD/MW	L_i^{T}	10	years
L^{P}	30	years	ξ	1%	-
θ_i	3,066	hours/year	r	0.94	-
T_i	23	USD/MWh	K_+^5	1.1	-

Table 2 defines the parameters of the PPA strike price of §3.1. Following NREL (2010), we use a functional form for C_i^{INV} that decreases over time by a fixed percentage ξ; specifically, it evolves according to a learning model $C_i^{\mathrm{INV}} = C_0^{\mathrm{INV}}(1-\xi)^i$. We chose the initial cost (C_0^{INV}) based on 2015–2016 wind projects in the U.S. (EIA 2018b) and the learning rate (ξ) based on the range of values in NREL (2010). Wind turbines are usually designed to operate for 20–25 years but many remain operational for a longer period of time (Ziegler et al. 2018), thus we select the lifetime (L^{P}) to be 30 years also to account for improving technology. The capacity factor (θ_i) of 35%

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is representative of the observed average for wind farms in the U.S. (EIA 2018a) and is assumed to be fixed throughout the planning horizon. The duration of the production tax credit (L_i^{T}) is based on United States policy in 2023, and its amount (T_i) within the range of values offered to wind energy projects (EPA 2023). Moreover, we assume the tax credit expires in 5 years, i.e., it is only granted to renewable energy facilities commencing construction at stages $i < 5$. We set the generator discount factor (r) such that its respective return on investment is roughly twice the risk-free interest rate. We use a maximum risk factor $K_+^m = 1.1$ for $m = 5$, i.e. a 10% premium for the 5-year PPAs, which decreases linearly as m is increased.

Next, we briefly introduce the stochastic processes we employed to model the evolution of uncertain quantities. We model the power price with a mean-reverting stochastic process with seasonality and jumps and the REC price with a Jacobi diffusion process, as done in Trivella et al. (2023) (see Online Supplement D in that paper for details on the stochastic differential equations, the market data used for calibration, and the fitted parameters). The PPA availability follows Bernoulli random variables, where probabilities $p_m \in [0, 1]$ that contracts are available are set to $\{0.4, 0.5, 0.6, 0.7, 0.4\}$ for $m \in \mathcal{M}$ based on Baker McKenzie (2015) and Wiser and Bolinger (2017).

Additionally, the electricity demand of a company is treated as uncertain due to various factors including technology change, company expansion programs, energy efficiency programs, and environmental conditions. We model this uncertainty using a geometric Brownian motion, which is a common choice in the procurement literature to describe demand uncertainty (Berling and Rosling 2005, Kouvelis et al. 2013, Secomandi and Kekre 2014). The process is defined by

$$dD_t = \mu_D D_t \, dt + \sigma_D D_t \, dW_t, \tag{16}$$

where μ_D, σ_D, and W_t represent drift, volatility, and a standard Brownian motion, respectively. To estimate the parameters of (16), we use as reference the approximate power consumption of Google data centers in the United States (Google 2016a) and estimate the consumption of a facility with two data centers as 600,000 MWh/year, and use this value as D_0. We assume $\mu_D = 0$, that is zero demand drift because of two opposing factors: (i) the increasing size and demand for such centers, which would suggest a positive drift; (ii) improving technology and energy efficiency initiatives implying a negative drift. We chose $\sigma_D = 0.05$ based on Secomandi and Kekre (2014).

In Table 3, we perturb the parameters defining our baseline instance to obtain instance sets S1–S5, comprising of 17 instances in total. These instances allow us to analyze the behavior of procurement policies and the robustness of methods as market parameters change. We describe how these perturbed instances were obtained in §4.2.

All procurement heuristics discussed in §3.3 were coded using C++ and Gurobi as the math

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Table 3: Extended instance sets with the baseline instance parameter superscripted by B.

Set	Modified parameter	Values
S1	Renewable energy target α	$\{0.6, 0.7, 0.8, 0.9^{\text{B}}, 1.0\}$
S2	PPA availability p_m for all $m \in \mathcal{M}$	$\{-0.2, -0.1, 0^{\text{B}}, +0.1, +0.2\}$
S3	Long-term mean of power price	$\{20, 30, 39.7^{\text{B}}\}$ USD/MWh
S4	Long-term mean of RECs price	$\{5, 9.4^{\text{B}}, 20\}$ USD/MWh
S5	Generator discount factor r	$\{0.9, 0.91, 0.92, 0.93, 0.94^{\text{B}}\}$

programming solver. We use 1000 Monte Carlo sample paths to estimate the value of heuristic procurement policies (i.e., upper bounds on the optimal policy value), as this choice resulted in standard errors below 1% of the mean. For the IRH upper bound estimation process, we use 30 inner sample paths at each stage of an evaluation sample path and then computed the mean to back out a non-anticipative control as discussed in §3.3. In this setting, the time taken to compute a decision at a given stage and state is always less than 1 second under each policy. Computing the upper bound of a policy (i.e., applying it over the entire planning horizon in Monte Carlo simulation) takes on average 10 seconds for FBH_m, and 28 and 216 minutes for FRH and IRH (each variant), respectively. Thus, the computational burden of IRH is higher because this method solves at each stage 30 math programs. Finally, estimating the lower bound with IRH takes 10 minutes on average.

4.2 Comparison of physical and virtual contracts

Below we compare the methods in Table 1 on the instance sets S1–S5 considering both PC and SC variants of MDP (12). For each instance, we report the procurement cost (i.e., the expected discounted total cost over the planning horizon) and the optimality gap with respect to the IRH_+ lower bound, labeled LB_+. We omit showing the IRH_0 lower bound because it is worse than LB_+ on average by 5% and 50% for the PC and SC variants, respectively. In the remaining text, when discussing the performance of a method, we are referring to the quality of its procurement policy.

We begin by discussing the results for the instance set S1, which was obtained by varying the target α from 60% to 100%. The corresponding results are displayed in Figure 2. IRH_+ performs best on all the S1 instances and has average optimality gaps of 2.1% and 4.2% in the PC and SC contract settings, respectively. The IRH_0 optimality gap is similar to IRH_+ when using PCs (2.6% on average) but substantially worse under SCs (11% on average). While FRH optimality gaps are smaller than analogous IRH_0 gaps by 3.6% on average when using SCs, the former method is on average 3.2% worse than the latter method under PCs. The performance of short-term and block heuristics is largely inferior to reoptimization methods.

The S2 instances vary the probability p_m of each contract $m \in \mathcal{M}$ from its base value between

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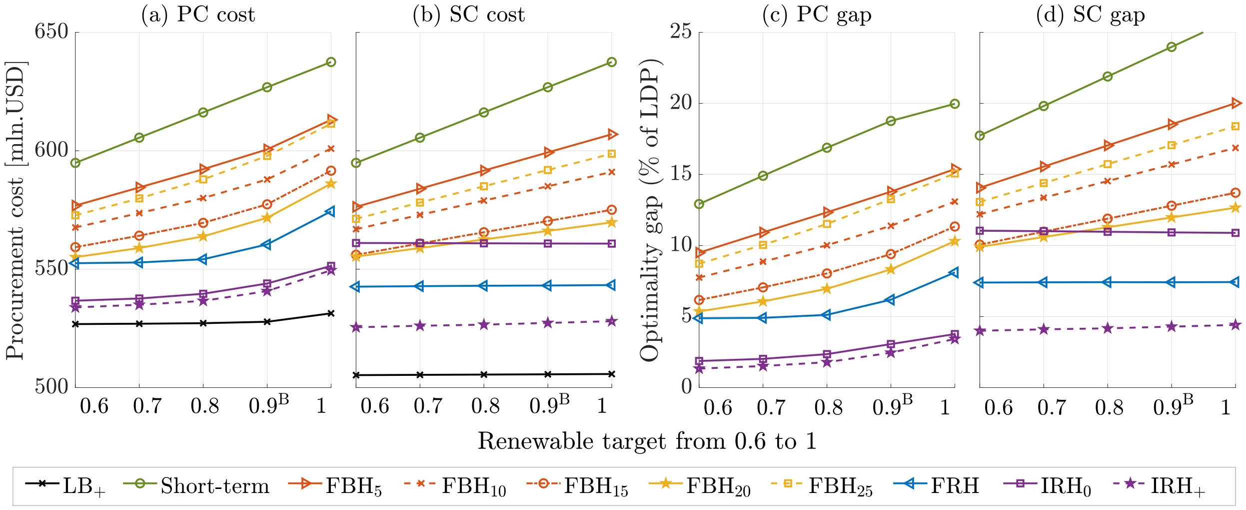
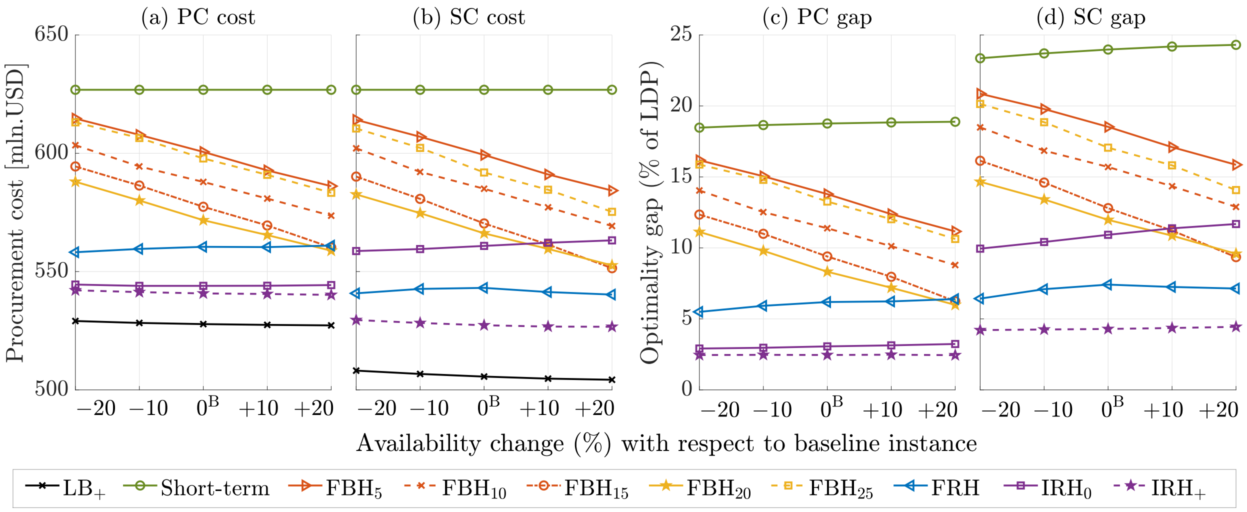
Figure 2: Procurement costs and optimality gaps for the S1 instance set.



−20% and +20% to understand the effect of changing the contract availability on methods. The results corresponding to these instances are reported in Figure 3.

Figure 3: Procurement costs and optimality gaps for the S2 instance set.



The relative ranking of methods is similar to the S1 instances: IRH$_+$ achieves the smallest optimality gaps across instances (2.4% and 4.3% on average for PCs and SCs, respectively). However, contract availability impacts single-contract methods (i.e., FBH$_m$) and multi-contract methods (FRH and IRH) in a markedly different manner. In particular, both the procurement cost and optimality gap of FBH$_m$ increase substantially in the presence of contract shortage. For instance, the BH$_{20}$ procurement cost increases by more than 5% when decreasing contract availability from +20% to −20% relative to the baseline. In contrast, the costs associated with both FRH and IRH, which consider multiple PPAs, are fairly stable under such availability changes (the maximum cost

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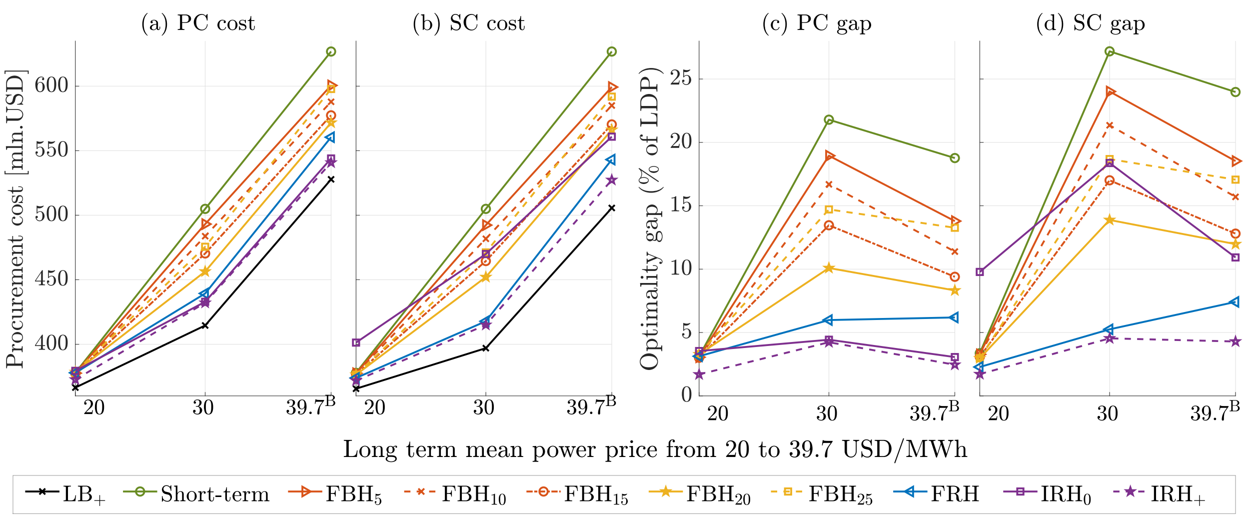
increase is less than 0.8%).

The instance sets S3–S4 are created by varying the long-term mean of power and REC prices. Specifically, the calibrated price model has long-term mean of power and RECs of 39.7 USD/MWh and 9.4 USD/MWh, respectively. Despite the spike in power prices observed in 2022, which is linked to specific geopolitical events, the increasing penetration of renewable energy suggests that in the long-term (e.g., 40 years) prices will decrease (Mills et al. 2017). To understand this effect, we consider the instances S3 in which the long-term mean power price is reduced to 30 USD/MWh and further to 20 USD/MWh. In contrast to the power price, the average REC price can increase or decrease in the long-term due to regulatory changes (EPA 2018). To account for this effect, the long-term mean of the REC price is decreased to 5 USD/MWh and increased to 20 USD/MWh in the instance set S4. Results for the S3 and S4 instance sets are displayed in Figures 4 and 5, respectively

Figure 4: Procurement costs and optimality gaps for the S3 instance set.



The relative performance of methods on these sets are consistent with our prior observations on the S1–S2 instances. If the long-term mean power price decreases, then the procurement cost decreases substantially under all policies. In contrast, when the long-term mean of the REC price changes, the procurement costs and optimality gaps of the spot and BH policies are affected substantially, while the reoptimization methods, FRH and IRH, are stable across instances. This behavior is due to FRH/IRH purchasing a considerable amount of PCs/SCs even when the REC prices are low, which insulates their procurement policies to REC price increases.

The instance set S5 considers changes in the PPA strike price as a result of the generator varying r from 0.94 to 0.9, which models the return on investment changing between 6.4% and 11.1%. The corresponding results displayed in Figure 6 show that the procurement cost increases under all

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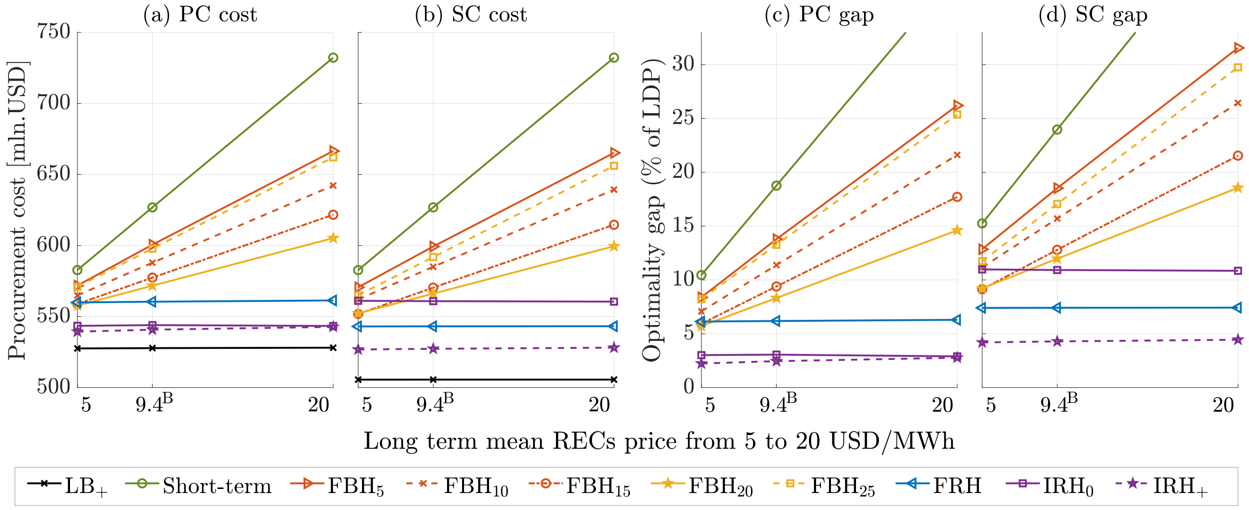
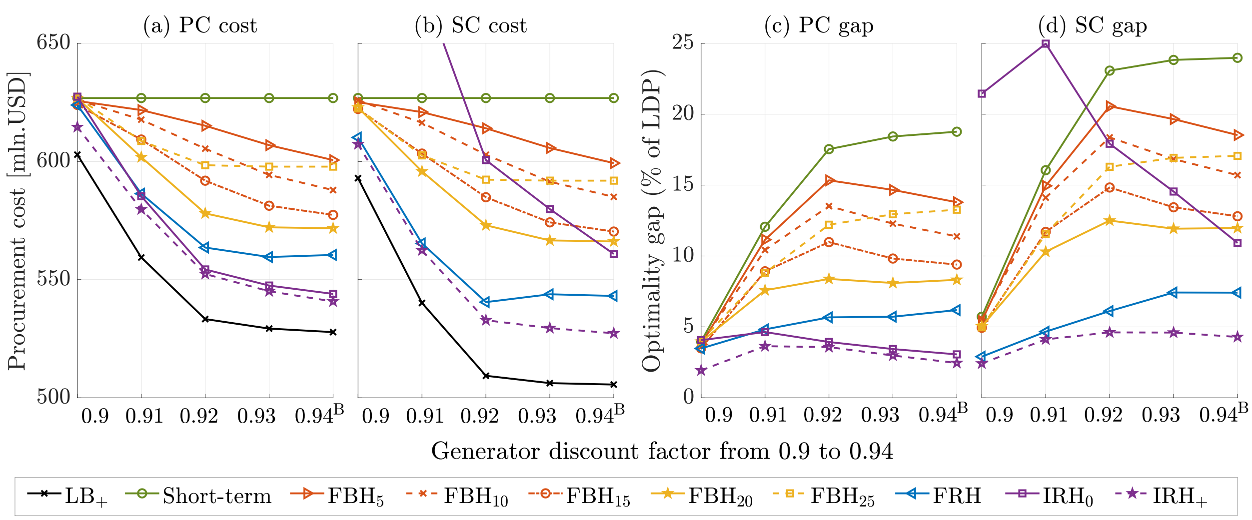
Figure 5: Procurement costs and optimality gaps for the S4 instance set.



methods when r decreases, except for the short-term policy. Equation (9) helps understand this behavior. From this equation it follows that reducing r raises the NPV component of the PPA strike price and potentially the PPA strike price itself. Therefore, FBH_m, FRH, and IRH_+ use less PPAs and rely more on short-term purchases as r decreases, which results in the performances of different methods becoming closer to each other.

Figure 6: Procurement costs and optimality gaps for the S5 instance set.



Overall, using simple procurement heuristics, such as short-term and FBH_m, on our instances results in higher procurement costs, whereas reoptimization methods work well, with IRH_+ outperforming FRH. Consistent with the literature, this superior performance of IRH confirms that obtaining non-anticipative decisions by averaging away future information from anticipative actions is better than computing decisions based on a single (non-anticipative) forecast, as done in

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FRH. The near-optimal and stable performance of IRH_+ across instance sets also indicates that the mean, a simple and inexpensive decision measure, is effective for use with IRH. Similarly, simple linear dual penalties appear sufficient to obtain high quality procurement decisions using IRH, in addition to good lower bounds.

4.3 Procurement insights

Figure 2 shows that the procurement cost increases as expected with the target α. Specifically, the procurement costs under PCs and SCs both vary linearly for $\alpha \leqslant 0.8$ and in a strictly convex and linear manner, respectively, for $\alpha > 0.8$. The procurement costs under SCs are in general lower than analogous costs under PCs (especially for high α values), which provides a cost incentive for using SCs in addition to their well-known advantage of being free from the physical delivery constraints associated with PCs. For instance, the procurement cost incurred under SCs is on average 2.5% lower than under PCs when using IRH_+. This comes at the expense of increased cash flow volatility under SCs as the coefficient of variation is on average 3.5% higher than PCs. These findings are largely in sync with our analytical results in §2.

Consistent with the extant procurement literature, we find that the inclusion of long-term contracts (i.e., PPAs in our case) in procurement portfolios helps hedge against price uncertainty and reduces procurement costs. In addition, our results indicate that the use of PPAs is more valuable to companies that have committed to a renewable power purchase target. This is seen, for instance in Figure 2, where the difference between the procurement costs of the short-term and IRH_+ policies increases with the target. Therefore, as corporations become more aggressive with procuring renewable power, the use of PPAs, in particular SCs, is likely to be higher, which is consistent with trends observed in practice (BNEF 2018). Constructing portfolios with PPAs is non-trivial as shown by the poor performance of the block heuristics that use a single contract type. The fairly flat procurement cost of the IRH_+ policy to changes in contract availability suggests that dynamically constructed portfolios containing multiple PPAs are robust to such variability, which is a useful property. This observation suggests some level of substitutability between different subsets of PPA contracts. However, individual contracts are not fully substitutable for another; if they were, the procurement costs of the block heuristics and IRH_+ would be similar.

Portfolios computed by IRH_+ on our baseline instance contain PCs with lengths 5, 10, 15, 20, and 25 years in the proportions 8.7%, 17.8%, 23.1%, 29.2%, and 21.2%, respectively, on average across the evaluation samples and stages. Analogous proportions when using IRH_+ with SCs are 3.4%, 10.8%, 15.2%, 31.4%, and 39.2%, which shows that longer contracts are used more often

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due to SCs having lower over-procurement risk than PCs as also discussed in §2.1. We observed that the stage-averaged mix of PPAs did not change significantly across instances. For example, on the S1 instances, the proportion of PPAs of different lengths varied by at most 4%. However, this mix does change substantially over time, in a manner that is more pronounced in the reach period, with these changes remaining significant in the rest of the planning horizon. For example, the proportion of 5, 10, 15, 20, and 25 year PPA contracts under IRH_+ in the baseline instance fluctuates by as much as 11.8%, 18.4%, 17.6%, 22.8%, 13.9%, respectively, between years 10 and 40. Thus, the near-optimal portfolios computed by IRH_+ indeed change dynamically over time.

Finally, using PPAs for procurement is not always beneficial. For instance, if these contracts become expensive due to generators expecting a higher rate of return (see Figure 6) then spot procurement would displace signing PPAs and the multi-stage procurement problem will reduce to procuring power and RECs as needed from the short-term market. This seems unlikely given the increasing use of PPAs (IEA 2021) and decreasing production costs associated with renewable power (IRENA 2021). Under low production costs, the strike price that a generator charges is likely to be highly correlated with the spot market, in particular, the expected spot price over the tenor of the contract, a feature that we try to capture in the strike price model of §3.1. In such an environment, generators would remain profitable even after current production tax credits expire resulting in PPAs continuing to play an important role in a firm's renewable power procurement strategy. We find support for this statement in additional experiments that we conducted, where we removed the production tax credit in our base instance and found procurement costs to increase by only 1.3% when using PCs and by 1.7% with SCs.

5. Conclusion

In this paper, we studied the corporate procurement of renewable power to meet a renewable target using physical and virtual fixed-volume PPAs when demand and (REC and power) prices are uncertain. By analyzing a two-stage stochastic model, we characterized optimal procurement quantities and costs in closed form when using SCs and PCs and different strike price structures following the practitioner literature. In particular, we showed that an SC lowers the procurement cost compared to a PC without necessarily increasing volatility. We also established the conditions under which adopting an interval strike price and/or a deterministic target is advantageous. Overall, our analysis provided strategic insights into favorable PPA structures and procurement behavior.

To facilitate tactical power procurement planning, we formulated a multi-period MDP model with short-term and long-term procurement options, which is challenging to solve. Heuristic pro-

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curement decisions can nevertheless be obtained by leveraging an easy-to-implement reoptimization method, FRH, and a state-of-the-art scheme that combines reoptimization and the information relaxation and duality approach, IRH, which also provides a lower bound on the optimal cost. Our numerical findings showed that IRH procurement decisions are near-optimal on realistic instances and outperform FRH and problem-specific heuristics. Moreover, procurement portfolios with multiple PPAs reduce power purchase costs significantly in the presence of a target compared to using a single PPA. In addition, such portfolios are effective at hedging against uncertainty in contract availability and REC prices. Finally, consistent with our two-stage analysis, the results from our multi-period planning model underscored the benefit of using SCs over PCs, as the former contract type leads to procurement costs that are 2.5% lower on average across the 17 instances we tested.

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SUPPLEMENTARY MATERIAL TO:

Physical vs. Virtual Corporate Power Purchase Agreements: Meeting Renewable Targets Amid Demand and Price Uncertainty

Seyed Danial Mohseni Taheri, Selvaprabu Nadarajah, Alessio Trivella

Proofs

To ease notation, in this appendix we represent $\mathbb{E}_0$ by $\mathbb{E}$ and the random variables D_1, P_1, and R_1 by D, P, and R, respectively. Furthermore, we denote by $\mathcal{N}(\mu, \sigma^2)$ a normal distribution with mean μ and variance σ^2, and by ϕ and Φ the probability distribution function (PDF) and cumulative distribution function (CDF), respectively, of a standard normal distribution $\mathcal{N}(0, 1)$.

Lemma 1 is used in the proof of Proposition 1. Define $Q_{\rm PC}(z;\alpha,K) := \mathbb{E}[\widetilde{C}_{\rm PC}(z, w_1;\alpha,K)]$ and $Q_{\rm SC}(z;\alpha,K) := \mathbb{E}[\widetilde{C}_{\rm SC}(z, w_1;\alpha,K)]$ as the expected procurement costs under a PC and an SC, respectively.

Lemma 1. *Under Assumption 1, there exist optimal procurement decisions z^*_{PC} and z^*_{SC} minimizing, respectively, $Q_{PC}(z;\alpha,K)$ and $Q_{SC}(z;\alpha,K)$, that belong to the interval $[\alpha a, \alpha b]$.*

Proof. We start by considering PCs, and then move to SCs. In both cases, we will show that no unique optimal solution exists in $\Omega := [0, \alpha a) \cup (\alpha b, b]$, which establishes the desired result. Recall that the power demand D follows a uniform distribution in the interval $[a, b]$, and its CDF is equal to 0 for $z < a$, $(z-a)/(b-a)$ for $z \in [a, b]$, and 1 for $z > b$.

The derivative of $Q_{\rm PC}(z;\alpha,K)$ with respect to z exists in the interval $[0, \alpha a]$ and is equal to

$$\frac{dQ_{\rm PC}(z;\alpha,K)}{dz}\Big|_{z\in[0,\alpha a]} = K - \mathbb{E}[P]\,Pr\,(D \geqslant z) - \mathbb{E}[R]\,Pr\,(\alpha D \geqslant z) = K - \mathbb{E}[P] - \mathbb{E}[R] \leqslant 0,$$

where the second equality holds since $Pr\,(D \geqslant z) = Pr\,(\alpha D \geqslant z) = 1$ for $z \in [0, \alpha a]$, and the inequality follows from the strike price upper bound (i.e., $K \leqslant \mathbb{E}[P] + \mathbb{E}[R]$) in Assumption 1. Therefore, $Q_{\rm PC}(z;\alpha,K)$ is non-increasing in $[0, \alpha a)$, has no unique optimal solution in this half-open interval, and we can focus on $z \geqslant \alpha a$ to search for an optimal solution. Similarly, the derivative exists in $[\alpha b, b]$ and is equal to

$$\frac{dQ_{\rm PC}(z;\alpha,K)}{dz}\Big|_{z\in[\alpha b,b]} = K - \mathbb{E}[P]\,Pr\,(D \geqslant z) \geqslant K - \mathbb{E}[P] \geqslant 0,$$

where the last inequality follows from the strike price lower bound (i.e., $K \geqslant \mathbb{E}[P]$) under Assumption 1. Therefore, $Q_{\rm PC}(z;\alpha,K)$ is non-decreasing in $(\alpha b, b]$, has no unique optimal solution in this interval, and an optimal solution satisfies $z \leqslant \alpha b$. We thus can conclude that $z^*_{\rm PC}$ belongs to the interval $[\alpha a, \alpha b]$.

We proceed to show that an analogous result holds for the solution $z^*_{\rm SC}$ of $\min_{z\geqslant 0} Q_{\rm SC}(z;\alpha,K)$. The derivative of $Q_{\rm SC}(z;\alpha,K)$ with respect to z exists in the two intervals $[0, \alpha a]$ and $[\alpha b, b]$ with values, respectively, equal to

$$\frac{dQ_{\rm SC}(z;\alpha,K)}{dz}\Big|_{z\in[0,\alpha a]} = \mathbb{E}[K - P] - \mathbb{E}[R]\,Pr\,(\alpha D \geqslant z) = K - \mathbb{E}[P] - \mathbb{E}[R] \leqslant 0,$$

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$$\frac{dQ_{\text{SC}}(z;\alpha,K)}{dz}\Big|_{z\in[\alpha b,b]} = \mathbb{E}[K-P] = K - \mathbb{E}\left[P\right] \geqslant 0,$$

where both inequalities follow from Assumption 1. Following the same argument used above in the case of PCs, we can conclude that z^*_{SC} belongs to the interval $[\alpha a, \alpha b]$. □

Proof of Proposition 1. We start by determining the optimal procurement quantity under PCs, and then move to SCs.

From Lemma 1, we know that there exists an optimal solution to $\min_{z\geqslant 0} Q_{\text{PC}}(\cdot;\alpha,K)$ in the interval $[\alpha a, \alpha b]$, thus we can limit the search to this interval. The expected cost function $Q_{\text{PC}}(z;\alpha,K)$ is convex in the procurement size z in the interval $[\alpha a,\alpha b]$ since the second-order derivative is positive:

$$\frac{d^2Q_{\text{PC}}(z;\alpha,K)}{dz^2}\Big|_{z\in[\alpha a,\alpha b]} = \mathbb{E}[P]\frac{\mathbf{1}_{\{z\geqslant a\}}}{b-a} + \mathbb{E}[R]\frac{1}{\alpha b-\alpha a} > 0.$$

Therefore, the optimal quantity z^*_{PC} can be calculated using the first-order condition

$$\frac{dQ_{\text{PC}}(z;\alpha,K)}{dz} = K - \mathbb{E}\left[R\right] Pr\left(\alpha D \geqslant z\right) - \mathbb{E}\left[P\right] Pr\left(D \geqslant z\right) = 0. \tag{E.1}$$

We proceed by considering two cases.

Case 1: $Pr(D \geqslant z) = 1$. Hence we have $z^*_{\text{PC}} \leqslant a$ in this case. The first-order condition (E.1) simplifies to

$$K - \mathbb{E}[R]\,\frac{b-\frac{z}{\alpha}}{b-a} - \mathbb{E}[P] = 0 \iff z^*_{\text{PC}} = \alpha\Big(b - \frac{K-\mathbb{E}[P]}{\mathbb{E}[R]}(b-a)\Big). \tag{E.2}$$

Enforcing $z^*_{\text{PC}} \leqslant a$ using solution (E.2) results in an upper bound on α equal to

$$\bar{\alpha} = \frac{a\mathbb{E}[R]}{b\mathbb{E}[R] - (K-\mathbb{E}[P])(b-a)}. \tag{E.3}$$

Case 2: $Pr(D \geqslant z) < 1$. Using $Pr(D \geqslant z) = (b-z)/(b-a)$, we obtain $z^*_{\text{PC}} > a$ in this case. Moreover, solving the first order condition (E.1) gives

$$z^*_{\text{PC}} = \frac{-K(b-a) + \left(\mathbb{E}[R] + \mathbb{E}[P]\right) b}{\frac{1}{\alpha}\mathbb{E}[R] + \mathbb{E}[P]}.$$

Note that $z^*_{\text{PC}} > a$ is satisfied when $\alpha > \bar{\alpha}$. In conclusion, the optimal PC procurement z^*_{PC} is given by

$$z^*_{\text{PC}} = \begin{cases} \alpha\Big(b - \frac{K-\mathbb{E}[P]}{\mathbb{E}[R]}(b-a)\Big), & if\ \alpha \leqslant \bar{\alpha}, \\ \frac{-K(b-a)+(\mathbb{E}[R]+\mathbb{E}[P])b}{\frac{1}{\alpha}\mathbb{E}[R]+\mathbb{E}[P]}, & if\ \alpha > \bar{\alpha}. \end{cases}$$

Next, consider an SC. As in the PC case, by Lemma 1, there exists an optimal solution to $\min_{z\geqslant 0} Q_{\text{SC}}(z;\alpha,K)$ in the interval $[\alpha a, \alpha b]$, thus we can limit our search space. This function is

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strictly convex in z because its second derivative is positive by Assumption 1:

$$\frac{d^2 Q_{\mathrm{SC}}(z;\alpha,K)}{dz^2}\Big|_{z\in[\alpha a,\alpha b]} = \mathbb{E}[R]\,\frac{1}{\alpha b-\alpha a} > 0.$$

Thus, the optimal SC quantity z^*_{SC} can be calculated by applying the first-order condition:

$$\begin{aligned}\frac{dQ_{\mathrm{SC}}(z;\alpha,K)}{dz}\Big|_{z\in[\alpha a,\,\alpha b]} &= 0\\ \iff\ & K-\mathbb{E}[P]-\mathbb{E}\,[R]\,Pr\,(\alpha D \geqslant z) = 0\\ \iff\ & z^*_{\mathrm{SC}} = \alpha\Big(b-\frac{K-\mathbb{E}[P]}{\mathbb{E}[R]}(b-a)\Big).\end{aligned}$$

□

Proof of Proposition 2. We characterize the behavior of $C_{\mathrm{PC}}(\alpha,K)$ and $C_{\mathrm{SC}}(\alpha,K)$ as functions of α, respectively, in part (**a**) and part (**b**) of the proof, and compare them in part (**c**).

(**a**) When $\alpha\in[0,\bar{\alpha}]$, we have $z^*_{\mathrm{PC}}\in[\alpha a,a]$, which implies $Pr(D\geqslant z^*_{\mathrm{PC}})=1$, and the optimal cost becomes

$$C_{\mathrm{PC}}(\alpha,K) = K\,z^*_{\mathrm{PC}} + \mathbb{E}[P(D-z^*_{\mathrm{PC}})] + \mathbb{E}[R(\alpha\,D-z^*_{\mathrm{PC}})_+].$$

Using the characterization of z^*_{PC} from Proposition 1, the derivative of this function with respect to α is

$$\begin{aligned}\frac{dC_{\mathrm{PC}}(\alpha,K)}{d\alpha}\Big|_{\alpha\in[0,\bar{\alpha}]} &= K\frac{z^*_{\mathrm{PC}}}{\alpha}+\mathbb{E}[P]\Big(-\frac{z^*_{\mathrm{PC}}}{\alpha}\Big)+\mathbb{E}\Big[R\Big(D-\frac{z^*_{\mathrm{PC}}}{\alpha}\Big)_+\Big]\\ &= (K-\mathbb{E}[P])\frac{z^*_{\mathrm{PC}}}{\alpha}+\mathbb{E}\Big[R\Big(D-\frac{z^*_{\mathrm{PC}}}{\alpha}\Big)_+\Big].\end{aligned}$$

Since z^*_{PC} is linear in α within the interval $[0,\bar{\alpha}]$, z^*_{PC}/α and thus $dC_{\mathrm{PC}}(\alpha,K)/d\alpha$ are independent of α. Moreover, $dC_{\mathrm{PC}}(\alpha,K)/d\alpha$ is non-negative because $K\geqslant\mathbb{E}[P]$ by Assumption 1. Therefore, $C_{\mathrm{PC}}(.,K)$ is a linear function of α in the interval $[0,\bar{\alpha}]$.

We show next that $C_{\mathrm{PC}}(\cdot,K)$ is convex increasing with the target when $\alpha\in(\bar{\alpha},1]$. In this case, $z^*_{\mathrm{PC}}\in(a,\alpha b]$ and $\Pr(D\geqslant z^*_{\mathrm{PC}})<1$. Expanding the definition of $C_{\mathrm{PC}}(\alpha,K)$ gives

$$\begin{aligned}C_{\mathrm{PC}}(\alpha,K) &= K\,z^*_{\mathrm{PC}}+\mathbb{E}[P(D-z^*_{\mathrm{PC}})_+]+\mathbb{E}[R(\alpha D-z^*_{\mathrm{PC}})_+]\\ &= K\,z^*_{\mathrm{PC}}+\mathbb{E}[P]\int_{z^*_{\mathrm{PC}}}^{b}\frac{D-z^*_{\mathrm{PC}}}{b-a}\,dD+\mathbb{E}[R]\int_{z^*_{\mathrm{PC}}/\alpha}^{b}\frac{\alpha D-z^*_{\mathrm{PC}}}{b-a}\,dD\\ &= K\,z^*_{\mathrm{PC}}+\frac{\mathbb{E}[P]}{b-a}\Big(\frac{b^2}{2}-\frac{z^{*2}_{\mathrm{PC}}}{2}-z^*_{\mathrm{PC}}(b-z^*_{\mathrm{PC}})\Big)+\frac{\mathbb{E}[R]}{b-a}\Big[\alpha\Big(\frac{b^2}{2}-\frac{z^{*2}_{\mathrm{PC}}}{2\alpha^2}\Big)-z^*_2\Big(b-\frac{z^*_{\mathrm{PC}}}{\alpha}\Big)\Big]\\ &= K\,z^*_{\mathrm{PC}}+\mathbb{E}[P]\frac{1}{2\,(b-a)}(z^*_{\mathrm{PC}}-b)^2+\mathbb{E}[R]\frac{1}{2\,\alpha(b-a)}(z^*_{\mathrm{PC}}-\alpha b)^2.\end{aligned}$$

The first derivative of $C_{\mathrm{PC}}(\alpha,K)$ is

$$\frac{dC_{\mathrm{PC}}(\alpha,K)}{d\alpha}\Big|_{\alpha\in(\bar{\alpha},1]} = \frac{\mathbb{E}[R]\,(-aK+b\,(K+\mathbb{E}[P](\alpha-1)))\,(a\,K+b\,(-K+\mathbb{E}[P]+2\mathbb{E}[R]+\alpha\,\mathbb{E}[P]))}{2\,(b-a)(\mathbb{E}[R]+\alpha\mathbb{E}[P])^2}.$$

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The denominator as well as the third term of the product in the numerator are strictly positive due to the bounds on the strike price in Assumption 1. We show that the second term in the numerator is also strictly positive using the following chain of inequalities:

$$\begin{aligned}-aK + b\,(K + \mathbb{E}[P](\alpha - 1)) &\geqslant -aK + b\,(K + \mathbb{E}[P](\bar{\alpha} - 1))\\ &= \frac{(b-a)\,((bK - b\mathbb{E}[P])(\mathbb{E}[R] - K + \mathbb{E}[P]) + aK(K - \mathbb{E}[P]))}{b\mathbb{E}[R] - (K - \mathbb{E}[P])(b-a)}\\ &> 0.\end{aligned}$$

The first inequality follows by lower bounding α by $\bar{\alpha}$, the first equality by replacing $\bar{\alpha}$ with its full expression given in (E.3) and simplifying the resulting terms, and the second inequality results from both the numerator and denominator being positive under the bounds on the strike price in Assumption 1. In addition to the first derivative being positive, the second derivative is also positive as it is equal to

$$\frac{d^2 C_{\mathrm{PC}}(\alpha, K)}{d\alpha^2}\Big|_{\alpha\in(\bar{\alpha},1]} = \frac{\mathbb{E}[P]\mathbb{E}[R](aK + b\,(-K + \mathbb{E}[P] + \mathbb{E}[R]))^2}{(b-a)(\mathbb{E}[R] + \alpha\mathbb{E}[P])^3} > 0,$$

where the strict inequality holds due to Assumption 1. Thus, the procurement cost is a strictly convex increasing function in the target level for $\alpha \in [\bar{\alpha}, 1]$.

(**b**) From Proposition 1 we know that z^*_{SC} is linear in α. The slope of $C_{\mathrm{SC}}(\alpha, K)$ with respect to α is

$$\frac{dC_{\mathrm{SC}}(\alpha, K)}{d\alpha}\Big|_{\alpha\in[0,1]} = (K - \mathbb{E}[P])\frac{z^*_{\mathrm{SC}}}{\alpha} + \mathbb{E}\Big[R\Big(D - \frac{z^*_{\mathrm{SC}}}{\alpha}\Big)_+\Big],$$

which is independent of α. This implies that $C_{\mathrm{SC}}(\alpha, K)$ is linear in α.

(**c**) From the proofs of parts (**a**) and (**b**) it follows that $dC_{\mathrm{SC}}(\alpha, K)/d\alpha$ equals $dC_{\mathrm{PC}}(\alpha, K)/d\alpha$ when $\alpha \leqslant \bar{\alpha}$. Therefore, since $C_{\mathrm{PC}}(\alpha, K)$ and $C_{\mathrm{SC}}(\alpha, K)$ coincide when $\alpha = 0$, these two costs are the same for $\alpha \in (0, \bar{\alpha}]$. Furthermore, $C_{\mathrm{PC}}(\alpha, K)$ is strictly convex increasing in α for $\alpha \in (\bar{\alpha}, 1]$, while $C_{\mathrm{SC}}(\alpha, K)$ remains linear with the same slope, which implies that the former cost is higher than the latter cost in this interval. □

Proof of Proposition 3. Recall that $\widetilde{C}_{\mathrm{PC}}(z, w_1; \alpha, K) = Kz + P(D-z)_+ + R(\alpha D - z)_+$. Defining $Y(z) = -P(z-D)_+$, we have $\widetilde{C}_{\mathrm{SC}}(z, w_1; \alpha, K) = \widetilde{C}_{\mathrm{PC}}(z, w_1; \alpha, K) + Y(z)$. We start by showing that $\mathrm{Var}[\widetilde{C}_{\mathrm{SC}}(z, w_1; \alpha, K)] \geqslant \mathrm{Var}[\widetilde{C}_{\mathrm{PC}}(z, w_1; \alpha, K)]$. Since $\mathrm{Var}[\widetilde{C}_{\mathrm{SC}}(z, w_1; \alpha, K)] = \mathrm{Var}[\widetilde{C}_{\mathrm{PC}}(z, w_1; \alpha, K)] + \mathrm{Var}[Y(z)] + 2\,\mathrm{Cov}[\widetilde{C}_{\mathrm{PC}}(z, w_1; \alpha, K), Y(z)]$, and $\mathrm{Var}[Y(z)] \geqslant 0$, the proof reduces to showing that $\mathrm{Cov}[\widetilde{C}_{\mathrm{PC}}(z, w_1; \alpha, K), Y(z)] \geqslant 0$, which we illustrate using the following chain of inequalities:

$$\begin{aligned}&\mathrm{Cov}[\widetilde{C}_{\mathrm{PC}}(z, w_1; \alpha, K), Y(z)]\\ &\quad= \mathrm{Cov}[Kz + P(D-z)_+ + R(\alpha D - z)_+, -P(z-D)_+] &\text{(E.4a)}\\ &\quad= \mathrm{Cov}[P(D-z)_+ + R(\alpha D - z)_+, -P(z-D)_+] &\text{(E.4b)}\\ &\quad= -\,\mathrm{Cov}[P(D-z)_+, P(z-D)_+] - \mathrm{Cov}[R(\alpha D - z)_+, P(z-D)_+] &\text{(E.4c)}\\ &\quad= -\,\mathbb{E}[P^2(D-z)_+(z-D)_+] + \mathbb{E}[P(D-z)_+]\mathbb{E}[P(z-D)_+]\end{aligned}$$

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$$- \mathbb{E}[PR(z-D)_+(\alpha D - z)_+] + \mathbb{E}[R(\alpha D - z)_+]\mathbb{E}[P(z-D)_+] \tag{E.4d}$$

$$= \mathbb{E}[P(D-z)_+]\mathbb{E}[P(z-D)_+] + \mathbb{E}[R(\alpha D - z)_+]\mathbb{E}[P(z-D)_+] \tag{E.4e}$$

$$\geqslant 0, \tag{E.4f}$$

where (E.4a) follows from the definition of $\widetilde{C}_{\text{PC}}(z, w_1; \alpha, K)$, (E.4b) is a consequence of Kz being a constant, (E.4c) follows from the linearity of the covariance, and (E.4d) from the well-known property that $\text{Cov}(A, B) = \mathbb{E}[AB] - \mathbb{E}[A]\mathbb{E}[B]$ if A and B are two random variables. The equality in (E.4e) follows from $(D-z)_+ \cdot (z-D)_+ \equiv 0$ and $(z-D)_+ \cdot (\alpha D - z)_+ \equiv 0$, and (E.4f) is a consequence of all expectations involving only non-negative random variables.

If $\Pr(z > D) > 0$, then $\text{Var}[Y(z)] > 0$, which combined with the non-negative covariance (E.4f) gives $\text{Var}[\widetilde{C}_{\text{SC}}(z, w_1; \alpha, K)] > \text{Var}[\widetilde{C}_{\text{PC}}(z, w_1; \alpha, K)]$. Instead, if $\Pr(z > D) = 0$, then $Y(z) \equiv 0$, which implies that $\text{Var}[\widetilde{C}_{\text{SC}}(z, w_1; \alpha, K)] = \text{Var}[\widetilde{C}_{\text{PC}}(z, w_1; \alpha, K)]$. □

The proof of Proposition 4 relies on Lemma 2.

Lemma 2. *Under Assumption 1, it holds that:*
(a) *The optimal PC procurement quantity $z^*_{PC,D}$ with deterministic RPPT, $\alpha\bar{D}$, is given by*

$$z^*_{PC,D} = \min\left\{\alpha\bar{D}, b - \left(\frac{K - \mathbb{E}[R]}{\mathbb{E}[P]}\right)(b-a)\right\};$$

(b) *$C_{PC,D}(\alpha, K)$ is linear and increasing in α for $\alpha = [0, a/\bar{D}) \cup [z'/\bar{D}, 1)$ and strictly convex and increasing in α for $\alpha \in [a/\bar{D}, \min\{z'/\bar{D}, 1\})$, where $z' := b - \frac{K-\mathbb{E}[R]}{\mathbb{E}[P]}(b-a)$.*

Proof. **(a)** The expected procurement cost for a PC with deterministic RPPT is

$$Q_{\text{PC,D}}(\alpha, K, z) = Kz + \mathbb{E}\left[P\left(D - z\right)_+\right] + \mathbb{E}\left[R\left(\alpha\bar{D} - z\right)_+\right]. \tag{E.5}$$

Similar to $Q_{\text{PC}}(\alpha, K, z)$, (E.5) is also convex in the procurement quantity z. Therefore, the optimal quantity $z^*_{\text{PC,D}}$ minimizing (E.5) can be determined by considering its first derivative:

$$\frac{dQ_{\text{PC,D}}(\alpha, K, z)}{dz} = K - \mathbb{E}\left[P\right] Pr\left(D \geqslant z\right) - \mathbb{E}\left[R\right] \mathbf{1}_{\{\alpha\bar{D} \geqslant z\}}.$$

If $z \geqslant \alpha\bar{D}$, then the indicator function in the last term is zero and the derivative is non-negative since $K \geqslant \mathbb{E}[P]$. Thus, the quantity $z^*_{\text{PC,D}}$ minimizing (E.5) lies in the interval $[0, \alpha\bar{D}]$. In this interval, the indicator function is equal to 1 and the solution to $dQ_{\text{P,D}}(\alpha, K, z)/dz = 0$ is

$$z' = b - \left(\frac{K - \mathbb{E}[R]}{\mathbb{E}[P]}\right)(b-a).$$

This quantity is independent of α. Moreover, z' is greater than a because the strike price K is upper bounded by $\mathbb{E}[P] + \mathbb{E}[R]$ in Assumption 1. Since the optimal procurement quantity $z^*_{\text{PC,D}}$ must lie in the interval $[0, \alpha\bar{D}]$, we conclude that $z^*_{\text{PC,D}} = \min\{z', \alpha\bar{D}\}$.

(b) We distinguish three cases for the expected cost (E.5) depending on the value of α.

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Case I): $\alpha\bar{D} < a$. Since $a < z'$, then $\alpha\bar{D} < z'$ and $z^*_{\text{PC,D}} = \min\{z', \alpha\bar{D}\} = \alpha\bar{D}$. The optimal cost is

$$\begin{aligned} C^I_{\text{PC,D}}(\alpha, K) &:= Q_{\text{PC,D}}(\alpha, K, z^*_{\text{PC,D}} = \alpha\bar{D}) \\ &= K\alpha\bar{D} + \mathbb{E}\left[P(D - \alpha\bar{D})_+\right] \\ &= K\alpha\bar{D} + \mathbb{E}\left[P(D - \alpha\bar{D})\right], \end{aligned}$$

where the last equality is due to $\alpha\bar{D} < a$. $C^I_{\text{PC,D}}(\alpha, K)$ is linear in α because its first derivative is independent of α:

$$\frac{dC^{\text{I}}_{\text{PC,D}}(\alpha, K)}{d\alpha} = (K - \mathbb{E}[P])\bar{D}.$$

Case II): $a \leqslant \alpha\bar{D} < z'$. In this case again it holds that $z^*_{\text{PC,D}} = \min\{z', \alpha\bar{D}\} = \alpha\bar{D}$, and the optimal procurement cost is

$$\begin{aligned} C^{\text{II}}_{\text{PC,D}}(\alpha, K) &:= Q_{\text{PC,D}}(\alpha, K, z^*_{\text{PC,D}} = \alpha\bar{D}) \\ &= K\alpha\bar{D} + \mathbb{E}\left[P\left(D - \alpha\bar{D}\right)_+\right]. \end{aligned}$$

The first and second derivative of this expression with respect to α are

$$\frac{dC^{\text{II}}_{\text{PC,D}}(\alpha, K)}{d\alpha} = \left[K - \mathbb{E}[P]\frac{b - \alpha\bar{D}}{b - a}\right]\bar{D} > 0,$$

$$\frac{dC^{\text{II}2}_{\text{PC,D}}(\alpha, K)}{d^2\alpha} = \mathbb{E}[P]\frac{\bar{D}^2}{b - a} > 0.$$

Therefore, the optimal procurement cost is convex increasing in α.
Case III): $z' < \alpha\bar{D}$. In this case, $z^*_{\text{PC,D}} = \min\{z', \alpha\bar{D}\} = z'$ and the optimal cost is

$$\begin{aligned} C^{\text{III}}_{\text{PC,D}}(\alpha, K) &:= Q_{\text{PC,D}}(\alpha, K, z^*_{\text{PC,D}} = z') \\ &= Kz' + \mathbb{E}[P]\frac{(z' - b)^2}{2(b - a)} + \mathbb{E}[R](\alpha\bar{D} - z'). \end{aligned}$$

Since the first derivative

$$\frac{dC^{\text{III}}_{\text{PC,D}}(\alpha, K)}{d\alpha} = \mathbb{E}[R]\bar{D}$$

does not depend on α, the procurement cost $C^{\text{III}}_{\text{PC,D}}(\alpha, K)$ increases linearly in the target level. Finally, the following relations hold between the slopes of the procurement costs in the aforementioned three cases.

$$\frac{dC^{\text{I}}_{\text{PC,D}}(\alpha, K)}{d\alpha} < \frac{dC^{\text{II}}_{\text{PC,D}}(\alpha, K)}{d\alpha} < \frac{dC^{\text{III}}_{\text{PC,D}}(\alpha, K)}{d\alpha}.$$

□

Proof of Proposition 4. **(a)** The expected SC procurement cost with deterministic RPPT is

$$Q_{\text{SC,D}}(\alpha, K, z) = \mathbb{E}[PD] + \mathbb{E}[(K - P)z] + \mathbb{E}[R(\alpha\bar{D} - z)_+].$$

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This function is continuous for $z \in \mathbb{R}_+$, and is differentiable for $z \in \mathbb{R}_+ \backslash \{\alpha \bar{D}\}$ with derivative

$$\frac{dQ_{\text{SC,D}}(\alpha, K, z)}{dz} = K - \mathbb{E}[P] - \mathbb{E}[R] \mathbf{1}_{\{\alpha \bar{D} \geqslant z\}}.$$

This expression as a function of z is a non-positive constant if $z \leqslant \alpha \bar{D}$, and a non-negative constant otherwise. In fact, in the former case the indicator function is 1 and it holds that $K - \mathbb{E}[P] - \mathbb{E}[R] \leqslant 0$ due to Assumption 1, whereas in the latter case the indicator function is zero and it holds that $K - \mathbb{E}[P] \geqslant 0$ also due to Assumption 1. It follows that $z^*_{\text{SC,D}} = \alpha \bar{D}$ is an optimal procurement quantity, and the associated optimal cost is

$$C_{\text{SC,D}}(\alpha, K) = Q_{\text{SC,D}}(\alpha, K, z^*_{\text{SC,D}} = \alpha \bar{D}) = \mathbb{E}[PD] + \mathbb{E}[(K - P)] \alpha \bar{D}.$$

The slope of the optimal cost function with respect to the target is

$$\frac{dC_{\text{SC,D}}(\alpha, K)}{d\alpha} = (K - \mathbb{E}[P]) \bar{D}.$$

Given that both functions $C_{\text{SC}}(\alpha, K)$ and $C_{\text{SC,D}}(\alpha, K)$ are linear increasing in α (see also Proposition 2) and are equal at $\alpha = 0$, it follows that $C_{\text{SC}}(\alpha, K) < C_{\text{SC,D}}(\alpha, K)$ in $\alpha \in (0, 1]$ if and only if the analogous condition on their slope holds, i.e., $dC_{\text{SC}}(\alpha, K)/d\alpha < dC_{\text{SC,D}}(\alpha, K)/d\alpha$. Below we establish a necessary and sufficient condition for this relation to be true.

$$\begin{aligned} \frac{dC_{\text{SC}}(\alpha, K)}{d\alpha} < \frac{dC_{\text{SC,D}}(\alpha, K)}{d\alpha} &\iff (K - \mathbb{E}[P]) \frac{z^*_{\text{SC}}}{\alpha} + \mathbb{E}\left[R \left(D - \frac{z^*_{\text{SC}}}{\alpha}\right)_+\right] < (K - \mathbb{E}[P]) \bar{D} \\ &\iff \bar{D} > \frac{(a - b)(K - \mathbb{E}[P])}{2\mathbb{E}[R]} + b = \frac{1}{2\alpha} z^*_{\text{SC}} + \frac{b}{2}. \end{aligned}$$

where the second implication is obtained by replacing z^*_{SC} with its expression given in Proposition 1 and simplifying the resulting term.

(b) The functions $C_{\text{PC}}(\alpha, K)$ and $C_{\text{PC,D}}(\alpha, K)$ have been characterized in Proposition 2 and Lemma 2, respectively. Both $C_{\text{PC,D}}(\alpha, K)$ and $C_{\text{PC}}(\alpha, K)$ are convex and increasing functions of α, and it holds that $C_{\text{PC,D}}(1, K) = C_{\text{PC}}(1, K)$ for

$$\bar{D} = D' := (b - a) \frac{\left(K - (\mathbb{E}[R] + \mathbb{E}[P])\right)^2 + \mathbb{E}[R](\mathbb{E}[P] + \mathbb{E}[R]) - (\mathbb{E}[P] + \mathbb{E}[R])^2}{2\mathbb{E}[P](\mathbb{E}[R] + \mathbb{E}[P])} + b.$$

This expression for D' belongs to the interval $[a, b]$ and can be derived as follows:

$$\begin{aligned} & C_{\text{PC}}(1, K) = C_{\text{PC,D}}(1, K) \\ &\iff K z^*_{\text{PC}} + \mathbb{E}[P] \frac{(z^*_{\text{PC}} - b)^2}{2(b - a)} + \mathbb{E}[R] \frac{(z^*_{\text{PC}} - b)^2}{2(b - a)} = K z' + \mathbb{E}[P] \frac{(z' - b)^2}{2(b - a)} + \mathbb{E}[R](\bar{D} - z') \\ &\iff \mathbb{E}[R] \bar{D} = K(z^*_{\text{PC}} - z') + \mathbb{E}[P] \frac{(z^*_{\text{PC}} - b)^2 - (z' - b)^2}{2(b - a)} + \mathbb{E}[R] \left(\frac{(z^*_{\text{PC}} - b)^2}{2(b - a)} + z'\right) \\ &\iff \bar{D} = \frac{z^*_{\text{PC}} - z'}{\mathbb{E}[R]} \left(K + \frac{\mathbb{E}[P](z^*_{\text{PC}} + z' - 2b)}{2\,(b - a)}\right) + \frac{(z^*_{\text{PC}} - b)^2}{2(b - a)} + z' \end{aligned}$$

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$$= (b-a)\frac{\big(K-(\mathbb{E}[R]+\mathbb{E}[P])\big)^2+\mathbb{E}[R](\mathbb{E}[P]+\mathbb{E}[R])-(\mathbb{E}[P]+\mathbb{E}[R])^2}{2\mathbb{E}[P](\mathbb{E}[R]+\mathbb{E}[P])}+b \quad \text{(E.6)}$$
$$= D',$$

where (E.6) is obtained by replacing z^*_{PC} and z' by their expressions based on Proposition 1 and Lemma 2, respectively, and simplifying. Since $C_{\mathrm{PC,D}}(\alpha, K)$ is strictly increasing in the value of $\bar{D}$, we can claim that if $\bar{D} > D'$, $C_{\mathrm{PC,D}}(1, K) > C_{\mathrm{PC}}(1, K)$. Therefore, there exists $\alpha \in (0, 1]$ that satisfies $C_{\mathrm{PC,D}}(\alpha, K) > C_{\mathrm{PC}}(\alpha, K)$.

Finally, the value of D' can be bounded from above as follows:

$$\begin{aligned} D' &\leqslant (b-a)\frac{-\mathbb{E}[P]\big(\mathbb{E}[P]+\mathbb{E}[R]\big)+\mathbb{E}[R]^2}{2\mathbb{E}[P](\mathbb{E}[R]+\mathbb{E}[P])}+b \\ &\leqslant -(b-a)\frac{\mathbb{E}[R]}{2(\mathbb{E}[R]+\mathbb{E}[P])}+b \\ &= \mathbb{E}[D]+\frac{(b-a)}{2}\frac{\mathbb{E}[P]}{\mathbb{E}[R]+\mathbb{E}[P]}, \end{aligned}$$

where the first inequality holds by replacing K with $\mathbb{E}[P]$ since $K \geqslant \mathbb{E}[P]$, and simplifying, and the second inequality results from the assumption $\mathbb{E}[P] \geqslant \mathbb{E}[R]$. Therefore, there exists $\alpha \in (0, 1]$ such that $C_{\mathrm{PC,D}}(\alpha, K) > C_{\mathrm{PC}}(\alpha, K)$ if $\bar{D} > \mathbb{E}[D] + \frac{(b-a)}{2}\frac{\mathbb{E}[P]}{\mathbb{E}[R]+\mathbb{E}[P]}$. □

Lemmas 3, 4, and 5 are used in the proof of Proposition 5.

Lemma 3. *Let P be a log-normal random variable with parameters μ and σ^2, and denote with $f(P;\mu,\sigma^2)$ its PDF. Moreover, consider $Y = log(P)$, i.e. $Y \sim \mathcal{N}(\mu, \sigma^2)$, and a scalar $B > 0$. It holds that:*

$$\mathbb{E}\left[P\mathbf{1}_{\{P<B\}}\right] = e^{\mu+\frac{\sigma^2}{2}}\,\Phi\left(\frac{\ln(B)-(\mu+\sigma^2)}{\sigma}\right)$$

Proof. Using the relationship between P and Y, we establish the following equalities:

$$\begin{aligned} \mathbb{E}\left[P\mathbf{1}_{\{P<B\}}\right] &= \int_{-\infty}^{B} P f(P;\mu,\sigma^2)dP = \int_{-\infty}^{B} e^{\ln P} f(P;\mu,\sigma^2)dP \\ &= \int_{-\infty}^{\ln(B)} e^Y \phi(Y;\mu,\sigma^2)dY = \int_{-\infty}^{\ln(B)} e^Y \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(Y-\mu)^2}{2\sigma^2}}dY. \end{aligned}$$

By adding and subtracting $\sigma^4 + 2\sigma^2\mu$ to the numerator of the second exponential term, we obtain:

$$\int_{-\infty}^{\ln(B)} e^Y \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(Y-\mu)^2}{2\sigma^2}}dY = e^{\mu+\frac{\sigma^2}{2}}\,\Phi\left(\frac{\ln(B)-\mu-\sigma^2}{\sigma}\right),$$

which proves the desired result. □

Lemma 4. *Given three scalars $B > 0$, $\mu \geqslant 0$, and $\sigma > 0$, it holds that:*

$$\frac{\exp(\mu+\sigma^2/2)}{\sigma}\left[\frac{1}{B}\phi\left(\frac{\ln(B)-(\mu+\sigma^2)}{\sigma}\right)\right] = \frac{1}{\sigma}\left[\phi\left(\frac{\ln(B)-\mu}{\sigma}\right)\right].$$

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Proof.
$$\begin{aligned}
&\frac{\exp(\mu+\sigma^2/2)}{\sigma}\left[\frac{1}{B}\phi\left(\frac{\ln(B)-(\mu+\sigma^2)}{\sigma}\right)\right] \\
&\qquad = \frac{\exp(\mu+\sigma^2/2)}{\sigma}\left[\frac{1}{B\sqrt{2\pi}}\exp\left(\frac{-(\ln(B))^2-(\mu+\sigma^2)^2+2\ln(B)(\mu+\sigma^2)}{2\sigma^2}\right)\right] \\
&\qquad = \frac{\exp(\mu+\sigma^2/2)}{\sigma}\left[\frac{1}{B}\phi\left(\frac{\ln(B)-\mu}{\sigma}\right)\exp\big(-\sigma^2/2-\mu+\ln(B)\big)\right] \\
&\qquad = \frac{\exp(\mu+\sigma^2/2)}{\sigma}\left[\frac{1}{B}\phi\left(\frac{\ln(B)-\mu}{\sigma}\right)\exp\big(-\sigma^2/2-\mu\big)B\right] \\
&\qquad = \frac{1}{\sigma}\phi\left(\frac{\ln(B)-\mu}{\sigma}\right).
\end{aligned}$$
□

Lemma 5. *Given $\alpha > 0$ and $\delta, \delta' > 0$, if $\mathbb{E}[K^{\text{INT}}(P;K,\delta)] < \mathbb{E}[K^{\text{INT}}(P;K,\delta')]$, then it holds that $C_{SC}^{\text{INT}}(\alpha,K,\delta) < C_{SC}^{\text{INT}}(\alpha,K,\delta')$.*

Proof. Let $z^*_{\text{SC}}(\alpha,K,\delta)$ be an optimal solution to the expected procurement cost $Q^{\text{INT}}_{\text{SC}}(z;\alpha,K,\delta)$, which is defined by

$$\begin{aligned}
Q^{\text{INT}}_{\text{SC}}(z;\alpha,K,\delta) &:= \mathbb{E}[\widetilde{C}_{\text{SC}}(z,w_1;\alpha,K^{\text{INT}}(P;K,\delta))] \\
&= \mathbb{E}[K^{\text{INT}}(P;K,\delta)-P]z + \mathbb{E}[PD] + \mathbb{E}[R(\alpha D - z)_+].
\end{aligned}$$

From the hypothesis it follows that

$$\begin{aligned}
&Q^{\text{INT}}_{\text{SC}}(z^*_{\text{S}}(\alpha,K,\delta');\alpha,K,\delta) - Q^{\text{INT}}_{\text{SC}}(z^*_{\text{S}}(\alpha,K,\delta');\alpha,K,\delta') \\
&\quad = \big(\mathbb{E}[K^{\text{INT}}(P;K,\delta)] - \mathbb{E}[K^{\text{INT}}(P;K,\delta')]\big)\cdot z^*_{\text{S}}(\alpha,K,\delta') < 0.
\end{aligned}$$

As a consequence, the following relation between optimal procurement costs hold

$$\begin{aligned}
C^{\text{INT}}_{\text{SC}}(\alpha,K,\delta) = \min_{z\geqslant 0} Q^{\text{INT}}_{\text{SC}}(z;\alpha,K,\delta) &\leqslant Q^{\text{INT}}_{\text{SC}}(z^*_{\text{S}}(\alpha,K,\delta');\alpha,K,\delta) \\
&< Q^{\text{INT}}_{\text{SC}}(z^*_{\text{S}}(\alpha,K,\delta');\alpha,K,\delta') = C^{\text{INT}}_{\text{SC}}(\alpha,K,\delta'),
\end{aligned}$$

which proves the lemma. □

Proof of Proposition 5. Recall that the power price P follows a log-normal distribution and that its natural logarithm follows $\ln(P) \sim \mathcal{N}(\mu_P, \sigma_P^2)$, i.e., it is a normal random variable with mean μ_P and standard deviation σ_P. Note that consequently $(\ln(P)-\mu_P)/\sigma_P \sim \mathcal{N}(0,1)$. The expected unit cost with the interval strike price is defined for $\delta \in [0,K)$ and can be expressed using Lemma 3 as

$$\begin{aligned}
\mathbb{E}[K^{\text{INT}}(P;K,\delta)] &= (K+\delta)\Pr(P > K+\delta) + (K-\delta)\Pr(P < K-\delta) + \mathbb{E}\left[P\mathbf{1}_{\{K-\delta\leqslant P\leqslant K+\delta\}}\right] \\
&= (K+\delta)\left[1-\Phi\left(\frac{\ln(K+\delta)-\mu_P}{\sigma_P}\right)\right] + (K-\delta)\,\Phi\left(\frac{\ln(K-\delta)-\mu_P}{\sigma_P}\right) \\
&\quad + \exp\left(\mu_P + \frac{\sigma_P^2}{2}\right)\left[\Phi\left(\frac{\ln(K+\delta)-\mu_P-\sigma_P^2}{\sigma_P}\right) - \Phi\left(\frac{\ln(K-\delta)-\mu_P-\sigma_P^2}{\sigma_P}\right)\right].
\end{aligned}$$

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The first derivative of $\mathbb{E}[K^{\text{INT}}(P;K,\delta)]$ with respect to δ is

$$\frac{\partial \mathbb{E}[K^{\text{INT}}(P;K,\delta)]}{\partial \delta} = 1 - \Phi\left(\frac{\ln(K+\delta)-\mu_P}{\sigma_P}\right) - \frac{1}{\sigma_P}\phi\left(\frac{\ln(K+\delta)-\mu_P}{\sigma_P}\right) - \Phi\left(\frac{\ln(K-\delta)-\mu_P}{\sigma_P}\right)$$
$$- \frac{1}{\sigma_P}\phi\left(\frac{\ln(K-\delta)-\mu_P}{\sigma_P}\right) + \frac{\exp(\mu_P+\sigma_P^2/2)}{\sigma_P}\left[\frac{1}{K+\delta}\phi\left(\frac{\ln(K+\delta)-(\mu_P+\sigma_P^2)}{\sigma_P}\right)\right.$$
$$\left. + \frac{1}{K-\delta}\phi\left(\frac{\ln(K-\delta)-(\mu_P+\sigma_P^2)}{\sigma_P}\right)\right].$$

By reformulating the last term in this derivative using Lemma 4 and canceling opposite terms out, we obtain a simpler expression:

$$\frac{\partial \mathbb{E}[K^{\text{INT}}(P;K,\delta)]}{\partial \delta} = 1 - \Phi\left(\frac{\ln(K+\delta)-\mu_P}{\sigma_P}\right) - \Phi\left(\frac{\ln(K-\delta)-\mu_P}{\sigma_P}\right). \tag{E.7}$$

We structure the rest of proof as follows. We first show that $\exists\delta > 0$ such that $\mathbb{E}[K^{\text{INT}}(P;K,\delta)] < \mathbb{E}[K^{\text{INT}}(P;K,0)] = K$ if and only if $K > \mathbb{E}[P]\exp(-\sigma_P^2/2)$. By Lemma 5, this implies the validity of the first claim of the proposition, i.e., that $\exists\delta > 0$ such that $C_{\text{SC}}^{\text{INT}}(\alpha,K,\delta) < C_{\text{SC}}^{\text{INT}}(\alpha,K,0) = C_{\text{SC}}(\alpha,K)$ if and only if $K > \mathbb{E}[P]\exp(-\sigma_P^2/2)$. We subsequently prove that $\mathbb{E}[K^{\text{INT}}(P;K,\cdot)]$, and thus $C_{\text{SC}}^{\text{INT}}(\alpha,K,\cdot)$, attain global minimum at $\delta^* = \sqrt{K^2-\exp(2\mu_P)} > 0$ if $K > \mathbb{E}[P]\exp(-\sigma_P^2/2)$.

Suppose $\mathbb{E}[K^{\text{INT}}(P;K,\delta)] < K$ for some δ. Then $\exists\bar{\delta} > 0$ such that the derivative (E.7) is negative at $\bar{\delta}$ because $\mathbb{E}[K^{\text{INT}}] = K$ at $\delta = 0$ and is a continuous function. The following implications hold.

$$\begin{aligned}\frac{\partial \mathbb{E}[K^{\text{INT}}(P;K,\delta)]}{\partial \delta}\Big|_{\delta=\bar{\delta}} < 0 &\Longrightarrow 1 - \Phi\left(\frac{\ln(K+\bar{\delta})-\mu_P}{\sigma_P}\right) < \Phi\left(\frac{\ln(K-\bar{\delta})-\mu_P}{\sigma_P}\right)\\ &\Longrightarrow \ln(K+\bar{\delta})-\mu_P > \mu_P - \ln(K-\bar{\delta})\\ &\Longrightarrow K^2 > \bar{\delta}^2 + \exp(2\mu_P)\\ &\Longrightarrow K > \exp(\mu_P) = \mathbb{E}[P]\exp(-\sigma_P^2/2).\end{aligned}$$

To prove the reverse direction of the iff result, suppose $K > \mathbb{E}[P]\exp(-\sigma_P^2/2) = \exp(\mu_P)$. The derivative (E.7) evaluated in $\delta = 0$ is negative because

$$\frac{\partial \mathbb{E}[K^{\text{INT}}(P;K,\delta)]}{\partial \delta}\Big|_{\delta=0} = 1 - 2\Phi\left(\frac{\ln(K)-\mu_P}{\sigma_P}\right) < 1 - 2\Phi\left(\frac{\ln(\exp(\mu_P))-\mu_P}{\sigma_P}\right) = 0,$$

where the last equality is a consequence of $\Phi(0) = 0.5$. Therefore, the expected unit cost is lower than K in the proximity of 0. In other words, there exists a sufficiently small $\delta > 0$ such that $\mathbb{E}[K^{\text{INT}}(P;K,\delta)] < K$. The first claim of the proposition is thus proven by invoking Lemma 5.

Next, we want to determine the global minimum δ^* of $\mathbb{E}[K^{\text{INT}}(P;K,\delta)]$ when $K > \exp(\mu_P)$. To this end, we provide a characterization of $\mathbb{E}[K^{\text{INT}}(P;K,\delta)]$ as a function of δ. First, we characterize a region for δ in which the function is convex as follows.

$$\frac{\partial^2 \mathbb{E}[K^{\text{INT}}(P;K,\delta)]}{\partial \delta^2} = \frac{-1}{(K+\delta)\sigma_P}\phi\left(\frac{\ln(K+\delta)-\mu_P}{\sigma_P}\right) + \frac{1}{(K-\delta)\sigma_P}\phi\left(\frac{\ln(K-\delta)-\mu_P}{\sigma_P}\right) \geqslant 0 \tag{E.8}$$

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$$\iff \frac{1}{\sqrt{2\pi\,\sigma_P^2}}\left[\frac{-1}{(K+\delta)}\exp\left(\frac{-(\ln(K+\delta)-\mu_P)^2}{2\sigma_P^2}\right)+\frac{1}{(K-\delta)}\exp\left(\frac{-(\ln(K-\delta)-\mu_P)^2}{2\sigma_P^2}\right)\right] \geqslant 0$$

$$\iff -\frac{1}{(K+\delta)}\exp\left(\frac{-(\ln(K+\delta)-\mu_P)^2}{2\sigma_P^2}\right)+\frac{1}{(K-\delta)}\exp\left(\frac{-(\ln(K-\delta)-\mu_P)^2}{2\sigma_P^2}\right) \geqslant 0$$

$$\iff \exp\left(\frac{-\ln(K+\delta)^2+\ln(K-\delta)^2+2\mu_P\ln(K+\delta)-2\mu_P\ln(K-\delta)}{2\sigma_P^2}\right) \leqslant \frac{K+\delta}{K-\delta}$$

$$\iff \exp\left(\frac{-\ln(K^2-\delta^2)+2\mu_P}{2\sigma_P^2}\ln\left(\frac{K+\delta}{K-\delta}\right)\right) \leqslant \frac{K+\delta}{K-\delta}$$

$$\iff -\ln\left(K^2-\delta^2\right)+2\mu_P \leqslant 2\sigma_P^2$$

$$\iff \delta \leqslant \sqrt{K^2-\exp(2\mu_P-2\sigma_P^2)} =: \hat{\delta}.$$

Therefore, the function $\mathbb{E}[K^{\mathrm{INT}}(P;K,\cdot)]$ is convex for $\delta \in [0,\hat{\delta}]$. Moreover, since $2\mu_P - 2\sigma_P^2 > -\infty$ and $\exp(\cdot)$ is a strictly increasing function, we have $\hat{\delta} < K$. Hence, $\mathbb{E}[K^{\mathrm{INT}}(P;K,\cdot)]$ is concave for $\delta \in [\hat{\delta}, K)$. We further characterize $\mathbb{E}[K^{\mathrm{INT}}(P;K,\delta)]$ by showing that it is increasing in δ when $\delta > \hat{\delta}$. Since the second derivative (E.8) is negative in the interval $[\hat{\delta}, K)$, then the first derivative (E.7) is a decreasing function in $[\hat{\delta}, K)$. As a result, (E.7) attains its infimum for $\delta \to K$ and the following inequalities hold:

$$\begin{aligned}\frac{\partial\mathbb{E}[K^{\mathrm{INT}}(P;K,\delta)]}{\partial\delta}\Big|_{[\hat{\delta},K)} &\geqslant \inf_{\delta\in[\hat{\delta},K)}\frac{\partial\mathbb{E}[K^{\mathrm{INT}}(P;K,\delta)]}{\partial\delta}\\ &= \lim_{\delta\to K} 1-\Phi\left(\frac{\ln(K+\delta)-\mu_P}{\sigma_P}\right)-\Phi\left(\frac{\ln(K-\delta)-\mu_P}{\sigma_P}\right)\\ &= 1-\Phi\left(\frac{\ln(2K)-\mu_P}{\sigma_P}\right)\\ &> 0,\end{aligned}$$

where the last inequality is strict because $\Phi\left((\ln(2K)-\mu_P)/\sigma_P\right) < 1$. Since $\mathbb{E}[K^{\mathrm{INT}}(P;K,\delta)]$ is strictly concave increasing for $\delta \geqslant \hat{\delta}$ and convex when $\delta \leqslant \hat{\delta}$, its global minimum can be found using the first-order condition.

$$\begin{aligned}\frac{\partial\mathbb{E}[K^{\mathrm{INT}}(P;K,\delta)]}{\partial\delta} = 0 &\iff 1-\Phi\left(\frac{\ln(K+\delta)-\mu_P}{\sigma_P}\right) = \Phi\left(\frac{\ln(K-\delta)-\mu_P}{\sigma_P}\right)\\ &\iff \ln(K+\delta)-\mu_P = \mu_P - \ln(K-\delta)\\ &\iff \delta = \sqrt{K^2-\exp(2\mu_P)} = \sqrt{K^2-\mathbb{E}_0[P_1]^2\exp(-\sigma_P^2)} =: \delta^*,\end{aligned}$$

where the second implication is a consequence of the symmetric property of the normal distribution, i.e. $\ln(K+\delta)$ and $\ln(K-\delta)$ have equal distance from the mean of $\ln(P)$. In conclusion, $\mathbb{E}[K^{\mathrm{INT}}(P;K,\delta)]$ has global minimum at $\delta = \delta^*$ if $K > \mathbb{E}[P]\exp(-\sigma_P^2/2)$. From Lemma 5, δ^* is also global minimum of $C_{\mathrm{SC}}^{\mathrm{INT}}(\alpha, K, .)$ in this case. □

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