EFFECTS OF ELEMENTARY GENERAL EDUCATION TEACHERS, SPECIAL EDUCATION TEACHERS, AND MATH INTERVENTIONISTS COLLABORATING IN A PROFESSIONAL LEARNING COMMUNITY

By

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To the Faculty of Washington State University:

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Abstract

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This qualitative research study examined how math interventionists and special education teachers collaborated with general education teachers. I focused on improving access to the core general education mathematics curriculum for elementary students with mathematics difficulty. Differing educational perspectives, divergent learning theories, and instructional approaches, as well as various curriculum materials, make collaboration between multiple teachers complex work. To foster collaboration and attend to the instructional needs of students with mathematics difficulty, I provided an eight-week professional development series. The series included the following topics: developing co-teaching roles and responsibilities, understanding principles of *Cognitively Guided Instruction*, understanding the *Five Practices*, creating rich math tasks, and using a multi-tiered system of support so students with mathematics difficulty could improve understandings. Data collected included interviews and observations before and after the professional development, as well as audio recordings during collaboration meetings and during professional learning community meetings. I found that for the teachers in this study, teachers' perception of student need and teachers' abilities to create a learning trajectory affected co-teaching relationships. I found that when participating teachers co-designed instruction by reframing math tasks and planned using the *Five Practices*, they focused on instruction to help students with mathematics difficulty attend to gaps in understandings. I also found that professional development supported math instruction through lesson summary enactment and lesson summary discourse patterns. Providing focused and intentional instruction, so students address and resolve mathematics difficulties is a worthy and important goal. This study endeavored to highlight ways mathematics educators and teacher leaders can support teachers and students who have mathematical difficulties.

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DEDICATION

First, I dedicate this dissertation to my mother, who passed away before I could graduate. Mom always thought I could do anything I set my mind to and supported me in my pursuit of achieving a doctorate with taking my girls each week for dinner while I attended classes, studied, or wrote papers. Second, I dedicate this dissertation to my late husband Matthew who also passed away while I was working on completing my degree. Matt always thought I would write a book someday and encouraged me to pursue my goal of getting a doctorate degree. Third, I dedicate this dissertation to my girls Abby and Anna. They were both patient and understanding for years with having a mom busy with school and work. I also dedicate this dissertation to my husband Derrik, who in the final stages of this process, made sure I did what I needed to do, every day, to complete this project. Last, I dedicate this dissertation to my twins Allison and Aiden, who never knew the struggle but who will grow up knowing their mom as a doctor.

CHAPTER ONE: STATEMENT OF PROBLEM

This study examined how math interventionists and special education teachers collaborated with general education teachers. I focused on opening access to the core general education mathematics curriculum by creating learning trajectories for elementary students with mathematics difficulty. Elementary math interventionists provide focused instruction to support students in addressing gaps in learning from previous grades and provide access to grade-level core mathematics content. Special education teachers have a similar goal. They endeavor to help students with gaps in understanding reach their full potential and support the learning of concepts and processes from previous grades. According to foundational special education research, students reaching full potential means students engage in grade-level content while simultaneously developing understandings for unfinished learning from previous grades and/or extending understanding when ready for a challenge (McLeskey & Landers, 2006). I endeavored to discover how these specialized teachers collaborated with the general education teacher in supporting students to attend to gaps in learning. Through my work as an instructional coach for special education teachers, then later, as an instructional coach for math interventionists, I realized the job complexity for both the learning support special education teacher, the math interventionist, and the general education teacher.

Before describing my research, I first describe key terminology and the definitions I used for this study. I considered a learning support teacher as someone who has worked with students identified as having a mild to a moderate learning disability, health impairment, or developmental delay. A student in learning support qualifies for special education when the student's cognitive assessment is within normal limits, but the student's academic performance is at least two years behind their grade-level peers. The discrepancy model or a Response to

Intervention (National Center for Learning Disabilities, 1989) model gave educators two different ways to identify students for special education (Fuchs & Fuchs, 2007; Fuchs et al., 2007). In the district in which the study occurred, students qualified for special education only by the discrepancy model. There are many different special education programs and therefore, different types of special education teachers. In this study, I use special education teacher to refer to a learning support teacher who worked with students identified with a learning disability, health impairment, or developmental delay and with at least a two-year discrepancy between academic performance and academic achievement testing as outlined in foundational special education literature (McLeskey & Landers, 2006). In an effort to use additive language, I use the terms gaps in understandings or unfinished learning to describe the difference between the student's current level of learning and the student's grade-level standards.

I considered a math interventionist as someone who worked with students identified at risk for a mathematics learning disability in the content area of math. These students usually demonstrate mathematics difficulty, but they have not met the criteria for a mathematics disability (Lewis, 2014; Stevens et al., 2018). For students identified as needing math intervention, three different data points must triangulate the need for intervention (Stevens et al., 2018). The Response to Intervention (National Center for Learning Disabilities, 1989) model also identifies students for math intervention and helps to prevent the need for a special education placement (Fuchs & Fuchs, 2007; Fuchs et al., 2007). Typically, students who receive math intervention have not completed a cognitive special education assessment, as math intervention occurs before a special education referral or assessment has taken place. I used the term mathematics difficulty (Fuchs et al., 2010; Lewis, 2014) to refer to students either receiving math intervention or special education in the area of math (Stevens et al., 2018).

One of the roles of the special education teacher has been to differentiate instruction in the general education setting for students in the areas of reading, writing, and math. Within the district in which this study took place, prior to 2010, learning support teachers typically pulled students to a separate location and supported students through remediation. This practice continues to occur for some learning support students in this district, as well as students who received math intervention. I refer to this model in special education and intervention as the pull-out model (Marston, 1996). Through this pull-out practice, students did not have access to core instruction (Fletcher, Lyon, Fuchs, & Barnes, 2007).

Research in special education indicated for some time the ineffectiveness of the pull-out model for helping students reach grade-level standards (Fuchs & Fuchs, 2007; Fuchs et al., 2007; Geary, 2004, 2005; Geary, Hoard, & Hamson, 1999; Ginsberg & Pappas, 2007; Griffin, 2007; Karp & Voltz, 2000; Lewis, 2014; Lewis & Fisher, 2016, 2018; Lewis & Lynn, 2016; Marston, 1996). Current special education research has shown a more promising approach for helping students in special education. This approach is referred to as *inclusion* and takes place when the special education teacher supports core content learning in the general education setting (Friend, Cook, Hurley-Chamberlain, & Shamberger, 2010; Solis, Vaughn, Swanson, & McCulley, 2012). Researchers and educators tend to prefer inclusive education for students receiving special education (Mulholland & O'Connor, 2016). A few studies indicated the success of inclusion and co-teaching. Murawski and Swanson (2001) asked, "Where is the data?" in their review of co-teaching (p. 258) and Friend et al. (2010) also asserted research that supports positive outcomes for co-teaching is limited, "but these glimmers of positive outcomes must be fortified to assert without equivocation whether or not co-teaching positively affects student outcomes. The sustainability of this instructional model depends on better quality and more research" (p. 22). As

Friend et al. (2010) indicated, many researchers have studied the foundation of co-teaching but few studies focused on understanding how this service delivery model impacts teaching and learning,

> Most inquiry on co-teaching has emphasized co-teachers' roles and relationships or program logistics rather than demonstrating its impact on student achievement and other key outcomes, and far more literature exists describing co-teaching and offering advice about it than carefully studying it. (p. 9)

Recent research on co-teaching indicates positive associations between confidence, interest, and attitudes on co-teaching with provided professional development (Pancsofar & Petroff, 2013). Other research has indicated the importance of co-teaching as a tool to construct new learning (Rytivaara, Pulkkinen, & Bruin, 2019). Co-teaching requires time, effort, and multiple opportunities to negotiate learning for both teachers and students (Rytivaara et al., 2019). This study focused on a gap in the literature by going beyond co-teaching roles and responsibilities (Sileo, 2011; Stainback & Stainback, 1984). I examined how collaboration between math interventionists, special education teachers, and general education teachers supported learning for students with mathematics difficulty. My study did not directly link teacher learning to improved outcomes for students with mathematics difficulty. I focused on how professional development for general education elementary teachers, math intervention teachers, and special education teachers can support instructional shifts for students with mathematics difficulty.

Math Intervention Teachers in Elementary Education

Math intervention teachers at the elementary level support students with mathematics learning gaps in kindergarten through fifth grade. Math interventionists support learning with tasks of counting, comparison of quantity, mathematics fluency, and word-problem performance (De Smedt & Gilmore, 2011; Stevens et al., 2018; Stock, Desoete, & Roeyers, 2010; Tolar, Fuchs, Fletcher, Fuchs, & Hamlett, 2016). In the past, math interventionists focused on acquiring skills rather than focusing on reasoning and sense-making (Stevens et al., 2018). Often this remedial instruction, regardless of location, was disconnected from classroom core instruction and skill generalization was poor (Fuchs & Fuchs, 2007; Fuchs et al., 2007; Geary, 2004, 2005; Geary et al., 1999; Ginsberg & Pappas, 2007; Griffin, 2007; Karp & Voltz, 2000; Lewis, 2014; Lewis & Fisher, 2016, 2018; Lewis & Lynn, 2016).

Not all math interventionists have had specialized training in the content area of elementary math instruction (Dyson, Jordan, & Glutting, 2013). Many interventionists at the elementary level have had extensive training in reading intervention (Al Otaiba et al., 2019). With the No Child Left Behind Act [NCLB] of 2001, Title One provisions allowed for adding math intervention to the list of support services offered by schools (NCLB, 2001). Some schools hired math interventionists with specialized training in elementary math instruction, but the majority of schools began asking reading interventionists to instruct two students or a small group with mathematics difficulty for math intervention (William McKenna, Shin, & Ciullo, 2015). The result was that students with mathematics difficulty did not receive high-quality, targeted instruction around the area of need.

Special Education Teachers at the Elementary Level

Elementary special education teachers support students in special education in grades kindergarten through fifth grade, in reading, writing, and math (Murawski & Hughes, 2009). This requires a single teacher to understand 18 different curricular domains (six grade levels multiplied by three content areas), and at times, can result in special education teachers with a fragmented knowledge of standards, curriculum, and content, at each grade level and in each core academic area. Special education teachers often lack a strong understanding of each grade level or with specific content areas (Council for Exceptional Children, 2004). Compounding the challenges faced by elementary special education teachers, each student in special education is entitled to specially designed instruction. This required feature of special education highlights the importance of special education teachers and general education teachers collaborating with each other (Polloway, 2002; Rytivaara et al., 2019). Special education teachers design individualized instruction for as many students as special education teachers have on their caseloads. Unless special education teachers receive extra professional development in understanding children's mathematical thinking, special education teachers may lack the understandings for how children attempted to solve problems (Polloway, 2002). Researchers found that special education teachers may not know how to further a student's mathematical thinking within the general education setting (Carpenter, Fennema, Franke, Levi, & Empson, 1999, 2015; Fletcher et al., 2007; Fuchs et al., 2007). When neither the general education teacher nor the special education teacher has research-based ideas or strategies to support math learning for a special education student, this often results in the special education teacher pulling students in special education to a back table to provide remediation of procedural learning (Geary, 2004, 2005; Geary et al., 1999). Remediation of procedural learning is an instructional focus only on skills and/or computations that should have been mastered in previous grade levels, rather than a focus on problem-solving that supports students with current grade-level learning (Carpenter et al., 2015).

Even when students with mathematics difficulty are included in the general education setting and supported by the math interventionist or the special education teacher, the learning opportunities for students with mathematics difficulty are typically void of opportunities to develop math concepts (Boaler, 2008; Brophy, 1999; Fletcher, 1971; Floden, 2002; Grouws, 2004). Essentially, the students in special education do not gain access to core content nor address gaps in learning from previous grades (Marston, 1996). As a result, the student with mathematics difficulty did not have the full opportunity to attend explicitly to concepts as their peers who have already completed previous and current math learning (Franke et al. 2007; Hiebert & Grouws, 2007; Lewis, 2014; Mason, 2008).

Professional development (PD) focused on research-based approaches such as complex problem solving and the use of high cognitive or rich math tasks (Carpenter et al., 1999, 2015; Smith & Stein, 2011) supported general education teachers, math interventionists, and special education elementary teachers (Fuchs & Fuchs, 2007; Fuchs et al., 2007; Lewis, 2014). Focused PD provided ideas on how to increase math achievement for students with mathematics difficulty and helped teachers provide learning opportunities, so all students reach their full potential (Fuchs & Fuchs, 2007; Fuchs et al., 2007; Lewis, 2014).

When general education math teachers and special education teachers collaborated and provided specially designed instruction, differentiated assignments, differentiated assessments, and any other tools or resources, the entire class benefited (Friend et al., 2010). It is increasingly more important that teachers work together so students with mathematics difficulty can reach their full potential (Lewis, 2012; Will, 1986).

When both teachers have research-based ideas on math instruction, co-teaching shows promise as a model that allows students with mathematics difficulty to address gaps in missed learning (Friend et al., 2010; Lewis, 2014; Solis et al., 2012). Friend et al. defined Co-teaching as “the sharing of instruction by a general education teacher and a special education teacher or another specialist in a general education class that includes students with disabilities" (Friend et al., 2010, p. 9). Co-teaching is a complex endeavor that requires several skill sets. Teachers need established roles and responsibilities, alignment in instructional beliefs, and the ability to collaborate between general education teachers, special education teachers, and math interventionists (Friend et al., 2010; Garmston & Wellman, 2009; Solis et al., 2012). The skills and understandings about co-teaching have varied depending on PD, understanding of collaboration, and the time teachers have had available for collaboration (Cook & Friend, 1995; Friend, 2007; Scruggs, Mastropieri, & McDuffie, 2007; Sileo, 2011; Will, 1986).

Theoretical Perspectives

I framed the work underlying this study using a transformative worldview (Mertens, 2003) in combination with sociocultural theories of learning (Vygotsky, 1978). The transformative worldview works with most research designs because of the grounded belief that utilization of a variety of research methods, techniques, and theories is appropriate when one cycle of inquiry designed to benefit a marginalized group creates the next (Mertens, 2010). Sociocultural theories of learning situate learning as an action in which individuals make sense of the world around them (Creswell, 2007). I believe knowledge is co-constructed, and students learn best when they are encouraged to reason and make meaning of mathematical ideas in an engaging context (Carpenter et al., 2015 Vygotsky, 1978). An optimal learning environment for students includes an appropriate zone of proximal development (ZPD) (Vygotsky, 1978). Because the range of learning needs varies greatly in a classroom, rich math tasks allow for a variety of student entry points into the tasks. All students have important and worthy mathematical ideas and inherent solution strategies for solving engaging problems in a context (Carpenter et al., 2015).

Overview of Study Design

This action research case study aimed to investigate the implementation of a professional development (PD) program, focused on co-teaching, and the impact of the PD on collaborative research-based instructional practice. The aims of the study were to advance the knowledge base of co-teaching (Friend et al., 2010; Murawski & Swanson, 2001) and provide PD that developed, supported, and sustained, effective co-teaching instruction (Aguilar, 2013; Sileo, 2011; Sweeney, 2011). This project examined the implementation of a twelve-week PD program with elementary general education teachers, special education teachers, and/or math intervention co-teachers. The PD comprised two main areas: research-based math instructional practice and collaboration. Co-teaching pairs participated in 12, ninety-minute sessions that I facilitated in a dual role as instructional coach and researcher (Jaworski, 1998). These instructional-based collaboration meetings supported learning as well as providing an intimate setting for co-teachers to engage in open dialogue. During the instructional portion of the PD the co-teaching partnerships learned (a) about roles and responsibilities of co-teaching (Friend et al., 2010; Sileo, 2011; Stainback & Stainback, 1984), (b) Cognitively Guided Instruction (CGI) (Carpenter et al., 1999, 2015), (c) a launch, explore, summary lesson structure (LES) (Lappan, Phillips, & Fey, 2007), and (d) how to orchestrate a summary math discussion (Smith & Stein, 2011).

Nine participants included four co-teaching partnerships at the elementary level. The nine teacher participants (a) planned for co-taught instruction, (b) implemented instruction together, and, (c) analyzed student work samples. Co-teaching partners planned lessons using the LES lesson structure (Lappan et al., 2007). Special education teachers, general education teachers, and/or math interventionists evaluated the effectiveness of collaborative lessons by analyzing growth on classroom-based math assessments and assignments for students with mathematics difficulty (Lewis, 2014).

Data sources included co-teaching surveys, interviews, observations, PD documents, and student work samples. Each teacher participated in a semi-structured interview prior to the PD and post PD. The co-teaching partners participated together in a semi-structured interview prior to the PD and post PD. As part of the study design, the co-teaching partners continued to participate in their professional learning communities (PLC). PLCs in this district were structured around general education grade-level teams that met once a week and focused on student data and needed instruction. Teachers not on a grade-level team may not have a consistent PLC. Some instructional positions, such as a learning support teacher or math interventionist only have one teacher at a specific location, and this is a disadvantage for teachers by not having a specific PLC. Because the administrative leadership at each school determined which teachers composed a PLC, not all co-teaching partners were on the same PLC. I attended PLC meetings of each general education teacher taking the stance of participant-observer (Spradley, 1980).

Qualitative data analysis methods used included the constant comparative method (Bogdan & Biklen, 2007; Corbin & Strauss, 2007; Miles, Huberman, & Saldaña, 2014) and analytic induction (Merriam, 2009). I developed a codebook (Appendix A) to generate initial codes, and used HyperRESEARCH®, a qualitative research, analysis software program, to assist in condensing and coding the data to identify patterns and themes.

Research Questions

1. How do special education teachers, general education elementary teachers, and/or math interventionists, develop their co-teaching relationships? What are the teachers' perceptions regarding the process of developing these relationships?
2. How do special education teachers, general education elementary teachers, and/or math interventionists co-design math instruction with a focus on supporting students with mathematics difficulty in accessing grade-level core content?
3. How does the PD support mathematical instructional practice for special education teachers, general education teachers, and/or math interventionists?

In my findings indicated for Research Question 1 (RQ1), I found that several factors influenced teachers in developing co-teaching relationships. Co-teaching relationships depended on teacher perceptions of students, and the goals teachers had for student learning. Co-teaching relationships influenced how the co-teaching partnerships thought about lesson planning and lesson plan grain size. Findings for Research Question 2 (RQ2) indicated that certain structures supported co-designed instruction for students with math difficulty. When the intervention team (general education teacher, math interventionist, and special education teacher) used the *Five Practices* (Smith & Stein, 2011) and principles of CGI (Carpenter et al., 1999, 2015), teachers provided all students access to core instruction. The use of instructional coaching and reframing of tasks helped the teachers advance their instructional practice. Findings for Research Question 3 (RQ3) indicated the PD supported mathematical instructional practice with lesson summary enactment and lesson summary discourse moves.

In the final chapter, I discuss multiple implications. First, teachers' perspectives of students influenced the co-teaching relationship. Teachers' views ranged from a strength-based perspective, a gap-based perspective, or an asset-based perspective (Celedón-Pattichis et al., 2018). Teams of teachers, utilizing different perspectives, planned, and implemented lessons that assisted students with mathematics difficulty to address gaps in learning. Teachers needed to understand the elements of effective intervention in order to implement intervention effectively. When teachers had a lack of knowledge about how to intervene, the intervention did not occur. A third implication was that when teachers engaged in PD that focused on supportive instructional strategies, teachers used these strategies and applied them to their instruction. Teachers learned how to reframe math tasks to leverage student thinking and solution strategies. A fourth implication discussed in this study involved the use of instructional materials. The curriculum resources used by teachers either supported or hindered their instructional practice. Teachers who used materials with a narrow focus or limited learning targets tended to plan and instruct in those ways. Teachers struggled to plan in ways divergent from the curriculum materials they used. Teachers needed support in the forms of coaching, PD, and curriculum materials to plan with a learning progression in mind. A fifth implication focused on the use of the *Five Practices* (Smith & Stein, 2011) in lesson planning. When teachers used the *Five Practices* in lesson planning, they were much more likely to understand the importance of the summary portion of the lesson. Teachers understood how to help students connect and generalize ideas. A sixth implication focused on discourse in the lesson summary. Even with the supportive help of the *Five Practices* in planning for a math discussion, teachers continued to struggle to enact a lesson summary. Teachers struggled to use discourse moves that encouraged horizontal, student-to-student classroom discourse. The final implication involved changes in instructional leadership. When a school had multiple different principals over the course of a few years, the PD lacked consistency and focus. Multiple changes in school leadership were a barrier to developing teacher capacity in math instructional strategies.

Overview of Study

Nine participants co-designed and co-implemented math instruction aimed at allowing students with mathematics difficulty to attend to gaps in learning. The PD offered a possible solution to address the problem through a provided learning opportunity for co-teachers. Teachers learned how to negotiate roles and responsibilities of co-teaching while also learning about research-based math instructional practices that allowed students with mathematics difficulty reach their full potential and attend to gaps in understanding. This research pursued how co-teachers experienced the PD in the development and implementation of co-taught math lessons through case study methodology by reporting, comparing, and analyzing the challenges and successes each co-teaching partnership experienced. I endeavored to understand ways instructional coaches or district teacher leaders could support co-teachers. Co-teachers learned how to be effective co-teachers as well as learned research-based instructional approaches for students with mathematics difficulty. This study aimed to contribute to the fields of math education, math intervention, and special education, and PD.

Literature informing this study included research and theory in the areas of math education, math intervention, special education, collaboration, and PD. I formed the theoretical framework for this study by merging these five bodies of literature and drawing on a transformative worldview.

The following structure describes the organization of the six chapters of this dissertation, the bibliography, and appendixes. Chapter 2 includes a discussion of the theoretical framework and review of related literature on special education, mathematics education, collaboration, PLCs, and PD. Chapter 3 includes a description of the research and methodology of the study. Chapter 4 outlines the PD program for teacher learning. Chapter 5 contains a description the findings of the study. Chapter 6 includes a discussion the interpretations of the findings and implications of the study.

CHAPTER TWO: REVIEW OF THE LITERATURE RELATED TO MATHEMATICS PROFESSIONAL DEVELOPMENT

I framed the work underlying this study using a transformative worldview (Mertens, 2003) in combination with sociocultural theories of learning (Vygotsky, 1978). Following a discussion of this framework, I present five bodies of literature that inform the study: (a) math intervention, (b) special education, (c) research and theories of general education mathematics instruction, (d) collaboration between general education, special education, and/or math intervention teachers and (e) effective PD strategies. I made an assumption that through my research, positive change and empowerment of a marginalized group was enacted (Mertens 2003).

Transformative Worldview

Tashakkori and Teddlie (2003a) suggest that the advocacy-participatory worldview, also known as the transformative-emancipatory paradigm aligns philosophically with the use of action research. Mertens (2003) agrees, commenting, "The transformative paradigm might involve quantitative, qualitative, or mixed methods, but the community affected by the research would be involved to some degree in the methodological and programmatic decisions" (p. 141). Researchers who use this transformative paradigm establish an explicit goal for the research to serve the ends through the creation of a more just and democratic society (Mertens, 2003). Researchers, who hold a transformative worldview, use a literature review to determine known information about a certain issue and define the research problem through that literature review (Mertens, 2003). The transformative-emancipatory perspective seeks to prevent marginalized groups and their depicted cultures represented as deficient in some way (Creswell, 2007; Mertens, 2003). The transformative researcher begins with a comprehensive literature review and then moves toward spending time with members of the population of interest to build trust (Mertens, 2003). Methods used in the transformative paradigm include a combination of observations, interviews, demographic data, preliminary surveys, or other empirical data (Mertens, 2003). Researchers with this stance focus on how the data collection process and outcomes benefit the intended population. Researchers monitor how valid data are collected but also how the data collection process advances the needs of the researched population (Mertens, 2003; Creswell, 2007; Plano Clark & Creswell, 2008). The language used in transformative research avoids deficit language to refer to the intended population in a negative way (Mertens, 2003). The philosophical underpinnings of the transformative worldview support qualitative case study action researchers in employing different research designs. These research designs illuminate a societal or educational problem and advance a policy change for a marginalized group (Bogdan & Biklen, 2007; Corbin & Strauss, 2007; Creswell, 2007; Mertens, 2003; Miles et al., 2014; Stake, 1995; Yin, 2014). An integral part of the theoretical framework for this study integrates existing research and theory about of math intervention, special education, and the requirements of math interventionists and special education teachers (NCLB, 2001; Individual with Disabilities Education Improvement Act [IDEA], 2004).

Sociocultural Learning Theory

Vygotsky's (1978) sociocultural learning theory utilizes concepts of constructivism and focuses on the social interactions of individuals. Vygotsky (1978) believed that knowledge is co-constructed. Students learn self-regulation skills through social interactions and development occurs through the transmission of language and symbols (Vygotsky, 1978). A key concept developed by Vygotsky was the zone of proximal development. This is defined as a difference between a person's instructional level and their independent learning level. The foundation of sociocultural learning theory means students move from an instructional level to an independent level because students interact with adults and peers (Vygotsky, 1978). These key features of constructivism and sociocultural learning theory provide the backbone in research on student learning and the development of teacher pedagogical content knowledge (Shulman, 1986).

Learning Disabilities and Service Delivery Models

Learning disabilities became a federally designated handicapping condition in 1968, and the population of students with a learning disability label continued to grow steadily (Fletcher, Reid Lyon, Fuchs, & Barnes, 2007; U.S. Department of Education, 1999). About one-half of identified students who have received special education services had a learning disability (U.S. Department of Education, 1999). Students are identified with a learning disability in the area(s) of reading, written expression, and/or mathematics. Federal regulations grouped learning disabilities into seven different areas: listening comprehension, oral expression, basic reading skills, reading comprehension, written expression, mathematics calculation, and mathematics reasoning (Fletcher, Foorman, Shaywitz, & Shaywitz, 1999). It is not surprising that with five out of seven categories of learning disabilities focused on literacy development, the area of reading led the way in terms of research and the instructional needs of students with learning disabilities (Fletcher et al., 2007). Less is known about students with mathematics learning disabilities, even though the field made substantial progress since the 1980s and 1990s (Fletcher et al., 2007; Gersten, Clarke, & Mazzocco, 2007; Ginsberg & Pappas, 2007; Lewis, 2010, 2014; Lewis & Fisher, 2016). Special education legislation helped mathematics researchers understand past and current special education service delivery models (Evans, 2007). This legislation also helped researchers understand instructional methods for students who received special education mathematics services.

Special Education Legislation

In 1975, the Education for All Handicapped Children Act, which later changed to, Individuals with Disabilities Education Act defined the concept of learning disability as the following:

> The term "specific learning disability" means a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, which may manifest itself in an imperfect ability to listen, speak, read, write, spell, or to do mathematical calculations. The term includes such conditions as perceptual handicaps, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia. The term does not include children who have learning disabilities, which are primarily the result of visual, hearing, or motor handicaps, or mental retardation, or emotional disturbance, or of environmental, cultural, or economic disadvantage. (IDEA Sec. 300.8(c)(10))

This definition created confusion among researchers in the field of special education over categorizations of learning disabilities and causes of learning disabilities (Fletcher et al., 2007; Gersten et al., 2007; Mazzocco, 2007). The language around learning disabilities used in the research base and in schools typically focused on a deficit perspective and identified ways students with learning disabilities were different than their general education peers (Fletcher et al., 2007; Gersten et al., 2007; Mazzocco, 2007). In the past, educators generally used deficit language to describe a child's ability. Parents advocated for a change in discourse and demanded more knowledge and research to understand the instructional needs of children with learning disabilities. Lewis (2014) advocated for a shift in language to describe students with mathematical learning disabilities as students with persistent and robust misunderstandings rather than students that are deficient in some way.

In 1969, child advocates lobbied Congress to enact Public Law 91-230 that authorized research and training programs that addressed the needs of children with specific learning disabilities (Fuchs et al., 2007). This public law allowed for the funding of many research studies focused on learning disabilities. Between the years of 1996 – 2005, only one in about 14 of these federally funded studies focused on mathematics learning disabilities (Gersten, Clarke, & Mazzocco, 2007). During this time, confusion remained regarding how to identify students with a mathematics learning disability (Fletcher et al., 2007; Gersten et al., 2007; Lewis & Fisher, 2016; Mazzocco, 2007). This persistent confusion among researchers on definitions, terms, and ways to identify mathematics learning disabilities inhibited research efforts. This confusion also impeded new ways of identifying students who needed more support and services in the general education classroom (Evans, 2007; Fletcher et al., 2007; Gersten et al., 2007; Lewis & Fisher, 2016; Mazzocco, 2007). Confusion on terms such as mathematics disability and mathematics difficulty still exist in the research base (Harbour, Adelson, Karp, & Pittard, 2018; Lewis & Fisher, 2016; Whitney, Lingo, Cooper, & Karp, 2017).

Educational policy. In 2004, the US Congress reauthorized the Individuals with Disabilities Education Act (IDEA, 2004). The reauthorization indicated that

> States could not require districts to use IQ tests for the identification of students for special education in the learning disability category, states had to permit districts to implement identification models that incorporated response to instruction and children could not be identified for special education if poor achievement was due to lack of

appropriate instruction in reading or mathematics, or to limited proficiency in English (Fuchs et al., 2007, p. 23).

The reauthorization of IDEA opened the door for new special education identification and service delivery models. The reauthorization also attempted to reduce the number of children identified for special education who were racially, culturally, or socioeconomically diverse from a white, suburban, middle class, population (Harry, 2007; Paul, Fowler, & Cranston-Gingras, 2007; Royer & Walles, 2007).

Shifting Special Education Identification and Service Delivery Models

The primary model used throughout special education history to identify students for special education has been the aptitude-achievement discrepancy model. With this model, if a student failed to learn in an expected way, it was because of the student's deficits (i.e., missing skills or abilities). Deficit thinking was fundamental to dominant special education discourses (Cochran-Smith & Dudley, 2013; Fuchs et al., 2007; Lewis, 2010; Murawski & Hughes, 2009). When a deficit perspective is used, teachers determine the missing skill and then teach towards the lagging skill rather than working from a strength-based perspective (Geary 2004, 2005). Attempting to test and identify specific mathematics skills has been a focus of Geary's research in the field of special education and mathematics disability (Geary, 2004, 2005; Geary, Hamson, & Hoard, 2000; Geary, Hoard, Nugent, Byrd-Craven, 2007). Historically, Geary was a leader in researching mathematics learning disabilities. More recently, Katherine Lewis has been researching the intersecting fields of math and special education and has shifted thinking of mathematics learning disabilities away from a deficit perspective (Lewis, 2010, 2014; Lewis & Fischer, 2016).

Geary's work used behaviorist, and cognitive learning theories focused on the identification and treatment of students' missing skills (Gersten et al., 2007). Behaviorist learning theory served as the backbone of special education research (Geary, 1993, 2004, 2005, Geary et al., 2000; Russell & Ginsberg, 1984; Shalev, Manor, & Gross-Tsur, 1993) and contributed to the reason special education teachers have typically approached teaching mathematics using divergent instructional approaches from general education teachers. Behaviorist learning theory focuses students on learning isolated skills and teachers using instructional approaches such as task analysis. Typically, many special education teachers used task analysis to break skills and procedures down into small parts. Students learned skills in pieces and then assembled the pieces together to create an understanding of the entire task or procedure (Boaler & Greeno, 2000; Schunk, 2012; Spillane, 2002). The heavy use of cognitive and developmental psychology models in studying mathematics disabilities created a focus on deficit perspectives and also resulted in a fragmented instructional approach towards mathematics skills (Geary, 1993, 2004, 2005; Geary et al., 2000; Geary et al., 2007; Lewis & Fisher, 2016; Russell & Ginsberg, 1984; Schunk, 2012; Shalev et al., 1993). "These [cognitive and developmental psychology] theoretical models and experimental methods have provided the foundation for the study of cognitive deficits in children with mathematics disabilities" (Geary, 2005, p. 306). In the past, special education teachers have worked with students in special education on task analysis procedures removed from the general education classroom (Gelzheiser, Meyer, & Pruzek, 1992; Routman, 2012).

Pull-out model. When special education instruction focuses on remediation, then the special education service delivery model tends to focus on removing the student from the classroom for specially designed instruction to remediate skills (Cochran-Smith & Dudley-

Marling, 2012; Cochran-Smith & Dudley, 2013). The pull-out model provides instruction removed from the general education classroom (Marston, 1996). Pulled-out instruction around remedial skills limits students in connecting concepts taught in the general education classroom with skills taught by the special education teacher in the pull-out setting (Gelzheiser et al., 1992; Routman, 2012). The discrepancy identification model often creates a divide between special education and general education teachers (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley, 2013; Washburn-Moses, 2010), and this divide leads to fragmented and disconnected instructional strategies and skills for the students in special education (Griffin, 2007). By focusing on other identification and service delivery models, special education and general education teachers can work together towards inclusive, collaborative, instructional practices (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley, 2013; Washburn-Moses, 2010).

Response to Intervention. In 1999, the U.S. Department of Education published federal regulations in response to IDEA (2004). These regulations asserted that states could use a discrepancy model or a Response to Intervention (RtI) model for determining whether a student had a learning disability. The states could use a process that evaluated if students responded to research-based intervention as a service delivery model. RtI is an alternate special education identification model that has an emphasis on proactive instruction, ongoing assessment, and data-based decision-making (Rouse & McLaughlin, 2007; Thomas, Cook, Klein, Starkey, & DeFlorio, 2018). Principles of RtI suggest targeted and aligned intensive instruction in the general education classroom positively affects the student with a mathematics learning difficulty and promotes an inclusive classroom (Murawski & Hughes, 2009; National Center for Learning Disabilities, 1989).

Constructivist learning theory is the foundation for RtI (Vygotsky, 1978) and assumes competence for the student. This assumption stands in contrast to a deficit perspective based on behaviorist learning theory. Behaviorist learning theory assumes a problem with the student (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley-Marling, 2013; Washburn-Moses, 2010). “A social constructivist rendering of school failure differs sharply from the deficit perspective that attributes failure to individual traits or abilities” (Cochran-Smith & Dudley-Marling, 2013, p. 280).

The shift in the identification of learning disabilities from a discrepancy-based model to a RtI model, or additive approach, is a major shift for most schools (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley, 2013). The identification process shifts the focus from an assumption that something is wrong with the individual child to an examination of the fit between the student and the learning environment and/or instructional practices. The new approach assumes that something is wrong with instruction for that student. The problem to address is with the instruction not matching student need (Fuchs et al., 2007; Murawski & Hughes, 2009).

As discussed earlier, the 2004 reauthorization of the Individuals with Disabilities Education Improvement Act (IDEIA) no longer stated that the use of the discrepancy or deficit formula was the sole method to identify students with disabilities. The use of data that demonstrated the child’s response (or lack thereof) to research-based interventions was now equally permissible to identify students with disabilities (Fuchs et al., 2007; Gersten et al., 2007; Mazzocco, 2007).

Another key feature of the additive approach paradigm shift was the shift away from providing specialized instruction only after a child had failed enough to qualify for services, which was reactive in nature. Using a proactive approach helped remedy an instructional problem before misconceptions or gaps in student understandings developed (Murawski & Hughes, 2009). Fuchs et al. (2007), Fuchs and Fuchs (2007), Griffin (2007), and Fuchs, Fuchs, and Gilbert (2018) have written extensively on early intervention and prevention of learning disabilities. Stevens et al. (2018) conducted a meta-analysis of 25 years of research on mathematics intervention and outlined elements of effective intervention. The RtI additive approach emphasized the use of intensive instruction that was collaboratively designed (Friend et al., 2010) to improve student learning before small gaps in students' mathematics understandings became larger gaps. This proactive RtI approach required both special education and general education classroom teachers to instruct students collaboratively (Friend et al., 2010), rather than waiting for a team to validate that a student had deficits in learning and needed special education services (Fletcher et al., 2007; Fuchs et al., 2007; Gersten et al., 2007; Griffin, 2007; Mazzocco, 2007).

Principles of RtI include identification of students who need help, data-based decision-making, teacher knowledge of interventions needed by the student, and knowledge of what resources are available at the school (Al Otaiba et al., 2019; Thomas et al., 2018). Team decision making and data-based documentation are foundational elements to RtI and is required at repeated intervals during intervention instruction (Fuchs et al., 2007). RtI focuses on six to eight-week cycles of instruction and data collection, also commonly referred to as progress monitoring. Although the principles of RtI are rooted in helping students to address learning gaps, RtI has also become a process of sorting and labeling kids (Al Otaiba et al., 2019). These labels revert to old ways of categorizing students by their deficits rather than focusing on student need (Lewis & Fisher, 2016). When a school utilizes an RtI model, teams of teachers use assessment data and collaborate to determine how to meet the instructional needs of the students.

Collaboration and RtI. When mathematics coaches help teachers analyze classroom data and determine student instructional need, students have improved academic performance (Harbour et al., 2018). Harbour et al. (2008) found that all students made significant growth on performance outcomes when a full time mathematics coach or specialist worked at the school. "Because the overall results of the current study indicated that fourth-grade students with disabilities, as well as students without disabilities, benefit from elementary schools providing full-time mathematics coaches and specialists" (Harbour et al., 2008, p. 673). Just having a mathematics coach or specialist is not enough for effective RtI implementation. Al Otaiba et al. (2019) found that teachers felt less prepared to make data-based instructional decisions when they reported having limited knowledge of RtI. Three out of six essential elements for effective RtI implementation directly connects to collaboration (Lembke et al., 2012). Effective RtI implementation requires a system of progress monitoring for data-based decision making, research-based instruction for core learning and intervention instruction, and finally, ongoing program evaluation (Lembke et al., 2012). These essential elements of RtI require the collective efficacy of a variety of teachers within the school (Harbour et al., 2008).

RtI has become an unpopular term with some educators as it emerged as another way to talk about students as high, medium, or low rather than focusing on student instructional need (Dickman, 2006). When used inappropriately, RtI identifies students as red, yellow, or green as a prediction of student achievement on state assessments. As concepts of RtI have become intertwined and burdened with ideas not helpful to actually intervening with students (Al Otaiba et al., 2019), a new term has emerged to clarify and remedy the problems that emerged with RtI.

Multi-tiered systems of support (MTSS) is essentially the same concept as RtI without the negative connotations associated with RtI (Shepley & Grisham-Brown, 2019).

Multi-tiered system of support. MTSS is a focused service delivery model ensuring instruction aligns with student need by preventing instructional problems before they arise. Essential elements of MTSS include core instruction and tiered interventions and supports, universal screening measures, progress monitoring, and data-based decision-making. A key feature of a MTSS framework is an integrated connection among all the possible supports a student could receive in school. No longer should core instruction, English language instruction, intervention instruction, social/emotional instruction, and/or special education instruction be contained to individual silos for the student. When a student receives interconnected services from several school departments, then the student has benefited from a MTSS service delivery model.

Although MTSS is essentially the same as RtI, MTSS is different in one significant way. MTSS assumes students could be receiving extra support services in multiple areas, and these supports should be aligned and working cohesively with each other (Al Otaiba et al., 2019; Shepley & Grisham-Brown, 2019). For a cohesive, aligned, instructional experience for the student, MTSS also assumes teachers would collaborate with each other to ensure a connection between different support services. The teacher focus is on fixing instruction rather than fixing the student (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley-Marling, 2013; Washburn-Moses, 2010). Special education teachers, academic interventionists, and general education educators collaborate to figure out how to build on the language, culture, experience, and background knowledge all students bring with them to school to support their learning (Fletcher et al., 2007; Paul et al., 2007; Royer & Walles, 2007; Shepley & Grisham-Brown, 2019). This work is not simple, nor easy and often requires new learning for the entire MTSS team. Shepley and Grisham-Brown (2019) found that for MTSS to be effective "significant time for PD and one-to-one coaching are provided to teachers" (p. 307). When a MTSS approach is utilized, teachers implement a variety of instructional strategies with the purpose of all students gaining access to core instruction.

Principles of Effective Intervention

Regardless of the service delivery model for intervention, there are key principles for the implementation of effective intervention. Although much has been written about reading intervention (Graham & Kelly, 2019), the research base for math intervention is lacking (Stevens et al., 2018). Stevens et al.'s (2018) meta-analysis of math intervention research found only 25 studies that met the inclusion criteria for research articles about math intervention. Regardless of the content area for intervention (reading, math, or behavior), researchers have identified seven key components of highly effective intervention (Lembke et al., 2012). Highly effective intervention represents an approach whereby a team collaborates to engage in a data-based decision-making process. This process provides intervention instruction aligned to classroom instruction, provides targeted lessons, includes quality assessments, incorporates progress-monitoring tools, and is responsive to the student's needed intensity level of intervention (Celedón-Pattichis et al., 2018; Fuchs et al., 2018; Graham & Kelly, 2019; Harbour et al., 2018; Lembke et al., 2012; Stevens et al., 2018; van der Scheer & Visscher, 2018).

Team and data-based decision-making. A key feature of effective intervention is that a team uses a data-based decision-making process to design an intervention (van der Scheer & Visscher, 2018). Both RtI and MTSS call for the intervention team to consist of a general education teacher, an academic interventionist, and a school administrator. Often intervention teams also will include a school's special education teacher and instructional coach. Once the assembled team evaluates student data, then the team makes decisions based on the data. Specifically, the team evaluates the data to determine the instructional needs of individual students as well as groups of students (Stevens et al., 2018). The team also makes decisions about the learning goals for each instructional group of students. Lastly, the team decides which instructional approach and resources are most promising for accomplishing the instructional goals (van der Scheer & Visscher, 2018). Using a team decision-making process requires teachers collaborate to design targeted intervention lessons aligned to core instruction (Harbour et al., 2018).

Targeted lessons aligned with core instruction. Effective intervention includes targeted lessons aligned to core classroom instruction. These are two tightly intertwined components of an intervention. During the team and data-based decision-making process, the team generates the targets for the intervention and discusses how these targets align with core instruction (Harbour et al., 2018). The research base supports intervention instruction connected to core classroom instruction. As Marston (1996) has indicated, when students are pulled-out for instruction, often instruction does not generalize to core content. The student may learn concepts in a pull-out model (Marston, 1996), but students later have difficulty applying those concepts in the general education setting (Fitzpatrick et al., 2019). In studies where intervention has made the most difference for students, this was because the instruction occurred in the general education classroom and was aligned to instruction occurring at that time (Graham & Kelly, 2019; Stevens et al., 2018). The intervention should include targeted lessons based on student need. During the team's data-based decision-making process, the general education teacher should know the overall instructional outcome of the intervention, as well as the daily instructional goal.

Mathematics coaches as well as mathematics specialists have shown to positively influence teacher's instructional practices and beliefs as well as positively affect student learning (Harbour et al., 2018). Knowing the specific needs of the student is a function of quality assessments.

A comprehensive assessment system. Another key feature of effective intervention is the use of a comprehensive assessment system (Buffum & Mattos, 2015). A comprehensive assessment system begins with the use of a screener to help identify students needing extra support (Lembke et al., 2012). Other components of a comprehensive assessment system include diagnostic assessments, formative assessments, and progress monitoring. A high-quality diagnostic assessment should be valid. Formative assessments are important in informing instruction but not considered a valid or reliable type of assessment (Small, 2019). Many different kinds of assessments are available, and each has a different purpose (Rouse & McLaughlin, 2007). Summative assessments and screening assessments serve a similar purpose as to rule out who does not need intervention in a given area. Neither summative assessments nor screening assessments help to inform instruction (Moss & Brookhart, 2012). The essential assessments needed for effective intervention are the use of diagnostic assessments, progress-monitoring assessments, and growth assessments (Buffum & Mattos, 2015; Stevens et al., 2018). Diagnostic assessments pinpoint the actual need for intervention but do not also work as a growth assessment (Rouse & McLaughlin, 2007). Growth assessments often come with an intervention curriculum. Primary growth assessments occur before and after instruction. Secondary growth assessments are progress-monitoring assessments. Progress-monitoring assessments also measure growth but at regular predetermined intervals and guide the plan for instruction. Depending on the growth a student makes as indicated from the progress-monitoring tool, the teacher may decide to change the intensity of the intervention.

Intensity of intervention. The intensity of the intervention is changed based on evidence of student learning. By adjusting a number of factors, a teacher can change intervention intensity (Buffum & Mattos, 2015). One way to change intervention intensity is for the teacher to change the number of students in the intervention group. To increase the intensity, a teacher reduces the number of students in the group. If a teacher wants to decrease the intensity of the intervention, then a teacher adds more students to the intervention group. A second option to vary the intensity is for a teacher to change the frequency of intervention meetings during a week. For example, a teacher increases the intensity of the intervention by requiring the group to meet five days a week instead of the four days a week. In a similar fashion, the teacher decreases the intervention intensity by reducing the frequency of the group meeting. As a third option, a teacher can change the duration of the intervention lesson. For example, a teacher can make the intervention more intense by changing the duration of the group from 20 minutes to 30 minutes or makes the intervention less intense by reducing the duration of the intervention group. As a final option, an intervention team shifts the intensity of the intervention by changing the teacher expertise of who implements the intervention. For example, the intervention might be more intensive for the student by changing the intervention instruction given by the classroom teacher to a specialized math interventionist.

Studies have found that interventions that made a difference in student learning often included collaboration, data-based decisions, quality assessments, aligned and targeted instruction, progress monitoring, and changes to intensity of intervention based on need (Buffum & Mattos, 2015). Many studies on interventions found interventions made little to no improvement to student learning because several elements of effective intervention was absent (Graham & Kelly, 2019). This is not surprising when most of the elements of effective intervention involve collaboration and time to plan for aligned instruction. Planning for intervention does not usually incorporate time for collaboration (Buffum & Mattos, 2015; Garmston & Wellman, 2009). Collaborating and providing instruction aligned with general education instruction, with intervention instruction, and special education instruction has been a focus on the research base since the passage of NCLB. Alignment of all types of instruction is a needed shift in instructional strategies.

Shifting Instructional Strategies

Gaining access to the general curriculum for those students with special needs has been a strong agenda in special education for many years (Fletcher et al., 1999). Prompted by the passage of NCLB act in 2001 and its requirements that all teachers are highly qualified, there was a strong emphasis on ensuring that all students, including those with disabilities, have access to the content of the general education curriculum and to meet the academic benchmarks associated with those standards. Unfortunately, the dominant model in special education was for the student in special education to work on remedial skills such as basic fact memorization or computational skills removed from the general education setting (Geary, 1995, 2004, 2005; Geary et al., 2000; Geary et al., 1999; Geary et al., 2007; Karp & Voltz, 2000; Lewis, 2014; Lewis & Fisher, 2016). Even special education teachers or math interventionists who reported the use of inclusive instructional practices in the general education classroom often ended up teaching disconnected content at a back table while the general education students participated in cooperative problem-solving tasks (Baglieri, Valle, Connor, & Gallagher, 2011; Buffum & Mattos, 2015; Fuchs et al., 2018; Stevens et al., 2018; Karp & Voltz, 2000). Intervention and instruction for students in special education have been approached in dichotomous ways by both general education and special education teachers (Cochran-Smith & Dudley-Marling, 2012;

Cochran-Smith & Dudley-Marling, 2013; Lewis, 2014; Mazzocco, Myers, Lewis, Hanich, & Murphy, 2013; Paul et al., 2007; Washburn-Moses, 2010).

Even though there has been a divide among special education and general education teachers working collaboratively, the special education community has engaged in multiple efforts to build partnerships with general education teachers (Friend et al., 2010; Reynolds, Wang, Walberg, 1987). Universal design for learning represents a framework for providing greater curricular flexibility in the presentation of information, in students' ways of demonstrating knowledge and skills, and in students' engagement (Rao, Wook Ok, & Bryant, 2014). The concept of universal design for learning was included in the reauthorization of IDEA in 2004. Universal design for learning intends to reduce obstacles in curriculum and instruction and provide appropriate supports so that all students have access to the general curriculum and achieve at high levels (Murawski & Hughes, 2009; Rao et al., 2014). The major assumption underlying universal design for learning is that no single approach ensures all students have access to the curriculum. In other words, multiple and flexible teaching methods, assignments, activities, assessments, technologies, and materials are essential for quality instruction (Polloway, 2002).

Utilizing different instructional approaches as a type of intervention is the key principle to universal design for learning (Polloway, 2002). Karp and Voltz (2000) outlined three different instructional approaches to weave together during interventions to help students build conceptual understanding in mathematics. These instructional approaches are explicit instruction, apprentice approach, and constructivist approach (Karp & Voltz, 2000). These three instructional approaches draw on behaviorist (Spillane, 2002) and constructivist learning theories (Boaler & Greeno, 2000). When special educators or math interventionists rely too heavily on one approach, students with mathematics difficulty often continue to struggle and lack confidence, and lack the needed skills to be successful in an inclusive general education setting (Karp & Voltz, 2000; Lewis, 2010, 2014). General education teachers tend to rely heavily on a constructivist approach, while special education teachers and math interventionists tend to rely heavily on an explicit direct instruction of skills approach based on behaviorist learning theory (Boaler & Greeno, 2000; Fuchs & Fuchs, 2007; Fuchs et al., 2007; Geary, 2004, 2005; Ginsberg & Pappas, 2007; Griffin, 2007; Karp & Voltz, 2000; Lewis, 2014). A mix of instructional approaches from both behaviorist and constructivist learning theories benefit the special education student, rather than one single approach (Fuchs & Fuchs, 2007; Fuchs et al., 2006; Geary, 2004, 2005; Ginsberg & Pappas, 2007; Griffin, 2007; Hunter, Bush, & Karp, 2014; Lewis, 2010, 2014; Vygotsky, 1978). A variety of instructional approaches for students with gaps in understanding is essential, and this premise is the foundation for collaboration between general education teachers, academic interventionists, and special education teachers (Reynolds et al., 1987). Often special education teachers and general education teachers need to build on their understanding of how mathematics reform efforts can shift instructional practice.

Reform Efforts and Instructional Shifts in Mathematics Education

There have been many calls for mathematics reform in US history. These reform efforts impact learning theories and instructional practices implemented in classrooms across the nation. One study that opened the eyes of American researchers and teachers was the comprehensive Third International Mathematics and Science Video Study (TIMSS, 2007). This study comprised 59 nations and assessed student achievement in fourth grade, eighth grade, and twelfth grade in mathematics and science. TIMSS also assessed and compared classroom instruction with recorded video lessons (Hiebert & Stigler, 2000; Gonzalez et al., 2008). In 1995, educational spending per student in America was at the top of the list of the countries that participated in the TIMSS, yet America's fourth graders ranked near the bottom in terms of mathematics and science achievement (Gonzalez et al., 2008; Hiebert et al., 2003). Even more troubling data from 2007, TIMSS showed that American students were not making much growth, despite continued reform efforts in mathematics instruction (Gonzales et al., 2008). Between 1995 and 2007, fourth graders averaged a gain of 11 points on the mathematics portion of the assessment, showing little growth in developing conceptual mathematics understandings (Gonzales et al., 2008). Students who do not develop conceptual mathematic understanding tend to lack transfer of skills and have a lower capacity to solve mathematics problems in an authentic situated context (Hiebert et al., 2003). Because the TIMSS linked teacher instruction to student achievement, the study also provided clear evidence that mathematics instruction in American classrooms continued to need improvement (Gonzales et al., 2008; Hiebert et al., 2003). The video portion of TIMSS indicated American mathematics instruction focused on the breadth of content, which resulted in memorization of procedures without conceptual understanding for the mathematics behind the procedures (Gonzales et al., 2008; Timmerman, 2003). Because the way teachers teach affects the way students learn, (Hiebert et al., 2003) TIMSS evidence indicated a need to study and learn more about effective math instructional approaches. There is a need to study mathematics education for students with mathematics difficulty (Lewis, 2014; Lewis & Lynn, 2016) as well as a call for reform efforts in how special education teachers, math interventionists, and general education teachers collaborate (Forman, 2003).

Recent research and reform efforts stemming from the TIMSS focused instruction away from a traditional, teacher-centered, direct instruction, skill-based approach (Hiebert & Grouws, 2007; Hiebert & Stigler, 2000). Three promising research efforts since the TIMSS include: (a) a focus on constructivist and sociocultural learning theories to develop student conceptual understanding and skill efficiency, (b) an emphasis on using cognitively demanding tasks, and (c) a change in classroom discourse toward more student interaction.

Mathematics Learning Theory

Sociocultural learning theory assumes the construct of teachers as facilitators of student learning as students develop ideas and construct meaning around mathematics concepts (Vygotsky, 1978). Constructivism is a philosophical perspective contending that individuals construct much of what they learn and understand (Bruning Schraw, Norby, & Ronning, 2004). Although constructivism is based in philosophy, its assumptions provide general predictions or theories that can be tested (Schunk, 2012). Key assumptions of constructivist learning theory include the interaction of persons and situations in the acquisition and refinement of skills and knowledge (Cobb & Bowers, 1999). People are active learners and develop knowledge for themselves (Hiebert & Grouws, 2007), and learners become actively involved with content through the manipulation of materials and social interaction (Vygotsky, 1978). Vygotsky's (1978) sociocultural learning theory based on concepts of constructivism focused on the social interactions of individuals, and that knowledge is co-constructed. Students learn skills of self-regulation through social interaction and development occurs through the transmission of language and symbols (Vygotsky, 1978). A key concept developed by Vygotsky is the zone of proximal development, which states there is a difference between a child's instructional level and their independent learning level. The basis of sociocultural learning theory is that students can move from an instructional level to an independent level through interaction with adults and peers (Vygotsky, 1978). These key features of constructivism and sociocultural learning theory provide the backbone of important research in mathematics education in terms of teachers facilitating students developing conceptual understanding.

Conceptual understanding. Conceptual understanding focuses on, the deep interconnected mental connection or understanding of mathematic facts, procedures, ideas, concepts, or principles (Bay-Williams, 2010; Carpenter et al., 2015; Hiebert & Carpenter, 1992; Hiebert & Grouws, 2007; Stein, Grover, & Henningsen, 1996). Teaching for conceptual understanding stemmed largely from reform efforts, and research indicated that development of conceptual understanding improves procedural fluency (Boaler & Greeno, 2000; Hiebert & Grouws, 2007; Hiebert & Stigler, 2000). Three promising features identified in the research base that can help students develop conceptual understanding: student struggle, attention to mathematical concepts, and CGI (Carpenter et al., 2015; Hiebert, 2003; Hiebert & Grouws, 2007).

Student struggle. The first key feature that helps facilitate conceptual development is allowing students to struggle with important mathematical concepts (Bay-Williams, 2010; Hiebert & Grouws, 2007). Often elementary teachers approach learning from a lens of nurturing students and caring for their needs (Noddings, 2012), which can seem opposed to allowing students to struggle with mathematical concepts (Bay-Williams, 2010; Boaler, 2008; Hiebert & Grouws, 2007). Without understanding that student struggle is an important feature to mathematics learning, teachers can unknowingly rescue students by telling a solution strategy. Hiebert and Grouws (2007) use the term struggle to mean allowing students to wrestle with mathematic concepts and to construct knowledge around those mathematical ideas. This means students engage in cognitively demanding, meaningful tasks in which the solutions are within their reach (Carpenter et al., 2015; Hiebert et al., 1996; Smith & Stein, 2011). Vygotsky's (1978) zone of proximal development is the space in which students grapple with concepts while a teacher pushes on student thinking to help the student reach a deeper understanding (Carpenter et al., 1999; Franke, Kazemi, & Battey, 2007). The lack of allowing students to struggle with concepts in the elementary mathematics classroom could be a contributing factor as to why American students have not made significant gains in mathematics conceptual understanding as measured on the TIMSS assessment. "Many findings suggest that some form of struggle is a key ingredient in student's conceptual learning" (Hiebert & Grouws, 2007, p. 388).

Attend to mathematical concepts. The second key feature to developing conceptual understanding is when teachers and students attend explicitly to mathematics concepts (Bay-Williams, 2010; Franke et al. 2007; Hiebert & Grouws, 2007; Mason, 2008; Smith & Stein, 2011, 2018). This is not surprising based on extensive research that demonstrated students learn when they have the opportunity to learn (Boaler, 2008; Brophy, 1999; Fletcher et al., 1999; Floden, 2002; Grouws, 2004; National Research Council, 2001). Teachers must help students attend explicitly to concepts (Franke et al. 2007; Hiebert & Grouws, 2007; Kazemi & Hintz, 2014; Mason, 2008; Smith & Stein, 2011, 2018). "By attending to concepts we mean treating mathematical connections in an explicit and public way" (Hiebert & Grouws, 2007, p. 383). Instructional practices that promote attending explicitly to concepts include classroom discussions of key mathematic ideas, as well as coherent, structured, scaffolded instruction (Brophy, 1999; Kazemi & Hintz, 2014; Munson, 2018; Smith & Stein, 2011, 2018). Teachers need to be intentional by demonstrating how ideas are connected and make all thinking visible (Mason, 2008; Munson, 2018; Smith & Stein, 2011, 2018). When teachers explicitly discuss and represent connections between the different solution pathways, they make the thinking visible. The purpose of this instructional strategy is to help students develop these concepts and not just tell students about the concept (Hiebert & Grouws, 2007; Hiebert & Stigler, 2000; Smith & Stein, 2011, 2018). Research on conceptual development has also indicated student improvement with procedural learning (Hiebert & Grouws, 2007). “The findings from several of the studies reviewed, suggest that instruction emphasizing conceptual development facilitated skill learning as well as conceptual understanding” (Hiebert & Grouws, 2007, p. 387).

Cognitively Guided Instruction (CGI). CGI is another instructional approach that helps develop a conceptual understanding (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Carpenter et al., 1999; 2015; Empson & Levi, 2011). A proven, effective, instructional approach that allows students to be creative in mathematical thinking and solution strategies has been termed Cognitively Guided Instruction (CGI) (Carpenter et al., 1997; Carpenter et al., 2015; Fennema, Carpenter, & Franke, 1996; Franke et al. 2007; Hiebert & Grouws, 2007). The CGI approach asks teachers to probe student thinking and listen to student explanations while teachers also require students to solve problems multiple ways (Carpenter et al., 1999, 2015; Empson & Levi, 2011; Kazemi & Hintz, 2014). CGI researchers produced a framework for how basic number concepts and skills develop in the early grades (Carpenter et al., 1999, 2015).

CGI provides evidence that teachers’ classroom practice (a) includes eliciting and making public student thinking, (b) involves eliciting multiple strategies, (c) focuses on solving word problems, and (d) uses what is heard from students to make instructional decisions led to the development of student understanding (Franke, Kazemi, & Battey, 2007, p. 243). Key ideas of how teachers utilize concepts of CGI include teachers being aware of different types of problems involving the four operations, as well as how children develop basic number facts (Carpenter et al., 2015). Understanding that some story problems are more difficult than others because of their structure is important for teachers when creating mathematics tasks and facilitating a discussion on a variety of solution strategies (Carpenter et al., 1997; Carpenter et al., 1999, 2015, Smith & Stein, 2011, 2018). Eleven different problem types exist for addition and subtraction (Carpenter et al., 1997). Each problem type can result in a child attempting to solve the problem in different or unique ways (Carpenter et al., 1999, 2015). How teachers scaffold children's thinking on problems and focus their thinking on solutions can have a direct impact on skill efficiency (Carpenter et al., 1997; Carpenter et al., 1999, 2015; Fennema et al., 1996; Franke et al. 2007; Hiebert & Grouws, 2007).

Developing skill efficiency. Skill efficiency is the "accurate, smooth, and rapid execution of mathematical procedures" (Hiebert & Grouws, 2007, p. 380). There is little empirical research to support the use of memorization techniques as a way to improve skill efficiency. As Henry and Brown (2008) discovered, memorization of basic addition and subtraction facts using timed tests, flashcards, worksheets, and games did little to help students' mastery of basic facts. Classroom time spent on these memorization techniques actually was time lost on conceptual understanding (Carpenter et al., 2015). This does not mean that skill efficiency teaching is completely inappropriate. A review of process-product studies focused on skill efficiency from Hiebert and Grouws (2007) revealed, "Teaching that facilitates skill efficiency is rapidly paced, includes teacher modeling with many teacher-directed product-type questions, and displays a smooth transition from demonstration to substantial amounts of error free practice" (p. 382). Many teachers begin a focus on skill efficiency with addition and subtraction of basic facts. Van de Walle, Karp, and Bay-Williams (2013) define addition and multiplication basic facts, as "Basic facts for addition and multiplication are the number combinations where either addends or both factors are less than ten" (p. 171). A student being able to respond correctly in under three seconds without resulting in inefficient strategies, such as counting by ones is basic facts mastery (Van de Walle et al., 2013). Prior to reform efforts, the traditional approach to mathematics instruction placed a high value on skill efficiency and the quick recall of basic facts, as well as the ability to reproduce algorithmic procedures independently during seatwork time (Porter, 1989; Stein et al., 1996; Stodolsky, 1988). Children can learn to recall basic number facts once an understanding of relationships among number concepts has developed (Carpenter et al., 1999, 2015). Conceptual understanding and skill efficiency can develop together (Russell, 2010). Developing skill efficiency is important, and this is a developmental process. Lack of conceptual understanding hinders skill efficiency (Hiebert & Grouws, 2007; Kilpatrick, Swafford, & Findell, 2001; Van de Walle et al., 2013).

The CGI instructional approach helps teachers focus on skill efficiency as well as how students attempt to solve problems (Carpenter et al., 2015; Kelemanik, Lucenta, & Janssen Creighton, 2016; Parrish, 2010; Parrish & Dominick, 2016). Using tasks that have a real-world context allows students to build on mathematical thinking because students can represent the action in the problem with pictures or manipulatives (Carpenter et al., 1997; Carpenter et al., 1999, 2015; Fennema, Franke, & Carpenter, 1993; Franke et al. 2007; Hiebert & Grouws, 2007).

Cognitively challenging tasks. The use of cognitively challenging, authentic, culturally relevant, problem-solving tasks is a promising instructional approach. This instructional approach develops both conceptual understanding and skill efficiency. The use of cognitively challenging tasks is an emphasis in NCTM's (2000) Principles and Standards for School Mathematics as well as the Common Core State Standards Mathematics (Cai, 2010; Moschkovich, 2010; National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). One of the most important roles of the classroom mathematics teacher is to select tasks that are cognitively challenging, allowing students the opportunity to wrestle with mathematical concepts (Stein, Remillard, & Smith, 2007). Other researchers refer to cognitively challenging tasks as rich math tasks (SanGiovanni, 2017; Smith & Stein, 2011). Cai (2010) defines mathematics tasks as "projects, questions, problems, constructions, applications, and exercises in which students engage with mathematical challenges" (pp. 11-12). Mathematical tasks are far more than just the typical story problem or sets of story problems from a mathematics textbook in which the same operation, problem type, and single solution pathway is expected and utilized (Carpenter et al., 2015). According to CGI researchers, cognitively demanding tasks should include at least five out of six main characteristics (Boaler, Munson, & Williams, 2018a, 2018b, 2018c; Carpenter et al., 2015; Empson & Levi, 2011; Kazemi & Hintz, 2014; SanGiovanni, 2017; Smith & Stein, 2011, 2018; Stein et al., 1996, 2007, 2008). Rich math tasks need five of the six criteria: (a) meaningful and engaging context, (b) allowing for a variety of solution pathways, (c) allowing for students to generalize ideas around a big math concept, (d) all students have access to the problem, (e) the use of manipulatives are encouraged and brings meaning to the problem, and (f) some tasks can have multiple correct solutions. Rich math tasks allow teachers to focus on the problem-solving process with students, rather than just demonstrating a procedure or an operation.

When teachers attend to key features in the problem-solving process, such as thinking about variables, students can be introduced to algebraic thinking in the absence of students knowing algebraic notation (Blanton & Kuput, 2003; Carpenter, Franke, & Levi, 2003; Carraher & Schliemann, 2006; Carraher, Schliemann, Brizuela, & Earnest, 2006). It is important for teachers to acknowledge that algebraic thinking can take place in the absence of algebraic notation and can assist with conceptual learning (Blanton & Kuput, 2003; Carraher & Schliemann, 2006; Carraher et al., 2006; Empson & Levi, 2011; Schroeder & Lester, 1989).

Teaching through problem-solving is a perspective based on socio-cultural and constructivist learning theories. Embedded in the task are relevant mathematic concepts and skills (Baek, 2008; Carraher & Schliemann, 2006). Cognitively demanding tasks that use arithmetic problem solving and include algebraic or relational thinking will have a variety of solution pathways (Baek, 2008; Carpenter, 1999; 2015; Carpenter et al., 2003; Stein et al., 1996; Stein, Engle, Smith, & Hughes, 2008).

In addition to high-leverage, cognitively demanding mathematics tasks having a variety of solution pathways (Smith & Stein 2011; 2018), these tasks, based on a context, also need to be authentic, and have meaning and relevance for children (Carpenter et al., 1999, 2015; Torres-Velasquez & Lobo, 2005). The tasks lose their value if students do not relate to the problem or do not see a need for the solution. Tasks also need to be culturally relevant (Moschkovich, 2010). Tasks that require a certain level or aspect of prior knowledge in which a subset of students may not have will result in marginalizing a subgroup of students (Bishop & Forgrass, 2007). When planning for tasks, teachers need to take into account the prior knowledge of all of the students and design tasks students would find interesting and engaging (Fennema et al., 1993; Henningsen & Stein, 1997). When teachers design the task to be engaging, and then do the work of anticipating solution pathways, they can ensure the task has a low entry point, and high ceiling (Carpenter, 1999; 2015; Carpenter et al., 2003; Kazemi & Hintz; 2014; Stein et al., 1996; Stein, Engle, Smith, & Hughes, 2008; Smith & Stein, 2011, 2018).

A cognitively demanding task provides a variety of solution pathways ranging in complexity from concrete to abstract (Baek, 2008; Cai, 2010; Carpenter et al., 2003; Smith & Stein, 2011, 2018; Stein et al., 2008). Teachers can ensure that the task is cognitively demanding with mathematical concepts addressed in the standards through completing the task itself and identifying the variety of ways students may attempt to solve the problem (Cai, 2010; Carpenter et al., 1999; Smith & Stein, 2011; Stein et al., 1996; Stein et al., 2008). Smith and Stein (2011; 2018) refer to this as anticipating solution strategies. Being able to plan for how students may attempt to solve the cognitively demanding task can help the teacher to organize and highlight a classroom discussion (Smith & Stein, 2011, 2018). In addition to students engaging with rich cognitively demanding tasks, students can improve conceptual development by engaging in classroom discourse (Smith & Stein, 2011, 2018; Stein et al., 1996, 2007, 2008).

Shifting classroom discourse. Whole group classroom discourse occurs during the mathematical discussion at the end of the three-part lesson of Launch-Explore-Summary (Lappan et al., 2007). “Classroom discourse refers to the ways of representing, thinking, talking, agreeing, and disagreeing that teachers and students use to engage in problem solving” (Cai, 2010, p. 12). The three-part lesson comes from research on the Launch-Explore-Summarize (LES) instructional model of the Connected Mathematics Project (Lappan et al., 2007; Schroyer & Fitzgerald, 1986). The teacher launches the lesson with an authentic problem for students to solve (Stein et al., 2008). This launch transitions students into group work, and as students begin to work together, the teacher monitors for the solution pathways utilized by different groups (Smith & Stein, 2008). The teacher also confers with students about the way the student is solving the problem (Munson, 2018). The teacher then uses this information to create a plan for organizing the upcoming mathematics discussion (Stein et al., 2008; Smith & Stein, 2011). During the mathematical discussion, students demonstrate multiple solutions to the task and the teacher holds students accountable for articulating the connections between the different solution strategies (Boaler et al., 2018a, 2018b, 2018c; Carpenter et al., 2015; SanGiovanni, 2017; Smith & Stein, 2011, 2018; Stein et al., 1996, 2007, 2008). A mathematical discussion described as horizontal discourse occurs when students do most of the talking with the teacher facilitating a few questions. Mostly horizontal discourse during a mathematical discussion is uncommon in the typical mathematics classroom.

An initiate, respond, evaluate, (IRE) pattern, or initiate, response, feedback (IRF) pattern is more common in a typical mathematics classroom (Cazden, 2001; Herbel-Eisenmann & Breyfogle, 2005; Mehan, 1979). Researchers found this type of discourse pattern does little to advance student understanding (Cazden, 2001; Henningsen & Stein, 1997). The type of classroom discourse that takes place during and after problem-solving both between teacher and student and among students is just as important as the design of the task itself (Cai, 2010; Kazemi & Hintz, 2014; Smith & Stein, 2011; Stein et al., 2008). Teachers need to know key discourse moves for orchestrating a meaningful mathematics discussion (Herbel-Eisenmann & Schleppegrell, 2010; Kazemi & Hintz, 2014; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010; Munson, 2018; Smith & Stein, 2008; 2011; 2018).

***Principles of a mathematics discussion*.** Mathematics discussions should be intentional, purposeful, and not based on a show and tell approach (Lampert & Cobb, 2003; Kazemi & Hintz, 2014). Four principles guide an effective mathematics discussion. First, the discussion should achieve a mathematical goal (Kazemi & Hintz, 2014). Different types of desired outcomes require planning for and leading the discussion in different ways. Second, students should know the parameters and requirements for sharing and listening to the ideas of others. Third, teachers need to know how to orient students to the thinking of others, highlighting ideas that advance students towards the mathematical goal (Lampert & Cobb, 2003; Smith & Stein, 2011). Last, teachers must communicate that all students can make sense of the problem, every student has valuable thinking to share, and students will be required to listen to and connect their ideas to the ideas of others in the class (Kazemi & Hintz, 2014; Smith & Stein, 2011, 2018). During the discussion, a teacher showcases student solution strategies beginning with the most concrete representation and systematically working towards the more abstract and algebraic representation of the solution (Smith & Stein, 2011; Stein et al., 2008). This planned analytic scaffolding from the classroom teacher allows all students to have access to the problem. Students build conceptual understanding from a gradual incline of concrete to abstract student demonstrations (Stein et al., 2008; Smith & Stein, 2011; Nathan & Knuth, 2003). Planning for the mathematics discussion to build in the sophistication of ideas is not an easy task for teachers to accomplish (Kazemi & Hintz, 2014). Research indicates the need for a change in the way teachers utilize classroom discourse and the effectiveness of orchestrating a mathematics discussion (Ball, Sleep, Boerst, & Bass, 2009; Boerst, Sleep, Ball, & Bass, 2011; Lampert et al., 2010; Smith & Stein, 2011, 2018; Stein et al., 2008).

Types of discourse moves. Research supports the importance of discourse moves in the classroom with development of conceptual understanding, skill efficiency, and mathematical reasoning (Herbel-Eisenmann & Schleppegrell, 2010; Lampert et al., 2010; Smith & Stein, 2011, 2018; Stein et al., 2008). Teachers employ multiple discourse moves to engage students in clarifying their thinking and to become owners of new co-constructed knowledge (Cai, 2010; Herbel-Eisenmann & Schleppegrell, 2010; Lampert et al., 2010; Smith & Stein, 2011; 2017. Discourse moves of stepping out (Herbel-Eisenmann & Schleppegrell, 2010) revoicing, (Herbel-Eisenmann & Schleppegrell, 2010; Smith & Stein, 2011, 2018) restating, (Herbel-Eisenmann & Schleppegrell, 2010; Smith & Stein, 2011, 2018) and wait time (Cai, 2010; Henningsen & Stein, 1997; Smith & Stein, 2011, 2018) hold promise in helping students to focus on concepts and learn how to reason mathematically.

Stepping out. Discourse moves have only recently gained traction in research and instructional practice communities (Kazemi & Hintz, 2014; Smith & Stein, 2011, 2018). The discourse move of stepping out dates back more than 20 years (Cobb, Yackel, & Wood, 1993; Rittenhouse, 1998) but has been acknowledged more recently (Herbel-Eisenmann & Schleppegrell, 2010) as an instructional approach for promoting student voice in solution strategies and problem-solving. Stepping out involves teachers acknowledging and encouraging student use of metacognition in terms of solution process and strategies. As students reflect on their thinking during the solution process, teachers draw student attention towards reflection and metacognition (Herbel-Eisenmann & Schleppegrell, 2010). The combined use of the instructional moves of stepping out and revoicing can help focus student thinking on mathematical concepts and connections between solution strategies (Kazemi & Hintz, 2014; Herbel-Eisenmann & Schleppegrell, 2010; Smith & Stein, 2011, 2018).

Revoicing. Revoicing as an instructional discourse move allows the teacher to draw attention to the important mathematical concepts discussed by the child (O'Connor & Michaels, 1993; 1996; Stein et al., 2008; Smith & Stein, 2011, 2018). The teacher is able to clarify or extend what the child has said with the intended goal for other students to focus on the mathematical importance of what the student was trying to communicate (O'Connor & Michaels, 1993; 1996; Stein et al., 2008; Smith & Stein, 2011, 2018). Teachers need to take caution not to change the intended meaning of what the student was saying by taking too much control or ownership when revoicing. This discourse move is most effective after a series of students have shared mathematical thinking to draw attention to the connections and ideas discussed by the students (Kazemi & Hintz, 2014; Smith & Stein, 2011, 2018). The discourse move of revoicing requires the teacher to repeat student thinking.

Restating. Restating is another discourse move in the same arena as revoicing (Smith & Stein, 2011, 2018). Restating, as an instructional discourse move requires other students to revoice what a student has shared during a mathematics discussion (Smith & Stein, 2011, 2018). Requiring a student to restate the thinking from another student means students will need to actively listen and follow the mathematical reasoning and problem-solving process. When students restate thinking from another student, this also encourages students to make connections between mathematical concepts, and perhaps approximate a new solution pathway during the next group work session.

Wait time. Smith and Stein (2008; 2011; 2017) found more wait time also increases cognitive demand. The use of increased wait time as an instructional discourse move enables students to reengage in the discussion and conjectures underway. Teachers can increase connections and concepts through leveraging discourse via the teachers' instructional moves (Cai, 2010; Turner, Bogner Warzon, & Christensen, 2011; Smith & Stein, 2018). All of these discourse moves can increase the cognitive demand and rigor during mathematics instruction (Smith & Stein, 2011, 2018). Not only do teachers need to be aware of how to increase meaningful, rigorous discourse in the classroom, they also need to be aware of inadvertent measures that can decrease the cognitive demand. Chapin, O'Connor, and Anderson (2003) found lack of wait time lowered cognitive demand. Teachers can also downgrade a task by telling of solutions or by simplifying the task (Cai, 2010; Turner et al., 2011; Smith & Stein, 2018).

Through using intentional discourse moves that increase cognitive demand, teachers can help students focus on developing conceptual understanding, develop problem-solving processes and strategies, as well as develop efficient skills and procedures. Not all students will actively participate in classroom discourse. Teachers need to know discourse moves and how to implement them to increase student participation in a mathematics discussion (Smith & Stein, 2011, 2018). Participation in classroom discourse is a predictor of student learning (Bishop & Forgrass, 2007; Presmeg, 2007). Students who participate less in a classroom discussion are more likely to struggle with mathematics concepts (Lewis, 2010). When special education teachers, math interventionists, and general education teachers engage in collaborative work, (Friend et al., 2010) a combined effort can be made to increase discourse from students who are less likely to independently participate in a classroom discussion and therefore help students with mathematics difficulty (Gutiérrez & Dixon-Román, 2008; Lewis, 2014; Lubienski, 2002; Stevens et al., 2018).

Collaboration Among Different Types of Teachers

Depending on the district, some special education teachers and general education teachers were not required to work closely together prior to the No Child Left Behind Act (NCLB; 2001). Also, prior to 2001, most elementary schools across the nation had not conceptualized the role of math interventionist (Stevens et al., 2018). In the past, educational professionals in general education and special education viewed these two educational domains as completely separate areas within the field of education (Robinson & Buly, 2007). The lack of collaboration between general education teachers and special education teachers in the past has been compounded by a focus on different learning theories and separate certification programs at the university level (Carter, Prater, Jackson, & Marchant, 2009; Robinson & Buly, 2007). All students by law

(IDEA, 2004; NCLB, 2001) must have access to the core curriculum in the general education setting, therefore special education teachers, general education teachers, and interventionists are required to work closely together to meet the learning needs of students receiving multi-tiered supports (Fisher & Frey, 2001).

One primary way special education teachers, general education teachers, and interventionists have been working together to meet the demands of their new shared population is through collaboration. The co-construction of knowledge with an emphasis on teachers taking an inquiry stance towards the instructional needs of the students to achieve a common goal is how collaboration is defined (Cook & Friend, 1995; Friend & Cook, 2006; Friend et al., 2010; Fuchs, Fuchs, & Fernstrom, 1993; Fuchs et al., 2006; Fuchs et al., 2007; Scruggs et al., 2007). This increased collaborative approach to instruction for students receiving multi-tiered levels of support creates a need for researchers as well as practitioners to understand how general education teachers, special education teachers, and math interventionists collaborate to improve student achievement (Solis et al., 2012). For effective collaboration, special education teachers, general education teachers, and math interventionists need to understand roles and responsibilities, different co-teaching models, and PLC engagement.

Roles and Responsibilities

Developing a collaborative space between general education teachers, special education teachers, and math interventionists, begins with how teachers have defined their job-specific roles (Cochran-Smith & Dudley-Marling, 2012; Murawski & Hughes, 2009). Special education teachers working with children with mild to moderate learning disabilities have often depended on the general education teacher for gaining access to the student in special education and for determining how and when the special education teacher will work with the special education student (Youngs, Jones, & Low, 2011). Math interventionists often define their roles as helping students to gain skills and procedures. Sometimes this help occurs outside the general education classroom. General education teachers have access to all the students all of the time, and consequently, an inequitable relationship between general education teachers, special education teachers, and math interventionists can develop (Youngs et al., 2011). Special education teachers and math interventionists often end up in a subservient role to the general education teacher. The special education teacher or math interventionist can function more as an instructional assistant rather than another classroom teacher (Griffin, Kilgore, Winn, & Otis-Wilborn, 2008). Youngs, Jones, and Low (2011) found that new special education teachers needed support from their principals in negotiating roles and responsibilities with their general education peers to enable an equal partnership to develop. New special education teachers depended on the building administration to communicate the belief about inclusion and to ensure special education teachers receive the same training on core content and instructional strategies as their general education peers (Youngs et al., 2011). Because so few special education teachers and/or math interventionists work in each building, these teachers can become isolated and marginalized within their own school community unless the administrative leadership helps to foster an understanding of equal teacher status, collaboration, and co-teaching, as well as a focus on student needs (Murawsk & Hughesi, 2009).

Administrative leadership. The administrator at the building must want and encourage inclusive practices (Murawski & Dieker, 2013). Administrators demonstrate this desire through systems and structures they put in place that can make it easier for teachers to be inclusive and collaborate (Murawski & Dieker, 2013). Schedules, often crafted at the hands of the administrator, can allow for teacher collaboration through an overlap of common planning times

(Murawsk & Hughesi, 2009). Creating a schedule that allows for large blocks of instructional time also encourages inclusion (Murawski & Dieker, 2013). Having sufficient time for collaboration is an important obstacle to overcome as well as communicating the expectation for inclusion (Murawski & Dieker, 2013). Allowing the special education teacher or intervention team to have their own large classroom space sends the message that removing students from daily core instruction is not only acceptable but encouraged (Friend et al., 2010). There needs to be an alignment between what the administrator communicates about collaboration and what the administrator does to support inclusive, collaborative practices (Murawski & Dieker, 2013). Administrators who understand the purpose of inclusion can establish systems and policies to promote the success of inclusive instructional practices (Murawski & Dieker, 2013). In the past, most general education teachers with special education students in their classes felt that they were not primarily responsible for the academic progress of the included students (DeSimone &Parmar, 2006). Furthermore, when these general education teachers named the most important responsibilities of the general education teacher, no teachers raised the issue of accommodating or modifying instruction for students struggling to understand core curriculum (DeSimone & Parmar, 2006). The general education teachers in the DeSimone and Parmar study felt differentiation and instruction for students in special education as well as student access to the core curriculum was the primary job of the special education teacher. In the last thirteen years, shifting viewpoints on collaboration and co-teaching is evident in the literature.

Recent studies found that teachers perceived collaborative work as an important dimension to inclusive instructional practice and that this collaborative work not only benefitted teachers professionally, but also improved educational outcomes for students with special education needs (Mulholland & O'Connor, 2016). Teachers understand the importance of co-teaching to support students with special education needs within inclusive classrooms, but in recent studies on co-teaching, teachers also highlight the importance of professional development and time built into the work day to collaborate (Pancsofar & Petroff, 2013; Rytivaara et al., 2019). Professional development on co-teaching is associated with greater teacher confidence and interest in co-teaching and more positive teacher attitudes about this instructional practice (Mulholland & O'Connor, 2016; Pancsofar & Petroff, 2013). Effective communication about inclusive instructional practices during professional development from administrative leadership can prevent many problems from developing, such as unbalanced teacher status or divergent views of student instructional need.

Teacher status. Even when the administrator has effectively communicated inclusive practices through words and actions, collaboration may not occur unless the general education teacher, the special education teacher, and the math interventionist have equal status (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley, 2013; Solis et al., 2012; Washburn-Moses, 2010). The special education teacher, as the differentiation specialist will have many ideas on how to alter lesson presentation, process, or product (Tomlinson, 2014). The math interventionist will have ideas on how to connect core instruction to the student learning needs through the Common Core State Standards (CCSS) math progression documents (www.achievethecore.org). The general education mathematics teacher can also develop the curricular vision using curriculum, standards, and planning for student need (Roth McDuffie & Mather, 2009). The three expert knowledge bases should come together on equal footing for effective collaboration to take place (Friend et al., 2010; Solis et al., 2012). Teachers need to view each other in terms of having expert knowledge, and all three teachers need to have equal status for collaboration and inclusive instructional practices to occur. Even with administrative support and perceived equal status, a collaborative partnership can still struggle if the general education teacher, special education teacher, and math interventionists have divergent beliefs on the learning needs of the students receiving multi-tiered levels of support (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley, 2013; Washburn-Moses, 2010).

Negotiation of student needs. Perception of student learning needs affects inclusive practices between general education and special education teacher (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley, 2013; Washburn-Moses, 2010). Collaborative, inclusive practices may struggle to gain traction if the general education teacher, the special education teacher, and/or the math interventionist view the student through the lens of divergent learning theories. The special education teacher, math interventionist, and the general education teacher need to negotiate and agree on student strengths and instructional needs for inclusive instructional practices to work. Without this discussion, the special education teacher, the math interventionist, and general education teacher can employ divergent instructional approaches that are counterproductive for the student with mathematics difficulty (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley, 2013).

For general education teachers, math interventionists, and special education teachers to incorporate new collaborative instructional practices, it is important they understand the personal qualities of each other (Brownell, Adams, Sindelar, Waldron, & Vanhover, 2006). When special education teachers, math interventionists, and general education teachers have knowledge about curriculum, pedagogy, behavior management, and the ability to reflect on student learning then teachers are much more likely to take on a collaborative partnership and be a high adopter of new collaborative instructional practices (Brownell et al., 2006). These characteristics can be a determining factor if teachers are in a position to adopt instructional strategies taught by the new collaborative teaching partners. A willingness to adopt new instructional strategies is the foundation for working with students in an inclusive co-taught environment (Brownell et al., 2006).

Co-Teaching Models

Researchers have struggled to develop a common understanding of co-teaching and qualifications of a co-teaching model (Friend et al., 2010; Murawski & Swanson, 2001). In the past ten years, a clearer description of co-teaching has emerged (Bouck, 2007; Scruggs et al., 2007; Wilson & Blednick, 2011). Co-teaching is based on a collaborative relationship between a general education teacher and a specialized teacher in which both teachers are teaching in tandem within the classroom with the purpose of helping students access core curriculum to approach or meet state and national benchmarks (Bouck, 2007; Dieker & Murawski, 2003; Friend & Cook, 2006; Hehir & Katzman, 2012).

Co-teaching, the pairing of general and special educators in a general education classroom, is one of the supportive structures to ensure an appropriate education for a student with disabilities in an inclusive setting. Co-teaching is the most popular inclusive educational model to meet the educational needs of students with disabilities previously enrolled in exclusive, segregated settings. (Wilson & Blednick, 2011, p. 7)

Co-teachers are required to negotiate a highly complex relationship in which teachers are forced to discuss the affordances or constraints of the co-teaching model on a variety of levels, from as simple as sharing a space to more complex tasks such as designing differentiated instruction (Bouck, 2007). Co-teaching also requires that teachers are willing to discover and design the co-teaching service delivery model. The purpose of the model is to create instructional and classroom freedom so that both teachers are not feeling isolated or forced into the choices of the co-teaching partner (Bouck, 2007).

Consulting, the use of a supportive resource room, and the use of instructional assistants were in the past considered co-teaching models (Idol, 2006). In recent research, the co-teaching description has been limited to the sharing of instruction with a general education teacher, an interventionist, and/or a special education teacher in a general education setting that includes students with learning difficulties (Friend et al., 2010). Therefore, consulting or the use of instructional assistants are not considered co-teaching. When the general education teacher seeks help or advice from the special education teacher or interventionist, consulting has occurred. The special education teacher or interventionist then offers suggestions and instructional strategies as the differentiation specialist, but the general education teacher carries out the suggested instructional practice (Idol, 2006; Wilson & Blednick, 2011).

Co-teaching is not taking students to a resource room for the implementation of specially designed interventions created by both the special education and general education teacher (Idol, 2006; Wilson & Blednick, 2011). The general education teacher, in conjunction with the special education teacher, decides upon the needed intervention. Students pulled-out of the general education setting to focus on intensive direct instruction aimed at students reaching the instructional benchmark set forth in the standards is the resource room model (Marston, 1996). The resource room model is not co-teaching, even though teachers collaboratively designed instructional strategies. Instruction removed from the general education setting is not considered co-teaching (Friend et al., 2010).

Co-teaching is not the use of instructional assistants. Usually, instructional assistants do not plan with the general education teacher to design the co-taught instruction or the shared small group responsibility (Leatherman, 2009). The special education teacher or the interventionist usually supervise and provide lesson plans for the instructional assistant. Usually, instructional assistants develop their skills at the worksite (Idol, 2006). Instructional assistants carry out the instruction designed by the general education teacher, the math interventionist, the special education teacher, or all of the teachers. Even though two or three people are working together to design and implement instruction, the use of instructional assistants is no longer included in the description of co-teaching (Friend et al., 2010). Regardless of who comprises the partnership, co-teachers have to understand the need for collective decision making when designing instructional strategies for student with mathematics difficulty (Wilson & Blednick, 2011). Time dedicated to discussing student need and instructional strategies is important for the success of co-teaching and one prevalent structure for this dialogue to take place is within the professional learning community.

Professional Learning Communities

Teachers collaborating in a mathematics professional learning communities (PLCs) is one way to develop instructional strategies to change instructional practices (Baek, 2008; Desimone, Porter, Garet, Yoon, & Birman, 2002; Desimone, 2011; Garet, Porter, Desimone, Birman, & Yoon, 2001; Hill & Ball, 2004 Stanulis, Little, & Wibbens, 2012; Supovitz, 2002; Timmerman, 2003; Tunks & Weller, 2009). PLCs provide a setting for learning from practice, development of teacher identity, and fostering a sense of belonging (Grossman, Wineburg, & Woolworth, 2001; Sowder, 2007). PLC teams form either through the same grade levels or similar content areas or with a cross-school section of teachers. The most important feature of a high functioning PLC is that the community comes together easily and often (Hollins & McIntyre, 2004; McLaughlin & Talbert, 2006).

PLC Participation

Participation in PLCs in many schools and districts is voluntary, but as the research base grows on the benefits of PLCs as a type of PD (Hollins & McIntyre, 2004; Sisk-Hilton, 2011), many schools and districts are requiring participation (McLaughlin & Talbert, 2006; Sowder, 2007). Voluntary or required participation can have a direct relationship with PLCs effectiveness (McLaughlin & Talbert, 2006). Voluntary membership in a PLC means members of the group want to participate in an academic conversation, and they share a common purpose (Lord, 1994; Secada & Adajian, 1997; Sowder, 2007). Beyond establishing the common purpose, members of a PLC believe they are responsible to each other for accomplishing their goals, and they work together to improve teacher and student learning (Lord, 1994; McLaughlin & Talbert, 2006; Secada & Adajian, 1997; Sowder, 2007; Supovitz, 2002). To accomplish the goal of a focus on student and teacher learning, a skilled leader either from within or outside the school will facilitate the PLC meeting (McLaughlin & Talbert, 2006). Students benefit the most when teachers situate the PLC work within instructional practices (Cobb, McClain, Lamberg, & Dean, 2003; Stein, Smith & Silver, 1999; Wenger, 1998). Once a PLC has a common goal, there are several ways to begin the work (McLaughlin & Talbert, 2006).

The Work of PLCs

One way for the PLC team to focus on collaborative work is through common assessment data. Having teachers create, score, and analyze common formative assessments and end of unit assessments can help guide discussions of student learning expectations (Holmlund Nelson, 2008, Holmlund Nelson, Deuel, Slavit, & Kennedy, 2010; Holmlund Nelson, Slavit, & Deuel, 2012; Nave, 2000). The assessment data then drives instructional decisions, and the PLC group can plan for varying levels of student need (DuFour, DuFour, Eaker, & Many, 2006).

Focusing on student work is another way to approach PLC work. Rather than beginning with assessments, the PLC team comes to the meeting with examples of student work at varying levels (Nave, 2000). Through the examination of student work, the PLC group makes instructional decisions or makes suggestions for interventions for the most struggling students (Buffum & Mattos, 2015; DuFour et al., 2006; McLaughlin & Talbert, 2006; Holmlund Nelson et al., 2012). A third approach to entering the work of a PLC is through engaging teachers in learning content in the disciplinary area. Having teachers focus how students learn mathematics concepts can develop teacher's mathematical knowledge for teaching and be the central focus of the PLC (Ball, Thames, & Phelps, 2008; Carpenter et al., 2015; Empson & Levi, 2011; Hill et al., 2004; Hill & Ball, 2008; Hill, 2010a). Of course, all of these are intertwined approaches. One approach will certainly lead to the others as the PLC progresses in sophistication.

PLCs have become a way for a similar job-alike group of teachers to go through a structured process to create shifts in teaching and learning. This process includes focusing on a problem of practice, creating a theory of action, designing and implementing new instructional practices, and collecting data to determine if the new instructional practices resolved the initial problem of practice (Buffum & Mattos, 2015; DeFour, DuFour, Eaker, & Many, 2006). Holmlund Nelson et al. (2010) found that teachers could struggle with asking critical questions about their instructional practice. Because PLC teams usually meet each week, there is more opportunity for teachers to learn important content than through a regular staff meeting that occurs monthly. Teachers get to focus on how to teach students with various instructional needs during a PLC meeting (Rigelman & Ruben, 2012). Job-specific teams engage in reflective dialogue and share instructional strengths and needs, and, in this way, PLCs become a structure to enable teachers to work together and de-privatize practice (Rigelman & Ruben, 2012). The

PLC becoming a structure in which teachers have the opportunity to learn how to improve their craft is just one of many benefits of PLCs.

The Benefit of PLCs

Increased student learning is a benefit when teachers use collaborative inquiry, focused on student need, to inform instruction (Holmlund Nelson et al., 2012). The nature of a student-centered focus in PLC meetings means teachers feel more comfortable to discuss what is happening in their classrooms because the focus is not on the teacher but on the student (Key, 2006 Sweeney, 2011). The nature of the dialogue in the PLC meeting is what contributes to transformative learning for teachers, impacts classroom practice, and therefore results in learning changes for the student (Holmlund Nelson et al., 2010, 2012; Key, 2006).

Effective PLCs

Most PLCs cannot jump right into dialogic inquiry and the resulting transformative learning without developing an atmosphere of trust (Holmlund Nelson, 2008). Trust is not the only essential element to a successful inquiry-based PLC. The PLC must also establish common goals, and each member needs to have a shared commitment to the problem of practice and the work generated by the PLC in the problem's resolution (Butler & Schnellert, 2012). The quality of relationships in the PLC can be a determining factor for how well the PLC is able to define the goals and engage in inquiry cycles based on student work (Butler & Schnellert, 2012). Relationships and safety become critical to the de-privatization of practice in the PLC in order for teachers to focus on student work samples to develop an instructional-based inquiry question (Holmlund Nelson, 2008). Developing a foundation for the PLC based on common goals is critical to an effective, cohesive team (DuFour et al., 2006, 2008). Fleming and Monda-Amaya (2001) found that beyond trust and quality of relationships, outside support and recognition for the work of the PLC was just as important of a factor in determining team effectiveness (Amabile & Kramer, 2011).

Members of the PLC must also understand the perspectives of other team members for the PLC to take on an inquiry stance (DeFour, Eaker, & DuFour, 2008). For teachers to move their own thinking forward, they must suspend their beliefs and values enough to consider the perspectives of those they have not yet heard (Buffum & Mattos, 2015; Snow-Gerono, 2005). By putting aside their own personal beliefs, teachers recognize different perspectives within the PLC and how these competing views could cause collaborative tension if teachers do not find some commonalities within the differing perspectives (Snow-Gerono, 2005). Structured protocols can help teachers find commonalities in defining the problem of practice addressed by the PLC as well as focusing the team on student instructional needs rather than on teacher deficits (Buffum & Mattos, 2015; Williamson & McLeskey, 2011). Using an interwoven instructional strategies approach can help integrate differing perspectives stemming from different learning theories held by individuals of the PLC (Van Garderen, Scheuermann, Jackson, & Hampton, 2009). These few strategies may be sufficient for some PLCs to shift towards a collaborative inquiry-based stance but, as Holmlund Nelson (2008) indicates, these strategies alone will more than likely not be enough.

Teachers need support for both the processes of inquiry and for the creation of an environment that models, nurtures, and embeds an inquiry stance. Second, targeted support is critical to move teachers past problematic areas: refining ambiguous inquiry questions, developing the trust needed to share student work, making sense of that student work in relation to their inquiry question, and promoting a willingness to wonder and ask critical questions about instructional decisions, classroom practices, and student learning.(p. 579).

Summary of PLCs

PLCs are complex structures critical to the development of collaborative inquiry focused on student instructional needs and the success of equal co-teaching partnerships (DeFour et al., 2008). PLCs can be a source of embedded PD to help shift instructional practice to meet the needs of students with learning difficulties (Quate, 2008). The continued use of ongoing PD either as a part of a PLC meeting or in the structure of a whole staff is essential for teachers to develop effective mathematics instructional and intervention approaches (Buffum & Mattos, 2015; Sisk-Hilton, 2011).

Effective Mathematics Professional Development

Prior to reform efforts teacher PD often consisted of whole staff meetings, one-day workshops, or guest presentations that were unconnected to instructional practice and unrelated to school goals (Curry, 2008; Little, 1993; Murry, 2014, NCTM, 2000). Research has shown these brief PD opportunities do little to change instructional practice and therefore, do not improve student learning (Curry, 2008; Desimone et al., 2002, Desimone, 2009; 2011; Little, 1993; Murry, 2014). Too many district-based initiatives or fad-based initiatives also result in little school improvement (Schmoker, 2016). Sustained, ongoing PD is just as important as the PD content in shifting instructional practice (Loucks-Horsley et al., 2010; Wayne, Yoon, Zhu, Cronen, & Garet, 2008). Effective, high-quality, PD for mathematics teachers should focus on developing specialized knowledge needed for mathematics teaching and developing knowledge of the curriculum (Goldschmidt & Phelps, 2010; Smith & Stein, 2018). There are several elements that ensure PD is effective and of high-quality (Wayne et al., 2008).

Quality Mathematics Professional Development Elements

Researchers have found that quality PD increases teachers' mathematical knowledge for teaching (MKT) and changes teachers' instructional practices (Borko, Jacobs, Eiteljorg, & Pittman, 2004; Desimone et al., 2002; Jacobs, Lamb, & Philipp, 2010; Hill & Ball, 2004; van Es & Sherin, 2008; Smith & Stein, 2018). Quality PD has been described as having a clear focus on content, active learning opportunities, a connection to high standards, active engagement in leadership roles for teachers, collective participation, and takes place across time (Desimone, 2011; Desimone et al., 2002; Garet et al., 2001; Murry, 2014; Zepeda, 2007).

Mathematics PD implemented for at least a year or more results in instructional practice change (Tunks & Weller, 2009; Garet et al., 2001; van Es & Sherin, 2008; Zepeda, 2007). Although extended duration for quality PD is an important factor, the duration of PD is perhaps not as essential as once thought for increasing teachers' MKT (Hill et al., 2004). In these studies by Hill and colleagues, teachers increased their MKT by participating in a three-week-long summer institute, but these studies did not focus on the relationship between an increase in mathematical knowledge for teaching and a resulting change in instructional practices (Hill et al., 2004; Hill & Ball, 2008; Hill, 2010a). Although increasing mathematical knowledge for teaching can occur during summer PD institutes, this does not mean that teachers will shift instructional practices because of the increased mathematical knowledge (Borko et al., 2008; van Es & Sherin, 2008).

The most effective mathematics PD practice in creating instructional change is PD implemented at the grade or school level (Borko et al., 2008; Jacobs et al., 2010; Knapp, 1997; van Es & Sherin, 2008). Even though PD designed and implemented at a grade level or school level is more effective in creating instructional change, some of these forms of PD require teachers to change many aspects of mathematics instructional practices at once (Carpenter et al., 1999; Fennema et al., 1996; Franke et al., 2001; Knapp, 1997). For example, when a teacher employs a new instructional practice of using rich mathematics tasks, the teacher also is learning about the launch-explore-summary lesson format, using the *Five Practices*, (Smith & Stein, 2011) implementing discourse moves to increase horizontal discourse, and principles of CGI (Carpenter et al., 2015; Smith & Stein, 2008; 2011; 2018). Learning about and approximating one new instructional practice of how to implement a rich mathematics task, leads to using other instructional practices, which might also be new. Implementing new instructional practices requiring mathematical knowledge for teaching is the desired outcome of high-quality PD (Loucks-Horsley et al., 2010; McLaughlin & Talbert, 2006; Murry, 2014).

Mathematical knowledge for teaching. Shulman's (1986) foundational research on pedagogical content knowledge generated new thinking about the knowledge teachers need to develop for teaching. Researchers have begun to study what specialized knowledge is needed for teaching mathematics and how this knowledge is developed and held (Ball, Thames, & Phelps, 2008; Hill, 2010b; Hill & Ball, 2004; Hill, Schilling, & Ball, 2004; Hill, Rowan, & Ball, 2005). In the last decade, researchers have built on Shulman's (1986) pedagogical content knowledge by focusing on teachers' specialized content knowledge, referred to as mathematical knowledge for teaching (MKT) (Ball, Thames, & Phelps, 2008; Hill et al., 2004). To respond to student mathematics productions teachers need a highly developed level of MKT. Teachers also need MKT to select accurate representations of mathematics concepts. This knowledge helps teachers decide if novel solution strategies will generalize to all problems of similar type. MKT allows teachers to explain why an algorithm or mathematics procedure works and what the algorithm or procedure means (Ball et al., 2008; Hill & Ball, 2004; Hill 2010; Smith & Stein, 2018).

Teachers' levels of MKT can be a predictor of student achievement (Hill & Ball, 2004; Hill, 2010a; Hill et al., 2008; Hill et al., 2005). There has been clear evidence of the relationship between intentional PD, the MKT that teachers develop through the PD, and their resulting instructional practices (Hill & Ball, 2004; Hill et al., 2008; Hill et al., 2004; Smith & Stein, 2018).

The way teachers hold and understand mathematical knowledge influences the way they instruct (Hill et al., 2005; Hill et al., 2004). Teachers' MKT can be based on a situated (Boaler & Greeno, 2000) or behavioral perspectives (Spillane, 2002), connected to large mathematics domains, or isolated knowledge void of mathematic connections (Hill & Ball, 2004; Hill et al., 2004). How teachers use and hold MKT is more important than the amount of mathematics the individual teacher may know (Hill & Ball, 2004). This means that individuals with a mathematics degree may have little or no MKT (Hill & Ball, 2004; Hill 2010). Organization of MKT is content-specific (Hill & Ball, 2004) and teachers need solid understandings of mathematics in the content domains of number sense, algebraic thinking, and operations (Hill & Ball, 2004).

The ability of teachers to understand and teach mathematics content has a direct relationship with student achievement. This relationship justifies the need to study MKT and teacher development of this knowledge (Hill, 2010a). According to TIMSS 2007, students in grade four continued to struggle with long division and the subtraction algorithms (Gonzales et al., 2008). This is not surprising when assessments of teachers' MKT showed teachers had a difficult time representing these mathematical ideas conceptually. Teachers had an even harder time explaining how the standard algorithm for long division or subtraction functions (Hill, 2010a, 2010b, Hill & Ball, 2004). To increase student performance with difficult mathematics concepts, teachers need some way to develop and expand their MKT further (Hill et al., 2005; Hill et al., 2008). One promising way shown to increase teachers' MKT is through focusing on student work and mathematical thinking during PD opportunities (Hill & Ball, 2004; Jacobs et al., 2010).

Focus on student work and mathematical thinking. A key PD strategy shown to increase teachers' MKT and provide a shift in instructional practice occurs when teachers focus on the mathematics thinking of their own students (Fishman, Marx, Best, & Tal, 2003; Franke & Kazemi, 2001; Munson, 2018). Any strong PD plan should be rooted in what teachers want students to know and be able to do independently at the end of the grade as well as desired student behaviors and attitudes (DuFour et al., 2006; DeFour et al., 2008; Loucks-Horsley et al., 2010; Murry, 2014). By focusing on what students think and do, an important aspect to a PD plan centers on student work (DuFour et al., 2006;; DeFour et al., 2008; Heller, Daehler, Wong, Shinohara, & Miratrix, 2012; Loucks-Horsley et al., 2010; Munson, 2018; Murry, 2014; Smith & Stein, 2011, 2018). When PD focuses on student work, teachers are more likely to realize student misconceptions and next step instructional needs (Carpenter et al., 1999, 2015; Murry, 2012; Sparks, 1998). One way to do this is to require teachers to go through the same learning experiences they would require of their students (Murry, 2014; Smith & Stein, 2011, 2018). Elementary mathematics teachers understand the different types of problems within all four operations because of teacher MKT. Teachers shift instructional practice when they understand the different strategies children use to solve these types of problems (Carpenter et al., 1999, 2015; Empson & Levi, 2011; Hill & Ball, 2004; Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010; Parrish, 2010; Parrish & Dominick, 2016; Sparks, 1998).

There are many different formats and protocols focusing teachers on student work and thinking (DiRanna, Osmundson, Topps, & Gearhart, 2008; Loucks-Horsley et al., 2010; Love et al., 2008; Smith & Stein, 2018). Even though different guidelines exist for how teachers should focus on student work, these guidelines all have similar processes (Kelemanik et al., 2016; Loucks-Horsley et al., 2010; Smith & Stein, 2018). Setting a clear common goal for the PD centered on student work helps keep the PD on track and reminds teachers the purpose of the work (Desimone, 2009; DuFour et al., 2006; Loucks-Horsely et al., 2010; Murry, 2014). Next, teachers select student work that helps achieve the goal of what teachers want to learn about student thinking or understanding. For example, it is meaningless for a teacher to select work samples from all high achieving students if the goal was to learn how to better instruct the students with the most mathematics difficulty on a district mathematics assessment. Third, the PD discussion should have a facilitator to guide the discussion and help promote deep analysis of student learning and the relationship to teacher practice (Heller et al., 2012; Loucks-Horsely et al., 2010; Love et al., 2008; Murry, 2014). This third step of a knowledgeable instructional leader is also important because the leader can ensure the discussion is not centered on a student deficit perspective but rather on how instruction can be improved (Aguilar, 2013; Baglieri et al., 2011; Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley, 2013; Loucks-Horsely et al., 2010; Sweeney, 2011). Finally, teachers should reflect on their own professional learning and how a focus on student work changes instructional practice (DuFour et al., 2006; DeFour et al., 2008; Murry, 2014). Certainly, teachers could follow this process and examine student work from their own classroom in isolation. Overwhelmingly, PD research supports teachers from the same building and same grade working together to increase their MKT. When teacher MKT increases so does student mathematical thinking (Baek, 2008; Desimone et al., 2002; Desimone, 2009; Desimone, 2011; DuFour et al., 2006; DeFour et al., 2008; Garet et al., 2001; Hill & Ball, 2005; Loucks-Horsely et al., 2010; Murry, 2014; Timmerman, 2003; Stanulis et al., 2012; Tunks & Weller, 2009). This PD approach uses student work as the primary method to develop teacher thinking. There is also strong evidence that PD focused on curriculum can also be a vehicle to support ongoing teacher development (Sowder, 2007).

Focus on curriculum. Many professional developers come from the perspective that curricular materials and teacher noticing should be a focus of PD (Ball & Cohen, 1996; Brown, Smith, & Stein, 1996; Jacobs et al., 2010; Loucks-Horsley et al., 2010; Schifter, Russell, & Bastable, 1999; Sowder, 2007; McLaughlin & Talbert, 2006; Murry, 2014). Specially designed PD opportunities help teachers implement newly adopted curriculum (Sowder, 2007). The PD facilitator can use the curriculum a variety of ways in creating PD opportunities (Borasi & Fonzi, 1999, 2002; Remillard & Bryans, 2004; Remillard & Kaye, 2002; Sowder, Philipp, Armstrong, & Schappelle, 1998). Some researchers use the curriculum to drive the PD (Borasi & Fonzi, 1999, 2002) while other researchers focus on learning MKT during PD sessions. Some teachers develop their own curriculum using a variety of materials, and this provides unique opportunities for PD experiences (Remillard & Bryans, 2004; Remillard & Kaye, 2002). Still, other professional developers use a process of inquiry to help teachers explore the mathematical content they taught (Sowder et al., 1998; Sowder, 2007). Regardless of the approach of curriculum use in PD, the research has shown this process to be effective in promoting teacher learning and instructional change (Ball & Cohen, 1996; Borasi & Fonzi, 1999, 2002; Brown, Smith, & Stein, 1996; Loucks-Horsley et al., 2010; Remillard & Bryans, 2004; Remillard & Kaye, 2002; Schifter, Russell, & Bastable, 1999; Sowder, 2007; McLaughlin & Talbert, 2006; Murry, 2014).

Grain size in lesson planning. When planning for math instruction teachers fluidly think about, plan for, and implement a learning progression or learning trajectory (Boaler et al., 2018a, 2018b, 2018c). A learning trajectory refers to the large generalizable idea or math concept students should generate down to the small daily rich math task that helps students formulate or generate ideas about the math concept being explored (Boaler et al., 2018a, 2018b, 2018c; San Giovanni, 2016; Sztajn & Wilson, 2019). How teachers think about a learning trajectory or the grain size of lesson planning that comes naturally to a teacher has an impact on the types of lessons that teachers will plan and implement (Sztajn & Wilson, 2019). When teachers plan with only a small learning target in mind, then the larger math concept can become lost (Moss & Brookhart, 2012; Sztajn & Wilson, 2019). When teachers tend to think in small grain size lessons, then they tend to focus teaching on small skills or math concepts without connections (Smith & Stein, 2011, 2018). When teachers can fluidly move up and down the learning trajectory while planning, then they tend to plan lessons that are more focused and intentional on getting students to generalize a large math concept (Boaler et al., 2018a, 2018b, 2018c; San Giovanni, 2016; Sztajn & Wilson, 2019).

Less Effective Professional Development

Just as there are proven effective PD strategies, there are also PD strategies known to be less effective (Loucks-Horsley et al., 2010; McLaughlin & Talbert, 2006; Murry, 2014). Professional developers need to pay close attention to reasons that PD could inhibit growth and prevent teachers from making instructional shifts. Shifts in instruction can take place when the climate and culture are right for learning (Amabile & Kramer, 2011). Many factors contribute to less effective PD. Some of these factors include the culture for learning, coherence, sustainability, and equity (Loucks-Horsley et al., 2010; McLaughlin & Talbert, 2006; Murry, 2014).

A culture for learning. The culture and climate in the building must be safe and respectful for learning to take place (Loucks-Horsley et al., 2010; McLaughlin & Talbert, 2006; Murry, 2014; Sowder, 2007). If the building climate is not primed to be receptive towards learning and instructional change, then the best plans, goals, content, and process will result in limited teacher learning and little to no instructional shifts (Amabile & Kramer, 2011). Beyond a safe and nurturing learning environment, there must be time allowed for the learning to take place (Buffum & Mattos, 2015; Loucks-Horsley et al., 2010; McLaughlin & Talbert, 2006; Murry, 2014). Administrators who always expect learning to take place outside of the workday or expect immediate implementation after an initial induction are also rendering the PD ineffective due to lack of time (DuFour et al., 2006; DeFour et al., 2008). Change takes time, patience, and a coherent instructional plan. Without these elements, the PD will not be successful (Loucks-Horsley et al., 2010; McLaughlin & Talbert, 2006; Murry, 2014; Sowder, 2007).

Coherence. PD that lacks coherence in terms of goals, content, or process will be ineffective at changing instructional practices (Loucks-Horsley et al., 2010; McLaughlin & Talbert, 2006; Murry, 2014). A clear set of goals for PD is like the road map for the learning journey. If the map is blank, how will teachers know where they are going (Loucks-Horsley et al., 2010; McLaughlin & Talbert, 2006; Murry, 2014)? The PD leader or facilitator needs to have a clear vision and an established set of goals to be the anchor of each PD opportunity (Penuel, Fishman, Yamaguchi, & Gallagher, 2007). This provides coherence to the PD and keeps the group on track. Coherence is also important because any group of teachers will more than likely have at least someone in the group who tries to derail the learning opportunity (Penuel et al., 2007). By having a written, established list of goals or outcomes for the PD, the facilitator can refocus the group on the goals if or when things might go awry (DeFour et al., 2008; Loucks-Horsley et al., 2010; McLaughlin & Talbert, 2006; Murry, 2014). Effective PD needs a coherent plan, but the plan must also be sustainable (Penuel et al., 2007).

Sustainability. Lack of a plan for PD sustainability is another way PD opportunities become ineffective (Schmoker, 2016). Sustainability can be thought of in terms of time or money, but an even deeper issue of sustainability needs to take into account how new teachers or teachers new to the school and district will be brought up to speed on prior PD efforts (Loucks-Horsley et al., 2010; McLaughlin & Talbert, 2006; Murry, 2014; Schmoker, 2016). Many times school initiatives stemming from PD work loses momentum because of the lack of a sustainable plan to train teachers on the work the school has done prior to teachers joining the staff (Loucks-Horsley et al., 2010; McLaughlin & Talbert, 2006; Murry, 2014; Schmoker, 2016). If there is a high staff turnover rate from one year to the next in a school, all the work could be lost. A well-crafted plan to educate new staff on the work completed in prior years is a way to sustain the past work as well as future work (Loucks-Horsley et al., 2010; McLaughlin & Talbert, 2006; Murry, 2014). Just as some forms of PD are more sustainable, some PD approaches provide more equity than other PD approaches do.

Equity. Equity in PD is a concept many professional developers should remember when planning for PD opportunities (Loucks-Horsley et al., 2010; McLaughlin & Talbert, 2006; Murry, 2014). PD facilitators should make sure access to the PD opportunity is available to all teachers within the school context or examine how the PD opportunity could favor teachers from certain locations, lifestyles, gender, or racial groups (Loucks-Horsley et al., 2010; Murry, 2014; Sweeney, 2011). Equity also means professional developers need to determine if the PD content will provide teachers an opportunity to "examine and challenge their beliefs about who can learn and how diverse groups of students best learn" (Loucks-Horsley et al., 2010, p. 136). Professional developers also need to question if the content of the PD experience will ensure equitable opportunity for all students to learn mathematics and science content (Buffum & Mattos, 2015; Loucks-Horsley et al., 2010; Murry, 2014). Finally, design equity occurs when participation and engagement from all teachers is possible. Providing equitable PD opportunities and an available instructional coach is essential for special education teachers, math interventionists, and general education teachers to develop effective co-teaching relationships (Friend et al., 2010). "High-quality PD should be accompanied by coaching, and other supports demonstrated to change teacher practice" (Friend et al., 2010, p. 20).

Instructional Coaches and Professional Development

The concept of instructional coaching began to emerge in the educational literature in the 1990s as innovative support in teacher change (Flaherty, 1999; Senge, 1990). Since that time, research on instructional coaching has continued to develop. Even though some districts in the US may still consider instructional coaching a relatively new concept. An effective approach in creating instructional change is through instructional coaching (Sailors & Shanklin, 2010). Instructional coaching is a professional act facilitated by teacher leaders within a school or district and helps teachers focus on specific goals for student learning (Sweeney, 2011). Instructional coaches focused on student-centered coaching take a divergent stance from fixing teachers towards a stance of shifting instructional practice based on student data (Sweeney, 2011). Student-centered instructional coaches facilitate learning for teachers to allow teachers to set specific targets for students grounded in standards and curriculum. Instructional coaches taking this student-centered stance work collaboratively with teachers to set goals and gather measurable data (Aguilar, 2013; Sweeney, 2011). In their work with teachers, instructional coaches provide ongoing PD as a model to sustain conditions over time in which there is a focus on productive learning for students (Sarason, 1990). "Effective PD provides continued follow-up, support, and pressure that can only be delivered by a school-based coach" (Sweeney, 2011, p. 31).

One way to provide the necessary support as an instructional coach is to formalize coaching cycles between the instructional coach and the teacher being coached (Aguilar, 2013; Sweeney, 2011). Coaching cycles have three primary characteristics. First, the coaching cycle involves in-depth high cognitive work with a teacher or pair of teachers and an instructional coach lasting at least six to nine weeks (Aguilar, 2013; Loucks-Horsley et al., 2010; Sweeney, 2011). Second, coaching cycles include regular collaborative planning sessions such as an hour once per week as well as time in the classroom for either modeling instruction, observing teaching and learning together, or implementing co-teaching lessons (Aguilar, 2013; Loucks-Horsley et al., 2010; Sweeney, 2011). Lastly, coaching cycles focus on a third point, which is working with either formal or informal student data rather than having a teacher-centered focus (Aguilar, 2013; Loucks-Horsley et al., 2010; Sweeney, 2011). Effective PD, when partnered with instructional coaching, "create(s) the conversation that leads to behavioral, pedagogical, and content knowledge change" (Aguilar, 2013, p. 9).

Summary of Literature Review

The review of the literature indicates the importance of special education teachers, math interventionists, and general education teachers co-teaching to enable students with mathematics difficulty reach their full learning potential (Friend et al., 2010; Gutiérrez & Dixon-Román, 2008; Lubienski, 2002). The use of cognitively high demand tasks (Smith & Stein, 2011, 2018) in an inclusive general education setting (Friend et al., 2010) can help students develop conceptual understanding as well as procedural fluency (Empson & Levi, 2011; Carpenter et al., 2015; Smith & Stein, 2011, 2018). Special education teachers, math interventionists, and general education teachers may not intrinsically know how to negotiate the co-teaching relationship or how to develop a conceptual understanding for students with mathematics difficulty. High-quality PD (Loucks-Horsley et al., 2010) is needed to provide special education teachers, math interventionists, and general education teachers an avenue for learning about collaboration and co-teaching (Buffum & Mattos, 2015; Friend et al., 2010; Sztajn & Wilson, 2019). Student-centered PD based on coaching cycles (Aguilar, 2013; Loucks-Horsley et al., 2010; Sweeney, 2011) brings the fields of math intervention, special education, and general education mathematics instruction together.

CHAPTER THREE: METHODOLOGY

Helping students who have mathematics difficulty reach state or national benchmarks in mathematics has been a continuing challenge in the intervention, special education, and mathematics fields for years (Geary et al., 2007; Harbour et al., 2018; Lewis, 2010; Lewis, 2014; Lewis & Fisher, 2016; Lewis & Fisher, 2018; Steven, Rodgers, & Powell, 2018). Research has shown that a focus on remediation of skills for students with mathematics difficulty does not actually address gaps in understanding. By remediation of skills, I am referring to concentrating on fluency of basic addition and subtraction facts in the upper elementary grades or focusing on accuracy of procedures removed from conceptual understanding.

Remediation of skills is detrimental to mathematics achievement and only serves to prevent students from gaining access to grade-level curriculum (Lewis & Fisher, 2016; Murawsk & Hughesi, 2009). Research has shown that remediation is not a viable solution for helping students to reach their full learning potential (Geary, 1993, 1995, 2004, 2005; Geary et al., 2000; Lewis, 2010; Lewis & Fisher, 2016; Russell & Ginsberg, 1984; Schunk, 2012; Shalev et al., 1993). Therefore, researchers have tried to find innovative instructional models towards helping students with mathematics difficulty gain access to core content and reach their full learning potential (Whitney et al., 2017; Wilson & Blednick, 2011). Teachers need PD on co-teaching (Friend et al., 2010), PD with using CGI and cognitively demanding tasks (Carpenter et al., 2015; Smith & Stein, 2011), and experience using the Launch-Explore-Summary lesson structure (Lappan et al., 2007). This study endeavored to discover ways that co-teachers developed their co-teaching relationships, and how PD assisted in creating co-teaching instructional shifts, which enabled students with mathematics difficulty, reach their full potential.

Research Questions

1. How do special education teachers, general education elementary teachers, and/or math interventionists, develop their co-teaching relationships? What are the teachers' perceptions regarding the process of developing these relationships?
2. How do special education teachers, general education elementary teachers, and/or math interventionists co-design math instruction with a focus on supporting students with mathematics difficulty in accessing grade-level core content?
3. How does the PD support mathematical instructional practice for special education teachers, general education teachers, and/or math interventionists?

Research Approach

Through the critical perspective of action research (Mills, 2011), I designed and implemented PD that focused on co-teachers using the teaching and learning cycle. Math interventionists, special education teachers, and general education teachers learned: (a) the models, roles, and responsibilities of co-teaching (Friend et al., 2010), (b) how to notice children's mathematical thinking when solving math problems (Carpenter et al., 1999, 2015), and (c) how to use a lesson summary discussion (Smith & Stein, 2008; 2011) to advance student mathematical thinking. Teachers learned about developing a co-teaching relationship and new mathematical instructional approaches. Teachers analyzed student work samples, planned new instruction, and designed assessments to improve instruction (Lewis & Tsuchida, 1998).

I studied how the participating teachers used this foundational knowledge while working through the teaching and learning cycle with a co-teaching partner. Each partnership consisted of a general education teacher, a math interventionist, and/or a special education teacher. The instructional PD sessions built upon each other to provide collaborative experiences in which the teachers learned to:

- Create or adapt rich math tasks with entry points appropriate for all intended students (Smith & Stein, 2011),
- Anticipate how students with mathematics difficulty will enter the rich math task and work on the task (Smith & Stein, 2011),
- Anticipate needed scaffolds for students with mathematics difficulty to be successful with grade-level tasks (Smith & Stein, 2011),
- Plan for students with mathematics difficulty to build on and connect mathematical ideas (Smith & Stein, 2011)
- Enact a Launch-Explore-Summary (Lappan et al., 2007) instructional format,
- Plan for ways to build student independence during a math unit (Smith & Stein, 2011),
- Create and implement differentiated math assessments (Tomlinson, 2014),
- Plan for math concepts to be revisited in a variety of contexts and representations (CCSSM), and
- Analyze summative and formative assessment data to begin the teaching/learning cycle again (Jones, 2008)

The PD sessions were flexible in the amount of content covered in each session and the instructional approach based on the needs of the teachers. I gained entry to each site by emailing all of the district math interventionists and asking if the math interventionist would be interested in joining the study. Math interventionists then recruited general education teachers and/or special education teachers. Data collection began in March of 2017 and by June 2017 data collection completed. However, additional communication with the participants did continue in person and via email into the 2017-2018 school year. This communication provided continued support to the co-teaching partnerships. On occasion, I asked for additional documents related to collaboration and co-teaching. I also asked for additional student work samples and followed up on the successes and challenges of co-teaching and co-taught lessons.

Case Study Methodology in an Action Research Context

Research in which the primary strategy is usually to capture the deep meaning and experiences in the participants' own words involves a form of a case study (Stake, 1995; Yin, 2014). A case study entails the immersion in a setting and rests on both the researcher and participants' worldviews (Stake, 1995; Yin, 2014). Miles, Huberman, and Saldaña (2014) and Merriam (2009) describe case study research as bound to a context in which any set of appropriate methods and data are used. Case studies and action research used in conjunction allow the researcher to understand a problem situated in a context (Stake, 1995; Yin, 2014). Action research is an inquiry by or with insiders in an organization but not inquiry done to or on other insiders within the organization (Herr & Anderson, 2005). In action research, the researcher identifies a problem of practice and then designs a solution for the problem (Herr & Anderson, 2005).The intent of an action research study is to form an action to improve or resolve a problem of practice (Marshall & Rossman, 2006). The action in action research is the designed intervention to solve the problem (Mills, 2011). My study used PD as the intervention and included two parts that occurred concurrently. The first part of the intervention involved eight to 12 instructional sessions. The second part of the PD involved eight to 12 collaboration meetings. There is not enough known about co-teaching or the use of PD as a way to develop a co-teaching relationship (Friend et al., 2010).

My transformative worldview and the framed action research became important in how I designed my study (Mertens, 2003; 2010). The two primary theoretical perspectives in action research are practical action research and critical action research (Mills, 2011). Practical action research has a less philosophical emphasis and focuses on the procedures of action research (Mills, 2011). Critical action research comes from the postmodern worldview and has a focus on liberation through knowledge gathering (Mills, 2011). Critical action research is participatory, democratic, socially responsive, and placed within a context (Mills, 2011). Critical action research examines ways professional practice liberates students, teachers, and/or administrators through changes to teaching, learning, and instructional policy in the organizational context (Greeno, 1998). My critical action study attempted to solve an instructional problem involving math interventionists, special education teachers, and general education teachers. Through helping math interventionists, special education teachers, and general education teachers understand co-teaching and collaboration, we helped students with mathematics difficulty reach their full potential. I positioned myself as a district insider with other district insiders to improve the knowledge base and to create professional and organizational transformation (Greeno, 1998; Herr & Anderson, 2005; Mills, 2011).

As an instructional coach for math intervention at the elementary level, and a former instructional coach for special education at the kindergarten through high school level, I have provided instructional support for math interventionists, special education teachers, and general education teachers in the area of collaboration, co-teaching, and utilizing student data to inform instruction. The district used student-centered coaching (Sweeney, 2011) in combination with coaching cycles (Aguilar, 2013; Sweeney, 2011) as a framework that guided the work of instructional coaches. Student-centered coaching practices helped teachers set specific learning targets for students based on standards and curriculum (Sweeney, 2011). Student-centered coaching also framed coaching based on collaborative conversations between the coach and teacher being coached (Sweeney, 2011). This coaching framework was not about trying to fix teachers, (Aguilar, 2013) but is more about the value of sociocultural learning theory (Vygotsky, 1978) and developing co-constructed knowledge. As an instructional coach who used student-centered coaching, I met with teachers and had a conversation around a teacher's voiced problem of practice (Jaworski, 1998). Together we explored the problem of practice and designed instruction to advance student learning. Because coaching work is emergent and based on teacher need and because the coaching is based on co-constructed knowledge, as the coach, my job was to ask questions and help teachers discover more about their own instructional practice (Jaworski, 1998). My job was not to tell teachers what to do or be an enforcer of district curriculum or policy. When working with co-teaching partners, I listened to how the partnership functioned and offered PD in the form of individual or partnership coaching cycles (Aguilar, 2013; Sweeney, 2011).

In this action research study, I worked with math interventionists, special education teachers, and general education teachers at three schools within the district to discover ways to improve math co-teaching instruction. I positioned myself as an insider working with other insiders to make an improvement to instruction for students with mathematics difficulty (Jaworski, 1998).

Research Setting and Context

The research setting and context was a large school district in the Northwest part of the U.S. in which I was a former special education instructional coach for special education teachers K-12 and more recently, a math intervention coach. During the 2016-2017 school year, this school district had approximately 30,000 students with over 5,000 students with multiple years of below standard performance on state assessments. In 2010, the district moved from a special education self-contained instructional approach towards an inclusive co-teaching instructional model. When the district transitioned from the pull-out model (Marston, 1996) to a push-in model, teachers had to redefine their work. General education teachers and special education teachers suddenly found themselves trying to negotiate new relationships, roles, and responsibilities as being part of a co-teaching relationship (Friend & Cook, 1995; Friend et al., 2010). No formal, district-provided PD established how to do this work. Each school was encouraged to design and develop the co-teaching model based on the needs of the students at that school. Some schools struggled to design and implement a co-teaching model. Other schools had no problem with the design or implementation of the new co-teaching model. Some schools discovered that without PD support, strained relationships resulted, and old ways of pulling students from core instruction began to reemerge. At the time of the study, there were 33 schools in my district. Most administrative leaders asked for PD for their interventionists, special education teachers, and general education teachers on inclusive practices and the roles and responsibilities using a co-teaching model.

Participants

The nine research participants were self-selected elementary math interventionists, elementary special education teachers, or elementary general education teachers. These teachers shared students with mathematics difficulty. The three math interventionists self-selected to participate in the research study by replying to an email that I sent to all the elementary math interventionists in the district. In this email, I outlined the PD opportunities for math interventionists, special education teachers, and general education math teachers who used a co-teaching model to support students with mathematics difficulty. The email asked the math interventionist to reach out to a general education teacher and then let me know if both the general education teacher and the math interventionist were interested in participating in the PD and the study. When I had a math interventionist and a general education teacher interested in the PD and the study, I also invited the special education teacher, at that same building, to join the study. I then made sure the administrative leadership at the school agreed with the co-teaching partners in joining the study. I also confirmed first individually and then with partners that the co-teaching partners were interested in joining the study together as participants.

Once I confirmed that both the general education teacher and the math interventionist had volunteered to participate in the study and the PD, I emailed the volunteers and asked the volunteers to take the co-teaching survey. It was difficult to recruit volunteers. I had erroneously assumed that math interventionists and general education teachers would be clamoring for a PD opportunity that came with free books, clock hours, and learning around co-teaching and CGI practices (Carpenter et al., 2015). After I received International Review Board approval (December 2017), I spent the first two months recruiting volunteers for my study. I ended up working with all the teachers that showed any interest in the PD. I had four teams of teachers at three different locations. Each team had either a math interventionist, or a special education teacher, and a general education classroom teacher. The participants consisted of three math interventionists, two special education teachers, and four general education teachers. In Chapter 5, I describe the teachers in more detail.

Data Sources and Collection

In this section, I describe the data sources and data collection procedures that I used for this study. I created a table that outlines when data collection and analysis occurred (Appendix B). The data sources included:

- Co-Teaching survey (Appendix C)
- Reports from an online co-teaching survey (Appendix D & Appendix E)
- Individual teacher interviews prior to PD (Appendix F)
- Individual teacher interviews after PD (Appendix G)
- Co-Teaching partnership interviews prior to PD (Appendix H)
- Co-Teaching partnership interviews after PD (Appendix I)
- Observations and notes from the PLC meetings that either one or both teachers attended (Appendix J)
- Observations and notes from the collaboration meetings (Appendix J)
- Documents used to create or implement the PD,
- Documents or artifacts that teachers brought to the PD,
- Student work samples

The data collected intended to support an understanding of co-teaching relationships of the participants. Initial data came from the co-teaching survey teachers completed (Appendix C) and the reports that the survey generated (Appendix D & Appendix E). Collected transcript data came from the PLC meetings, from the collaboration meetings, and from co-teaching lessons. Interview data were collected prior to and after the PD as individuals and as partners. I arranged a time to conduct semi-structured interviews with each individual teacher prior to (Appendix F) and after the PD (Appendix G) concluded. I used this same process for the partnership interviews that occurred prior to (Appendix H) and after the PD (Appendix I) but with a different semi-structured interview protocol. I collected observational data during collaboration meetings, PLC meetings, and PD instructional sessions. I used an observational tool for the collaboration meetings and PLC meetings (Appendix J). Table 1 highlights the numbers and types of transcribed data sources at each school site.

Table 1

Transcribed Data Sources at Each School Site

School Sites	PD Instructional Sessions	Collaboration Meetings	Lessons	PLC Meetings	Interviews	Observational Notes
Hillview	12	12	35	6	8	6
Oakview	12	12	34	10	12	10
Three Peaks	9	9	15	7	6	7

To increase the credibility of the findings, I used these sources to triangulate the data. Data triangulation is a strategy used by qualitative researchers to increase the credibility of emergent findings (Merriam, 2009). As Merriam (2009) informs, "Probably the most well-known strategy to shore up the internal validity of a study is what is known as triangulation." (p. 215). Triangulation uses at least three sources of data to verify the findings (Miles et al., 2014). Data triangulation can occur in a variety of ways in the form of multiple methods, multiple sources of data, and/or multiple investigators (Merriam, 2009). The combination and comparison of multiple complementary data sources was an important aspect of this study (Tashakkori & Teddlie, 2003b). Tashakkori and Teddlie describe data triangulation as, "the combinations and comparisons of multiple data sources, data collection, and analysis procedures, research methods, and/or inferences that occur at the end of the study" (2003b, p. 674). Data triangulation is a term in which the meaning can be lost. Data sources, data collection, analysis procedures, and research methods describe triangulation. “The aim is to pick triangulation sources that have different foci and different strengths, so that they can complement each other.” (Miles et al., 2014, p. 299). This study used data triangulation through multiple data sources, as outlined above. For each data source listed, I described the data source and analysis methods.

Co-Teaching Survey

Both teachers created a free account and completed a short five-minute survey at http://www.noboxinc.com/tact.html (Appendix C). After both teachers completed the online survey, both teachers received an initial report on co-teaching behaviors (Appendix D). The online survey helped facilitate a conversation around strengths and next steps needed in a co-teaching relationship. The report included links so that if a teacher wanted more information or suggestions on how to improve a specific co-teaching behavior, the report also served as a resource to find more information on co-teaching.

The online co-teaching survey generated two separate reports. The survey generated a co-teaching behavior analysis report (Appendix D). This report allowed the math interventionist, the special education teacher, and/or the general education teacher an opportunity to investigate 20 essential co-teaching behaviors. The report ranked the behaviors as either emerging, developing, or established for each teacher. The second report was a co-teaching action improvement plan (Appendix E). The behavior analysis report generated the improvement plan report. This second report provided the co-teaching partnership an entry point into discussing the alignment in thinking around planning, instruction, classroom management, and assessment. The improvement plan report indicated, and prioritized items or issues teachers needed to discuss right away and provided suggestions for ways teachers might strengthen the co-teaching relationship.

Interviews

I conducted four types of interviews for this study. Interviews were either individual or with the co-teaching partner. The interviews occurred either prior to or after the PD. To review, the four types of interviews were individual interview prior to PD, individual interview after PD, partnership interview prior to PD, and partnership interview after PD. I conducted 26 semi-structured interviews. I organized the individual semi-structured interview protocol prior to PD by research question (Appendix F). I also organized the individual semi-structured interview protocol questions by research question to use after the PD concluded (Appendix G).

I conducted eight semi-structured interviews with the co-teaching partners in which I interviewed the co-teaching partners together. The first initial interview, (Appendix H) occurred prior to the PD. The second post-interview (Appendix I) with the co-teaching partners occurred after the PD concluded. Semi-structured interview protocols (Merriam, 2009) facilitated collection of descriptive data related to each participant's perceptions of PLC meetings, perceptions of the collaboration meetings, and their perceptions of how these discussions impacted lesson design using a co-teaching model. Each interview question indicates how that data supported a specific research question.

Specially crafted interview questions generated data that were useful for each set of research questions. The prior to PD interview protocols served as a foundation for the post PD interview protocols. There was an intentional parallel construction between interview questions prior to PD and the post PD interview questions. Some modified interview questions captured data related to how the PD supported instructional practice. Some added questions to the post PD interview protocol captured data related to how the PD supported mathematical instructional practice for math interventionists, special education teachers, and/or general education teachers.

I captured interviews using a standard digital audio recorder. I transcribed audio files to rich text files. I sent 210 files to a third party for transcription to keep the amount of transcription I needed to complete reasonable. I transcribed two of each data source listed in Table 1 myself and sent the remaining to a third party for transcription.

Observations

I conducted observations using an observation tool (Appendix I). The observations occurred during PLC and collaboration meetings. I observed each team at that team's PLC meeting prior to the PD to gather baseline data. I was interested in learning if the teachers currently used the teaching and learning cycle as part of the PLC planning process. I also wanted to learn if student work informed instructional decisions, and if so, the extent to which student work informed instructional decisions. During the PD instructional sessions, the teachers learned these skills. Baseline data were important in determining how many of these skills were already included in the teacher's practice before participating in the PD. I observed four different PLCs over the course of the 12-week PD. I observed the Hillview PLC six times, the Oakview PLC ten times, and the Three Peaks PLC seven times. I gathered observation notes at every collaboration meeting, even if the observation notes were only for a portion of the meeting. Once the teachers began working and collaborating with each other, I became a careful observer of teachers and their discussions. Each observation lasted a minimum of 30 minutes and a maximum of 90 minutes. While collecting data, I assumed the stance of participant-observer (Merriam, 2009). As the participant observer, I focused my observations on the following seven areas:

- Evidence of building a collaborative relationship – Research Question 1
- Designing instruction - Research Question 2

- Evidence of planning for students needing math intervention or in special education – Research Question 2
- Using CGI (Carpenter et al., 2015) principles – Research Questions 2 and 3
- Using *Five Practices* (Smith & Stein, 2011)- Research Questions 2 and 3
- Analyzing student data – Research Question 3
- Creating assessments – Research Question 3

These seven areas guided what I noticed during the collaboration and PLC meetings. The focused observation data related to specific research questions. During each observation, I recorded field notes of what I heard, keeping as much inference out of the observational notes as possible (DeWalt & DeWalt, 2011; Spradley, 1980). After each observation, I added to my field notes by writing down ideas I had in my head (DeWalt & DeWalt, 2011). I converted transcribed observational notes to a plain text file, and uploaded those to HyperRESEARCH®.

Student Work Samples

Student work samples served in two distinct ways. First, I analyzed student work to assess how the PD supported a shift in mathematical instructional practice. The district-created curriculum had limited, cognitively demanding, rich math tasks (Carpenter et al., 1999, 2015; Boaler et al., 2018a, 2018b, 2018c; Flynn, 2017; San Giovanni, 2016). I was looking for evidence of teacher learning and a shift in instructional practice by analyzing changes in student work. Second, work samples helped teachers plan for instruction. I asked teachers to bring initial math work samples to gain an understanding of the curriculum being used within each particular school and the instructional strategies employed by each teacher. Each building in my district used the district-created math curriculum, developed their own mathematics curriculum, or purchased their own commercial-based mathematics curriculum. With the baseline of student work samples, I analyzed these samples for evidence of teacher learning and implementation of new instructional strategies. I compared baseline work samples with work samples brought to PD instructional session number six, seven, or eight. Besides student work samples used to assess how math instruction supported students with mathematics difficulty, I also used student work samples as a formative assessment to guide my instructional plans. Through work sample analysis, I prompted teachers with questions about student strengths and next steps for instruction. I asked teachers questions about how the student work was evidence of the student's developmental thinking regarding the solution strategy the student used.

I analyzed work samples to discover how the PD supported an instructional practice shift based on CGI. The teachers used work samples to assess student instructional need with different problem types and sophistication of solution strategies. Teachers placed student names on a solution strategy matrix (Appendix K). The work samples served as a formative assessment for the teachers and helped guide co-designed lessons based on student instructional need or next steps. I prompted teachers to examine student work by asking these questions:

- How did the student enter the task?
- In this work, what evidence is there of various solution strategies?
- Does the solution strategy demonstrate a basic or complex understanding of the operation?
- What do you notice the student is attempting to do with this solution strategy?
- What is the student strength and next instructional step?
- Was the solution strategy student-developed, or was that strategy explicitly taught?
- How could this work inform instructional planning?
- How will you advance student thinking to the next level?

- What would a solution look like that used a compensation strategy?
- What additional supports could ensure the student with mathematics difficulty receives access to the mathematics content?
- How can this work inform formative or end of unit assessment design?

Through the analysis of student work samples, I knew how well the PD was functioning to improve student learning. The work sample artifacts provided evidence on how the instructional approaches used by the co-teaching partnership allowed access to grade-level math instruction for students with mathematics difficulty. When I examined student work samples at the start of the PD, and after the PD, I determined the extent to which the teacher had any shifts in instructional approach. I also determined the extent to which students gained access to grade-level mathematics content through the newly designed work.

Data Analysis

I applied the constant comparative method of data analysis (Miles et al., 2014) in this qualitative case study as I compared "one segment of data with another to determine similarities and differences" (Merriam, 2009, p. 30). I used analytic induction to test formulating hypotheses about perceptions of co-teaching and the impact the PD had on student learning. As sorted and analyzed data form and refine, hypotheses emerge through a rigorous process of analytic induction (Merriam, 2009). For the development of the initial codes, I thought about what I commonly heard when working with math interventionists, special education teachers, and general education teachers in my district around concepts of co-teaching. Miles, Huberman, and Saldaña (2014) referred to this as first cycle coding. I used a combination of descriptive and process codes (Miles et al., 2014). I used nouns, verbs, and nouns and verbs for code names to enable stanza labeling (Saldaña, 2013). As I read the transcript for the first time, I marked off sections of text that I referred to as a stanza. Then I labeled that stanza with a code. For example, I used the code of collaboration to label stanzas with teachers working together with equal and shared decision making to work towards a common goal (Friend & Cook, 1995). I separated out collaboration from co-teaching, so I created separate codes for co-teaching and a corresponding code definition (Appendix A). I created a code for co-teaching to include any situation in which two or more teachers shared instructional responsibility during a lesson. Friend et al. (2010) make the distinction that co-teaching and collaboration are two separate concepts. I analyzed the data for differences between collaboration and co-teaching.

At the start of second round coding, I developed a codebook (Appendix A) in which I described and organized coding decisions and examples of stanzas that exemplify that code (Saldaña, 2013). The codebook provided a way to be mindful of the decisions I was making about codes and their meaning. I documented coding decisions and examples of data that matched that code in my codebook. I used the codebook as a reference as coding continued. I used the codebook to help keep me accountable, and to maintain calibration between when coding began and when coding ended. When I coded stanzas, I kept the flexibility of generating new codes, renaming codes, or combining codes after initial coding. For example, all passages initially coded as designing math instruction (RQ2) I recoded with new codes of large generalizable goal or small daily goal. I also recoded with (a) used CCSS progression documents in planning, (b) used district-created math curriculum in planning, or (c) used EngageNY (New York Education Department, 2012) in planning. This allowed me to ebb and flow with the data, which maintained the emergent nature of qualitative research (Merriam, 2009). I used HyperRESEARCH®, a qualitative analysis software program, to assist with data analysis. I used HyperRESEARCH® to code sections of the text based on the codebook I developed (Appendix

A). During the first round of coding with HyperRESEARCH® I tagged each stanza of text using the pre-established codes. During the second round of data analysis, I went through the data again looking for new codes (Stake, 1995). During the third round of data analysis, I created HyperRESEARCH® reports in which I searched for interesting code combinations. For example, I searched for patterns between large generalizable goal, small daily goal, and the types of materials teachers used in planning. I wrote research memos during the second and third phase of data analysis to reduce the data even more, and then I generated claims and supported the claims with data-based evidence. Three layers of analysis and coding continued for all 222 transcripts as well as other documents and artifacts. Throughout this process, I used the constant comparative and analytic induction method (Merriam, 2009; Miles et al., 2014).

The next analytic strategy I used was explanation building as a type of pattern matching (Yin, 2014). The goal of this analytic strategy was to analyze data in a way that explained the case (Yin, 2014). Each co-teaching partner or team constituted one case. With data analysis of coded interviews, student work samples, and observational data, I identified links between the PD, collaboration meetings, and co-teaching lesson design. I did this in narrative form, as Yin describes (2014). I also used a data display as is favored by Miles, Huberman, and Saldaña (2014). For each co-teaching team, I aligned the student work data and the observational data using a table to display the data (Appendix L). I turned the table into a matrix to look for phenomenon within each co-teaching partner case (Appendix L). For example, with the Oakview team I record ideas from student work samples of EngageNY (New York Education Department, 2012) worksheets and observation notes of planned lessons that did not initially include EngageNY materials. I also compared and aligned the student work data and the observational data and searched for rival interpretations against other data sources. For example, I explored possible explanations for reasons why the Oakview planned lessons did not result in enacted lessons.

According to Yin (2014), four principles are the foundation for assuring high-quality data analysis. These principles of data analysis include attending to all the evidence, searching for all plausible rival interpretations, addressing the most significant aspect of the case study, and using my own prior expert knowledge (Yin, 2014). As I worked through the intended rounds of data analysis, I attempted to meet the expectations for assuring high-quality data analysis.

Consideration of the Research Value

Friend et al. (2010) asserted the need for research to understand the impact co-teaching has on student learning. Other researchers such as Murawski and Swanson (2001) have indicated the limited evidence between co-teaching and student outcomes. Katherine Lewis (2014) called for a shift in thinking about mathematics learning disability away from deficits to focus on student understandings. In consideration of the value of my research, a discussion of trustworthiness, internal validity, external validity, measurement validity, outcome validity, process validity, democratic validity, catalytic validity, and dialogic validity enhances the credibility of my study.

Trustworthiness

According to Lincoln and Guba (1985), a study's trustworthiness involves the demonstration of the researcher's interpretation that the data are credible to those individuals who provided the data. The traditional quantitative research concept of trustworthiness includes different types of validity (Herr & Anderson, 2005). Because action research has different goals than qualitative or quantitative research, different ways to measure validity are needed (Mills, 2011). Both Mills (2011) and Herr and Anderson (2005) agree that action research has five primary indicators for action research. Indicators of validity criteria are internal validity, outcome validity, process validity, democratic validity, catalytic validity, and dialogic validity (Herr & Anderson, 2005; Mills, 2011).

Internal Validity

Internal validity refers to the credibility, believability, and trustworthiness of the findings (Miles, Huberman, &Saldaña; 2014). Lincoln and Guba (1985) suggest seven different techniques for establishing credibility. For this study, I used four of the seven techniques. The four techniques I used were prolonged engagement, triangulation, member checking, and peer debriefing.

Prolonged engagement. I used prolonged engagement as a form of internal validity. Prolonged engagement is the act of being with the participants for an extended period. As a district instructional coach, I was with the participants more than just the 12 weeks of the PD. I spent time at all three locations presenting whole staff PD and working with math interventionists in coaching cycles. The staff viewed me as a member at all three locations. Because of the time I spent at all three study locations, I developed a rapport with the teachers and developed trust as we co-constructed meaning during the PD.

Triangulation. The second internal validity technique I used was triangulation. Triangulation refers to the idea of searching across three different data points to confirm findings and/or using three different data analysis techniques to confirm findings. When analyzing the data, I used recorded transcripts from all PD meetings, observation notes from team PLC meetings, and transcript and observation notes from lesson enactments. When analyzing this data, I used open coding when transcribing the meeting data. I used HyperRESEARCH® to refine my findings and generate reports, and I used analytic memos to make claims and support my claims with evidence. I showed my analytic memos to my participants, and this technique is another form of internal validity known as member checking.

Member checking. The fourth approach I used to establish credibility included the use of member checking. Considered one of the most important ways to establish credibility, member checking allows the participants to voice their thoughts during and after data collection (Lincoln & Guba, 1985). Member checking can occur during the study as a natural part of the conversation to check perceptions and understanding of preliminary findings. During the PD meetings with individual teams, I often articulated and summarized the ideas from the prior meeting and asked if any of the members viewed the meeting in a different way.

I used member checking when I restated issues for teams when there was a misalignment between teacher ideas. For example, at Three Peaks, when Sally and Megan were setting goals for the students in special education, the two teachers were having difficulty arriving at agreed-upon goals. Sally wanted the goals to focus on big math concepts while Megan wanted to focus on IEP goals and have a narrower focus. As the researcher and coach during this meeting (Jaworski, 1998; Lewis, 2014), I restated the problem and asked how the two teachers could find common ground. As I was highlighting the different viewpoints on goal setting for the two teachers, they had an opportunity to change or restate their opinion about goal setting. With member checking in this way, each teacher had the opportunity to correct the assertion that the two teachers viewed goal setting in divergent ways.

I also used member checking at the Hillview location after the study concluded. I met with the team and presented several transcripts from lesson enactment, in which all three members were present. I asked the teachers to read the transcripts, and then we had a conversation about what they found interesting in the transcript data. At that time, I shared my initial findings and what I thought was important from my study standpoint. I gave the Hillview team an opportunity to challenge or correct my initial findings. At the Oakview location, member checking came into play when coaching in the moment of need occurred. After observing a lesson, I asked the teacher questions about the lesson. I probed the teacher with questions that would give the teacher an opportunity to correct my interpretation of the lesson. I met with Cheryl and asked her about lesson summary enactment, as an example. My questions gave her an opportunity to discuss how she was unsure of lesson summary importance and how she was still unsure of what was to be included in the lesson summary. This coaching conversation after the lesson (Aguilar, 2013; Loucks-Horsley et al., 2010; Sweeney, 2011), provided Cheryl with an opportunity to correct my interpretation of the lesson I had observed. Member checking is controversial because participants can shift findings or alter the strength of the findings based on their feedback. Besides using the participants in the study to voice their ideas on data and data analysis to establish internal validity, I used professional peers as another way to establish internal validity.

Peer debriefing. The third internal validity technique I used was peer debriefing. With peer debriefing, the researcher engages in analytic discourse with a disinterested peer with the idea that the peer can help the researcher uncover biases and assumptions made during data analysis. I utilized my office mate as my disinterested peer as she is responsible for coaching reading interventionists, and I am responsible for coaching math interventionists. We often found parallels in our work or challenged each other in our thinking. Questions we pose to each other extend beyond our knowledgeable content areas. During data analysis for my study, I would give my office mate a copy of a lesson transcript or of a meeting transcript. I would have her read the transcript and then tell her about how I interpreted the data. She would ask questions and challenge my thinking with alternative ways to interpret the data. Using peer debriefing as a way to establish study credibility was helpful when I made claims about special education teachers, general education teachers, or math interventionists. My office mate did not know these teachers and did not know the position they held at the different schools. My office mate was able to challenge my thinking and assertions about the different math instructional approaches and the teaching positions that the teachers held. Even though my office mate and I viewed data in similar ways, does not mean there was external validity.

External Validity

External validity in qualitative research is transferability (Lincoln & Guba, 1985) or how well the findings transfer to other contexts, situations, or populations. There were only nine teachers in my study so it would be inappropriate to claim that readers could transfer findings to all teachers or even to subgroups of teachers such as special education teachers, or elementary math interventionists. Even though findings cannot transfer to other populations, another researcher may be able to use these findings to look for and discover similarities in other contexts or situations. Besides other researchers using my findings, researchers can assume my findings are valid.

Measurement Validity

Measurement validity refers to how valid are the findings. Researchers should ask if the findings make sense and if the findings can be trusted. One way to ensure the trustworthiness of the research is ensuring the researcher checked personal bias and expectations before conducting the study. The researcher needs to be open to new or alternative findings other than the potential hypothesized findings. I expected to find that the teacher role (general education teacher versus special education teacher) affected the teacher's worldview and therefore, instructional practice.

However, I did not expect to find that the materials teachers used in planning influenced the way teachers thought about lesson planning to create a lesson trajectory. Before I conducted my study, I hypothesized that special education teachers would typically focus on small objectives, and these small objectives would not reach the point of a large generalizable idea. I also hypothesized that general education teachers would be more flexible in their thinking with lesson planning grain size, moving from daily lesson to large generalizable idea. What I discovered through data analysis is that the materials teachers used to plan had an impact on how the teacher thought about lesson planning, more than the assigned position of the teacher. Although I had generated informal hypothesizes prior to conducting my study, after data analysis, one possible explanation for the way teachers thought about lessons and planned for lessons was in the structure and types of materials they used to plan.

Outcome Validity

The fundamental principle of action research is an action that works towards a solution to a complex problem (Mills, 2011). One way to determine if action research is of high-quality is to examine the effectiveness of the researched action and if new learning is applied to another action research cycle (Herr & Anderson, 2005; Mills, 2011). Outcome validity measures the researched action in terms of helping to address the initial problem of practice (Herr & Anderson, 2005; Mills, 2011). I was not able to demonstrate outcome validity in my study because the teachers in my study tended to revert to old ways of thinking when presented with new mathematical content. The teachers needed more continued support to sustain instructional shifts longer than the 12 week PD allowed. Outcome validity was determined at the end of the study when a discussion of findings raised new questions (Mills, 2011). “Outcome validity also acknowledges the fact that rigorous action research, rather than simply solving a problem, forces the researcher to reframe the problem in a more complex way, often leading to a new set of questions or problems.” (Herr & Anderson, 2005, p. 55). As I will discuss in Chapter 6, after the study concluded, I had more questions about the length of time needed for PD to make long-term instructional shifts. I also had questions about how to sustain learning from year to year, without teachers losing focus. When I analyzed outcome validity, I also analyzed process validity.

Process Validity

Process validity affected outcome validity. If the process of the action research were flawed or superficial, then the outcome would reflect the problems of the process (Herr & Anderson, 2005). Process validity connects with the recursive nature of action research. The data I collected answered the research questions I posed, but not as I expected. I felt my study had process validity because data analysis began while I was still collecting data. It was important to begin data analysis before data collection concluded in to ensure the process for analyzing the data were reasonable and sound. It was important to examine the data as I was collecting it and reflect on the suitability of the data collection techniques (Herr & Anderson, 2005; Mills, 2011). By scheduling data collection and analysis activities (Appendix B), I helped ensure process validity. The table I created on data collection and data analysis activities (Appendix B) allowed for the recursive nature of analyzing data while I was still in the process of collecting data. By being intentional with simultaneous data collection and data analysis, I also increased the democratic validity.

Democratic Validity

Democratic validity occurred when I collaborated with the participants to ensure that I included perspectives from all stakeholders (Herr & Anderson, 2005; Mills, 2011). Democratic validity depends on the inclusion of multiple voices from an ethical and social justice stance

(Herr & Anderson, 2005; Mills, 2011). As I approached my research from a transformative worldview, it was important that I collaborated with the co-teachers and administrators to ensure that the PD sessions and collaboration meetings voiced the needs and concerns of the co-teachers. I attempted to provide democratic validity in my study, and this was evident during the PD instructional sessions. I provided opportunities to set goals and outcomes for the PD sessions. During every PD session, I provided opportunities to the teachers to give feedback about the process and content of the prior PD session. I also asked teachers to provide feedback about what the co-teachers were learning and how teacher learning influenced instructional growth for students. I used a researcher journal and audio recorder to capture quotes and the voices of the participants. This was another strategy to ensure democratic validity and to examine catalytic validity.

Catalytic Validity

"Catalytic validity requires that the participants in the study are moved to take action on the basis of their heightened understanding of the subject of the study" (Mills, 2011, p. 109). I saw evidence of catalytic validity from attending and observing the collaboration meetings and PLC meetings. During the PD instructional sessions, teachers learned about CGI (Carpenter et al., 2015), the Launch-Explore-Summary lesson structure (Lappan et al., 2007; Schroyer & Fitzgerald, 1986), and the *Five Practices* (Smith & Stein, 2011). Teachers also learned about co-teaching models, roles and responsibilities, and the teaching-learning cycle (Carpenter et al., 2015, Friend et al., 2010; Jones, 2008; Lappan et al., 2007; Schroyer & Fitzgerald, 1986; Smith & Stein, 2011). I looked for evidence of teachers approximating new learning through their collaborative discussions, their PLC meetings, and through observation of co-teaching. I noticed different teams took more action than other teams. I utilized my research journal and created research memos based on trends I noticed in the data in terms of change in my thinking for how I taught the PD content or change in my participants' understandings. As Herr and Anderson (2005) state,

> The most powerful action research studies are those in which the researcher recounts a spiraling change in their own and their participants' understandings. This reinforces the importance of keeping a research journal in which action researchers can monitor their own change process and consequent changes in the dynamics of the setting. (p. 57)

The use of a researcher journal was not only helpful with documenting catalytic validity but was also instrumental with dialogic validity.

Dialogic Validity

I used dialogic validity to ensure a well-rounded view of the research. Dialogic validity helps to avoid the potential for groupthink (Herr & Anderson, 2005; Mills, 2011). In this form of validity, analysis of the quality of research occurs through some form of a peer-review process (Herr & Anderson, 2005; Mills, 2011). To include this form of validity, I had critical conversations with others about my research findings and practices (Herr & Anderson, 2005; Mills, 2011). My work and office mate served in this role. During data collection and analysis, I talked with my office mate about the PD instructional sessions and my observational notes. My office mate would provide feedback to me about how to improve the instructional sessions and how to enhance catalytic validity. I had meetings with Dr. Roth McDuffie during data collection and analysis. Our conversations helped me to focus on emergent findings. Even with increased validity of the action research with the five validity criteria, limitations to action research still exist.

Limitations

This study is not without limitations. The size and type of participant sample limit the findings and eliminates any possibility of generalizable research findings. Findings that apply to this research context may not apply to other similar contexts. Although action research findings are not generalizable, this does not necessarily equate to a limitation of action research. Generalizable results are often wide and thin to fit the entire population (Herr & Anderson, 2005; Mills, 2011) while qualitative action research provides thick, rich descriptions from a local reality (Geertz, 1983; Herr & Anderson, 2005; Mills, 2011). Regardless of the perspective taken on generalizable results, researchers need to be aware of personal bias issue (Herr & Anderson, 2005; Mills, 2011). Personal bias often can creep into the investigation. One way to get in touch with personal bias is to record expected findings prior to the investigation (Herr & Anderson, 2005; Mills, 2011). One way I recorded expected findings is through the creation of the initial analytic framework, which then turned into the codebook (Appendix A). I generated codes for what I expected I would discover in the data. I acknowledge my own personal bias with the data (Herr & Anderson, 2005; Mills, 2011). I know what I expected to find in the data. I was willing to search for discrepant data. Discovering data that does not match my initial personal propositions can result in my own catalytic validity. The recursive nature of noticing change uncovers new understanding for my participants and myself (Herr & Anderson, 2005; Mills, 2011). Discovering and acknowledging personal bias within research is important for maintaining valid and reliable research results. In the next chapter, I outline six primary findings.

Another limitation of my study included the 12-week PD and lesson observations in capturing instructional shifts. Twelve weeks was enough time to identify instructional patterns and learn about teacher instructional practices. Twelve weeks did not seem to be enough time to capture instructional shifts. Instructional shifts could have occurred after the 12 weeks and had not yet been evident with my daily observations. My study did not include a detailed follow-up with my participants to discover if the PD learning translated across academic years. I do not know if the learning that Maggie and Chris took on during the PD resulted in long-term instructional shifts. Research that follows participants for a longer duration across multiple school years might give more insight into changes in instructional practice. Teachers developed expertise around collaboration, creating learning progressions, utilizing the *Five Practices* (Smith & Stein, 2011) and enacting the lesson summary. However, was that new learning sustained over time?

A third limitation of my study was my role as both math intervention instructional coach for the district and researcher (Jaworski, 1998; Lewis, 2014). At times meetings driven by my coaching role prohibited me from attending grade-level PLC meetings or lesson observations. With juggling three different school locations and schedules between buildings, teachers, and my other job responsibilities, there were times that I missed meetings and had gaps in my data. On the other hand, my dual role also helped me to build relationships with the teachers. The teachers I worked with were willing to open up and share their practice and perspectives in ways that might not have occurred if I was strictly a researcher and not an instructional coach. However, as a district-level coach, teachers often viewed me as a representative of the district. Sometimes this translated into searching for approval for certain instructional approaches, or not being fully transparent for fear of what I might say back at the district office. I had to work carefully and intentionally to allow the participants to feel safe. Several times teachers asked that I remove excerpts from certain audio recordings from my data set. In compliance with International Review Board approval, I complied with any such request without hesitation or questions.

A fourth limitation of my study was the solitary nature of conducting the data analysis. I was the only researcher gathering the data, transcribing the audio recordings, completing open coding (Miles et al., 2014), and then conducting second and third round coding using HyperRESEARCH®. In an ideal research situation, multiple coders would analyze data and meet to establish inter-coder agreement. In the absence of multiple coders, I used multiple techniques to ensure internal validity. In addition, I regularly met with Amy Roth McDuffie, my dissertation advisor, to discuss patterns, questions, and emerging theories. I shared early drafts of my analysis and writing for her feedback, which helped in questioning and/or seeing more evidence, gathering literature to understand emerging themes, and delving deeper into my data.

CHAPTER FOUR: PROFESSIONAL DEVELOPMENT PROGRAM

Research on co-teaching, instructional math approaches, and PD demonstrates a need for PD for math interventionists, general education teachers, and special education teachers (Aguilar, 2013; Carpenter et al., 2015; Friend et al., 2010; Loucks-Horsley et al., 2010; Smith & Stein, 2011; Sweeney, 2011; Wilson & Blednick, 2011). The purpose of the PD that I designed was to create a collaborative learning environment in which teams of teachers planned for access to core instruction for students with mathematics difficulty. The PD I designed and implemented focused on: (a) learning how to engage in co-teaching (b) learning how to implement high cognitive tasks, and (c) learning to notice student solution strategies. These instructional outcomes benefited math interventionists, general education teachers, and special education teachers' instructional practice and student learning (Aguilar, 2013; Carpenter et al., 2015; Friend et al., 2010; Loucks-Horsley et al., 2010; Smith &Stein, 2011; Sweeney, 2011; Wilson & Blednick, 2011).

The research on instructional coaching (Aguilar, 2013; Sweeny, 2011) indicates that focused PD on a systematic coaching cycle is an effective approach for shifting teacher beliefs and instructional approaches to benefit all students (Aguilar, 2013; Sweeny, 2011; Murry, 2014). Coaching cycles use a three-part process. First, coaching cycles include specific PD goals with intended teacher learning outcomes (Aguilar, 2013; Sweeny, 2011). Second, coaching cycles also include collaborative planning meetings in which teachers have time to discuss, plan, and implement ideas learned during the PD (Aguilar, 2013; Sweeny, 2011). Third, the coaching cycle also includes the instructional coach taking part in classroom instruction in either modeling instruction or observing instruction (Aguilar, 2013; Sweeny, 2011). My PD used this coaching cycle as a way to help math interventionists, general education teachers, and special education teachers learn how to collaborate. During the collaboration meetings teachers learned how to implement effective co-teaching practices while focusing on high cognitive tasks and student solution strategies (Aguilar, 2013; Carpenter et al., 2015; Friend et al., 2010; Murry, 2014; Smith & Stein, 2011; Sweeny, 2011; Wilson & Blednick, 2011).

Providing the PD Program Sessions

As a participant in this action research study, I designed eight and implemented up to 12, 90-minute PD sessions (Jaworski, 1998). I planned for eight instructional sessions, but some teams moved at a slower pace than I had planned. I adjusted instruction as needed on a team-by-team basis. I allowed the instruction to be fluid, and each week we picked up where we left off. Because some teams moved at a slower pace, they actually participated in 12 instructional sessions that contained the same content as the eight planned sessions.

The co-teaching partners engaged in learning about roles, responsibilities, student ownership, and other factors contributed to highly functional co-teaching partnerships (Friend et al., 2010). Beyond learning about how to be an effective co-teacher, the teachers also learned about how to notice and attend to student instructional needs based on theories and principles of CGI (Carpenter et al., 2015) as well as, how to orchestrate a productive math discussion (Smith & Stein, 2011). The PD helped establish and provided a stronger foundation for the co-teaching relationship. The PD provided instruction for the co-teachers on diagnosing student instructional next steps and allowed co-teaching partners to design rich math tasks that advanced the mathematical thinking of all learners within the general education math classroom (Carpenter et al., 2015; Smith & Stein, 2011). All four teams of teachers that participated in the study attended the same two-day summer institute. Then after the start of the school year, the eight, ninety-minute PD sessions occurred at three different building locations (Oakview Elementary, Hillview

Elementary, and Three Peaks Elementary) for the four teams of teachers that participated in the study.

Beginning Summer Institute Session

The teachers met each other and completed team-building exercises during a two-day summer institute prior to the eight-week PD sessions beginning. The two-day summer institute sessions comprised co-teachers from all school levels and all content areas across the district and were not only the teachers who participated in the research study. During the work sessions, the teachers learned about co-teaching practices and began building their co-teaching relationships. The teachers who attended the summer institute sessions also learned about the district definition of intervention. The district definition of intervention was, "Intervention is a collaborative, diagnostic, decision-making process that emphasizes aligned instruction and targeted lessons with degrees of intensity and progress monitoring so students can complete unfinished learning." (School District, Federal Programs Office, 2016). The teachers learned about intervention as a way of thinking about how students can complete unfinished learning and have access to core instruction. The two workdays allowed for seven, two-hour work sessions in which the learning focused on each aspect of the district definition. The seven work sessions included:

1. What it means to be collaborative,
2. How to use diagnostics for reading and math intervention,
3. How to use a team decision-making process,
4. How to align intervention instruction to core instruction,
5. How to provide targeted interventions,
6. How to change the degree of intensity of intervention, and
7. How to use progress-monitoring tools to measure growth.

Teachers learned how the use of these seven intervention principles allowed students to have access to core instruction while also attending to learning gaps. These workdays included time to generate ideas for needed future PD around co-teaching practices and allowed teachers opportunities to engage in small group discussions around inclusive intervention practices taking place in schools across the district. The summer institute set the stage for teachers to join the study in January of 2017.

Session One: Fundamentals of Co-Teaching Population

The first PD session involved the co-teaching partners completing an existing online survey on co-teaching at http://www.noboxinc.com/tact.html (Appendix C). The Teachers Analyzing Co-Teaching website and survey provided a report to both of the teachers in the co-teaching relationship on their alignment of 20 essential co-teaching behaviors. After both partners took the survey, a generated report listed the 20 essential co-teaching behaviors as either emerging, developing, or established for each teacher. A second report put together an action plan based on identified emerging or developing co-teaching behaviors. During this session, a discussion of the secondary co-teaching analysis and action plan occurred. The purpose of this was to allow partners to move forward together in a productive collaborative manner. As an initial PD activity, the teachers compared survey results with their partner and made a two-column chart. The chart listed the strengths of the co-teaching partnership in one column and three next steps or needs of the partnership in the other column.

As a participant and researcher (Jaworski, 1998), I used the survey, the reports, and the two-column chart to understand how well the teachers in the co-teaching partnerships were aligned in their co-teaching thinking and behaviors. A discussion occurred using the secondary action plan report on critical issues embedded within a co-taught context. This first PD session allowed the partners to complete the survey, generate the reports, complete a two-column strengths and needs chart, and brainstorm issues they found critical to co-teaching. I asked the teachers to generate a list of topics partners wanted addressed over the course of the PD. A reading passage about co-teaching from Friend et al. (2010) occurred, and then partners discussed the reading. To anchor our thinking in the second session, I provided another article about co-teaching and collaboration. Through this first PD session, teachers understood their own co-teaching experience or their beliefs about co-teaching on a deeper level through the results of the survey, the two-column strengths and needs chart, and the sample reading passage.

Session Two: Productive Co-Teaching Models

The second PD session facilitated learning about the different forms of co-teaching (Friend et al., 2010; Wilson & Blednick, 2011), language regarding student ownership (DeSimone & Parmar, 2006), and identifying students by instructional needs (Holmlund Nelson, 2008). An important aspect of the foundation of the PD provided language and understanding of the different models of co-teaching (Wilson & Blednick, 2011). The co-teaching partners learned about the three primary models of co-teaching. These models consisted of consultation, lead and support, and parallel co-teaching (Wilson & Blednick, 2011). I provided a handout and gave a brief presentation on the three primary models of co-teaching.

Consultation is the lowest level of co-teaching support in which the general education teacher is the primary teacher in the classroom and is alone with students most of the instructional day. In this model, occasionally the math interventionist or special education teacher will come into the general education classroom and offer support or guidance, but this occurs on an as-needed basis (Bouck, 2007; Scruggs et al., 2007). With the lead and support model, the math interventionist or special education teacher is with the general education teacher for a larger block of time (at least 30-60 minutes) in which both teachers are enacting the curriculum around a content area. In this model, the general education teacher is the primary facilitator of instruction while the math interventionist or the special education teacher may be off to the side or sitting next to the students with mathematics difficulty (Wilson & Blednick, 2011).

The role of the math interventionist or special education teacher in the lead and support model is to provide some sort of instructional support to the students with mathematics difficulty, while the general education teacher is presenting the problem or providing instruction around new content (Wilson & Blednick, 2011). Forms of support could entail the math interventionist or special education teacher drawing a picture, creating a graphic organizer, providing a vocabulary word and definition box, using a highlighter to highlight keywords or text features, or creating any other tool in which the student with mathematics difficulty would gain access to the content because of the additional support. This additional support varies based on the needs of the student and the ability of the math interventionist or special education teacher to think and differentiate in the moment (Tomlinson, 2014).

In the parallel teaching model, both teachers are the primary facilitators of instruction. The co-teachers "ping pong" back and forth with instruction, providing extra supports to the entire class rather than just the identified students (Wilson & Blednick, 2011). This third model of co-teaching requires the most amount of collaboration, as well as the most time without students for collaboration. Providing language and structure around the three primary types of co-teaching allowed the co-teachers to advance and adapt their views on co-teaching. Learning about the models of co-teaching brought unity to each co-teaching team that participated in the study.

Each school in my district has a different understanding and approach to co-teaching. In 2010, the director of federal programs and the director of special education mandated the implementation of inclusion and co-teaching in every school across the district. Each school was responsible for the implementation of intervention or special education services in the general education setting. Each school selected its choice of an inclusive service delivery model. At that time, some schools were already using the lead and support model and were ready to advance towards the implementation of parallel teaching. Other schools had only ever used a pull-out model (Marston, 1996) in which the delivery of special education services occurred in a special education classroom. Schools that were only accustomed to a pull-out model (Marston, 1996) were slower to attempt a new inclusive model and began with the consultation. In some cases, schools attempted to move towards a lead and support model. With the participants coming from different schools across the district, I am aware the teachers were in different places with their thinking and understanding about co-teaching. When the teachers learned about the different co-teaching models, the teachers had a new vocabulary to describe the inclusive instructional practices used at their own school. The teachers developed common understandings about the meaning of terms such as co-teaching, consultation, lead and support, and parallel teaching.

After the co-teaching partners learned about the different co-teaching models, the partners also learned about the critical nature of student ownership (DeSimone & Parmar, 2006), student instructional needs (Tomlinson, 2014), and the danger of student labels placed by the teacher (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley-Marling, 2013). It was essential that the co-teaching partners viewed the students that they work with as belonging to both of them (DeSimone & Parmar, 2006). I immediately addressed any observed discourse between the teachers that identified students other than belonging to both teachers. Discussions of co-teaching relationships function most productively when students are "ours" not "mine" and "yours" (DeSimone & Parmar, 2006).

Another critical aspect of co-teaching is how each teacher views the student need or student ability (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley-Marling, 2013). Unified co-teachers perceived student need the same, and then the underlying cause of the student need is addressed (Cook & Friend, 1995; Friend & Cook, 2006). Dissension can arise between co-teachers when divergent approaches to student need are utilized (Scruggs et al., 2007). Both teachers need to come from the stance that the child is inherently fine and the instruction has yet to fit the need of the student (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley-Marling, 2013). When co-teachers agree on finding instructional methods that will work for the needs of their children, then co-teaching can be a successful model for helping students in special education reach their full potential (Cook & Friend, 1995, Friend & Cook, 2006; Fuchs et al., 1993; Fuchs et al., 2006; Fuchs et al., 2007; Scruggs et al., 2007.

During session two, the teachers and I discussed reading passages from Friend et al., (2010), Cochran-Smith and Dudley-Marling (2012), and an article about difference not deficit from Lewis (2014). These passages highlighted the above ideas and built schema for inclusive instructional practices and ways math interventionists, general education teachers, and special education teachers can work together. Teachers discussed the reading at the opening of session two. The ideas in the reading provided the anchor to the ideas covered in the remaining time. By having a similar structure at each session, teachers knew to complete reading prior to each session, and each session would with a discussion on the reading. Getting different ideas and viewpoints out helped to build trust and cohesion between the teachers at each location. Next, co-teaching partners created a visual representation of the different co-teaching models they learned about and the current model used at their building. By creating this visual representation, teachers solidified language around different co-teaching models. This image of co-teaching and inclusive practices helped teachers create a plan for their school's current reality and the desired outcome. Finally, I provided teachers time to develop two goals they would like to accomplish with their co-teaching partner during the course of the PD.

Session Three: Principles of Cognitively Guided Instruction

The third PD session provided an overview and introduced the teachers to ideas and concepts of CGI (Carpenter et al., 2015; Empson & Levi, 2011). These concepts helped math interventionists, special education, and general education teachers begin to think about different problem types (Carpenter et al., 2015; Smith & Stein, 2011) and identify children's different solution strategies (Carpenter et al., 2015; Empson & Levi, 2011). Each teacher received several different texts. The texts given to the teachers included:

- *Children's Mathematics, Second Edition: Cognitively Guided Instruction* (Carpenter et al., 2015),
- *Five Practices for Orchestrating a Powerful Math Discussion* (Smith & Stein, 2011, 2018),
- *Mine the Gap for Mathematical Understanding Grades K-2: Common Holes and Misconceptions and What To Do About Them* (San Giovanni, 2016),
- *Mine the Gap for Mathematical Understanding Grades 3-5: Common Holes and Misconceptions and What To Do About Them* (San Giovanni, 2016) and
- *Extending Children's Mathematics: Fractions & Decimals: Innovations in Cognitively Guided Instruction* (Empson & Levi, 2011).

During this session, co-teaching partnerships learned about the Launch-Explore-Summary lesson format (Lappan et al., 2007; Schroyer & Fitzgerald, 1986). We discussed the different problem types for all four operations (Carpenter et al., 2015). We discussed the concept of different solution strategies children progress through as they become more sophisticated in their mathematical thinking (Carpenter et al., 2015; Smith & Stein, 2011). This session provided an overview of these concepts, and the next three sessions went into specific detail on lesson structure, problem types, and student solution strategies.

The session began with the teachers working like students and mathematicians. I modeled the Launch-Explore-Summary (Lappan et al., 2007; Schroyer & Fitzgerald, 1986) lesson format by giving the teachers a problem that had a variety of entry points and different solution pathways. I told the teachers a story about our then, new superintendent. The story was about the superintendent wanting to start a new "green" transportation department that used solar-powered golf carts and solar-powered bicycles as a method for transporting students. I read the problem aloud and provided the problem on paper.

> There were 84 wheels but only 25 seats in the new 'green garage.' How many solar-powered golf carts and bicycles were there in the 'green garage?' Use pictures, numbers, and words to explain your thinking. Try to solve the problem at least two different ways.

I told the teachers that on the table, there was large chart paper, small white paper, lined paper, markers, red and blue rainbow tiles, and red and white flip chips for them to use to track their thinking. I then asked the teachers if they had any clarifying questions about the problem. Right away, the teachers wanted to know if the story was true and how I found out about this new "green" program. I responded with, "I see that you are highly engaged in the context of this problem, any other questions?" One teacher asked if the bicycles had two wheels, and the golf carts had four wheels. I confirmed that was the case. Another teacher asked if there was more than one correct solution. I responded with, “That is a good question,” and left my answer at that. Another teacher asked if manipulative use was required. I responded with, “Use what makes sense to you.” I made a chart of these questions and made sure to come back around to why these questions and responses have important implications during a launch-explore-summary (Lappan et al., 2007; Schroyer & Fitzgerald, 1986) lesson format.

As teachers were working together to find a solution to the problem, I took monitoring notes and collected data on a variety of solutions. After teachers had time to work, I pulled the group together and modeled a summary discussion. I asked four different teachers to demonstrate and discuss their solution strategy. I began with a teacher who used manipulatives and slid the rainbow tiles around to find the solution. I then selected a teacher who drew a picture and used a guess-and-check strategy to find the solution. I then selected a teacher who made a table to find the solution. Finally, I selected a teacher who used a substitution algebraic equation to solve the problem. I used discourse moves during the summary to require teachers to make sense of each other’s work. I concluded the summary discussion by asking the teachers to reflect on their learning by writing on a notecard. I asked the teachers if they were given a similar problem the following day what solution strategy they would approximate. I then shifted the discussion and asked teachers to identify the different lesson components they just experienced and how the summary discussion helped build on their mathematical thinking.

I highlighted ideas I wanted teachers to think about in terms of knowledge needed for mathematics teaching. After our discussion on the Launch-Explore-Summary (Lappan et al., 2007; Schroyer & Fitzgerald, 1986) lesson structure, we watched a short video in which several different students solved the same problem the teachers had just solved in a variety of ways. We then discussed the complexity of the solutions we watched and discussed if we were to rank order them in terms of complexity, which solution would come first, second, and so on. Teachers provided a rationale behind their thinking. The teacher thinking and rationale became my formative assessment for upcoming work and future sessions. I ended this session by talking about the different problem types for the four operations and different solution strategies students' use. Reading chapters two, three, and four from the Carpenter et al. (2015) text was homework for the next session. These chapters built background knowledge of problem types and solution strategies. The use of the video and discussing different problem types set the stage for the work in the next PD session. Teachers were asked to bring student work samples to the next PD session.

Session Four: Student Work That Guides Instruction

During the fourth PD session, we began by talking about the reading in the Carpenter et al., (2015) text. We discussed what the teachers learned about problem types and solution strategies. I began by passing out addition and subtraction problems on index cards. Teachers used the addition and subtraction problem type chart (Carpenter et al., 2015, p. 14) to figure out and label the problems by their type. We then discussed if the problem was too hard for the student, and the student did not have access to the work, what were possible strategies to provide access to the work? We then moved on to examining common student work as a way to anchor our experience together. We examined two different problems solved four different ways. The first problem (Figure 1) was in context, and the second problem used naked numbers (Figure 2). We discussed if the teachers could identify the problem type and then identify the solution strategy that demonstrated direct modeling, counting-on, or strategies that used some sort of relational thinking (Figure 1). I asked the question if you only evaluated student work for correct or incorrect answers what would you assume from this sample (Figure 1) of students. If you analyzed this work (Figure 1) looking for conceptual understanding, what would you then assume about student learning?

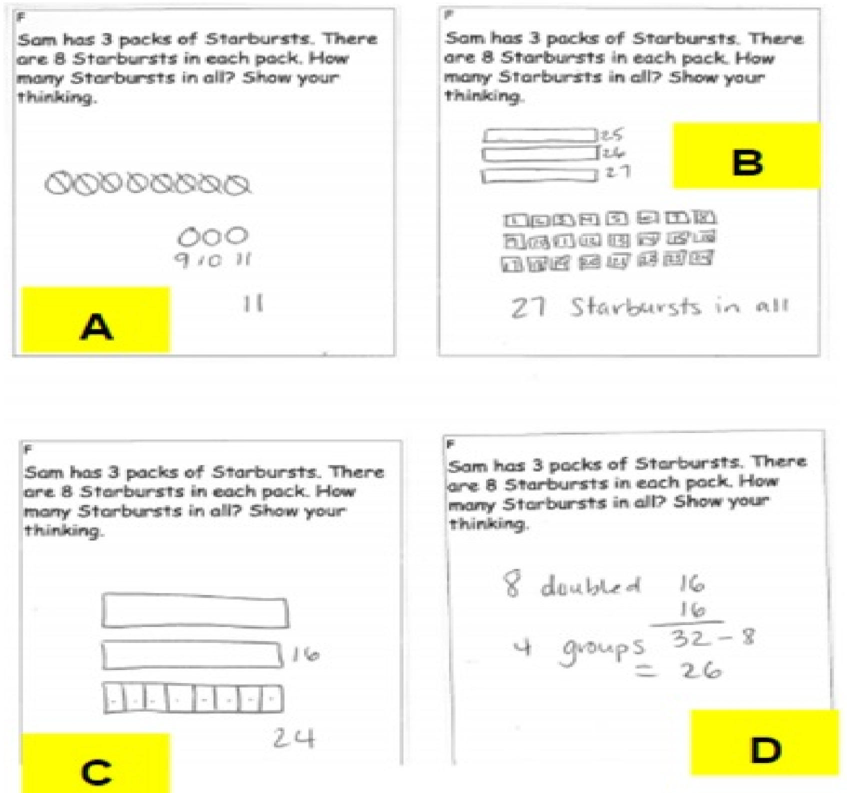

Figure 1. Starburst story problem solved four ways.

Student A clearly did not understand the work and just grabbed the numbers and added. The student got the wrong answer but also did not show conceptual understanding. Student B demonstrated more understanding about the problem but had some lapses in thinking when continuing to label the Starburst packs with the numbers 25, 26, and 27. This student showed a conceptual understanding of multiplication but lost track of thinking. Student C used a strategy of a double and then partitioned the last pack into eight sections and counted on from 16 to arrive at 24. This student demonstrated conceptual understanding, used a double strategy, and a counting-on strategy to solve. Student D used a more sophisticated strategy, knowing that double the double was four groups of eight, but that was too many groups of eight. The student then subtracted a group of eight. This solution is more complex and clearly demonstrated an understanding of multiplication. Student D made an error in computation and arrived at an incorrect answer. If a teacher only evaluated the work for correct and incorrect responses, three out of four answers were wrong. Using incorrect answers as student feedback, the teacher could inappropriately provide remedial lessons on multiplication as a result. If the teacher evaluated student work looking for the student to demonstrate a conceptual understanding of multiplication, three out of four solutions demonstrated an understanding of multiplication. Using this information, the teacher could assume most students were ready for more advanced concepts of multiplication. We then looked at a subtraction problem and discussed the four different solution strategies presented in the handout (Figure 2).

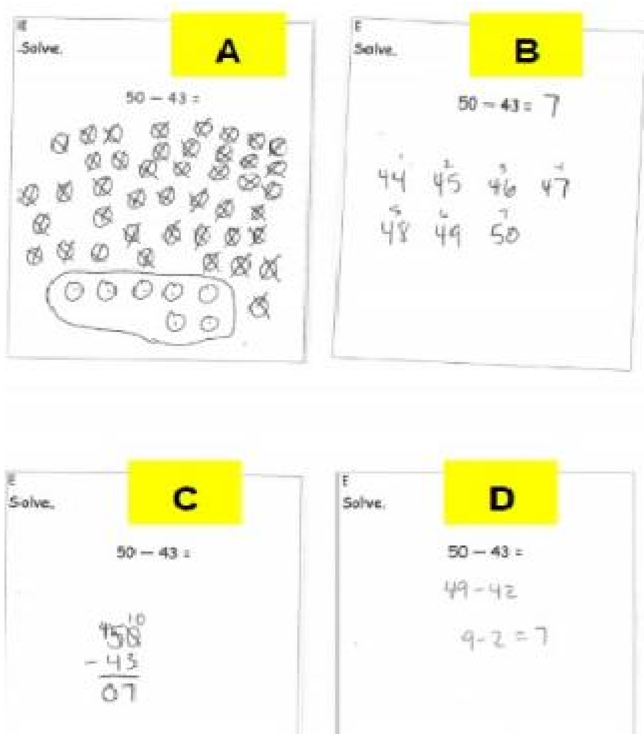

Figure 2. Solving for 50 – 43.

Student A used a direct modeling strategy (Carpenter et at., 2015). It is evident in the drawing that Student A counted and drew 50 circles. Then student A crossed off 43 of the circles, then counted what remained. This student counted three times to solve this subtraction problem. Student B used a counting-on strategy (Carpenter et al., 2015). This student also used some relational thinking, knowing that it is possible to count-on and add to complete subtraction. Student C used the traditional algorithm, and this solution does not demonstrate a conceptual understanding of subtraction. Student C could have a conceptual understanding when it comes to subtraction but based on the work sample alone, the teacher would not be able to make that determination. Student D used a compensation strategy, knowing that the difference between two numbers is the same when you adjust both numbers by the same quantity. Student D then noticed the tens was the same quantity, so only looked at the ones and solved the problem with a difference as seven. When the teachers and I looked at these examples, I asked the teachers which solutions demonstrated a conceptual understanding of subtraction. I also asked teachers which solutions showed a low level of conceptual understanding for subtraction and which solutions showed a higher level of understanding. After we discussed these student work samples, I then asked teachers to examine the work samples that they brought.

The student work samples the teachers brought varied in type and content area because of various levels of learning. Classroom teachers brought work that contained grade-level content. Math interventionists brought work samples from students who had understandings of these concepts that were at least a grade below the classroom co-teaching partner. The special education teachers brought work samples from 2-3 years below grade-level standard or the general education co-teaching partner. The teachers engaged with the student work to identify the problem type (if a problem type was evident). If teachers brought worksheets or other samples that did not contain story problems, we analyzed student work for solution strategy rather than problem type. I also provided work samples for them with various problem types and solution strategies if the teacher did not want to engage with their own classroom work. Understanding the varying complexity of the 11 different addition and subtraction problem types (Carpenter et al., 2015) or examining different solution strategies (Carpenter et al., 2015) was an entry-level place for co-teaching partners to begin to develop and advance their math pedagogical content knowledge (Hill & Ball, 2008; Hill et al., 2004; Hill, 2010b). While the teachers examined the student work, the co-teaching partners identified and labeled, varying levels of solution strategies. The teachers ordered identified solution strategies in terms or most concrete representation to the most abstract representation. The identifying and ordering of solution strategies allowed the co-teaching partners to begin to notice how students thought about and solved various problems. Teachers then used the provided problem type and solution strategy matrix to look for small groups or individual trends in student data (Appendix K). Utilizing this knowledge, teachers began to think about how to prompt students to attempt the next level in solution sophistication. Reading Chapters one and two in the Smith and Stein (2011) text was homework for the next session. Teacher learning during this PD set the foundation for the next three sessions.

Session Five: Organizing Student Data

During the fifth PD session, we revisited the co-teaching survey and discussed emerging ideas about co-teaching between the partners. We began the session by answering sentence starters of, "In regards to co-teaching, I have learned ________________. I am wondering about _____________. My new thinking around math instruction is ______________." After answering the sentence starters, I gathered these data, quickly read the results, and then posed questions to partners around ideas of co-teaching and math instruction. I then presented a graphic organizer as a support tool to help teachers think about individual student interests and strengths, conceptual and procedural fluency needs, and ways co-teaching partners can support each other with math instruction. I designed a graphic organizer through my work with other co-teaching partnerships in my district. I found, as an instructional coach, that having a third point as a focus for a discussion (Aguilar, 2013; Loucks-Horsley et al., 2010; Sweeney, 2011). In the field of instructional coaching, the third point is a focus on either data or an instructional tool rather than a focus on the teacher or coach (Aguilar, 2013; Loucks-Horsley et al., 2010; Sweeney, 2011). The third point of the graphic organizer made identifying and describing student instructional need easier for the special education teacher, the general education teachers, and/or the math interventionist to do together. With student strengths, interests, and instructional needs identified, designing differentiated math tasks was the next focus for the co-teaching partners to do together. Understanding the progression of standards is a prerequisite skill for planning differentiated math tasks.

Use of the student support tool (Appendix M) provided entry into examining the learning progressions of the Common Core State Standards for Mathematics (CCSSM) (2010). Some students in special education with large gaps in understanding may not be ready to meet grade level in CCSSM. However, as math interventionists, special education teachers, and general education teachers examine the CCSS learning progressions (insert citation here) together through the lens of student need, then it is possible to identify a student's entry into grade-level mathematics content. When the co-teachers know student strengths and needs then identifying the student's instructional entry point with the Common Core learning progressions becomes the next logical step in collaborative planning.

Session Six: Implementing Mathematics Discussions

The co-teaching partners have already learned about and become familiar with different student solution strategies building from concrete to abstract by looking at different student work samples and through reading the Children's Mathematics text (Carpenter et al., 2015; Empson & Levi, 2011). This PD session, teachers learned how the *Five Practices* for Orchestrating a Productive Mathematics Discussion (Smith & Stein, 2011) advances student thinking. Smith and Stein (2011) identified *Five Practices* for a productive math discussion as anticipate, monitor, select, sequence, and connect. Teachers learned to anticipate how students might solve a problem prior to the launch of the problem. Teachers learned how to design and create tools for monitoring student solution strategies during the explore phase of the lesson. Teachers learned how to select student solution strategies presented during the summary portion of the lesson. Teachers learned how to sequence solution strategy presentations from concrete to abstract. Finally, teachers learned how to connect the various solution strategies. During this PD session, teachers learned how to engage with student need, CGI (Carpenter et al., 2015), and the *Five Practices* (Smith & Stein, 2011) to bring about a purposeful shift in math instruction. This shift allowed all students to have access to grade-level mathematics content while also allowed students to address gaps in understanding.

To facilitate teacher learning with the *Five Practices* (Smith & Stein, 2011), I presented the teachers with a simple multiplication story problem. I told the teachers that I love Starburst candies. I buy them by the pack, and eight Starburst come in one pack. My daughter gave me three packs for my birthday. How many Starbursts did I have altogether? I asked the teachers to think about this problem and wonder how students with mathematics difficulty would attempt to solve this task. I asked the teachers to think of at least four different ways students might attempt to solve the problem. I prompted with questions such as:

- How would a student solve using a direct modeling strategy (Carpenter et al., 2015)?
- How would a student solve using a counting-on strategy (Carpenter et al., 2015)?
- How would a student solve using a strategy with relational thinking (Carpenter et al., 2015)?
- How would a solution strategy look that used compensation?
- What would an incrementing solution strategy look like?
- What kinds of manipulatives do you think kids might use effectively?

Working as a co-teaching partnership, teachers created a tool or graphic organizer to collect student data on the various student solution strategies. After teachers designed a tool, I had them explain how they planned collaboratively to use the tool. I asked how this tool supported their thinking in moving students forward while also allowing students to address learning gaps. I then showed examples of graphic organizers I used in the past to collect student data. Next, I asked teachers to examine the different solution strategies they came up with and determine how they would select and sequence the various solution strategies. I then modeled how to facilitate a discussion in which the various solution strategies built from concrete to abstract. The teachers, as students, connected one solution strategy to the next. I concluded the session by asking the teachers to complete an exit ticket in which they were required to reflect on the *Five Practices* and set a goal as to which practices they will approximate in their class over the next two weeks. By reflecting on the learning and setting a goal to accomplish, the teachers were more accountable for attempting new mathematical instructional strategies.

Session Seven: The Teaching-Learning Cycle

In the seventh PD session, teachers completed a full teaching and learning cycle (Jones, 2008) utilizing the ideas in the prior six PD sessions. Teachers completed the teaching-learning cycle by first examining student data. Then the teachers used the student data to plan for instruction. The teachers used the CCSS (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) and the CCSS progression documents (Common Core Standards Writing Team, 2013) (available at www.achievethecore.org) and created a lesson trajectory. Finally, the teachers planned for and created a classroom-based assessment (Jones, 2008). Teachers examined and analyzed student work samples and monitoring notes on the various solution strategies students used (Carpenter et al., 2015; Smith & Stein, 2011). The co-teachers sorted students by solution strategy representation, (Carpenter et al., 2015), discussed student strengths, and needed next steps. The teachers designed a new rich math task or reframed a task from the district-created math curriculum and transformed the task into a rich math task (Carpenter et al., 1999, 2015; Boaler et al., 2018a, 2018b, 2018c; San Giovanni, 2016; Smith & Stein, 2011, 2018; Stein et al., 1996; Stein et al., 2007). Teachers used the student data and designed questions or probes to push on student thinking during the explore portion of the lesson (Lappan et al., 2007; Schroyer & Fitzgerald, 1986; Smith & Stein, 2011). The co-teachers engaged in this intentionally structured teaching-learning cycle (Jones, 2008). The teachers articulated what they learned and how this learning helped to shift instruction. Through knowing the significance of children’s mathematical thinking, teachers designed math assessments that highlighted student problem-solving strategies or highlighted potential student misconceptions (Carpenter et al., 2015). Once teachers designed a math assessment that highlighted student thinking, then an introduction to the concept of leveled assessments occurred.

Leveled assessments provide different entry points, so the level of support matches student need. Each version of the math assessment contained supports such as simplified language or easier numbers. When teachers are able to design and implement leveled assessments that maintain the rigor of grade-level core content, then students are able to register on an assessment and demonstrate what they know about mathematical concepts.

Session Eight: Assuring Sustainability of New Learning

I designed the final PD session to help the co-teachers build sustainability around co-teaching, collaboration, CGI, the *Five Practices*, and the teaching-learning cycle (Carpenter et al., 2015; Empson & Levi, 2009; Friend et al., 2010; Jones, 2008; Murawsk & Hughesi, 2009; Smith & Stein, 2011; Wilson & Blednick, 2011). Teachers used the assessment data to design small group interventions as well as begin to design math tasks within the next math unit. Co-teachers constructed an action plan on how they intended to maintain the work after the PD concluded. I encouraged teachers to include continued support from myself as an instructional coach in the district.

Collaboration Meetings

Part of the PD included separate collaboration meetings between the co-teaching partnership and me. These meetings provided a chance to address partnership concerns identified in the co-teaching survey as well as provided for an opportunity to supplement learning taking place during the PD instructional sessions. Collaboration meetings took place once a week, in addition to the PD instructional sessions. I created the agendas for the collaboration meetings, facilitated these meetings, and acted as a guide. I probed teacher thinking by asking questions, but I did not provide instruction during this time. If teachers specifically requested support or instruction, then I provided it. The collaboration meetings provided an opportunity for the math interventionist, the special education teacher, and the general education teacher to discuss what they learned in the PD. Teachers applied their recent learning to current student needs at these meetings.

Professional Learning Community Meetings

I audio recorded meetings in which either one or both teachers from the co-teaching partnerships were working with other teachers in planning, implementing, or analyzing instruction. These PLC meetings provided data on how learning from the PD transferred. Some of the grade-level PLCs included a special education teacher, but that was not always the case. Typically, special education teachers joined a general education grade-level PLC or had their own PLC if there were multiple special education teachers at that school. For example, a special education PLC at the elementary level may comprise a learning support special education teacher, a special education teacher who teaches in a more restrictive special education environment, the speech and language pathologist, and the school counselor. These are general trends, and there are no set rules in the district as to which teachers comprise a given PLC.

In this district, scheduled PLC meetings occurred once a week. The PLC data I collected was not uniform across the co-teaching teams due to scheduling. In my district, there was early release every Wednesday afternoon for PLC meetings. I had to take turns, rotating my attendance at PLC meetings with different co-teaching teams because most scheduled PLC meetings occurred Wednesday afternoons at a similar time.

During the PLC meetings, I took the stance of participant-observer (Spradley, 1980), and I only contributed to the PLC meeting when directly questioned. As a participant-observer, I closely watched the interactions among the teachers. I listened carefully to what the teachers discussed. I also listened carefully to what the teachers did not discuss. I took notes and attempted to make sense of how the teachers perceived collaboration and how the collaboration affected student mathematical understanding. The use of an observation tool (Appendix J) helped guide me to focus on what was pertinent and relevant to my study. The observation tool allowed me to focus on: (a) evidence of the teaching-learning cycle, (b) evidence of using CGI principles (Carpenter et al., 2015), (c) evidence of the *Five Practices* (Smith & Stein, 2011) and, (d) evidence of intentional planning for students with extensive mathematics difficulty (Lewis, 2014).

The PD sessions, the co-teaching survey, any student work samples, any artifacts the teachers brought to the PD, PLC meetings, collaboration meetings, individual teacher interviews, co-teaching partnership interviews, observations from the PLC meetings, and observations from the collaboration meetings all served as data sources for this action research study.

Summary of PD

This PD was a two-prong approach. Teachers received instruction and had time to work with their co-teaching partner(s) and other co-teachers during the eight PD instructional sessions. The second part of the PD experience involved eight to 12 collaboration meetings. During the collaboration meetings, co-teaching partners had an opportunity to refine their work as they collaborated to design co-taught mathematics lessons.

The PD experience allowed math interventionists, special education teachers, and general education teachers to learn about co-teaching models as well as roles and responsibilities for co-teaching partnerships (Friend et al., 2010). The focus of the PD instructional sessions shifted from learning about co-teaching to constructing new knowledge about CGI (Carpenter et al., 2015). After learning about CGI, teachers learned about the Launch-Explore-Summary lesson structure (Lappan et al., 2007; Schroyer & Fitzgerald, 1986). Teachers learned how to design a high cognitively demanding task or how to reframe a task in the district-created math curriculum (Carpenter et al., 1999, 2015; Boaler et al., 2018a, 2018b, 2018c; San Giovanni, 2016; Smith & Stein, 2011, 2018, Stein et al., 1996; Stein et al., 2007). Teachers also learned how to implement the *Five Practices* (Smith & Stein, 2011) while using a Launch-Explore-Summary lesson structure (Lappan et al., 2007; Schroyer & Fitzgerald, 1986). Finally, teachers learned how to apply all the newly acquired knowledge through a comprehensive teaching and learning cycle (Jones, 2008). The purpose of the PD was to study how PD affects collaboration and co-teaching instructional practice for math interventionists, special education teachers, and general education teachers.

CHAPTER FIVE: FINDINGS

I explored how general education teachers worked with special education teachers and/or math interventionists to provide access to core content for students with years of unfinished learning. Often special education teachers or math interventionists work with the general education teacher to help students complete the unfinished learning from previous grade levels. The purpose of this study was to provide a professional development intervention for the teachers in the study who worked in the capacity as either a general education teacher, a special education teacher, or a math interventionist. The professional development intervention gave the teachers an opportunity to collaborate to generate a plan for students who had unfinished learning to complete.

Organization of this chapter begins by describing my role as a researcher. I then provide the contexts of the three study locations of Hillview, Oakwood, and Three Peaks Elementary schools. Under the heading of each school site, I provide the context for each school's intervention model and information about each participating teacher. The remaining sections focus on relevant themes, evidence with data, to illustrate the experience of the teachers at different locations.

Role of the Researcher

As a district-level math intervention coach, I provide instruction and coaching (Aguilar, 2013; Loucks-Horsley et al., 2010; Sweeney, 2011) for math interventionists to plan and implement math intervention (Fuchs et al., 2010; Lembke et al., 2012). I help general education teachers plan for and implement intervention within core instruction and outside of core instruction. I help special education teachers connect IEP goals to core classroom content. The more my work with general education teachers evolved, the more I realized general education teachers needed theory and application of researched-based math instructional practices (Boaler et al., 2018a; Carpenter et al., 2015; Empson & Levi, 2011; San Giovanni; 2016; Smith & Stein, 2011, 2018). General education teachers also needed tools and support to align the district-created math curriculum with the known researched-based math instructional practices (Boaler et al., 2018a, 2018c; Carpenter et al., 2015; Empson & Levi, 2011; San Giovanni; 2016; Smith & Stein, 2011, 2018).

As a district coach supporting a workshop model of Launch-Explore-Summary (L-E-S) lesson structure, (Lappan et al., 2007; Schroyer & Fitzgerald, 1986) I facilitated and participated in coaching cycles (Aguilar, 2013; Loucks-Horsley et al., 2010; Sweeney, 2011) with general education teachers and/or math interventionists. Coaching cycles (Aguilar, 2013; Loucks-Horsley et al., 2010; Sweeney, 2011) are voluntary for teachers and consist of 6-8 week cycles that include lesson planning sessions, lesson observation, and then reflective coaching conversations.

During the study, lines blurred between researcher and coach, which then necessitates a description of researcher-as-instrument (Jaworski, 1998; Lewis, 2014). I conducted collaborative lesson planning meetings with each team. I also observed lessons that teams reframed from the district-created math curriculum to align with research-based instructional practices (Boaler et al., 2018a; Carpenter et al., 2015; Empson & Levi, 2011; San Giovanni; 2016; Smith & Stein, 2011, 2018). During lesson observations, sometimes teachers elicited my help. When teachers requested assistance with content knowledge or with enacting any part of the L-E-S lesson structure, (Lappan et al., 2007; Schroyer & Fitzgerald, 1986) I shifted from researcher to coach and supported teacher learning. At times, teachers mentioned tools or coaching support that I provided when my job of a coach also blended with my role as a researcher.

Context of School Sites and Teachers

All three elementary school sites were located in the same large school district in southwest Washington. Two of the schools qualified as school-wide Title 1 schools. The third location did not meet the criteria for school-wide Title 1 and therefore was a Learning Assistance Program school. There were at least two teachers at each location. Below is a summary of the school contexts (Table 2) and a summary of the teachers who participated in this study (Table 3).

Table 2

School Context and Data Across Three Years (2014-2017)

School	Number of GET[1]	Number of SET[2]	Number of MI[3]	Title 1 MR[4]	School Demographics	SBA Scores	SBA Analysis
Hillview	1	1	1	Yes MR – 32%	57.8 % White 21 % Hispanic 3% African American 56 % FRL[5]	3rd – 61.8% 4th – 61.2% 5th – 61.6%	Average of.2 % fewer students passed in 5th grade than in 3rd
Oakview	2	0	2	Yes MR- 56%	46% White 39% Hispanic 9% African American 69% FRL	3rd – 46.0% 4th – 44.6% 5th – 22.4%	Average of 23.6% fewer students passed in 5th grade than in 3rd
Three Peaks	1	1	0	No MR – 3%	57% White 19% Hispanic 4% African American 40% FRL	3rd- 68.0% 4th- 55.4% 5th- 55%	Average of 13% fewer students passed in 5th grade than in 3rd

At Hillview, there were three teachers. The teachers at Hillview consisted of a general education teacher, a special education teacher, and a math interventionist. At Oakview, there were four teachers. The teachers at Oakview consisted of two general education teachers and two

[1] GET abbreviation for general education teacher
[2] SET abbreviation for special education teacher
[3] MI abbreviation for math interventionist
[4] Mobility Rate
[5] Free and Reduced Lunch percentage

math interventionists. At Three Peaks, there were two teachers, a general education teacher and a special education teacher.

Table 3

Participating Teachers

School/Teacher	Role	Number of Students	Education	Years Teaching	Pedagogical Stance/Beliefs
Hillview /Maggie	GET[6]	25	Masters + Reading Endorsement	10	Some students need big ideas while others need content broken down into small pieces
Hillview /Holly	MI[7]	37	Masters + Math Endorsement	30	Students need to think of math in terms of big connected ideas
Hillview /Chris	SET[8]	35	BA in Business MIT[9]	1	Students need content broken down into small pieces
Oakview /Cheryl	GET	44	Masters in Special Ed.	18	Students need content broken down into small pieces
Oakview/Joan	GET	41	Masters	22	Students need content broken down into small pieces
Oakview/Allison	MI	40	Masters +	25	Students need to think of math in terms of big connected ideas
Oakview/Paige	MI	42	Masters + Math Endorsement	22	Students need to think of math in terms of big connected ideas
Three Peaks/Sally	GET	18	Masters	15	Students need to think of math in terms of big connected ideas
Three Peaks/Megan	SET	35	Masters in Special Ed.	10	Students need content broken down into small pieces

The state average for 3rd-grade students passing the math Smarter Balance Assessment was 57%. The state average for fourth-graders passing the math Smarter Balance Assessment was 54%, and the average for 5th-grade students was 48.6%. The district average for passing the

[6] General Education Teacher
[7] Math Interventionist
[8] Special Education Teacher
[9] Masters in Teaching

3rd-grade Smarter Balance Assessment was 53.4%, the fourth-grade average for passing was 54.2%, and the 5th-grade average was 45.9%.

At Hillview, approximately 60% of students passed the math portion each year, and the number of students passing the assessment stayed consistent across the grades. At Oakview and Three Peaks, as students advanced grades, fewer students met benchmark each year. Implied in this data is that, as there are more standards to master in the upper grades, and as content ideas build on each other, students at Oakview and Three Peaks struggled to make connections to ideas within and between grade-level content. In addition, it is important to note the difference in the mobility rate between Oakview and Three Peaks. Oakview had a much higher mobility rate than Three Peaks. Even though Three Peaks had a decline of 13% fewer students passing in fifth grade than in third grade, the mobility rate at Three Peaks is only 3%.

Hillview Elementary School

Hillview is located within the city limits of a large Pacific Northwest urban area. The free and reduced lunch rate at Hillview is 56.4 %, and 57.8 % of the school population is White. Twenty-one percent of the school population is Hispanic, while the African American population at Hillview is three percent. The remainder of the school population identifies as a mix of two or more races.

The state and national test to measure growth in Washington is the Smarter Balance Assessment (SBA) (www.smarterbalanced.org). This assessment is digital and adaptive for grades three through eight. The math assessment is broken into two components. The first section is a multiple-choice test portion that covers content standards. The second section is a performance task assessment designed to assess the mathematical practice standards. The performance task items build on each other and require students to explain and demonstrate mathematical practice standards. Smarter Balance Assessment data for math in the past three years at Hillview has hovered right around 60% meeting expectation for 3rd grade. In 4th grade, the SBA for three years prior to the 2016-2017 school year was close to 60% of students meeting expectation or proficiency level three. In 5th grade, the three-year data prior to 2016 showed almost the same percentage each year meeting expectation for proficiency. The 5th grade stayed consistent with 61% of students passing the math Smarter Balance Assessment.

Hillview had the same principal and instructional coach for ten years prior to the study. The school leadership at Hillview was forward-thinking in terms of identifying the need for and providing intervention. In the four years prior to the study, the principal required teachers to participate in six to eight-week data cycles in grade-level PLCs. Each data cycle day consisted of teachers working with the building leadership and instructional coach to evaluate student data and design instruction. During this time grade-level teams of teachers evaluated recent data for reading and math, made decisions about which students needed intervention, decided on the intervention lesson content, and then planned the six- to eight-week cycle of intervention lessons including assessment items for progress monitoring. Different teams structured their intervention time differently as some teams had a “walk to read” or “walk to math” type of structure. In a “walk to math” type of intervention structure, students walked to another teacher’s classroom to receive instruction on a focused idea. Sorted students went with a different grade-level teacher taking a specific group. Each group comprised a mixture of students from each grade-level class. Other grade-level teams prioritized intervention needs for only their own students and the students with the highest need received intervention within the classroom.

Hillview’s intervention model. Historically Hillview had a pull-out model (Marston, 1996) for reading intervention. The Hillview teachers used the pull-out model (Marston, 1996) to address gaps in learning. Marston (1996) described the pull-out model as students removed from the general education classroom setting to receive special education or math intervention instruction in a separate location. Over several years prior to the 2016-2017 school year, Hillview teachers noticed that the pull-out model (Marston, 2996) for reading intervention was not successful with helping students to complete unfinished learning from prior grades. Typically, students who received reading intervention through a pull-out model (Marston, 1996) were no better off after a year than students who remained in core instruction and qualified for intervention. These two groups of students made about the same amount of growth each year. This data indicated that the pull-out model (Marston, 1996) for reading intervention did not have an effective impact on students' reading growth. From the context of the reading intervention room to the core classroom, skills students learned did not generalize. Data over time revealed a need for a change in the structure of the reading intervention model. With the introduction of math intervention by Hillview's administration, there was an immediate high interest from classroom teachers to begin the structure for math intervention as an inclusive, push-in, collaborative model between the math interventionist and the classroom teacher. With a collaborative, push-in model, the special education teacher and/or the math interventionist goes to the general education classroom and provides support for the students with gaps in understanding (Friend et al., 2010). For the 2016-2017 school year, Hillview added Holly, a half-time math interventionist to the staff.

Participating teachers at Hillview. The teachers participating at Hillview consisted of the math interventionist, the special education learning support teacher, and a 5th-grade general education classroom teacher.

Hillview's Math Interventionist: Holly. Hillview's math interventionist, Holly, joined the staff at Hillview in the 2016-2017 school year. Prior to working at Hillview, she taught elementary, middle school, and high school mathematics courses. Holly also worked as a consultant for a large, not-for-profit math consulting company that provided math professional development to teachers around ideas of CGI (Carpenter et al., 1999, 2015). Holly worked with students and teachers for the past 30 years and believed that inclusion was a powerful approach to co-teaching and collaboration as a way to allow students to attend to gaps in missed learning from previous grades. This was clearly her belief about working with students who needed math intervention or were in special education, even though in her initial interview she indicated that she had no extended or formal training in working with special education students or students needing math intervention. Holly explained, "I never received any specialized training working with students who struggled with math concepts. I do think that one of the best ways to help struggling mathematicians is to work with the classroom teacher and the special education teacher."

Holly expressed the belief that math instruction should be focused on using rich math tasks in which students have a context that is engaging and authentic for the student, and the student can generalize a big math concept. Holly said this about students learning math, "Here's what I think about students learning math. They do not learn procedures in isolation with any meaning. The learning that sticks with the student is when the student develops their own understanding about numbers and number relationships." As a new staff member to Hillview, Holly was excited to join the research study to get to know other staff members and to have an impact on the staff with core math instruction as well as implementing math intervention within the classroom setting.

Hillview's Special Education Learning Support Teacher: Chris. Chris was a first-year special education teacher at Hillview during the 2016-2017 school year. Chris completed his Masters in Teaching with a special education endorsement from a local university the year prior to working at Hillview. Working as a special education learning support teacher was a second career for Chris. In his initial interview, he described math instruction in terms of breaking ideas down for students and wanting students to have fluency and accuracy with addition, subtraction, multiplication, and division facts. "I think when students can learn skills step-by-step, they will feel more successful and be more successful in the class because they know the steps to do." Chris also spoke about students' Individual Education Plan (IEP) goals as measuring fluency and accuracy with facts across the operations using timed tests. The structure for learning support at Hillview was mostly an inclusive model with some pull-out (Marston, 1996) for students with significant mathematics difficulty from more than four previous grade levels. In his learning support classroom, Chris posted several anchor charts hanging on the walls outlining steps for the addition and subtraction algorithm.

During his initial interview, Chris expressed excitement to join the research study to work with other classroom teachers, including the math interventionist, and to learn other strategies for math instruction other than teaching a systematic process for learning algorithms or procedures. At the beginning of the study, Chris questioned if the use of rich math tasks could make a difference for students he worked within special education as evidenced when he said, "I'm not really sure…I mean, I think teaching steps is more helpful than trying to teach a large concept all at once. My students need it broken down for them."

Hillview's Fifth-Grade Teacher: Maggie. Maggie taught ten years in a small farming community in a neighboring state prior to coming to the district in the 2016-2017 school year.

The change in teaching location and context was a large shift for her, coming from a school with one classroom per grade level to teaching at a school with five classes at 5th grade. The community Maggie came from was overwhelmingly White, and the free and reduced lunch percentage was under 15%. Maggie came from a district that supported direct math instruction. Students learned specific solution strategies and expected to memorize and reproduce the solution strategies presented during math instruction. With a different population of students, Maggie soon realized that the way of teaching math from her past was no longer working for her students at the new location. Maggie joined the research study to learn about rich math tasks and to learn how to notice and name student math behaviors related to CGI (Carpenter et al., 1999, 2015).

During Maggie's initial interview, she often spoke about students not retaining certain strategies taught in class or students automatically applying strategies to problems without regard to making sense of the problem. "Look at this work here. This student clearly was not thinking when she multiplied instead of subtracted. There is no making sense of the problem. It's just do the procedure I learned today in class."

Maggie talked about manipulatives as a tool for students to use who struggled with math content and not as something that might help support all students. "I never realized how powerful math manipulatives could be. I thought they were just for struggling students but actually, the math manipulatives help all students make sense of the problem, not just the struggling ones." Maggie also openly talked about her master's degree in literacy, with a reading endorsement, and her fear of math in not knowing how to help her students who had the most mathematics difficulty. Maggie was enthusiastic about joining the research study to learn about new math manipulatives, to learn about new instructional strategies, but most of all, to help her students.

Maggie’s 5th-grade class comprised 25 students. Eight of these students had IEPs for a math learning disability. Several of these special education students also received English Language Learning support. In addition to the eight students in special education, another eight students had not made a benchmark in 4th grade on the Smarter Balanced Assessment (SBA). This second group had not been receiving any extra math support prior to the 2016-2017 school year. The remaining nine students in Maggie’s class were doing well with math content and performance on the SBA. Maggie had four students placed in the gifted and talented program, located at Hillview. Four other target students had the potential to qualify for the gifted and talented program. To recap the composition of Maggie’s class, 16 students worked below grade level, and another nine students worked significantly above grade level. Both Maggie, Chris, and Holly were excited to learn side by side to help all the student’s in Maggie’s class be successful with fifth-grade math content.

Intervention team at Hillview. The composition of this team was unique because I served as a district instructional coach. I was the researcher but also a participant (Jaworski, 1998; Lewis, 2014). The math interventionist was a highly respected district teacher-leader new to elementary math intervention. The special education teacher was a first-year teacher. The general education teacher had ten years of experience but in a very different setting. This team of four met together each week to discuss classroom data, plan a new week of core math lessons, plan a week of intervention lessons as well as decide on progress-monitoring tools. Each member of the team was highly motivated to learn and was eager to learn about the other members of the team and valued collaboration and learning from each other. Each member of the team acknowledged the knowledge base and expertise of the other team members. Chris and Holly both adjusted their schedules with the support of the school principal to allow both Chris and

Holly to be available during Maggie's core math instructional block. With all four adults on hand during core math instruction, each adult helped facilitate the launch, explore, and summarize a portion of math workshop.

Hillview collaboration meetings. The teachers met for 90 minutes each week. Prior to the meeting, the teachers read either journal articles or sections from provided texts. The texts given to the teachers included:

- *Children's Mathematics, Second Edition: Cognitively Guided Instruction* (Carpenter et al., 2015),
- *Five Practices for Orchestrating a Powerful Math Discussion* (Smith & Stein, 2011),
- *Mine the Gap for Mathematical Understanding Grades K-2: Common Holes and Misconceptions and What To Do About Them* (San Giovanni, 2016),
- *Mine the Gap for Mathematical Understanding Grades 3-5: Common Holes and Misconceptions and What To Do About Them* (San Giovanni, 2016) and
- *Extending Children's Mathematics: Fractions & Decimals: Innovations in Cognitively Guided Instruction* (Empson & Levi, 2011).

The reading completed outside of the collaboration meetings helped build background knowledge or helped guide instructional decisions for the collaboration team meetings. We focused on relationship building and talking about our personal lives or struggles within our unique positions. The next 20 minutes of the collaboration meeting focused on examining student data and identifying which students needed additional learning opportunities. After identifying where students stood with the most recent big math idea, we planned the next set of core math lessons with all students in mind. As a team, we designed or reframed given tasks in the district-created math curriculum. After we designed four to five tasks, we anticipated how students could enter the task and how each subgroup of students could move their thinking forward or generalize a big math concept. We created graphic organizers (Table 4) to help us manage what we wanted to see from each subgroup of students.

Table 4

Graphic Organizer Used in Planning

Sub Group with # of students	Monday Task 1	Tuesday Task 2	Thursday Task 3	Friday Task 4
	Task:	Task:	Task:	Task:
Math IEP Students Group 1 (4)				
Mary, Jon, Lois, Jose (4)				
Math IEP Students Group 2				
Math Intervention Students (8)				
Target Students				
Excel Students (4)				

Whenever possible, we decided to highlight the thinking of someone who had a fixed mindset as a mathematician (Boaler, 2016). We assigned competency to some students by highlighting their thinking as being highly efficient or exceptionally organized during the summary portion of the lesson (Boaler, 2016; Boaler et al., 2018a, 2018b, 2018c). As a team, we attempted to shift the thinking of the class from “these are the good math students” to “we are all capable mathematicians.” Middleton and Spanias (1999) found that student motivation and views as being a capable mathematician is important and linked to mathematics achievement.

We also designed small group lessons to occur outside of the core instructional time while Chris and Holly were still in the classroom. We designed simple progress-monitoring collection tools (Table 5) to be able to measure growth and progress for each student with mathematics difficulty moving towards the generalization of a big math concept. Table 6 highlights progress monitoring for four students in special education. As a collaborative team, we filled in the progress-monitoring tool for all students with mathematics difficulty (Table 5).

Using student data, the team focused on students having access to core content while also addressing gaps in learning from previous grade levels. These collaboration meetings generated multi-tiered systems of support while keeping all students in the general education setting. At this location, these collaboration meetings occurred for 12 weeks.

Table 5

Progress-Monitoring Collection Tool – Cutting 12 Brownies Task

Student	Entered the task	Developed strategy to solve	Learning Progression Step A	Learning Progression Step B	Learning Progression Step C
	How did the student begin?	How much support was given?	Identified ways to cut brownies as factors for 12?	Identified relationship between brownie size and number of brownies?	Identified connection between 12, 24, and 48 brownies?
Mary	Drew a picture first then moved to using paper and tiles	With minimal support	Yes	Yes	Yes
Jon	Used tiles to make a model	Without support	Yes	Yes	Yes
Luis	Used tiles to find only some factors (2 & 6, and 3 &4)	With continued support	Yes- with support. Did not see connection with 1 x 12 or 12 x 1	Yes	Yes
Jose	Made a T chart and listed factors beginning with 1	Without support	Yes	Yes	Yes

Oakwood Elementary School

Oakwood is in the city limits of a large Pacific Northwest urban area. Oakwood is one of the oldest schools in the district, built in the 1950s. The percentage of students who received free or reduced lunch at Oakwood is 69%. Hispanic students comprised 39% of the students at Oakview, 46% of the students are identified as White, and 9% are identified as being two or more races. During the 2016-2017 school year, 46% of third-graders met standard on the math portion of the Smarter Balance Assessment. Of fourth-graders, 44% met standard while only 22% of fifth-grade students met standard during the 2016-2017 school year. Prior to the 2016-2017 school year, the past three-year trend in math assessment data at Oakwood was that as students' progressed through the grades, the percentage of students meeting standard decreased (Table 1).

Prior to the 2016-2017 school year, Oakwood had three different principals and four different instructional coaches within the last five years. With the frequent change in leadership, teachers had an inconsistent focus on professional development with all content areas. While Principal A focused on teachers learning instructional strategies for English Language Learners, Principal B focused on the use of personalized learning and the use of technology in the classroom. The most current principal, Principal C, wanted teachers to learn about using the workshop as an instructional delivery model. The teachers jumped from one instructional priority to the next without much coherence or connection between the different instructional priorities.

In the 2012-2013 school year, the staff used EngageNY (New York State Education Department, 2012) as a math curriculum. EngageNY (New York Education Department, 2012) was created and is currently maintained by the New York State Education Department to support the implementation of key aspects of the New York State Board of Regents reform agenda. The use of EngageNY (New York State Education Department, 2012) at Oakwood made this school an outlier within the district, as this was not a district adopted or even supported math curriculum. The teachers used EngageNY (New York State Education Department, 2012) from 2012-2017 as the primary math curriculum or resource.

All three principals required PLCs to use release time to evaluate student assessment data. Inconsistent implementation of data days occurred at Oakwood. Principal A used a large pocket chart with color-coded cards to indicate how every student performed on reading and math district assessments. Each student had two cards indicating meeting benchmark with a green card, close to meeting benchmark with a yellow card, and significantly below benchmark with a red card. Principal B expected grade-level teams generate a weekly report about the work teachers planned to do with students who were close to but not meeting the benchmark. Principal C expected teachers create lessons plans demonstrating leveled lessons for different instructional groups. The process of evaluating data and the use of teacher release time for data analysis varied greatly over the five years prior to 2016-2017. Teachers became accustomed to trying to prove teaching occurred rather than evaluate student learning.

Even with inconsistent teacher expectations at Oakwood, the intervention model remained unchanged across the years. Oakwood consistently used a pull-out model (Marston, 1996) for special education and reading intervention.

Oakwood's intervention model. Oakwood has a long-standing tradition of students pulled-out of core instruction to receive an intervention or special education support. Similar to Hillview, even though classroom teachers found that the pull-out model (Marston, 1996) did not help students' to address learning gaps, this practice continued at Oakwood. This pull-out model remained unchanged in five years prior to 2016-2017. The trend in data from Oakwood not only indicated that the pull-out model (Marston, 1996) did not help students attend to missed learning, but the amount of unfinished learning only grew as students got older. By the time students were in the 5th grade at Oakwood, the number of students not meeting grade-level expectations (as measured by the SBA) was greater than any previous year. Only 22% of fifth-graders met the benchmark standard. As a result, the district provided Oakwood with the first math interventionist.

Even with the progressive addition of a specialized teacher in math intervention, the program remained a pull-out model (Marston, 1996) in which students went to the intervention room to receive instruction on lagging skills from previous grades. According to the math interventionists at Oakview, these skills were often separate from grade-level content, and students were not able to connect skills taught during the intervention to the content taught during core instruction. The math interventionist Allison highlighted this idea in her exit interview, “It really is a waste of instructional time if math intervention is not connected to and delivered with core instruction for the student. When the kid walks from place to place, it is like all learning in the other location leaves their brain.” In the 2016-2017 school year, Oakwood added another half-time math interventionist to the staff. During this school year, Oakwood had two half-time math interventionists. The participating teachers at this school site (Table 2) consisted of both math interventionists and two, second-grade classroom teachers.

Oakwood’s interventionists: Allison and Paige. Allison was the first math interventionist within the district. Because Allison was one of the original math interventionists in the district, she has respected professional knowledge. She was a reading interventionist at another school in the district when recruited to be a math interventionist at Oakwood. Even though Allison worked full time, she split her time between two elementary schools. She spent two and a half days a week at Oakwood. Allison was an elementary classroom teacher ten years prior to 2016-2017, and at that time; she transitioned to providing reading and math intervention. Allison had her reading endorsement and no formal training for math intervention. Allison participated in coaching cycles with me during the 2016-2017 school year. During this school year, I introduced

Allison to the ideas of CGI (Carpenter et al., 1999, 2015) and she began noticing the mathematical thinking students brought with them to her math intervention instruction. Prior to learning about CGI, Allison viewed math intervention instruction as the teaching of explicit skills and specific solution strategies. Quick and accurate recall of not only facts across all four operations, but quick and accurate recall of strategies, skills, and procedures was Allison's perception of expected intervention instruction.

At the time of the study, Allison stated that she believed in collaboration and working with classroom teachers. Across the district, provided collaboration time was not a priority. As she stated in her initial interview, "I think to work smarter and not harder, we need to work together more. Much of what we do with kids can be in isolated silos if we aren't careful and make time to work together." Allison viewed her role as a support person for the classroom teacher. Because she saw herself in a supportive role, she worked with classroom teachers at their request and as they specified. Often, this involved teachers asking her to take students away from the classroom, and Allison worked with the student on a specific skill in another location. Allison was excited to join the research study because she was seeking professional development in understanding how children think about mathematics, and she wanted to learn new instructional approaches towards math instruction. Allison was pleased that the study included collaboration time with classroom teachers in her building. With another half-time math interventionist joining the team at Oakwood, Allison was eager to begin a collaborative relationship with her new team member.

Paige spent the past 20 years teaching middle school mathematics. After a year away from the classroom, she returned to work in 2016-2017 as an elementary math interventionist. This was her first year in this position. Paige had a math endorsement and an extensive background teaching middle school mathematics. Paige worked full time but only spent two and a half days of the week at Oakwood. Paige was a strong proponent of CGI (Carpenter et al., 1999, 2015), and she advocated for teachers to identify what mathematical ideas students brought with them to the classroom each day. Paige believed in working with classroom teachers closely to provide solid intervention instruction connected to core classroom content and focused on addressing gaps in learning from previous grade levels for her students. In her exit interview, Paige said, "The work we did together reaffirmed the importance of connecting intervention to classroom instruction. The students in second grade made significantly more growth than any other students I worked with. I attribute this growth specifically to the well thought out lessons we created with the classroom teachers." Paige was excited to join the study because as a new staff member to Oakwood, she wanted time to collaborate with classroom teachers she would be working with closely, and to learn instructional approaches for elementary students. Paige was a strong advocate for using data to inform instruction and encouraged the teachers she worked with to identify growth with the students they shared, as well as areas where gaps in learning persisted. During one of the collaboration meetings with the team, Paige said,

> We need to focus on this money and counting assessment. I see these common errors. What do we want to do about that? I see here that Sarah and Michael both demonstrated solid understanding of how counting coins and adding with two-digit numbers is connected. I think we need to leverage their thinking at the next workshop summary.

Both Paige and Allison collaborated with a different second-grade math and science teacher. Allison collaborated with Cheryl, and Paige collaborated with Joan (Table 2). Because both math interventionists worked closely with the two, second-grade math teachers, the math interventionists recruited the classroom teachers to join the study.

Oakwood's second-grade math team: Cheryl and Joan. Oakwood had four, second-grade classrooms. Two of the teachers taught only math and science, and two teachers taught English Language Arts only. Each teacher saw two groups of students for two and a half days during the week. The students spent the entire day with one teacher immersed in the content areas taught by that teacher. The teachers found that the dispersed content allowed each teacher to specialize in the specific content they taught.

Cheryl taught second grade for the past 18 years. Most of her career was at Oakwood. Prior to the 2016-2017 school year, in the last five years, she only taught second-grade math and science. Cheryl got her master's degree in special education and took courses towards her mathematics endorsement, although she did not complete that course work. Cheryl said she understood teaching mathematics in terms of planning lessons with games, songs, chants, and body movements to help students remember concepts and procedures. In her initial interview, Cheryl said, "Planning math lessons is not hard for me. I think about that skill I want students to know. I already have songs and chants to go with each skill. Then I pull the EngageNY [(New York State Education Department, 2012)] worksheets and make sure I am covering the content that the students will need in the worksheet. The kids love the songs, and that way they can remember the content." Cheryl believed students needed explicit instruction with concepts and strategies to complete procedures. At a collaboration meeting during the study, Cheryl said,

> We have to teach the students expected ways to add and subtract in the EngageNY curriculum. This means we have to teach the students how to use a hundreds chart for adding. Or how to use a number line to jump back. We have to teach the use of the hundreds, tens, and ones chart. When they know all of the strategies, then they can choose which one they use.

During her initial interview, Cheryl stated she preferred the EngageNY (New York State Education Department, 2012) curriculum to the district-created math curriculum because of the explicit strategy instruction. Cheryl stated she did not approve of rich math tasks as a way to teach concepts because students at Oakwood do not learn mathematics through rich tasks. “Our students come from high poverty and usually with many social-emotional problems. Rich tasks might work for more average or well to do kids, but our students need direct instruction to learn.” In the 2016-2017 school year, Cheryl had 23, second graders in the one section and 21 second-graders in the second section. One section had a high percentage of English Language Learners (ELL), and the other section had a high number of students on Individual Education Plans (IEP). The first section had ten students qualifying for ELL support, and the other section had five students on IEPs. Although both sections of Cheryl’s second-grade classes had a high need for extra support, this year was typical for any other year. Cheryl was excited to join the study to work closely with her math interventionist and begin a new model in which the math interventionist would be an active participant with instruction rather than pulling students out of the classroom. Cheryl was excited to have more collaboration time with her teaching partner Joan.

Joan taught second grade for the past 22 years. Joan worked at various elementary schools within the district and worked at Oakwood for the past eight years prior to the 2016-2017 school year. During this time, she collaborated with Cheryl for the past five years in which both teachers only taught math and science. During her initial interview, Joan stated that she preferred the EngageNY curriculum (New York State Education Department, 2012) to the district-created curriculum because of the workbooks the students used and because it was scripted. “The EngageNY curriculum [(New York State Education Department, 2012)] tells you what to teach step-by-step. The workbooks are helpful to know what to teach, as well. The only problem with Engage is the modules are so long. We have to rush to get through all the content." Joan also stated that she only agreed to teach math and science because someone had to do it, and she enjoyed working with Cheryl. Joan said that over time, she came to enjoy teaching math and science, and the content did not scare her as much as it had in the past. Joan's sections of second grade had just as high instructional need with the first section having 18 students and nine of those qualifying for ELL support. The other section had 23 students with seven on IEPs. Clustering occurred for all four sections of second grade. Two of the clusters had a high percentage of ELL students, with half or more than half the class qualifying for ELL support. The other two sections had a high percentage of students on IEPs. The school average for students in special education was 15% but these two sections of second grade had a percentage of either 25% or 30% of the class with IEPs. With a high percentage of students either needing ELL support or receiving special education, the second-grade teachers thought it was appropriate to utilize a math curriculum that provided explicit direct instruction aligned with CCSSM.

Intervention team at Oakwood. The composition of the intervention team at Oakwood was unique in the fact I was a district instructional coach for the math interventionists. I was the researcher but also a participant (Jaworski, 1998; Lewis, 2014). The two math interventionists ranged from being the most seasoned in this position to being the newest member of the intervention team. The two classroom teachers were unique because, as second-grade teachers, they only taught math and science all day long. This team of five met each week and discussed classroom data, planned a new week of core math lessons, planned a week of intervention lessons, and decided on progress-monitoring tools. We discussed which members of the team would use the various progress-monitoring tools to collect data for the following week's meeting. Although schedules aligned to allow for maximum collaboration, the ideas around the use of the EngageNY curriculum often got in the way of lessons designed to fit student need.

Oakview collaboration meetings. The research teachers met for 90 minutes each week. Prior to the meeting, the teachers read either journal articles or sections from provided texts. Oakview teachers received the same texts as the teachers at Hillview. The reading completed outside of the collaboration meetings helped build background knowledge or helped guide instructional decisions during the collaboration team meetings.

We spent the first five minutes on relationship building and talking about our personal lives or struggles within our unique positions. We spent the next 20 minutes of the collaboration meeting focused on student data analysis. We identified which students needed additional learning opportunities. Each math interventionist sat next to the classroom teacher she worked with during these meetings, and this helped build a relationship between the interventionist and the classroom teacher. After knowing where the class stood with the recent instructional skill, we planned the next set of core math lessons. We designed rich math tasks to implement with the whole class, but often, these lessons did not occur. A barrier towards lesson planning and implementation occurred because teacher beliefs on rich math tasks and math instruction were divergent from the provided PD instructional approach. This was also true for trying to focus on a big math concept rather than an isolated skill. Both general education teachers struggled to shift thinking away from small skills to large generalizable math concepts. During nine of the 12 collaboration meetings, the team discussion focused only on accuracy or proficiency with an isolated skill rather than discussing a large math concept. Examples of this include naming coins and identifying coin values or using the standard addition algorithm to add two, two-digit numbers versus what are different combinations of coins to equal the same total. Another example included how the addition operation behaves when there are more than nine ones.

Three Peaks Elementary School

The location of Three Peaks Elementary school is outside the city limits of a large Pacific Northwest urban area and is in the same district as Hillview and Oakwood. Three Peaks is a relatively newer elementary school built in 2003. When Three Peaks first opened, only 7% of students qualified for free or reduced lunch. In the last 15 years, the student population shifted dramatically, during the 2016-2017 school year, 39.7% of the student population qualified for free or reduced lunch. During the 2016-2017 school year, 19.1% of students identified as Hispanic, 9.1% of students identified as Asian, 4% of students identified as African American, while 57.3% of students identified as White. Three Peaks is one of seven schools in a district with 21 total elementary schools that is not school-wide Title 1. Stable leadership and staff was a strength at Three Peaks Elementary.

The same principal has been in place at Three Peaks for the past 13 years prior to 2016-2017, while the same instructional coach worked at Three Peaks for the past five years prior to the 2016-2017 school year. In the five years prior to the 2016-2017 school year, the staff at Three Peaks focused on literacy instruction and received little to no professional development in the content area of math. During the 2016-2017 school year, 42% of third-graders met standard on the math portion of the Smarter Balanced Assessment. Fifty-seven percent of fourth-graders met standard on the math Smarter Balance Assessment during that same year, 50% of fifth-graders met standard on the math portion of the Smarter Balance Assessment. In analyzing three years of Smarter Balance Assessment data, at Three Peaks prior to 2016-2017, fewer students meet standard with the assessment as they progressed through the grades (Table 2). Looking at the same group of students across time, in the 2014-2015 school year, 58% of third-graders met the standard. The next year those students were fourth graders, and 54% of the students met the standard. In the 2016-2017 school year, only 45% of those same students as fifth-graders met the standard. That was an overall decline of 13% while the percentage of student mobility was under 3%.

In the five years prior to the 2016-2017 school year, the district provided a district-created digital scope and sequence curriculum. With many units and lessons missing, or with links to lessons that did not function, teachers at Three Peaks openly and outwardly abandoned the district-adopted math resource, and this decision resulted in using alternatives for lessons from sources such as Pinterest (https://www.pinterest.com) or Teachers Pay Teachers (https://www.teacherspayteachers.com). The result was by the 2016-2017 school year there was no consistent or viable curriculum within even the same grade-level or across the grades. With each teacher having complete autonomy over math content and instruction, intervention for Three Peaks was almost nonexistent. Lack of a consistent curriculum within the same grade level meant teachers have nothing to discuss during PLC meetings. One of the many purposes of PLCs is the analysis of student data and development of lessons for when students need intervention or mastered content (DuFour et al., 2006; DeFour et al., 2008; Elmore, 2002). Without common resources, materials, or a curriculum, the work of the PLC at Three Peaks broke down because of a lack of any consistency in lesson planning and implementation. Without consistent lesson planning at Three Peaks, planning for intervention did not occur. Teachers worked in isolation of analyzing student data, planning, and implementing intervention lessons. Meaningful and effective implementation of intervention does happen in isolation (Al Otaiba et al., 2019; Stevens et al., 2018).

Three Peaks' intervention model. During the 2016-2017 school year, Three Peaks had three reading interventionists on staff and no interventionists providing math intervention. At Three Peaks, math intervention was up to each classroom teacher to decide which students needed intervention and what content would be included in the intervention. As the district coach that provided math intervention PD, I knew the staff at Three Peaks had not received any professional development regarding the seven needed components of an effective intervention (Fletcher et al., 2007; Fuchs et al., 2010; Lembke et al., 2012). With the staff at Three Peaks, not knowing these principles of intervention, the opportunity for quality intervention implemented by the classroom teacher was minimal.

Three Peaks' special education model. Three Peaks transitioned to a push-in model (Friend et al., 2010) for special education instruction during the 2010-2011 school year. The special education teacher and the general education teacher had to create their own collaboration time to complete this difficult work. Often, the result was the special education teacher was present during core instruction in the general education classroom, but the special education teacher was unsure how to support instruction in the classroom. The learning support teacher at Three Peaks had this to say about the lack of collaboration time:

> I work with more than 30 teachers. If I do not have time to talk with the teacher about in-class instruction, when I show up, I may not know how to help. Sometimes I end up sitting waiting for the teacher to tell me how to help our shared students. It is a waste of my time, and the student does not get the help they need. (Megan, initial interview)

The special education teacher also commented on how lack of collaboration time resulted in students that are more dependent on the special education teacher for work support rather than the student becoming more independent with core general education work. "Sometimes, I help too much. Kids rely on me to tell them what to do. My presence in the classroom is a hindrance. It does not foster independence if the level of support is not a good fit." Lack of collaboration time for the learning support teacher had a ripple effect on instruction (Cochran-Smith & Dudley-Marling, 2012; Cook & Friend, 1995; Wilson & Blednick, 2011; Youngs et al., 2011). The special education teacher at Three Peaks was eager for more collaboration time with one of the general education teachers, and this was a factor in why the teachers at Three Peaks joined the research study.

Participating teachers at Three Peaks. The teachers at Three Peaks made a unique team because the third-grade general education teacher was my former team member. The general education teacher and I taught third grade together for three years at Three Peaks. During the 2016-2017 school year, I was an instructional coach with the district for math intervention. However, my prior position was that of a special education instructional coach. I worked previously with the special education teacher as well. Because of the relationship already established between the special education teacher and the general education teacher and myself, the teachers wanted to join the study and learn more about math intervention. Both teachers at Three Peaks wanted to learn about how rich math tasks served as assessment and allowed students to address gaps in understanding.

Three Peaks Special Education Teacher: Megan. Prior to the 2016-2017 school year, Megan had taught special education for ten years in the district. Before becoming a learning support teacher, Megan worked in a self-contained classroom for students with intellectual disabilities. Megan's philosophical approach to math instruction focused on learning the basic facts of the four operations. Megan believed that students should memorize facts before working with problems in a context. Megan explained in her initial interview, "I just think students have to know basic facts before they can do higher-level thinking." Megan used many strategies to help students memorize facts within the operations. Megan had no advanced training in math education. Megan received her Master's degree in special education in 2010. Megan's special education Master's project focused on task analysis. This included breaking down skills into manageable pieces for students with intellectual disabilities. For her Master's program, Megan wrote lengthy detailed reports on how to teach a student to make a bed, wash dishes, or read a bus schedule by using an incremental instructional process. She used her learning in this area in her new position as a learning support teacher. She often supported her learning support students by giving them a list of steps to complete classroom work. Megan had little experience with the district-created math curriculum and openly admitted that she did not believe her students needed the content in core classroom instruction. Megan indicated in her initial interview that the students she worked with needed more of a focus on basic skills. She described her views as, "I mean my students, scaffolding the classroom instruction for them doesn't help them much. They need the basics. They need addition and subtraction facts. They need to learn key words for adding and subtracting in story problems. Often, time spent on core content is a waste. They won't get much from that learning."

Three Peaks 3rd-grade teacher: Sally. Sally had 15 years of teaching experience prior to the 2016-2017 school year. Sally worked at Three Peaks Elementary beginning in 2009. Sally attended multiple Teacher Development Group (www.teachersdg.org) conferences each year. Sally was well versed in CGI (Carpenter et al., 2015), and her math instruction focused on the Common Core Mathematical Practice Standards almost more than the third-grade content standards. In Sally's initial interview, she explained, "I think teaching kids how to reason and think about math problems is vital. If they learn the landscape of the operations, then developing new ideas or making a visual model isn't a struggle for them." Sally believed teaching the mathematical practices was a way for students to learn how to generalize large math concepts. Sally was experienced with a math workshop, lesson format. Her math workshop began with the launch of a rich task (Boaler et al., 2018a, 2018b, 2018c; Carpenter et al., 2015; Empson & Levi, 2011; Kazemi & Hintz, 2014; SanGiovanni, 2017; Smith & Stein, 2011, 2018). Students worked together to solve the task as Sally roamed the room, took conferring notes (Munson, 2018), and helped students formalize ideas for the summary portion of the lesson (Smith & Stein, 2011, 2018). During the summary portion of the lesson, Sally used approaches for selecting and sequencing student thinking (Smith & Stein, 2011, 2018) to maximize student learning. Sally did not fully trust the district-adopted scope and sequence, so she constructed her own learning trajectories for her class. Sally designed rich math tasks or used a resource to give her inspiration for a math task that would help students come to the big idea she wanted her students to realize or develop (Boaler et al., 2018a, 2018b, 2018c; SanGiovanni, 2017). Because Sally was an expert math teacher and well recognized for her leadership in math instruction, Sally had a high percentage of special education students placed on her class roster. Sally and Megan often shared students. They struggled to agree on what the special education students needed during classroom instruction. They also struggled to agree on how to support students with core instruction. Sally and Megan had very different instructional approaches as well as beliefs about what students in special education could accomplish.

Intervention team at Three Peaks. With the lack of a consistent math curriculum utilized across different grade levels, there was no planned, systemic math intervention that occurred at Three Peaks during the 2016-2017 school year. Individual teachers may have pulled a small group for instruction, but the intervention did not occur in the way as described in an RtI or MTSS model. In the 2016-2017 school year, students did not have any kind of extra math support other than a referral for special education. This could be a possible reason when examining three-year data prior to the 2016-2017 school year, the percentage of students at Three Peaks receiving special education for math was steadily on the increase.

Three Peaks' collaboration meetings. The research teachers met for 90 minutes each week. Prior to the meeting, the teachers read either journal articles or sections from provided texts. Megan and Sally received the same texts at teachers at Hillview and Oakview. The reading completed outside of the collaboration meetings helped build background knowledge or helped guide instructional decisions during the collaboration team meetings. We spent the first five minutes on relationship building and talking about our personal lives or struggles within our unique positions. We spent the next 20 minutes of the collaboration meeting focused on analyzing student data and identifying which students needed additional learning opportunities.

With this particular team, I worked with the teachers in finding reasonable or meaningful goals between the student's IEP goals and the instructional goals of core classroom instruction. Making sure progress was occurring on IEP goals was an important endeavor to Megan while demonstrating learning on grade-level standards was important to Sally. Helping these two teachers find the common ground on the two sets of goals became important work for this team.

Next, we examined classroom data. After knowing where the class stood with the most recent big math idea, we planned the next set of core math lessons with all students in mind. As a team, we designed or reframed given tasks in the district-adopted math curriculum. After we designed four or five tasks, we anticipated how students could enter the task and how each subgroup of students could move their thinking forward or generalize a big math concept (Smith & Stein, 2011). We created graphic organizers to help us manage what we anticipated from each subgroup of students. Whenever possible, we decided how to highlight the thinking of a student who had a fixed mindset as a mathematician (Boaler, 2008). We also attempted to assign a status to a student who had a low status in the class. We selected students with highly efficient or organized work during the summary portion of the lesson. As a team, we attempted to shift the thinking of the class from “these are the good math students” to “we are all capable mathematicians.”

We also designed small group lessons to occur outside of the core instructional time while Megan was still in the classroom. We designed simple progress-monitoring collection tools to be able to measure growth and progress for each special education student moving towards the generalization of a big math concept (Table 6). Using student data, the team focused on students having access to core content while also addressing learning gaps from previous grade levels.

Table 6

Progress-monitoring tools-Three Peaks

Student	Entered the task	Developed strategy to solve	Learning Progression Step A	Learning Progression Step B	Learning Progression Step C
	How did the student begin?	How much support was given?	Identified ways to solve for perimeter?	Identified ways to solve for area?	Identified connection between factors, area, and perimeter?
Mark	Counted squares on graph paper	No support	Yes	Yes	Yes
Hailey	Found sides with the same lengths and recorded amounts	Minimal support	Yes	Yes	No
Luke	Drew shapes on graph paper and counted squares	Consistent, ongoing support	Yes- only because of scaffolding	Yes – used repeat addition	No
Greg	Drew shapes on graph paper and used a counting-	Minimal support	Yes	Yes	No

on strategy

These collaboration meetings generated multi-tiered systems of support and kept all students in the general education setting. At this location, these collaboration meetings occurred for nine weeks until the end of the school year. The general education teacher and the special education teacher agreed to resume the work in the fall and finish out the remaining weeks with a new group of students. Unfortunately, Megan, the special education teacher, took a position in a different district. With a new special education teacher at Three Peaks, the general education teacher and I decided trying to resume the work was not a viable option.

Findings Related to the Research Questions

I will present six key findings that relate to the research questions. As a reminder, the research questions were:

1. How do math interventionists and/or special education teachers and general education elementary teachers develop their co-teaching relationships? What are the teachers' perceptions regarding the process of developing these relationships? (RQ1)
2. How do math interventionists and general education elementary teachers co-design math instruction with a focus on supporting students needing intervention in accessing grade-level core content? (RQ2)
3. How does the professional development intervention support mathematical instructional practice for math intervention and general education teachers? (RQ3)

Factors that Influence Teachers' Co-teaching Relationships (RQ1)

The first two findings helped explain how general education teachers and special education teachers and/or math interventionists developed their co-teaching relationship. One finding was on how the perceptions of teachers on student instructional need affected the way the co-teachers formed their relationships (Cook & Friend, 1995; Friend, 2007; Friend et at., 2010; Scruggs et al., 2007). Another finding indicated a connection between materials and lesson planning. How teachers perceived students and the way teachers thought about lesson planning affected their co-teaching relationship.

Teacher perceptions of students and goals for students' learning (RQ1-1). A theme that was prevalent throughout the data set was the way each teacher thought about or discussed student instructional needs had an impact on how the teachers developed a co-teaching relationship (Cochran-Smith & Dudley-Marling, 2012; Cook & Friend, 1995). When teachers had differing goals for students, differing beliefs about what students should learn, or differing ideas about what students needed to know, these differing perceptions needed to be discussed for the co-teaching relationship to move forward (Cochran-Smith & Dudley-Marling, 2012; Cook & Friend, 1995).

Hillview example: Reconciling differences. Below is an example from Hillview demonstrating how the general education teacher had a differing belief about what Mary, a 5th-grade student, could do. The example also shows how the co-teaching partnership attempted to reconcile the differences in beliefs about what students could do.

> Maggie (GET[10]): I just don't think Mary can learn these fraction concepts. She is just too far behind. She doesn't understand her multiplication facts, and division is a mystery to her. She is unsure about place value, so decimals is [sic] out of her reach.
>
> Chris (SET[11]): But look at how she was able to represent hundreds, tens, and ones, as a multiplicative relationship here. I'm wondering if we gave a task to Mary, Jon, Lois, and Jose that would highlight the pattern between whole numbers and decimals if then they

[10] General Education Teacher
[11] Special Education Teacher

could understand the idea of place value on a deeper level and therefore gain grade-level access to decimals.

Holly (MI[12]): I have an idea for a task we could pull from the *Mine the Gap* [(San Giovanni, 2016)] text that we got.

Maggie (GET[13]): Well, let's plan out this task and see what the students can do. We can always adjust next week if the task was too far out of their reach.

In this example, Maggie used a gap-based lens and described what she believed Mary could not do. I use this term "gap-based" to describe the way a teacher perceived a student. Maggie identified Mary's entry point into math content, and Maggie indicated the grade-level standard where Mary needs instruction. Maggie focused on the gap of what Mary did not understand, and so she was convinced that Mary did not have access to core content. However, Chris shifted the conversation and examined what Mary knew. Mary demonstrated in her work that she understood there is a multiplicative relationship between hundreds, tens, and ones. Chris used a strength-based lens and focused on how Mary could gain access to core content. Chris intentionally made the connection between multiplication and division and then connected to what Mary still needs to learn decimals. In this example, Chris constructed the bridge over the gap using Mary's strength and therefore gave her access to core content. Holly added to the conversation with a suggestion for a task designed to help students connect what they know to what they do not fully understand yet. While Maggie focused on the gap in student learning, Chris focused on the student strength, and Holly used an asset-based perspective (Celedón-Pattichis et al., 2018) and focused on the next instructional step. When the three teachers

[12] Math Interventionist
[13] General Education Teacher

discussed Mary's needs together, they developed a clear vision of how to provide access to grade-level math content.

The team at Hillview negotiated what the students with mathematics difficulty could do and how to move forward with work, so all the students in the class had access to core content. Finding a way to negotiate differing perceptions about students, strengthened the co-teaching relationship among the teachers at Hillview, as this exchange happened at the beginning of the study. By the end of the study, the alignment in teacher perceptions allowed for asset-based dialogue and less gap-based dialogue. Hillview was not the only team needing to negotiate goals for students.

Oakview example: Resolving goals for students. In the next example, the co-teaching team at Oakview also struggled with resolving differing goals for students.

> Cheryl (GET[14]): We have to keep in mind that the goal is for students to be able to pass the state test in 3rd grade. We need students to be fluent with facts and procedures for them to do well on that test.
>
> Allison (MI[15]): Is that really our goal, though? I think we need to focus on the result of students that have mastered mathematical practice number one to make sense of problems and persevere. If students leave second-grade making sense of what they are doing and making connections between math ideas, then even if they are the slowest at computation, I feel we would have succeeded.

[14] General Education Teacher
[15] Math Interventionist

Joan (GET[16]): I agree with both of you. I think we need to have both goals. Passing the test is all that the district really cares about, but we need to make sure kids can think and reason as well.

In this example, the team at Oakview had differing goals for students. Cheryl focused on test scores and what skills and strategies students needed to master to do well on the test. Allison shifted the conversation to focus on mathematical practices. Joan helped reconcile these two different goals, and she brought these ideas together by her suggestion of working towards both ideas. The team discussed the goals and outcomes they wanted. The team decided to focus on making sure students had the skills to pass the state test as well as the ability to think and reason about mathematical ideas. The Oakview team agreed on student goals before moving towards planning for instruction.

Three Peaks example: Unable to resolve scope of goals. The teachers at Three Peaks also worked to find common ground about the learning goals for the students with IEPs. Sally, the general education teacher (GET) wanted to focus on goals extending beyond the student's IEP goals while Megan (SET[17]) wanted to limit learning goals for the student to that of the IEP goal. This discussion reoccurred throughout the entire duration of the study. Each week at the collaboration meeting, Sally wanted to extend the goal for student learning generalizing a math concept rather than the performance of a computational skill.

Sally (GET): I would really like Greg to focus on the understanding with fractions that the size of the whole matters and that the line between the numerator and denominator the same as division. If he could talk about fractions in terms of division, I think he would have better access to grade-level content later on.

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[16] General Education Teacher
[17] Special Education Teacher

Megan (SET): I think that goal is not specific enough. We need to focus on his IEP goal that he can represent one-half and one-fourth in pictures, with a set, and on a number line.

Sally (GET): I think that goal is too small. I'm not sure that goal helps him think about fractions in a way that will connect with other ideas such as division. Can we set a goal for him in class that goes beyond his IEP goal?

Megan (SET): Yes, sure we can. I have to be honest with you. I am only going to focus on collecting data that aligns to his IEP goal. If you want to focus on that larger goal, you will need to think about how to get him there and then plan for data collection yourself.

In this example, Sally wanted a broader, more global goal for student learning. Megan wanted a narrow area of responsibility and therefore, narrowly defined the goal for student learning. The Three Peaks teachers struggled so much with the goals they wanted for students that their co-teaching relationship never progressed beyond goal setting. Megan and Sally were far apart in their thinking about what was important in student learning. Differing goals for students on IEPs got in the way of collaboration and planning for student learning. Throughout the time of the study, Megan and Sally were in constant negotiations about the learning goals for the students with IEPs.

With the Hillview team, the teachers learned to trust each other's perceptions of students as their co-teaching relationship developed. As this trust developed, the need to calibrate or discuss perceptions of students diminished (Cochran-Smith & Dudley-Marling, 2012; Cook & Friend, 1995). With the Three Peaks team, the teachers struggled with perceptions of important goals for the students with IEPs. The lack of working together on common goals for students was a barrier that prevented the co-teaching relationship progressing past student perceptions (Cochran-Smith & Dudley-Marling, 2012; Cook & Friend, 1995). Even though the Oakview team developed trust for each other's perceptions of students, and they agreed on the goals set for students, another factor seemed to get in the way for how teachers developed their co-teaching relationship.

Connections to materials and lesson planning (RQ1-2). Similar to past research, I found clear connections between how teachers thought about lesson planning, the types of materials they used for planning, and the types of lessons teachers planned (Remillard, 1999; 2000; Remillard & Bryans, 2004). These connections are evident when examining how the teachers thought about lesson planning and the types of lessons that the teachers planned. If the teacher thought fluidly, working back and forth between large math concepts and specific lesson goals, then the planned lesson tended to help the teacher achieve the goal of students generalizing a larger math concept. Also in fitting with this pattern, if the participant focused narrowly on day-to-day lesson planning, without thinking about the larger generalizable math concept, then the planned lesson focused on the small skill and this usually did not help the teacher achieve students generalizing a large math concept. The table below (Table 7) highlights all the teachers and the ways they thought about lesson planning, the types of lessons they planned, the materials used in lesson planning, and a description of the enacted classroom lessons.

Table 7

How Teachers Thought About, Planned, and Enacted Lessons

School Location, Participant, and Position	Thoughts about lesson planning and lesson focus	Materials used in lesson planning	Description of how enacted lesson aligned with planned lesson
Hillview Holly MI[18]	Fluid movement between big idea and daily tasks. Focused on big ideas	District scope and sequence, *Mine the Gap*, CCSS[19] Progression Documents	Lesson included all 3 parts Launch-Explore-Summary[20] (LES). Enacted lesson aligned with the plan
Hillview Chris SET[21]	Focus on small grain size – what should be accomplished tomorrow Focused on isolated skills	District scope and sequence	Lesson included launch and explore. Enacted lesson was downgraded from the plan in rigor and higher-order thinking skills
Hillview Maggie GET[22]	Ability shifted based on content to fluidly move between big ideas and daily tasks Focused on big ideas	District Scope and Sequence, *Mine the Gap*	Lesson included launch and some explore. Actual lesson resulted in whole group discussion with an Initiate-Respond-Evaluate[23] (IRE) funneling pattern
Oakview Allison MI	Fluid movement between big idea and daily tasks. Focused on big ideas	CCSS Progression Documents, *Mine the Gap*	Lesson included all 3 parts of the LES lesson structure. Enacted lesson aligned with the plan
Oakview Paige MI	Fluid movement between big idea and daily tasks. Focused on big ideas	CCSS Progression documents, *Mine the Gap*	Lesson included all 3 parts LES. Enacted lesson aligned with the plan
Oakview Cheryl GET	Focus on small grain size – what should be accomplished tomorrow Focused on isolated skills	Engage NY curriculum	Lesson included a launch. Enacted lesson was direct instruction on how to perform skill
Oakview Joan GET	Focus on small grain size – what should be accomplished tomorrow Focused on isolated skills	Engage NY curriculum	Lesson included a launch. Enacted lesson was direct instruction on how to perform skill

18 Math Interventionist
19 Common Core State Standards
20 Launch-Explore-Summary (LES) is a workshop based lesson structure.
21 Special Education Teacher
22 General Education Teacher
23 Initiate-Respond-Evaluate (IRE) is a discourse pattern between the teacher and student. The teacher initiates a question, the student responds, and the teacher evaluates the response. I-R-E patterns are often associated with narrow funneling types of questions (Cazden, 2001).

Three Peaks Sally GET	Fluid movement between big idea and daily tasks. Focused on big ideas	District scope and sequence	Lesson included launch and explore and sometimes summary. If summary occurred, whole class discussion with teacher in an I-R-E pattern
Three Peaks Megan SET	Focus on small grain size – what should be accomplished tomorrow Focused on isolated skills	Pinterest, Teachers Pay Teachers	Lesson was direct instruction on how to perform skill. No L-E-S

The materials teachers used also contributed to how they thought about and enacted lessons (Remillard, 1999; 2000; Roth McDuffie & Mather, 2009). Teachers who used the CCSS progression documents (Common Core Standards Writing Team, 2013) in planning tended to work fluidly between math content and the learning progression, and the goals for student learning. They focused on a large generalizable idea rather than the accurate performance of a skill or a procedure. NCTM outlines eight ambitious teaching practices, and the first ambitious teaching practice described establishes clear goals for the mathematics that students are learning, situating goals within learning progressions, and using the goals to guide instructional decisions (NCTM, 2014). When teachers planned fluidly with the learning progression and tied the progression to the daily goal, teachers planned a lesson that helped achieve the larger desired learning goal.

Hillview example: CCSS progression documents and a focus on big ideas. In the following example, Maggie clarified the learning goal in simple terms, and Holly followed up with how that beginning work linked to generalizing a big idea. Chris, who tended to think about lesson planning in a smaller grain size, (Table 6) was concerned for what students would do the next day. Chris also questioned if students reasoned to generate ideas about halving and doubling at the end of this lesson. Chris's pedagogical stance was that students needed ideas broken down (Table 6) so it seems logical that Chris would question if students could think about a big idea and generalize concepts. Holly reminded the team of two principles of CGI (Carpenter et al., 2015) that students inherently have strategies for solving problems, and the provided manipulatives allowed access to the problem. Holly focused on the task but also the big idea that students develop relational ideas between 12, 24, and 48.

> Maggie (GET): So the big goal in this brownie task is to be able to find different ways to partition either 12, 24, or 48? Right?
>
> Holly (MI): I think that's the starting place, but we really want to get the students to the place where they can reason proportionately and develop some bigger ideas with halving and doubling.
>
> Chris (SET): Is that the primary goal at the end of these lessons? What will we focus on tomorrow? What if some students can't come up with ways to get 12 equal brownie shares?
>
> Holly (MI): I think we need to assume the kids will make sense of this problem and develop ways to solve. I think having the rainbow tiles out will also help. I think it will be important at the end of the lesson to have students connect how 12, 24, and 48 are related to each other.

Holly often planned in terms of big ideas down to the daily work. In this example, she was clear about what ideas students should generate by the end of the lesson. Holly used the CCSS progression documents and the district-created curriculum to help her plan. The materials Holly used and the way Holly thought about lesson planning (Table 6) allowed her to be a support for her teammates in fluidly connecting daily work to the large generalizable idea. Other types of lesson planning materials inhibit planning for a large generalizable idea.

Oakview example: EngageNY and a focus on a narrow skill. In the next example, teachers from Oakview used the EngageNY curriculum (New York State Education Department, 2012) and planned lessons with a narrow focus. The Oakview team read the EngageNY materials (New York State Education Department, 2012) and the resulting lesson plan had a narrow scope in student learning and did not require students to generate or generalize any large concept.

Cheryl (GET): Grade 2, module 7, lesson 3 from Engage says the objective is for students to draw and label a bar graph and to relate the count scale to a number line.

Joan (GET): Ohh, yeah, that's right. I like these lessons! Are we going to do the same bar graphs as last year?

Cheryl (GET): I don't see why not. The students liked filling out the worksheets, and it wasn't too hard for them.

Allison (MI): What is the big idea you want students to walk away learning?

Cheryl (GET): The parts of a bar graph and how it relates to a picture graph and tally table.

Paige (MI): I see what you are saying, but the measurement and data domain in the CCSS is so small compared to the numbers in base ten domain, especially in second grade. I am wondering if we can shift the lesson focus to require more place value reasoning from students while they examine different bar graphs, picture graphs, and tally tables.

Cheryl (GET): This is your first time through Engage, so I think you'll see where place value and reasoning about number quantity comes in with other modules. For now, let's just leave these lessons as they are.

Joan (GET): Ok, let's do the direct instruction piece where we tell the students the 5 parts to a bar graph and then they do these three worksheets of turning tally tables or picture graphs into bar graphs.

Cheryl (GET): Sounds good to me. Do you want to send the copies off to the print shop?

Allison (MI): How would you like Paige and I to support your students with learning the required elements of a bar graph?

In this example, Allison and Paige attempt to get Cheryl and Joan to think about lesson planning in terms of a larger gain size of students generalizing a big idea. Allison asked about the goal of student learning. Cheryl responded with a narrow scope of learning the parts of the bar graph. Paige shifted the conversation to focus on place value within the context of a bar graph. Cheryl and Joan were familiar with the EngageNY (New York State Education Department, 2012) lessons, and continued to use those materials and planned with a limited student focus. Cheryl and Joan decided not to pick up the ideas from Paige, and therefore, the lessons and student work remained unchanged with a limited student learning focus. It is possible that Cheryl and Joan did not pick up the ideas from Paige because they fully did not understand the connection between bar graphs and addition concepts. This was very different from the Hillview team who knew the district-created fractions lessons needed revising.

The Oakview team relied heavily on the EngageNY (New York State Education Department, 2012) curriculum for five years prior to the 2016-2017 school year. In planning with a curriculum that provided a systematic script, the Oakview general education teachers developed rigid thinking about lesson planning. Lesson planning focused on the day-to-day skills students were to master. The EngageNY (New York State Education Department, 2012) curriculum did not help teachers to think about big ideas students should generate and generalize.

With a developed learning progression focused on big ideas missing from each EngageNY module, the teachers from Oakview who heavily relied on this curriculum struggled to think about big ideas. Instead, lesson planning by the Oakview general education teachers focused only on the small grain size of daily work. The Oakview math interventionists used the CCSS progression documents and the *Mine the Gap for Mathematical Understanding Grades K-2: Common Holes and Misconceptions and What To Do About Them* (San Giovanni, 2016) text in planning and were more familiar with thinking about learning progressions. The Oakview math interventionists were more likely to think fluidly from daily work moving towards generalizing a big idea, in part, because the materials and resources used in planning helped the math interventionists to think and plan using learning progressions. Curriculum materials were not the only factor in narrow lesson planning; IEP goals had a similar effect.

Three Peaks example: Using IEP goals to plan. In this next example, Megan and Sally struggled to plan lessons together using a larger grain size that went beyond the daily goal as well as the IEP goal. Sally often found the IEP goal to be too narrow of a focus for daily lessons. Megan thought the IEP goals were the correct grain size for lesson planning.

> Megan (SET): So if we look at John's IEP goal, he is to learn his multiplication facts up to 100 and fluently recall at least 30 facts in one minute. Let's focus on John memorizing multiplication of twos and threes up to two times ten and three times ten.
>
> Sally (GET): Yes, I know that's his IEP goal, and I see what you are thinking about him starting to memorize his facts, but I think we should focus on this standard for multiplication. In operations and algebraic thinking, there is a standard that gets at chunking multiplication to find an unknown total. Here it is 3OAB.5. It's the start of understanding the distributive property. Like to find 7 x6 is 7 x5 plus one more group of

seven. I think if John focuses on building arrays and learning multiplication this way, it will help him with his IEP goal of learning the multiplication facts and seeing the connection between multiplication and division.

Megan (SET): Jumping right into something so complex, will cause so much frustration for John. I think that is just too hard. After he learns some facts, then we can try having him learn the connection between multiplication and division. That kind of idea goes well beyond what he has to learn in his IEP.

Sally (GET): That's kinda my point. I think he is capable of building arrays and deconstructing them to see how 7 x 5 plus 7 x 3 is the same as 7 x8. This kind of learning will help him in the long run more than fact memorizing that he will probably end up forgetting soon as summer arrives. Heck, as soon as the weekend arrives.

Megan (SET): Since it's in his IEP, he needs to learn these facts. I will plan some games and flashcards for this next week so he can begin to memorize his twos and threes. Then if other kids want to join us while I'm here, I'm happy to help other kids learn those facts too.

Sally and Megan did not agree about the size of learning that was appropriate for John to take on. Megan wanted to use the IEP goals to help her plan, and she wanted to focus on memorizing of facts. Sally wanted to focus on a big idea in the CCSS of using the concept of the distributive property to help her plan. Sally wanted John to build and deconstruct arrays to begin to generalize the idea of the distributive property. In this example, the IEP goals kept the focus on learning to a narrow scope.

In these examples, the types of materials and teachers thought about in lesson planning influenced the types of lessons that they planned (Remillard & Bryans, 2004; Remillard & Kaye, 2002; Remillard, 1999; 2000). At Hillview, the teachers planned with the CCSS progression documents and used resources that helped them plan and think about big math concepts. The lessons they planned helped them to achieve that goal. At Oakview, the teachers used EngageNY (New York State Education Department, 2012) materials to help them plan. The curriculum materials had a narrow focus, and the planned lessons had a narrow focus. At Three Peaks, the special education teacher used the IEP goals to help her plan, and this resulted in a narrow lesson plan focus. The general education teacher used the CCSS to help her plan but was not able to convince the special education teacher that the special education student did have access to those standards. Curriculum materials alone may not affect lesson planning but it is clear that curriculum materials can influence lesson planning.

Approaches to Designing Instruction for Students with Mathematics Difficulty (RQ2)

The second research question focused on how teachers co-designed math instruction for students with mathematics difficulty. The first finding regarding RQ2 indicated that when teachers were provided time and learning opportunities they were able to analyze given tasks and then reframed tasks to include at least five out of six design criteria for rich math tasks (Carpenter et al., 2015; Boaler et al., 2018a, 2018b, 2018c; Flynn, 2017; San Giovani, 2016). Rich math tasks created access to core content while also allowed students to attend to gaps in understandings. The second finding regarding RQ2 indicated when teachers actively used the *Five Practices* (Smith & Stein 2011; 2017) when planning they were more likely to understand how the *Five Practices* (Smith & Stein, 2011) supported access to core instruction while also allowed students to address gaps in understanding.

Reframing student tasks (RQ2-1). Math interventionists and general education teachers reframed math tasks as a way to support students needing intervention in accessing grade-level core content. Teachers reframed student tasks from the district-created curriculum to transform the task into a rich math task (Carpenter et al., 2015; Boaler et al., 2018a; Flynn, 2017; San Giovani, 2016; Stein et al., 1996; Stein et al., 2007). A rich math task has at least five of the six following criteria:

- An engaging and meaningful context,
- Multiple entry points,
- Multiple correct solution pathways,
- Potentially multiple correct solutions,
- Generalizability about a big math concept,
- Utilization of math manipulatives brings meaning to the problem (Carpenter et al., 1999, 2015; Boaler et al., 2018a, 2018b, 2018c; Flynn, 2017; San Giovanni, 2016; Smith & Stein, 2011, 2018).

To be able to determine if the task can accurately be defined as a rich math task teachers needed to solve the task themselves and anticipate a variety of solution strategies (Smith & Stein, 2011, 2018).

It is important to note that not all rich math tasks need to have multiple correct solutions, but it is important for elementary students to be exposed to rich tasks that do have multiple correct solutions so that students develop a curiosity and wonder about finding all possible solutions (Boaler et al., 2018a, 2018b, 2018c). A rich math task can allow all students to advance their mathematical thinking and become more sophisticated in their approach towards solving other tasks with a similar landscape (Fosnot, 2016). Identifying when a math task was not a rich task served as the first step in being able to reframe the task at Hillview and Three Peaks.

Many of the district-created units, lessons, and tasks included direct instruction (Hiebert & Grouws, 2007; Hiebert & Stigler, 2000) and little to no exploration of ideas. Therefore, if students had gaps in understandings from previous grade standards, the student may not have had access to the daily content unless the teacher(s) reframed the task. The teachers in my study, who were aware of researched-based mathematical teaching practices, also planned from the perspective that the direct instruction approach was not as meaningful for students. The teachers in my study were more likely to reframe tasks when they were aware of a disconnection between the district-created math curriculum and researched-based best mathematical practice (Carpenter et al., 1999, 2015; Stigler & Hiebert, 1999; Vygotsky, 1978).

During the collaboration meetings, the general education teacher and/or math interventionist and special education teacher often engaged in conversation about how the district-provided math curriculum was not meeting the needs of most students. Students with mathematics difficulty from multiple previous grade levels also had limited access to the grade-level core content. These same students needed rich tasks that allowed for connections between ideas from previous years to current core learning.

Reframed tasks at Hillview: Using a coach-created tool. The teachers at Hillview knew that students needed well-crafted rich math tasks, and they needed to reframe the district-created tasks. Reframed tasks allowed for more exploration of concepts and allowed all students to access to the math concept of the math lesson. The collaboration meetings provided practice for the teachers at Hillview to become quite proficient at reframing tasks (Figure 3) and allowed practice with anticipating possible solution strategies. I developed the Reframing a Math Task tool based on literature (Boaler et al., 2018a; Carpenter et al., 2015; San Giovani, 2016) to support teacher thinking on the needed elements of a rich math task. The full tool contained common questions, or concerns teachers had with reframing tasks (Appendix N). The following excerpt came from the third collaboration meeting as the team was beginning a fraction unit and wanted all students to generalize ideas around fractions.

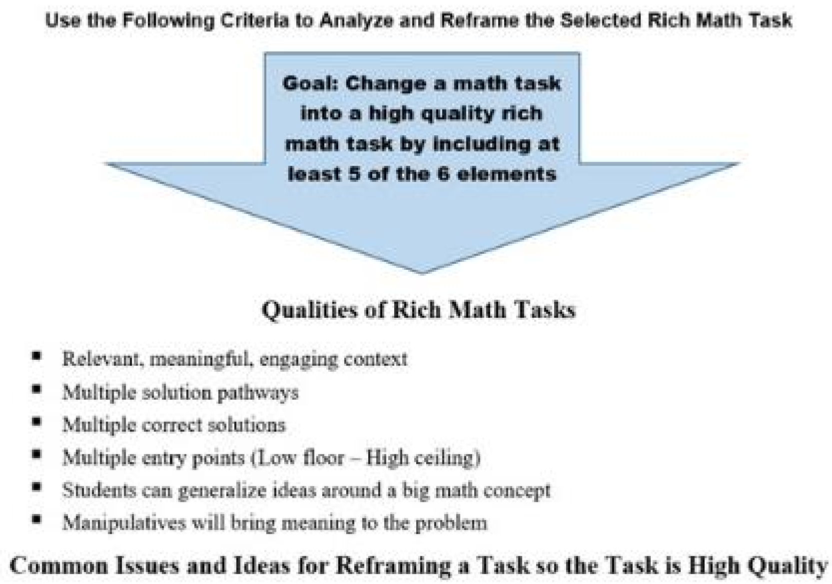

Figure 3. Reframe a Math Task.

Maggie (GET): If we are thinking about changing this task to be more engaging, what is [this task] missing? Looking at this handout Jenn gave us (Figure 3 and Appendix N). We should be thinking about a math task as being:

- Engaging, meaningful context
- Use of manipulatives and models can bring meaning to the problem
- Multiple solution pathways
- Potentially multiple correct solutions
- Students can generalize a large math concept
- All student can access the problem and develop their math thinking/reasoning

I think we should change the context from planting corn in rows to cutting a pan of brownies. I also think we allow students to select different numbers like 12, 24, or 48.

Holly (MI): Yes, cutting brownies is more engaging than planting rows of corn. I mean, why would our fifth graders care about that? But if we tell them that we are planning a party and need help figuring out all the different ways to cut the brownies, I think they will be much more engaged. And, I'll even bring in the pan of brownies for the kids to cut and share at the end of the week! This is a great way for the students to start generalizing ideas around fractions, unit fractions, and when the denominator doubles.

Maggie (GET): Ok, sorry, silly question here. What is a unit fraction? I guess I am confused about what we want the students to be thinking about over the course of this work.

Holly (MI): I'm thinking about the relationships between 12, 24, and 48 when the size of the whole, the pan of brownies, is unchanging. Like what happens to the pieces when the pan of brownies went from 12 to 24. Is there a quick way to go from 12 brownies to 24? A unit fraction would be 1/12, 1/24, or 1/48.

Jenn (Researcher/Coach): How about we take five minutes and solve this task as many ways as you can think of and then we will share our solution strategies with each other. The task can be something like; we need to share a pan of brownies for some people coming to a party. Decide if you are sharing the pan of brownies with either 12, 24, or 48 people. Then show all the ways you could equally share the brownies for the number of people you selected. We can decide on the exact wording of the task in a bit. Can you each tell me what number you selected so if there is a number left out I can solve for that one?

Chris (SET): So, we have the engaging context. How would Jose or Jon solve this problem?

Holly (MI): Maggie, I'm thinking the rainbow square tiles will help the kids make sense of the problem and persevere. Do you have those on hand? I want to use them now as I anticipate what Jose or Jon would do.

After this discussion, the Hillview team demonstrated different solution strategies, used the manipulatives as students may attempt to do, and discussed easiest to the most complex solution strategies.

This excerpt illustrated key patterns in how the teachers at Hillview unpacked the learning together. The teachers naturally clarified student work, clarified the big math concept, and moved from anticipating solution strategies to thinking about how to sequence solution strategies so all students could access different solution strategies (Smith & Stein, 2011, 2018). In this example, Maggie was unsure about unit fractions and was brave enough to ask for an explanation. Holly supported Maggie's thinking by discussing what exactly students should be learning. Holly also provided examples of unit fractions. I made an instructional move to take private think time and anticipate student solution strategies. The Hillview team supported each other in understanding the math content, and this was an essential element for the rest of the collaboration meeting.

This example also demonstrated my role as a coach in supporting the teachers with their thinking (Jaworski, 1998; Lewis, 2014). After we had discussed rich math tasks at the prior collaboration meeting, I made a handout (Figure 3) to support their thinking about the required elements for a math task to be considered a high-quality rich math task (Boaler et al., 2018a, 2018c; San Giovani, 2016). I created this handout as a future reference and as an anchor to analyze and evaluate tasks in the district-created math curriculum. As a coach, I find it is important to support teacher thinking to build capacity and independence for a new way of thinking, as I will not always be available at the time of need (Aguilar, 2013; Sweeney, 2011).

At this collaboration meeting, the team also spent a considerable amount of time discussing which students needed more status in the class by highlighting their thinking. Student perceptions of the composition of the class were that there were two groups of students, one group who did well in math, and the other group that did not do well in math. The team focused on equalizing status and helping all students to realize everyone in the class had powerful mathematical ideas worth sharing. All of this work stemmed from reframing a task to allow for an engaging context, multiple entry points, multiple solution pathways, multiple correct solutions, the use of manipulatives, and generalization of a big math concept. When teachers reframed a math task, teachers naturally wanted to solve the task. After solving the task, the teachers then thought about different solution strategies. Teachers discussed how those solution strategies leveraged mathematical thinking.

Reframed tasks at Oakview: Rebooting lesson design. During a collaboration meeting at Oakview, the teachers were reviewing an EngageNY (New York State Education Department, 2012) lesson on money. The lesson included 15 to 20 minutes of direct instruction provided by the teacher on coin names and values and then a worksheet in which students calculated the total value of coins presented in a row on the paper. There were seven such items on this worksheet (Figure 4). The team quickly realized this lesson met none of the criteria for a rich math task and began to reframe the task and the lesson to shift the focus away from direct instruction and more towards discovery and constructing knowledge about coins.

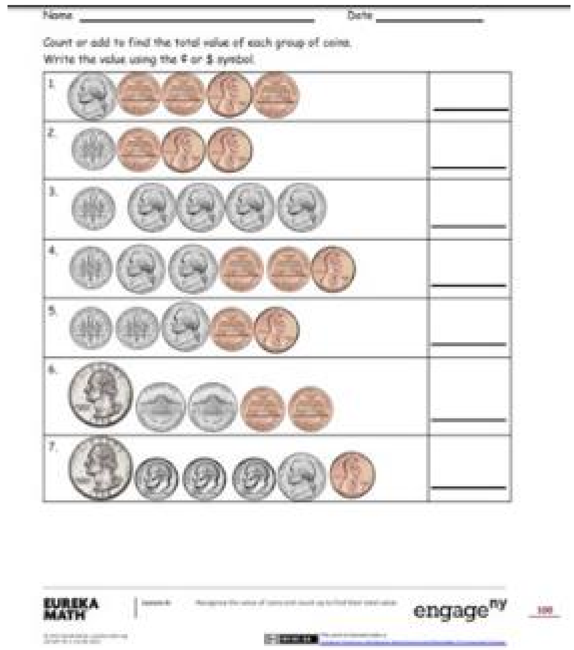

Figure 4. Engage NY Money Worksheet.

As the team talked, they decided to begin the lesson with students coming up to the front to share what they knew about different coins and values. After establishing some background knowledge, then teachers planned to model a quick way to draw and label coins with the values and initial of the coin name in the center of a circle. For example, to represent a quarter, the teacher and students would draw a circle with a Q and a 25 written in the center. The teachers planned to have students at the carpet in front of them and model how to draw the circles with coin values and initials. The teachers planned how students would draw five coins and label with the initial of coin name and the value. This was an intentional scaffold to help all students have access to coin values and names. Then the teachers discussed how to create an engaging and meaningful context. Allison came up with a story about going to buy a chocolate bar at the store and only having change in her pocket to pay for the candy bar. Allison suggested to the team the following task:

Allison (MI): The task could be, “I have less than 10 coins in my pocket of different values, and I bought the chocolate bar for one dollar. What combination of coins could I have used to buy the chocolate bar?” Then the follow-up question could be, “Find at least two other combinations of coins that would make one dollar.” Ohh! I know! Then the third follow-up question could be, “I have exactly 10 coins in my pocket, and I have two of each one, but I used less than five of the coins, now how did I pay for the chocolate bar?”

After Allison suggested this task, the team immediately went to work drawing and solving the task and anticipating what second graders might do (Smith & Stein, 2011). Through learning about rich math tasks during the professional development intervention, the team became efficient at reframed tasks with an engaging context, multiple entry points, multiple solution pathways, and multiple correct solutions. The team discussed not only using the plastic coins as a manipulative, but also how base ten blocks could help students keep track of coin values, and how to know if the total value of the coins equaled one dollar.

It is also worth noting here that Allison came up with the task and the series of follow-up questions. Allison knew that the second-grade standard was for students to use a variety of coins for adding to a hundred. Allison had the learning progression in mind. She fluidly thought about the daily work, the learning trajectory, and achievement of the big idea that students use coins to add to one hundred.

Reframed tasks at Three Peaks: Creating more access. The team at Three Peaks also realized the importance of reframed tasks to create more access to grade-level math content for all students. At their fourth collaboration meeting, the teachers discussed how, in previous years, the introduction to fractions with unlike denominators through pattern blocks had not been helpful for the majority of students in the class. Both teachers expressed concern about students developing a dislike of fractions because the introductory lessons did not allow students to build on what they knew or allowed for enough exploration of fraction concepts with unlike denominators. The pattern block lesson was the first lessons in a unit in which students worked with the addition of fractions with unlike denominators to total one whole. The original district-created lesson used the pattern blocks of a yellow hexagon, red trapezoid, blue rhombus, and green triangle. This lesson required students to look at a picture and write a fraction addition equation based on the shapes represented in the picture with the given information as one hexagon as the whole. In using pattern blocks in this way, one green triangle was the unit fraction of 1/6 of the whole hexagon. The trapezoid was ½ of the hexagon, and the rhombus was 1/3 of the hexagon.

During the collaboration meeting, Sally and Megan decided that for students to have a positive experience and be able to use what they knew about fractions with different denominators, they needed to reframe the task and allow for more discovery and exploration. Sally and Megan also did specific goal setting for student learning as part of the work of the reframed tasks. First, they established the goal for students to discover that fractions with different denominators could total a whole. The second big goal was for students to generate this idea and then have students discover a short cut for finding fraction equivalents. After discussing how to reframe the pattern block lesson, Sally and Megan decided to reframe the task.

> Sally (GET): I have an idea, we can create the context like, "My two daughters are arguing over who is right. Abby says there are less than five possible combinations to make a whole hexagon while Anna says there are more than seven combinations possible to equal one whole. Who do you think is right? Explain your thinking with pictures,

numbers, and words. Be sure to label your pattern block pieces with the fraction amount and write an equation showing how the pattern block pieces equal one whole."

When Sally presented students with some insider knowledge about Sally's daughters, the context was engaging for students to explore and discover which daughter was correct in her thinking. All students had access to the problem because students could use the pattern block manipulatives and begin laying pieces on top of the whole hexagon to see what pieces could be used together to total one whole hexagon. When the teachers created a context, used the actual pattern blocks, and left the solutions open-ended, all students had more access to the grade-level content. Both teachers worked out all the possible combinations ahead of time and made predictions about which students would think of certain combinations. The teachers also anticipated which combinations of pattern blocks might be more difficult for students to generate. The teachers generated probing questions to elicit thinking about other combinations and added these questions to the student-recording sheet that the teachers made (Smith & Stein, 2011).

I found that the examples of reframed tasks from all three locations were interesting because reframing the task led to a new beginning in teacher planning, not an end. When teachers took the time and reframed a task, and the task met the criteria for a rich math task (Carpenter et al., 2015; Boaler et al., 2018a, 2018b, 2018c; Flynn, 2017; San Giovani, 2016), the next step teachers naturally did was solve the task as students. As found by Smith and Stein, (2011, 2017) when teachers anticipated solution strategies, this naturally led to thinking about selecting and sequencing student work for the summary portion of the lesson. The role of selecting and sequencing student work becomes a foundation of math intervention embedded in core instruction (Smith & Stein, 2011, 2018).

Planning with the *Five Practices* (RQ2-2). Math interventionists, special education teachers, and general education teachers learned how reframed tasks provided access to core grade-level content for students with gaps in understanding. These teachers also learned how planning with the *Five Practices* (Smith & Stein, 2011, 2018) in mind also served a critical role in advancing the thinking of students with learning gaps. When teachers anticipated solution strategies (Smith & Stein, 2011, 2018) of students with the most math difficulty, the logical progression of student thinking was more evident to teachers. When teachers thought about how to select and sequence student work (Smith & Stein, 2011, 2018) for the lesson summary, teachers designed instruction that not only supported core instruction but also supported math intervention. When teachers co-designed math instruction with a focus on students addressing learning gaps, while also having access to grade-level content, thoughtful learning progressions embed math intervention in the core curriculum.

Five Practices at Hillview: Building a coherent learning progression. At Hillview, an initial task was planting rows of corn students, and the reframed task prompted students to split equally a pan of brownies of either 12, 24, or 48 pieces. With this initial reframed task and modeling from Holly on how to think in terms of a learning progression, Maggie planned follow-up tasks that built in coherence and sophistication.

> Maggie (GET): We could then ask, "From the pan of brownies cut into 12ths, if 1/6 of the pan of brownies were served, how many individual brownies were served?" Another task could be, "All three pans of brownies (12, 24, and 48 servings) were 1/3 gone. How many brownie pieces are left in each pan?"

In this example, Maggie demonstrated that she was thinking about how tasks could build on each other and how to get students to generalize fraction concepts. The learning-progression planning provided access to core content while also supporting students with unfinished fraction concepts that needed to be addressed. Maggie was not the only teacher to benefit from Holly's thinking and modeling about coherent planning and learning progressions. Chris also applied ideas of coherent planning and learning progressions when he suggested this task below.

> Chris (SET): "If 3 pans of brownies that are all the same size are cut into 12, 24, and 48 servings, what is happening to the size of the brownies?" and "Richard says one-half can never be smaller than one-fourth, do you agree or disagree" and "Can you find three other numbers that have the relationship between 12, 24, and 48? How can that relationship help you work with fractions?"

Chris began his suggestion with an entry point that he was sure all students in Maggie's class could access, (if the whole remained unchanged and the number of pieces in the whole increased then the size of the pieces decreased). However, he quickly built in sophistication, suggesting that students are given a task to wrestle with the question of if ¼ could ever be larger than ½. Chris also planned a follow-up task that holds students accountable for the relationships between 12, 24, and 48 and suggested that students find three other numbers with a similar relationship. Each follow-up task Chris suggested met the requirements for a rich math task, built upon each other in sophistication, and helped the teacher focus on the big learning outcomes. As the special education teacher on the team, Chris thought about providing access to the core general education math curriculum for students while also addressing learning gaps with fraction concepts from previous grade levels. With Chris' ideas on the table, Holly suggested another task that would also advance student thinking.

> Holly (MI): Something like this would be great to build off of what Chris just said and really get the students thinking about fraction equivalents. "Mrs. Maggie ate 3/12 of her

pan of brownies, Mr. Chris ate 6/24 of his pan of brownies, and Mrs. Holly ate 12/48 of her pan of brownies. Who ate the most? How do you know? What is the relationship between 3/12, 6/24, and 12/48?"

Reframing one task from planting rows of corn turned into a thoughtful learning progression that allowed for the teachers at Hillview to understand the goal of students generalizing fraction concepts. The teachers discussed the goals of the size of the whole matters, the ability to think proportionately, and understanding the relationship between fractions with different denominators. When teachers did not plan with a learning progression in mind, then ideas for students did not build in a meaningful way. Without a clear learning progression in mind, it is easy for teachers to forget the desired outcome of the lesson, which was a focus on students generalizing a big idea. In contrast, when teachers planned with a learning progression, the progression served as a road map for instruction.

Early into the professional development intervention, the Hillview teachers developed a trusting co-teaching relationship. The teachers shared similar goals for the students, similar goals in lesson planning, and the entire team took ownership in the learning-progression design process (Cochran-Smith & Dudley-Marling, 2012; Cook & Friend, 1995; Wilson & Blednick, 2011; Youngs et al., 2011). As a result, the Hillview teachers worked together and designed coherent learning progressions that provided access to core instruction and included embedded math intervention. However, I found that Holly was the only one on the team that consistently thought in terms of learning progressions and getting students to generalize the big idea. At Hillview, when the math content changed, Maggie and Chris reverted to old ways of thinking. In the following example, notice how Holly reminds Maggie and Chris to think about concepts and not skills.

Jenn (Researcher/Coach): Looking over the end of unit assessment, this fraction unit went very well. All students demonstrated a high level of understanding of fraction concepts as well as how to add and subtract fractions with different denominators. I mean, really, I don't think we even need an intervention group with this content. Holly, what do you think?

Holly (MI): I agree! Even the students with the most unfinished learning like Mary and Ester came so far! Let's focus our energy on the new unit like we did last time.

Jenn (Researcher/Coach): What's the next unit about in LearnZillion [the district-created math curriculum]?

Maggie (GET): Looking at the scope and sequence, the next unit is about multiplying and dividing fractions.

Chris (SET): Teaching how to multiply fractions is easy because you just show the kids how to multiply straight across and then they have the answer. But teaching students how to divide fractions is so hard! There are so many steps for them to learn. I made a chart for my other 5th graders that gives them a list of steps to follow.

Holly (MI): Have you guys looked very much at this book Jenn gave us? [Holly holds up Extending Children's Mathematics: Fractions & Decimals: Innovations in Cognitively Guided Instruction [(Empson & Levi, 2011)]. There are several chapters in here on helping students to develop conceptual understanding for how to divide fractions. The first big idea is dividing a whole by a fraction and then moving into dividing a fraction by a fraction. I have used these examples in the past, and it is pretty amazing how students can develop conceptual understanding for the invert and multiply algorithm.

Maggie (SET): Can you give me an example? I guess I am confused what you mean by dividing by a whole and the invert and multiple rule.

Holly (MI): Yes, say we have three pizzas, and we want to divide them in fifths. How many pieces would we have? The whole number is three divided by a unit fraction of 1/5. The equation would be 3 ÷ 1/5 =? Using the standard algorithm for dividing by a fraction (invert and multiply), this equation becomes 3 x 5 or 15. But given this context, students can draw three pizzas, cut each into five pieces, and see visually how there would be a total of [sic] 15 pieces of pizza. If you do enough tasks like this, students will generate the shortcut on how to divide fractions.

Maggie (GET): Thank you. That example really helped. So it looks like the first lesson in the district-created curriculum on dividing fractions is a whole number by a unit fraction. Well, that fits with what Holly was saying and the resource Jenn gave us.

Chris (SET): So we are going to show them how to invert and multiply? The worksheet for this lesson has a bunch of division problems on it.

Holly (MI): We may need to reframe this lesson into a rich math task like we did before if we are finding that the work doesn't really align with what we know about student learning. Let's reframe the task to have a context in which kids can draw a picture to represent what is happening. Maybe something like, "Six apples are divided in half. How many pieces are there? Represent the problem with an equation." Maybe some students will write 6 ÷ ½ = 12. Maybe some students will write 6 x 2 = 12. Then we can ask great questions during the summary to get students to think about the connection between these two equations. Not that apples are all that exciting to fifth-graders, but we could change the context to be something they care more about. Maybe there are 6 containers of fluffy slime split is half?!?

[The group laughs with the suggestion of an engaging context.]

When the addition and subtraction of fractions with unlike denominators unit ended, and a new multiplication and division fraction unit began, Chris and Maggie talked about how to break the learning down into manageable pieces for the students. With the introduction of new content, Chris and Maggie reverted to a prior way of thinking. Holly needed to remind Chris and Maggie to maintain the research-based math instructional approaches and apply these ideas to the new content of multiplying and dividing fractions.

Holly refocused the group to keep the ideas centered on big concepts, not isolated skills. Holly reminded Chris of the researched-based resources to use while planning the new math unit. Holly also reminded the group to design tasks with not only an engaging context but also where students could represent the problem with a visual model.

Five Practices at Oakview: Remaking a learning progression. At Oakview, the two math interventionists, Allison and Paige, tended to think in terms of learning progressions while the two general education teachers Cheryl and Joan tended to think in terms of the day-to-day lessons of the EngageNY curriculum (New York State Education Department, 2012) (Table 7). During the collaboration meetings at Oakview, the reframed task with counting money and adding up to 100 also served as the foundation for the creation of an intentional learning progression. Allison and Paige guided the work in the design of the learning progression. Table 8 is a summary of the learning progression crafted by the Oakview team. All four of the teachers had a voice in the created learning progression. Table 8 included the learning progression of the EngageNY materials (New York State Education Department, 2012) and then provided the learning progression of the reframed tasks. Notice the low-level addition problem types, as well as lower-level recall skills in the EngageNY (New York State Education Department, 2012) curriculum versus the high-leverage thinking required in the reframed tasks.

Table 8

Oakview Money Learning Progression

Entry Point	Lesson 1	Lesson 2	Lesson 3	Lesson 4	Generalizable Big Idea
EngageNY	Name coins and values	Add coins together to find a total	Select coins to equal a given total	Solve addition join result unknown story problems using coins	Students can name coins and find total values
Reframed Task	Less than 10 coins equal one dollar. Which coins were used to buy the chocolate bar for one dollar?	10 coins – two of each value but only 5 were used to buy the chocolate bar, which coins were used?	Allison used one dollar to buy 82 penny candies. What coins did she get back?	Allison had some money. She bought a 53 cent ball and a 29 cent pencil. She had 43 cents left. How much money did she start with?	Students connect coins to adding and subtracting in chunks up to 100. Students connect the inverse relationship of addition and subtraction

Over time and through lesson observation, it became clear that Cheryl and Joan did not have ownership or understanding of the newly created learning progression (Table 8). Even with a clear plan made during the collaboration meetings (Table 9), the newly reframed tasks situated in the learning progression never came to fruition in Cheryl's or Joan's classroom. Instead, the enacted lessons resembled the EngageNY (New York State Education Department, 2012) lessons familiar to Cheryl and Joan. Below is a table with the planned lessons from the learning progression and the enacted lessons from Cheryl and Joan classrooms.

Table 9

Planned and Enacted Lessons in the Classrooms of Cheryl and Joan

Planned and Enacted Lessons	Cheryl	Joan

Planned Lesson 1	Less than 10 coins equal one dollar. Which coins were used to buy the chocolate bar for one dollar?	Less than 10 coins equal one dollar. Which coins were used to buy the chocolate bar for one dollar?
Enacted Lesson 1	Direct instruction of coin values and names. Students completed the worksheets independently (see Figure 4).	Direct instruction of coin values and names. Teacher modeled and demonstrated as teacher and students completed the worksheets together (see Figure 4).
Planned Lesson 2	10 coins – two of each value but only 5 were used to buy the chocolate bar, which coins were used?	10 coins – two of each value but only 5 were used to buy the chocolate bar, which coins were used?
Enacted Lesson 2	Direct instruction on how to skip count and add coins together to find a total. Students complete three worksheets.	Direct instruction on how to skip count and add coins together to find a total. Students complete two worksheets.
Planned Lesson 3	Allison used one dollar to buy 82 penny candies. What coins did she get back?	Allison used one dollar to buy 82 penny candies. What coins did she get back?
Enacted Lesson 3	Direct instruction on selecting coins from largest value to smallest value for a given total. Students completed worksheets.	Direct instruction on selecting coins from largest value to smallest value for a given total. Students completed worksheets.
Planned Lesson 4	Allison had some money. She bought a 53 cent ball and a 29 cent pencil. She had 43 cents left. How much money did she start with?	Allison had some money. She bought a 53 cent ball and a 29 cent pencil. She had 43 cents left. How much money did she start with?
Enacted Lesson 4	Direct instruction on how to solve addition join result unknown story problems. Direct instruction on problem-solving strategies. Students completed worksheets independently.	Direct instruction on how to solve addition join result unknown story problems. Direct instruction on problem-solving strategies. Teacher and students completed worksheets together.

Although Cheryl and Joan planned reframed lessons with Allison and Paige that shifted instruction away from a direct instruction approach (Hiebert & Grouws, 2007; Hiebert & Stigler, 2000), the enacted lessons used direct instruction as the primary delivery method. The planned lesson included the use of an L-E-S lesson structure (Lappan et al., 2007; Schroyer & Fitzgerald, 1986); however, the resulting lessons used a heavy scaffolded gradual release of responsibility lesson format. In the planned lessons, students had opportunities to explore concepts and generalize ideas about coins and addition and subtraction. In contrast with the enacted lessons, the requirement of students was to reproduce the modeled and demonstrated strategy by the teacher. The learning progression from the enacted lessons did not help students construct valuable mathematics understanding, and also, the enacted learning progression did not support Cheryl or Joan in thinking about the big ideas their 2nd-grade students should learn.

Five Practices at Three Peaks: Downgrading a learning progression. At Three Peaks, Sally mostly planned in terms of learning progressions, while Megan often thought in terms of isolated skills or how to break the learning down into even smaller pieces (Table 7). The initial task that Sally and Megan reframed was a worksheet in which students were to determine the fraction amounts for a given pattern blocks within a hexagon, with the hexagon representing the whole. The reframed task was more open-ended in that students were to generate combinations of pattern blocks that totaled one whole hexagon. The newly created learning progression that Megan and Sally designed was more limited in scope because rather than focusing on big fraction concepts, their developed learning progression focused on fractions with different parts composing the whole, fractions as a set, and fractions on the number line. This narrow learning progression also met the narrow goals that Megan had established during the first collaboration meeting for her students on IEPs. The learning progression began with the pattern block lesson, moved into finding a fraction of a set, and then ended with finding and labeling fractions on a number line. At the first collaboration meeting, Megan clearly stated that the goal for students on IEPs was to represent fractions in shapes, fractions within a set, and fractions on a number line. The learning progression that Sally and Megan designed met Megan's learning outcomes for the students who received special education support but did not lead to students generalizing larger concepts about fractions. In this case, the learning progression was more about performing tasks with fractions in three different ways, rather than understanding fundamental concepts of fractions. Although the learning in the progression that Sally and Megan created used conceptual understanding, the way they approached the learning changed the learning from being student-created to more teacher-directed with a new set of procedures in working with fractions. For example, Megan created a systematic process for finding fractions within a set for students with IEPs. Systematic directions on how to find fractions within a set of objects reduced this conceptual math thinking to a new procedure. The way Sally and Megan designed the learning progression provided access to core grade-level content but did not allow students to generalize big fraction concepts from previous grade levels. The learning progression Sally and Megan designed lacked embedded math intervention for students with unfinished learning with fraction concepts. Even though the initial pattern block tasks were high-leverage, the result was a downgraded learning progression with newly introduced procedures.

At each location, the reframed tasks became an entry point into anticipating student solution strategies (Smith & Stein, 2011, 2018) and eventually led to creating a learning progression. The created learning progressions served as a guide for the teachers on the concept of how to push students during the "generalize and connect" portion of the lesson summary. It was evident in the data that when teachers knew the goal for student thinking in the summary portion of the lesson, and even when teachers enacted the summary portion of the lesson, effectively enacted lesson summaries do not automatically occur.

Support Provided for Teachers through PD (RQ3)

The first finding to RQ3 indicated that lesson summary enactment (Smith & Stein 2011; 2017) was difficult for teachers to approximate even with focused and intentional professional development (Aguilar, 2013; Sweeney, 2011). Lesson summary enactment occurred in 26% of lessons observed for this study. The second finding indicated that even with the professional development on lesson summary enactment, teachers tended to reduce the summary portion of the lesson to an Initiate-Respond-Evaluate (I-R-E) pattern with funneling types of questions (Cazden, 2001; Herbel-Eisenmann & Breyfogle, 2005; Mehan, 1979; Zrike & Connelly, 2015). Teachers struggled to implement the discourse moves from the *Five Practices* (Smith & Stein, 2011, 2018) they learned about during the professional development intervention.

Lesson summary enactment (RQ3-1). The third and final portion of a math lesson is the lesson summary (Lappan et al., 2007). Although the term summary is often used, the idea is not to review or summarize what occurred during the explore portion of the lesson but rather to have students generalize and connect mathematical ideas (Boaler et al., 2018a, 2018b, 2018c) through student presentations and horizontal discourse (Smith & Stein, 2011, 2018). The generalize and connect portion of the lesson is the heart of the math workshop and is when students have the opportunity to voice connections between solution pathways, articulate new thinking, and formalize ideas (Smith & Stein, 2011, 2018). Although the concept of having students generalize and connect ideas seems simple and easy enough, findings indicated that actual completion of the summary portion of the lesson is much harder for teachers to actualize during a math workshop lesson than would be expected.

The table below (Table 10) summarizes the number of collaboration meetings attended by each participant and the number of rich math tasks the team planned during the collaboration meetings. The number of lessons I observed at that location and the number of times I modeled the summary portion of the lesson is included in the table. I also included the number of times the teacher enacted the summary portion of the lesson independently. The total number of reframed or created math tasks was the same for each location because this work happened together as a team. The final three columns demonstrate how many times I supported the teacher with the lesson summary. I did this by either on the spot coaching or a co-taught lesson summary. The final column indicated the number of times the participant enacted the planned lesson summary independently without asking for support from myself or another co-teacher in the room. The total number of lessons I observed was 84.

Table 10

Overview of Lesson Summary Enactment at Three Locations

Teachers	Number of provided PD instructional sessions	Number of Collab. Meetings Attended	Total Number Reframed Rich Math Tasks	Number of Lessons I Observed/ Participate d in	Number of Times Participant had an opportunity to enact the Summary	Frequency of Summary Support for Participant	Number of Lessons with a complete LES sequence
Holly	12	12	42	35	12	0	4
Chris	12	11	42	35	6	3	2
Maggie	12	12	42	35	17	10	8
Cheryl	12	12	20	18	12	0	0
Joan	12	10	20	16	12	0	0
Allison	12	12	20	18	6	3	3
Paige	12	11	20	16	4	1	3
Sally	9	9	17	15	10	3	2
Megan	9	9	17	15	5	0	0

At the Hillview location with Holly, Chris, and Maggie, I observed 35 lessons because the three teachers co-taught math lessons together. At the Oakview location, I observed 18 lessons with Cheryl and Allison because they were a co-teaching pair. At that same location, I also observed 16 lessons with Joan and Paige because they were a co-teaching pair. The total number of independent lesson summary enactments was 22. Lesson summary enactment occurred in 26% of total lessons observed for this study. Five of the nine teachers elicited my support to enact the summary portion of the lesson either during the collaboration meeting or while the lesson was in progress. Of these five teachers, I supported lesson summary enactment for about half of the total opportunities that the teacher had for lesson summary enactment. Even with coaching at the time of need, teachers struggled with independently enacting the summary portion of the lesson.

Lesson summary enactment at Hillview: An exemplar demonstration. Below is an excerpt from a lesson summary facilitated by Holly at Hillview. In this example of the summary portion of the lesson, Holly facilitated student thinking in developing connections between the relationships of 12, 24, and 48 and the constraint of the brownie pan size or the brownies.

> Holly (teacher): Ok, we have just heard from several students and learned how they cut the pan of brownies for 12, 24, or 48 brownies. What I am wondering now is what connections did you make between the numbers 12, 24, and 48?
>
> Mary (student): I noticed that the pieces got smaller!
>
> Holly (teacher): Tell me more about that.
>
> Mary (student): Since the pan of brownies was the same size, when you had to have more brownies, the pieces got smaller.
>
> Holly (teacher): Interesting! What else did you notice?
>
> Luis (student): I was thinking about how 12 times 2 is 24 and 24 times 2 is 48.
>
> Jon (student): I noticed that too. The numbers were doubling.
>
> Holly (teacher): Mary do you agree with Jon and Luis?
>
> Mary (student): Yes! And now I think I see something new. I think when the amount of brownies doubled the size of each brownie got smaller. I think smaller by half. Yes, they are half as small when you go from 12 brownies to 24 brownies.
>
> Emily (student): I noticed something too that was tricky about the paper and the rainbow tiles. The paper was the constraint representing the pan of brownies, so it stayed the

same, but when we used the rainbow tiles as the brownies, the size of the brownies stayed the same, so the area of the pan grew.

Holly (teacher): Whoa, whoa, whoa! Hold the phone! You just said some important things here. Luis, can you tell us more about what Emily is saying?

Luis (student): Can I come up to the document camera and show it? [Luis comes to the front of the room and lays 12 rainbow tiles on top of the pre-cut paper.] See how these 12 brownies fit perfectly in this pan? But now let me use more rainbow tiles to get to 24 brownies. Now I have two pans of brownies because the rainbow tiles were like the brownies and stayed the same size. But that can't happen. We can't have 2 pans of brownies. What needed to happen was the brownies needed to get smaller. [Luis removes 12 tiles, so the original 12 are on top of the pre-cut paper and then uses a whiteboard marker to draw on the rainbow tiles.] See, how I cut the 12 brownies in half and now the pan is the same size, but I have 24 brownies.

Holly (teacher): Craig, can you tell a bit more about what Mary and Luis are noticing?

Craig (student): What I want to say, but I'm not sure if this is right or not, is that when you cut something in half, it doubles the amount. Each time the brownies got cut in half the number of brownies doubled. Like what Luis said about doubling.

Holly (teacher): Right now Mrs. Maggie and Mr. Chris are passing out post-it notes to you. I want you to write your name on the post-it, then write down what you think about Craig's idea that when something gets cut in half, it also doubles. Feel free to use pictures, numbers, and words to explain your thinking. Your post-it note turned into the back table is your ticket to line up for PE.

In this lesson summary example, Holly was masterful at moving student thinking along and getting all students to think about and connect the big idea. Notice how she leveraged student thinking and held other students accountable for articulating the thinking of others. Both Emily and Luis qualified for math intervention, and Holly elevated Emily's status by highlighting her thinking as something worth examining on a deeper level. Luis also had his status increased by demonstrating what Emily was saying about what happens when the brownie pan was the constraint, or the size of each brownie (rainbow tiles) was the constraint. Holly had a clear understanding of what ideas she wanted students to generate at the end of the summary portion of the lesson. The way Holly thought about lesson planning, the materials she used to help her plan, allowed her to enact a lesson with the desired outcome of students generalizing big ideas about half and double by the end of the lesson.

Lesson summary enactment at Three Peaks: A missed opportunity. In contrast, in this next example from Three Peaks, Sally was not as clear in her learning goals for the students, and this was evident in her lesson summary enactment. Notice how many questions Sally asked and how the questions are a depth of knowledge level 1 of recall, so student answers were short and basic.

Sally (teacher): Ok, so what did we do today?

Mark (student): We worked with different shapes.

Sally (teacher): Yep. Good. What did you learn?

Hailey (student): That you can figure out area and perimeter for different rectangles.

Sally (teacher): Yes! Good! What does area mean?

Luke (student): When you multiply the two outside edges to find the inside part.

Sally (teacher): How is that different than perimeter?

Mary Beth (student): That's adding the outside instead of multiplying.

Sally (teacher): So how is [sic] area and perimeter different?

Greg (student): One uses adding, and the other one uses multiplication.

Sally (teacher): Which one is which?

[Long pause with wait time.]

Sally (teacher): Perimeter is when you add up all the outside lengths to find out how long the shape is all the way around. Area is when you multiply the length times the width to find the space or area inside the rectangle. Ok, well it is time for us to go to [sic] library. Put your math journals away please and show me you are ready to line up.

In this lesson summary enactment excerpt from Sally at Three Peaks, an IRE pattern emerged (Cazden, 2001; Herbel-Eisenmann & Breyfogle, 2005; Mehan, 1979) as Sally prompted students to answer questions about area and perimeter. Notice how the amount and frequency of questions Sally asked compared to Holly. Sally spoke more than half the time in this excerpt while Holly spoke considerably less. Holly required students to listen and understand the thinking of anyone in the class. The lesson summary enactment in Sally's classroom was only vertical, teacher-student-teacher, discourse. Although this lesson summary enactment lacked focus and rigor, the lesson summary content did contain mathematical ideas.

Lesson summary enactment at Oakview: A non-example. In this next lesson summary enactment example from Oakview, the teacher provided positive feedback to students about stamina and growth mindset (Boaler, 2016). The lesson summary itself was void of student discourse and any presentation of mathematical ideas.

Cheryl (GET): Ok boys and girls, meet me at the carpet. I would like to talk to you about your math work you did today. [Students gathered at the carpet.] I am proud of how long you worked today on your money worksheets. I know they were hard, but you just tried your best, and you worked for a very long time. I was pleased with Roberto and Cruz for staying calm when the problems seemed too hard. I noticed both boys just kept trying different ideas or asking for help when they got stuck. I also saw Camila go to the peace table for a break when she was getting upset. These were all great strategies for working quietly for a long amount of time. I am proud of your effort today. Now, let us get ready to go to lunch.

In this example, Cheryl praised the students for working independently on their math worksheets, but she never mentioned any math content, and she also did not involve the students in the reflection on the independent work time. After the students went to lunch, I sat with Cheryl, and we discussed the lesson summary enactment. Below is an excerpt from what she had to say.

Jenn (Researcher/Coach): So tell me what you thought about the lesson summary today.

Cheryl (GET): Well, first off, I noticed I only had a few minutes before lunch. So I thought I needed to make it quick. So I decided to pull the students over to the carpet and praise them for the job they did in working quietly and independently on their money worksheets.

Jenn (Researcher/Coach): If you had more time, would you have done anything differently?

Cheryl (GET): I'm not really sure. Even though we have talked about the lesson summary in our collaboration meetings and I have watched Allison do it a few times I am always apprehensive. I don't know how to get the kids talking about their work and I'm not even convinced that it is important. I guess it seems kinda pointless to me. They know about the work they did, and I do too, so going over it again seems to be beating a dead horse a bit.

Jenn (Researcher/Coach): I understand what you are thinking. You mentioned that you aren't sure the lesson summary is important. Would you like me to put an article in your box about the impact of a lesson summary or flag some pages from the *Five Practices* [(Smith & Stein, 2011)] book that I gave you when we started this project together?

Cheryl (GET): Yes, that would be nice.

Jenn (Researcher/Coach): Ok, I will do that before I leave here today. Thank you for your time.

In this example, Cheryl developed a clear picture of why lesson summary enactment is not occurring in her classroom. In the above excerpt, she outlined first the constraint of time, but when pressed about this, she articulated that she was unsure of the point of the lesson summary. Cheryl implied the lesson summary was a review of the work students completed during the independent work time. Cheryl does not yet have the understanding of large grain size lesson planning or learning progressions that lead students to a big generalizable concept. Even with the professional development intervention, and the lesson summary being modeled by a peer, Cheryl is not ready to approximate the lesson summary. In this example, I also transitioned from researcher to coach (Jaworski, 1998; Lewis, 2014) by providing resources to support Cheryl's thinking about the reasons and importance behind a high-quality lesson summary.

With the three different lesson summary enactment examples presented, it was evident that lesson summary enactment looked many different ways. First, there was Holly's example, focused, clean, and swiftly moved students to the big idea. Next, there was Sally's example in which she asked multiple narrowly focused questions that left the mathematical conversation to wander aimlessly. Finally, there was Cheryl's example, which lacked any discussion of mathematical content. One reason that Holly's lesson enactment was so concise was that Holly had a clear and defined outcome in mind stemming from the reframed task.

Discourse in the lesson summary (RQ3-2). I examined the discourse in the lesson summary of the 22 lessons that contained an actual lesson summary. Consistent with the literature, a lesson summary is considered a lesson summary when the class is pulled together as a whole and students are required to reason, connect, and make sense of each other's thinking (Lappan et al., 2007; Schroyer &Fitzgerald, 1986; Smith & Stein, 2011, 2018). I defined a lesson summary in terms of the teacher pulling the class together as a whole and requiring students to discuss their work from the explore portion of the lesson. I did not require evidence of the high-leverage features of sharing student reasoning, connecting mathematical ideas, or making sense of thinking from another student when considering if the lesson did contain a lesson summary. If the class came together as a whole group, but no discussion of mathematical content occurred, I did not count the lesson as having a lesson summary.

Out of the 22 lessons with a lesson summary, I found that discourse in 14 of the lesson summaries contained only a vertical teacher-student-teacher discourse pattern. The other eight lesson summaries contained a mix of vertical discourse and horizontal student-to-student discourse. In those eight lessons, with vertical discourse, the teacher used a discourse move to elicit student thinking and hold students in the class accountable for understanding the assertions and claims of other students. The vertical teacher-student back and forth pattern resulted in the teacher asking low-level, mostly recall types of questions. In the vertical discourse pattern, students were not required to articulate or restate the thinking of others.

Lesson summary discourse at Hillview: Using high-leverage discourse moves. The example below was used prior in this chapter as a strong example for lesson summary enactment. Notice the discourse moves Holly made in holding students accountable for understanding the reasoning of others. Holly used three out of five discourse moves highlighted by Smith and Stein (2011; 2017). These discourse moves included asking a student to restate the thinking of another student, asking a student to apply their reasoning to the reasoning of another student, and prompting students for further participation.

> Holly (teacher): Ok, we have just heard from several students and learned how they cut the pan of brownies for 12, 24, or 48 brownies. What I am wondering now is what connections did you make between the numbers 12, 24, and 48?
>
> Mary (student): I noticed that the pieces got smaller!
>
> Holly (teacher): Tell me more about that.
>
> Mary (student): Since the pan of brownies was the same size, when you had to have more brownies, the pieces got smaller.
>
> Holly (teacher): Interesting! What else did you notice?
>
> Luis (student): I was thinking about how 12 times 2 is 24 and 24 times 2 is 48.
>
> Jon (student): I noticed that too. The numbers were doubling.
>
> Holly (teacher): Mary do you agree with Jon and Luis?
>
> Mary (student): Yes! And now I think I see something new. I think when the amount of brownies doubled the size of each brownie got smaller…I think smaller by half. Yes, they are half as small when you go from 12 brownies to 24 brownies.
>
> Emily (student): I noticed something too that was tricky about the paper and the rainbow tiles. The paper was the constraint representing the pan of brownies, so it stayed the same

but when we used the rainbow tiles were used as the brownies, the size of the brownies stayed the same, so the area of the pan grew.

Holly (teacher): Whoa, whoa, whoa! Hold the phone! You just said some important things here. Luis, can you tell us more about what Emily is saying?

Luis (student): Can I come up to the document camera and show it? [Luis comes to the front of the room and lays 12 rainbow tiles on top of the pre-cut paper.] See how these 12 brownies fit perfectly in this pan? But now let me use more rainbow tiles to get to 24 brownies. Now I have two pans of brownies because the rainbow tiles were like the brownies and stayed the same size. But that can't happen. We cannot have two pans of brownies. What needed to happen was the brownies needed to get smaller. [Luis removes 12 tiles, so the original 12 are on top of the pre-cut paper and then uses a whiteboard marker to draw on the rainbow tiles.] See, how I cut the 12 brownies in half and now the pan is the same size, but I have 24 brownies.

Holly (teacher): Craig, can you tell a bit more about what Mary and Luis are noticing?

Craig (student): What I want to say, but I am not sure if this is right or not, is that when you cut something in half, it doubles the amount. Each time the brownies were cut in half the number of brownies doubled. Like what Luis said about doubling.

Holly (teacher): Right now Mrs. Maggie and Mr. Chris are passing out post-it notes to you. I want you to write your name on the post-it, then write down what you think about Craig's idea that when something gets cut in half, it also doubles. Feel free to use pictures, numbers, and words to explain your thinking. Your post-it note turned into the back table is your ticket to line up for PE.

In this example, each time Holly spoke, she used a discourse move to require students to reason, connect, or make sense of each other's thinking. After Jon spoke, Holly asked Mary if she agreed with Jon's thinking. This discourse move is referred to as asking a student to apply their own thinking to another student's thinking (Smith & Stein, 2011, 2018). Holly used a different discourse move after Emily spoke, requesting Luis to make sense of Emily's thinking. Holly was asking Luis to restate Emily's thinking (Smith & Stein, 2011, 2018) and Luis responded by demonstrating what Emily said with the paper and manipulatives. When Holly called on Craig to restate Mary and Luis' thinking, Craig came out with an assertion that classifies as a large generalizable idea. Holly then required all students to write a note about what they were thinking about Craig's idea. This instructional move required all students to participate in Craig's claim that when something is cut in half, it doubles. Notice that this claim came from the culmination of student thinking and did not result from the teacher telling the students what to think.

Lesson summary discourse at Three Peaks: Using low-level discourse moves. I presented this example earlier in this chapter. Notice in contrast to Holly's example of how the entire excerpt is vertical discourse. The excerpt below identifies a classic IRE (Cazden, 2001; Henningsen & Stein, 1997) discourse pattern. This type of discourse pattern does little to advance student thinking or understanding (Cai, 2010; Cazden, 2001; Henningsen & Stein, 1997; Turner et al., 2011).

Sally (teacher): Ok, so what did we do today?

Mark (student): We worked with different shapes.

Sally (teacher): Yep. Good. What did you learn?

Hailey (student): That you can figure out area and perimeter for different rectangles.

Sally (teacher): Yes! Good! What does area mean?

Luke (student): When you multiply the two outside edges to find the inside part.

Sally (teacher): How is that different than perimeter?

Mary Beth (student): That's adding the outside instead of multiplying.

Sally (teacher): So how is [sic] area and perimeter different?

Greg (student): One uses adding, and the other one uses multiplication.

Sally (teacher): Which one is which?

[Long pause with wait time.]

Sally (teacher): Perimeter is when you add up all the outside lengths to find out how long the shape is all the way around. Area is when you multiply the length times the width to find the space or area inside the rectangle. Ok, well it is time for us to go to [sic] library. Put your math journals away please and show me you are ready to line up.

The discourse move that Sally used was that of wait time (Smith & Stein, 2011, 2018). However, after Sally asked and waited for students to clarify how to calculate area and perimeter, she told the students the math content and did nothing to require students to make sense of this information. Telling students of solution strategies actually downgrades the task (Cai, 2010; Turner et al., 2011).

Lesson summary discourse at Hillview: Using high-leverage and low-level discourse moves. This third lesson summary example came from Maggie at Hillview. This was one of the first lesson summaries that Maggie attempted facilitating herself. This excerpt took place after the students had been working with dividing a whole number by a unit fraction for the past four lessons. Maggie made an anchor chart (Table 11) with a list of the last four tasks and the equations students generated to solve those tasks. In this lesson summary, Maggie pulled all the students together and asked the students to notice and make sense of the equations.

Table 11

Maggie's Dividing with Unit Fractions Anchor Chart

Lesson Progression	Task	Student Equations
Task 1	12 jars of fluffy slime were split in half. How many halves were there? Write an equation that shows what happened in this story.	$12 \div \frac{1}{2} = 24$ $12 \times 2 = 24$
Task 2	8 pizzas cut into four slices per pizza. How many total pizza slices?	$8 \div \frac{1}{4} = 32$ $8 \times 4 = 32$
Task 3	7 cakes were cut with 8 equal pieces for each cake. Write an equation to show how the cake was divided. How many total pieces of cake were there?	$7 \div 1/8 = 56$ $7 \times 8 = 56$
Task 4	There were 112 bags of beads. Each person got 1/3 of the bag to make friendship bracelets. How many people could make friendship bracelets?	$112 \div 1/3 = 336$ $112 \times 3 = 336$

Maggie (teacher): Ok, look here, guys. I made this chart for you from the last several days of math work that we completed (Table 10).

[Maggie read the chart to the class.]

Maggie (teacher): Look carefully at these equations. What do you notice?

Jon (student): It doesn't matter if the equation is multiplication or division, the total ended up being the same number in each box.

Luis (student): Yeah, Jon, I see what you are saying, but it is weird. I mean. What is happening here? One equation is multiplication, and one is division. Both are using the same numbers. How are the two equations ending up to be the same total? I mean, shouldn't one equation be getting bigger if you are multiplying and one equation be getting smaller if you are dividing?

Maggie (teacher): What do you guys think about what Luis is saying?

Emily (student): Well, I get what Luis is saying. Usually, when you divide the number gets smaller. But we already kinda figured out that when you are working with fractions when things get divided into parts, then the number of parts gets bigger. Like in the fluffy slime example. We divided each container in half, so we ended up with 24 halves.

Maggie (teacher): So Emily, so you think that works only with the fraction of ½ or it works with all fractions?

Emily (student): Well, if you look at our chart, I think it works with all fractions, but it still seems hard to prove.

Maggie (teacher): Craig, what do you think about what Emily is saying?

Craig (student): I agree with Emily. I think we still need to test with more numbers. I am wondering about if something gets split by a weird fraction like 2/5, then what happens?

Maggie (teacher): Ohh, so what you are saying is we need to examine some other fractions? I guess I am not clear, what idea are we testing exactly?

Ester (student): Well, when you really look at the chart, it seems like there is this opposite – opposite pattern.

Maggie (teacher): Opposite – opposite pattern? Because multiplication and division are opposites?

Ester (student): Yes.

Maggie (teacher): And ½ and 2 are opposites?

Ester (student): Yes.

Maggie (teacher): ¼ and 4 are opposites?

Ester (student): Yes.

Maggie (teacher): 1/8 and 8 are opposites?

Ester (student): Yes.

Maggie (teacher): 1/3 and 3 are opposites?

Ester (student): Yes.

Maggie (teacher): So two opposites together end up being the same?

Ester (student): Yes….ummmm….maybe….I think so?

Maggie (teacher): Do you think this opposite – opposite pattern is true, Jon?

Jon (student): From the chart, it looks that way.

Maggie (teacher): Is multiplication and division opposite of each other?

Jon (student): Yes.

Maggie (teacher): [nods head, yes]. Are fractions and whole numbers opposite of each other?

Ester (student): It seems like they are.

Maggie (teacher): Ok Ester. Maybe we need to look at this pattern more. Right now let's get ready to go to music. Class, please line up at the door.

In this example, Maggie started out strong with the anchor chart (Table 10) she had made ahead of time to facilitate students generalizing a big math concept. The students began to notice patterns in the equations, and the students had some horizontal discourse. Maggie also used the discourse move of requesting a student to make sense of another student's thinking (Smith & Stein, 2011, 2018) when she asked Craig to make sense of Emily's thinking. When Ester talked about the opposite-opposite rule, Maggie lost her focus on the students being able to articulate the inverse rule. It was at this point in the lesson summary that the discourse pattern shifted to that of an IRE pattern (Cazden, 2001). Once the IRE pattern (Cazden, 2001) emerged, Maggie lost a sense of the direction of the lesson summary.

Teachers can increase connections and concepts through leveraging discourse via the teachers' instructional moves, as we saw in Holly's example. Consistent with other research, teachers can also downgrade a task through telling of solutions or by simplifying the task as we saw in Sally's example (Cai, 2010; Turner et al., 2011). The lesson summary does not necessarily have to fall into one of these two categories, as was evident in Maggie's example. Sometimes the lesson summary can be a mix of powerful discourse moves as well as include an IRE discourse pattern (Cazden, 2001).

Out of the 84 observed lessons, only eight used a high-leverage mathematical discussion in which students were required to make sense of other student's thinking. Even though the teachers participated in the professional development, and learned about high-leverage discourse moves in the Smith and Stein text (2011), this learning translated to unpredictable enactment of a lesson summary for all teachers at the end of the data collection period. These findings indicated that even with support, facilitating a summary discussion is challenging, and may require more time and support than included in this study.

Summary of Findings

I identified six key findings presented from the data set. Across the six findings, the role of a facilitator on each teacher team played an important part in how the team worked together. A strong co-teaching relationship included alignment of learning goals for the student by the general education teacher, the special education teacher, and/or the math interventionist, which aligned to the Friend, et al. (2010) research. The concept of lesson grain size was an overarching idea across many of the findings. When teachers thought about, planned for, and used resources that supported lesson progression planning, teachers were more likely to think fluidly and situate daily goals with larger learning progression. This thinking is aligned with productive instructional practices (NCTM, 2014). When teachers learned of research-based instructional practices for designing rich math tasks, they could reframe math tasks from the district-created math curriculum to align with researched-based instructional practices for rich math tasks (Boaler et al., 2018a, 2018b, 2018c; Flynn, 2017; San Giovani, 2016). The use of the *Five Practices* (Smith & Stein, 2011, 2018) supported teachers in coherent lesson progression planning. With professional development and coaching support, a high-quality lesson summary (Smith & Stein, 2011) is difficult to enact. With instruction on discourse moves during the professional development intervention (Smith & Stein, 2011, 2018), teachers learned to incorporate a variety of discourse moves; however, not all teachers learned how to leverage the discourse moves and remain focused on the goal of the lesson summary.

CHAPTER SIX: DISCUSSION AND IMPLICATIONS

In this chapter, I discuss the implications of teachers' perception of students in the fields of special education and math education. Teachers' worldview and learning theory beliefs influence the way teachers collaborate and plan for student learning (Mertens, 2003). Collaboration is essential for effective intervention for students (Lembke et al., 2012) My findings of reframed math tasks have implications in the fields of special education and math education. Special education teachers and general education teachers need to work together to create opportunities for students with mathematics difficulty (Lewis, 2014). The findings related to lesson plan materials and lesson planning that used the *Five Practices* (Smith & Stein, 2011) has implications for the field of math education and professional math development. The structure teachers used in planning, the materials teachers used in planning, and the way teachers thought about learning trajectories while planning had an impact on the way teachers enacted math lessons. These findings also have significance for designed and enacted professional development. The findings regarding teachers' enactments of the lesson summary and lesson summary discourse patterns could inform approaches for how teacher leaders support teachers in the most essential element of a math lesson. The lesson summary (i.e., connect and generalize portion of the lesson) is the heart of any math workshop (Munson, 2018). With more understanding about how teachers planned and interacted with each other, designing effective professional development can shift instructional practice for special education teachers, general education teachers, and math interventionists.

Teachers' Perceptions of Students

As described in Chapter 5, I found special education teachers', general education teachers', and/or math interventionists' perceptions of students affected collaborative lesson planning. Based on these findings, coaches and professional developers might consider three different types of perspectives teachers could hold in working with students with mathematics difficulty (Lewis, 2010, 2014; Lewis & Fisher, 2016, 2018). These different perspectives include a strength-based perspective (Fuchs et al., 2018), a gap-based perspective, and an asset-based perspective (Celedón-Pattichis, 2018).

A Strength-Based Perspective

The special education teachers who participated in the study were more likely to view students from a strength-based perspective (Fuchs et al., 2018). During collaborative planning meetings, they focused on what students already knew or had mastered. This strength-based lens may come from an underlying belief from behaviorist learning theory in which starting with what a student knows is a way to begin to plan for instruction. Behaviorist learning theory is grounded in the idea that learning begins with what is already mastered and builds in small, incremented, chunks (Fuchs & Fuchs, 2007; Fuchs et al., 2007; Geary, 2004, 2005; Ginsberg & Pappas, 2007; Spillane, 2002). From a strength-based perspective, students gain entry into the learning designed for the student by the special education teacher, but these students likely do not have entry into grade-level core math content (Fuchs et al., 2007; Murawski & Hughes, 2009). As the learning for the student with mathematics difficulty is broken down into insolated chunks, the student has less opportunity to connect big ideas (Boaler & Greeno, 2000; Fuchs & Fuchs, 2007; Fuchs et al., 2007; Geary, 2004, 2005; Ginsberg & Pappas, 2007; Griffin, 2007; Karp & Voltz, 2000; Lewis, 2014; Lewis & Fisher, 2016). Content that is broken down into smaller pieces also could possibly lead to a slower instructional pace, and this slower instructional pace potentially translates to more unfinished learning for the student by the end of the school year.

A Gap-Based Perspective

The general education teachers who participated in the study predominantly discussed students using a gap-based perspective during the collaborative planning meetings (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley, 2013; Washburn-Moses, 2010). With a gap-based perspective, the general education teacher tended to view the student in terms of what the student could not do or did not understand. This failure assumes the problem is with the child rather than a problem with instruction (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley-Marling, 2013; Washburn-Moses, 2010). This gap-based lens may come from the underlying belief that math instruction is linear, students must master certain skills and procedures before complex content is introduced (Geary, 2004, 2005; Geary et al., 2000; Geary et al., 2007).

In this study, when teachers considered math content in isolated bins, rather than as a subject with interwoven content and connected ideas (NCTM, 2014), teachers tended to use a gap-based lens for planning for students with mathematics difficulty. My findings indicate that if teachers believed the student must complete unfinished learning from previous grades before working on current grade-level core content, then students were likely to develop even more unfinished learning over the course of the year (Fuchs & Fuchs, 2007; Fuchs et al., 2007). Teachers need to shift away from solely utilizing a gap-based perspective and/or a strength-based perspective and consider utilizing an asset-based collaborative perspective (Celedón-Pattichis et al., 2018).

An Asset-Based Perspective

An asset-based collaborative perspective uses ideas from the gap-based perspective and a strength-based perspective. Grounded in an asset-based perspective is the idea that the special education teacher and the general education teacher can collaborate together to create access to core content for students with gaps in understanding (Celedón-Pattichis et al., 2018; Friend et al., 2010; Gutiérrez & Dixon-Román, 2008; Lubienski, 2002). When using an asset-based perspective, teachers leverage the knowledge of what the student controls, to help create access to content knowledge the student needs to gain (Celedón-Pattichis et al., 2018). By collaborating and designing the bridge from the student's current level of understanding to grade-level content together, students gain the most opportunity to address gaps in understanding while also completing grade-level learning (Friend et al., 2010; Gutiérrez & Dixon-Román, 2008; Lubienski, 2002).

The math interventionist, Holly, at Hillview, used an asset-based perspective when she worked with the other teachers at Hillview. Chris, the special education teacher, tended to use a strength-based perspective. Maggie, the general education teacher, used a gap-based perspective. When Holly helped merge their thinking towards using an asset-based perspective, the collaboration and co-teaching relationship strengthened (Celedón-Pattichis et al., 2018). With the use of an asset-based perspective, the teachers focused on content knowledge the student already knew and leveraged this knowledge towards learning the grade-level content. The collaboration allowed students to have entry into content at their individual, developmental level and allowed the student to address gaps in understanding along the way towards grade-level content (Celedón-Pattichis et al., 2018). When the teachers perceived their shared students in a similar way, they were more likely to work together towards the same goals for their students (Friend et al., 2010; Gutiérrez & Dixon-Román, 2008; Lubienski, 2002). District and school administrators need to build time into the daily schedule for collaboration and provide PD. Well-crafted PD can help teachers understand different perspectives and leverage these perspectives to benefit student learning. After teachers have engaged in PD, then teachers are likely to be prepared to maximize precious collaboration time.

Sally and Megan from Three Peaks struggled with developing their co-teaching relationship. Sally, the general education teacher, thought about teaching and learning math content in big generalizable ideas. Megan, the special education teacher, thought about teaching math in terms of instruction centered on small isolated skills. When working with students, Megan defined the goals for student learning with a narrow scope. When Megan had defined her role for student learning in a narrow way, then Megan's work with Sally had a limited scope. It is possible that Megan defined her work with a narrow focus because of district IEP goal-writing requirements. The findings suggest Megan wrote IEP goals in a way that simplified data collection because this practice was encouraged by leaders in the district. Megan might have been more willing to work on content goals with a deeper focus if ideas about IEP goal-writing and data collection did not have a narrow scope and focus. This suggests the state and district special education rules on IEP goal-writing potentially inhibited this special education teacher from focusing on big conceptual ideas that the students seemed to need. These implications are consistent with prior researchers' findings in that a misalignment on perspectives of IEP goal-writing, lesson planning, and instruction can negatively influence the co-teaching relationship (Friend et al., 2010; Gutiérrez & Dixon-Román, 2008; Lubienski, 2002).

Megan and Sally struggled with their co-teaching relationship because they lacked alignment in their view on student learning. Unlike the Hillview team, Megan and Sally did not have a math interventionist or a third team member to help bring the two views on student learning together. I attempted to fill in with this role as I led or coached the collaboration meetings (Jaworski, 1998; Lewis, 2014). However, the collaboration meetings only held once each week was not frequent enough to make meaningful change. It is also worth noting that the Three Peaks team also participated in the fewest collaboration team meetings than the teachers at the other two locations.

Building co-teaching relationships take time, effort, and potentially also, coaching support (Aguilar, 2013; Sweeney, 2011). A solid co-teaching relationship usually takes several school years to develop into an effective, sustainable, relationship, and practice (Friend et al., 2010; Gutiérrez & Dixon-Román, 2008; Lubienski, 2002). Megan and Sally had done some tough work over the year in calibrating their ideas about student learning to focus on how to change and align their instruction (Cochran-Smith & Dudley-Marling, 2012; Cochran-Smith & Dudley-Marling, 2013; Washburn-Moses, 2010). However, Megan took a position in another district at the end of the study, which required Sally to begin to build a new co-teaching relationship with the special education teacher who replaced Megan.

Effective Interventions for Students

Effective intervention serves the specific need of the student in the act of intervening and preventing the student from needing more support in tier three of special education (Fuchs et al., 2018; Stevens et al., 2018). If the intervention is successful, then students need less support, not more support in a higher tier (Fuchs et al., 2018; Al Otaiba et al., 2019; Stevens et al., 2018). The goal for students with mathematics difficulty is to attend to gaps in understanding, provide, and ensure access to core classroom instruction through quality intervention (Fuchs et al., 2018; Lembke et al., 2012; Lewis, 2010, 2014). High-quality, effective intervention has the following characteristics: (a) is collaborative, (b) uses a team decision-making process, (c) instruction is aligned to core classroom instruction, (d) uses multiple quality assessments, (e) uses progress monitoring, (f) uses targeted lessons, (g) and is responsive to student needs by changing intensity. When these seven elements are woven together, students have more opportunity to resolve mathematics difficulty (Celedón-Pattichis et al., 2018; Fuchs et al., 2018; Lembke et al., 2012; Lewis & Fisher, 2018; Al Otaiba et al., 2019; Stevens et al., 2018).

My findings indicated that the opportunity for students to benefit from quality intervention is low if teachers do not know or understand the elements of effective intervention (Al Otaiba et al., 2019; Lembke et al., 2012). The teachers in this study needed PD in the areas of how to use progress-monitoring tools when designing a learning trajectory (Sztajn & Wilson, 2019), and how to change the intensity of an intervention based on student need (McKenna, Shin, & Ciullo, 2015). When teachers know how to design a learning trajectory, then establishing benchmark understandings along the learning path is a natural progression in constructing the learning trajectory (Sztajn & Wilson, 2019). Teachers in this study established benchmark understandings, then they designed and/or selected progress-monitoring assessments. When the learning trajectory and progress-monitoring tools were in place for the teachers in this study before instruction began, they were more likely to be systematic about interventions during instruction (McKenna, Shin, & Ciullo, 2015). Using predetermined progress-monitoring tools was only one aspect of a comprehensive assessment system.

A comprehensive assessment system includes the use of a universal screener, diagnostic assessments, formative assessments, growth assessments, and progress-monitoring assessments (Lembke et al., 2012). I provided diagnostic assessments to the teachers but designing and implementing diagnostic assessments was beyond the scope of this study. I helped teachers analyze the diagnostic assessments so when we designed rich math tasks, teachers could be thinking about the range of needs within the classroom. We focused our energy on designing and implementing progress-monitoring tools so we could make data informed instructional decisions (Lembke et al., 2012).

Most teachers changed instruction in some way when data indicated students were not learning, but many teachers did not understand systematic ways to change and track the intensity of intervention prior to the PD. These findings suggest that district instructional leaders could provide more avenues for teachers to learn about effective intervention practices and teaching tools teachers to improve intervention instruction systematically. Potential methods for teachers to learn about effective intervention practices include building- or district-provided PD, web-based resources, and curriculum materials. This suggests districts need to ensure all three avenues of learning about intervention are accessible to teachers. After a variety of provided learning opportunities about effective intervention, district and building leaders need to utilize the PLC structure to maintain effective intervention practices.

PLCs are an effective structure to support teacher capacity with intervention instruction as well as support student instructional need (DuFour, DuFour, Eaker, & Many, 2006; DeFour et al., 2008; Lembke et al., 2012). The findings on effective intervention instruction suggests that students may benefit more from intervention instruction if there was provided PD about intervention and teachers were given time to discuss intervention instructional strategies during their weekly PLC meetings. Districts need to find a way to support teachers with implementation of intervention when district or building level PD is not an option (Schmoker, 2016).

One way I have attempted to support teachers in my district is through my own district math intervention website. My website linked to an internal district site about federal programs is easily accessible for teachers. On my website, I provide instructional information on intervention, research and practitioner articles, and planning tools to support teachers (Celedón-

Pattichis et al., 2018; Fuchs et al., 2018; Lewis & Fisher, 2018; Al Otaiba et al., 2019; Stevens et al., 2018). The planning tools available to teachers include not only the Reframe a Math Task tool (Appendix N) but also other tools to help teachers track when and how they change intervention instruction to meet student need.

Designers of intervention curricular resources could consider including information on the seven components of effective interventions (Celedón-Pattichis et al., 2018; Fuchs et al., 2018; Lewis & Fisher, 2018; Al Otaiba et al., 2019; Stevens et al., 2018). When teachers read these intervention materials with embedded intervention principles and best practices, teachers have an opportunity to build a stronger understanding of effective intervention. Curriculum materials continue to be powerful in how teachers approach instruction (Remillard, 1999; 2000; Remillard & Bryans, 2004; Roth McDuffie & Mather, 2009). By ensuring intervention teaching materials contain high-quality information on effective intervention, teachers can avoid downgrading the intervention instruction through telling of solutions or by simplifying the task (Cai, 2010; Turner et al., 2011; Smith & Stein, 2018). Understanding effective intervention is just one aspect of understanding the MTSS model.

When teachers know components to MTSS and how the MTSS model is similar but also different from the RtI model, they are prepared to align and connect all the instructional support systems (i.e. English Language Learner support, Social-Emotional Learning support, Special Education support, reading and math intervention) to core instruction (Al Otaiba et al., 2019; Shepley & Grisham-Brown, 2018). RtI was a sound instructional idea originally, but in my experience with districts in my area, RtI turned into a justification for sorting and tracking students – practices inconsistent with the original intentions of RtI (Al Otaiba et al., 2019). Valid instructional systems or techniques usually have student need at the center. RtI as a service delivery model lost respect in the education community when it became a way to predict how students would perform on state assessments rather than about student instructional need (Al Otaiba et al., 2019). MTSS assumes students will receive multiple types of support and intertwined additional supports become layered onto core instruction. Districts could allow the scheduling of multiple supports to occur at the same time. Districts also need to encourage principals to make scheduling a top priority. One reason the Hillview team was so successful was that the principal allowed an adjustment to the entire building schedule so the special education teacher and math interventionist could be in the same general education classroom at the same time. My study provides an example of how these support services can be effective when teachers collaborated to align support services with core instruction. One way intervention instruction can align to core instruction is with rich math tasks (Carpenter et al., 1999, 2015; Boaler et al., 2018a, 2018b, 2018c; Flynn, 2017; San Giovanni, 2016).

Reframed Math Tasks

I found that the theoretical underpinnings given to teachers during PD supported understanding of research-based instructional practices, and as a result, teachers applied those practices to classroom instruction. These practices included the use of rich math tasks (Carpenter et al., 1999, 2015; Boaler et al., 2018a, 2018b, 2018c; Flynn, 2017; San Giovanni, 2016) and an L-E-S lesson structure (Lappan et al., 2007; Schroyer & Fitzgerald, 1986). I created a tool (Appendix N) to help teachers apply the researched-based instructional practices when examining the district-created math curriculum. Once teachers were aware of research-based math practices (Carpenter et al., 2015; Smith & Stein, 2011, 2018), they also became aware of how the district-created math curriculum did not necessarily align with these practices. Many of the district-created math lessons needed reframing.

The Use of a Coach-Created Tool

I found that by giving teachers a structure to reframe math tasks (Appendix N), teachers readily applied the structure and reframed tasks to meet the rich math task criteria (Carpenter et al., 1999, 2015; Boaler et al., 2018a, 2018b, 2018c, Flynn, 2017; San Giovanni, 2016; Smith & Stein, 2011, 2018). I created a handout or tool to help teachers reframe tasks (Appendix N). This finding has implications for teacher leaders in planning and organizing professional development. Once teachers understood the characteristics of a rich math task (Carpenter et al., 1999, 2015; Boaler et al., 2018a, 2018b, 2018c, Flynn, 2017; San Giovanni, 2016; Smith & Stein, 2011, 2018), then teachers used this information to reframe tasks to leverage the desired student thinking and solution strategies (Carpenter et al., 1999, 2015; Smith & Stein, 2011, 2018). I found teachers valued this work and saw the benefit in student learning. In a follow-up conversation after the study had concluded, Chris made these comments to me in an email.

> Chis (SET at Hillview): Maggie and I were wondering if you would be able to help us "revise" a few Learn Zillion [district-created math curriculum] units before school begins? We, meaning teachers and students alike, were engaged and successful during the fraction units you helped us with last year. We are hoping to keep this momentum going this year. Please let us know when we can touch base with you.

The Hillview team used the handout that I provided to help reframe math tasks (Figure 3). The tool I designed included the components of a rich math task. The tool then provided suggestions for how to reframe the task once the analysis of identified missing components of the original math task occurred (Appendix N). This tool supported teachers in reframing a math task, even if I was not readily accessible. It is important for coaches to listen to the needs of teachers and then find, create, or provide resources that support teachers in their instructional endeavors (Aguilar, 2013; Sweeney, 2011). Teacher use of coach-created tools is one way coaches can measure their coaching impact. One area of the great need for my district is to develop structures so all building-level and district-level coaches can share coach-created tools. In my district, there are more than 30 elementary coaches. This suggests it is quite possible that various coaches construct the same tool or PD multiple times over. This is a district waste of coaching time and effort. For districts to maximize learning and coaching support, there must be ways that coaches can support each other in the work that they do. One way I can support coaches with math intervention resources and manipulatives is to travel from building to building, demonstrating how the curriculum resources and manipulatives can support teacher and student learning.

Understanding Manipulatives

One limitation of the tool was the assumption that teachers knew which manipulatives to use, that students and teachers knew how to use the manipulatives, and how those manipulatives can bring meaning to the problem for the student. I found that teachers needed support in understanding how students might use manipulatives and how manipulatives can highlight mathematical concepts. I knew teachers would need instruction on the intervention manipulatives of Cuisenaire rods, Digi Blocks®, and number balance scales. I was surprised to learn that teachers needed modeling and time to explore other manipulatives like pattern blocks, plastic fraction strips, and rainbow square tiles. The Hillview team needed time each week to work with the manipulatives and discover which manipulatives were the most helpful for the task. The team also needed time to explore how students might use the manipulatives or how the manipulatives may cause confusion for some students. Building content knowledge around the use of manipulatives was important, so teachers were ready to prompt with high-leverage questions during the explore portion of the lesson.

The teachers saw the value and wanted to continue with this work the following year. These teachers became more informed about research-based instructional practices and knew the district-created math curriculum did not necessarily align with researched-based instructional practices (Carpenter et al., 1999, 2015; Smith & Stein, 2011, 2018), L-E-S lesson structure (Lappan et al., 2007; Schroyer & Fitzgerald, 1986), and rich math tasks (Boaler et al., 2018a, 2018b, 2018c, San Giovanni, 2016; Smith & Stein, 2011, 2018). The Hillview teachers pursued coaching support to improve curriculum and instruction. The Hillview teachers witnessed the benefit of coaching and the use of research-based instructional practices for *all* students in Maggie's classroom.

Curriculum Materials

When teachers understand curriculum materials with constructivist lesson plans, they are more prepared to shift instruction away from telling and towards student discovery (Hiebert & Grouws, 2007; Hiebert & Stigler, 2000). Having flexibility in lesson planning is an important aspect of math instruction. When the curriculum materials do not meet the needs of the class or lack constructivist ideas, teachers can adjust the lesson plan to meet student instructional needs or increase ways students construct mathematical knowledge. Consistent with past research, even when math curriculum materials are strong with constructivist lesson plans, teachers could interpret and/or implement lessons with a focus on procedures rather than the high cognitive demand of doing mathematics (Smith & Stein, 2011). Not only are high-quality math curricula important to high-quality math instruction, how the materials are used, is equally important (Remillard, 1999; 2000; Remillard & Bryans, 2004). When teachers learned how to analyze and reframe a task, they were critical consumers of each task and learned what elements to change to meet the needs of the students in the class.

The Hillview team often changed the problem context to make the context more engaging for fifth-grade students. The Three Peaks team changed tasks to allow for more solution strategies and to allow for several correct answers. The Oakview team completely redid tasks to shift away from a direct strategy instructional approach in the EngageNY curriculum (New York State Education Department, 2012) towards an approach that required students to engage in the discovery of mathematical concepts. This finding was in contrast to other findings that suggested teachers rarely increased the cognitive demand with a task (Stein et al., 2000). This study was an example of the hard work teachers do across a district, with little support from other teachers at various locations. The findings in this district have implications for other districts with similar needs. If a district created a structure that allowed teachers to find and track how teachers reframed the district-created curriculum, then PLC teams could discuss various options or suggestions for reframing tasks rather than starting over reframing each task. More PLC time could then be devoted to examining and analyzing student data, which then drives the teaching and learning cycle (DuFour, DuFour, Eaker, & Many, 2006; DeFour et al., 2008).

I found that if teachers can describe the characteristics of a rich math task (Boaler et al., 2018a, 2018b, 2018c; Carpenter et al., 2015; Empson & Levi, 2011; Flynn, 2017; San Giovanni; 2016; Smith & Stein, 2011, 2018), and if teachers had a structure to analyze given curriculum-based tasks (Figure 3), then teachers were empowered and supported to redesign curriculum using research-based instructional practices. If districts could provide structures when teachers do the heavy lifting of aligning the curriculum to researched-based instructional practices, then teachers can examine and utilize the work of others. This suggests that reframed tasks would be in continuous improvement, and as a result, there would be noticeable improvement with instruction and student learning across a district.

I also found that when special education teachers, general education teachers, and math interventionists, had a strong co-teaching relationship, collaborated, and reframed tasks together, teachers planned with all students in mind. When teachers thoughtfully reframed the task, they naturally thought about students with mathematics difficulty. When teachers planned for how these students would enter the task and complete the work, this provided access to core instruction for all students (Carpenter et al., 2015; Empson & Levi, 2011). When teachers collaborated, learning for all students was intentional and thought through (Cochran-Smith & Dudley-Marling, 2012; Murawski & Hughes, 2009). The team at Hillview used graphic organizers to anticipate how different subgroups of students might enter the task and the teachers also recorded how or what the teachers expected to see in the development of student thinking (Table 3). The Three Peaks team had a narrow focus for the work they did together, and that translated into a narrow focus for collaborative, rich math task planning. The lack of alignment in thinking with the Oakview team resulted in the planned lessons never becoming enacted lessons. These findings have implications for teacher leaders and professional development designers that tools created by instructional coaches can establish a foundation for instructional shifts. Based on my findings, teachers learned to reframe math tasks that supported student's needs and allowed students to generalize math concepts (Roth McDuffie, Wohlhuter, Breyfogle, 2011). This work led teachers to design a learning trajectory.

Learning Trajectories

To construct a learning trajectory, teachers need to know the mathematical knowledge held by each student through assessments or individual clinical interviews (Lewis & Fisher, 2018). In my study, when teachers knew the overall goal of the big generalizable idea, they were more prepared to build a learning trajectory. This finding is consistent with other researchers who stated that when teachers are well versed in researched-based instructional approaches students with mathematics difficulty have more opportunity to achieve the desired instructional outcome (Barrett et al., 2019; Lewis & Fisher, 2018). Teacher knowledge and curriculum materials play a critical role in the design of learning trajectories (Sztajn & Wilson, 2019).

Consistent with other research, I found that teachers who thought about lessons in terms of small ideas or isolated skills, and used resources that supported small lesson plan grain size, planned lessons with a narrow focus (Remillard, 1999; 2000; Remillard & Bryans, 2004). Lessons with a narrow focus lacked the needed element of students generalizing a big idea. The desired outcome for any learning trajectory is a big idea worth generalizing (Sztajn et al., 2019). Planned lessons in which the daily goals nests in a larger more meaningful goal of a generalizable idea, as indicated by NCTM's (2014) ambitious teaching practices, is one of the desired outcomes of PLCs. This idea is the foundation of a learning trajectory (Sztajn et al., 2019). Similarly, in my study, teachers who thought about lesson planning and used materials that supported learning trajectory planning tended to plan lessons with daily goals, but also larger goals in which students generalized a big math concept. Teachers who planned using learning trajectories thought more fluidly during planning, moving up and down the learning trajectory continuum, from small grain size, the daily goal, to large grain size, the big generalizable idea. This suggests that PD focused on learning trajectory construction would benefit learning for all students, as teachers who know and understand learning trajectories can attend to instructional needs in the moment while also keeping the focus on the larger instructional goal. This also suggests that curriculum designers can provide teachers support in understanding the learning trajectory. Text boxes in the margins of curriculum materials can remind teachers of the daily instructional goal and provide information about the daily goal building towards the larger generalizable idea. I found that when teachers constructed the learning trajectory themselves, they are more likely to understand this thinking. If the learning trajectory developed by curriculum designers provides small reminders in the margins on the learning trajectory pathway, this information can be helpful to teachers.

Learning Trajectories at Hillview

During the second half of the PD, the Hillview team discussed a math task and then immediately asked each other about the generalizable idea. Clarifying the generalizable idea was another way the Hillview co-teaching relationship strengthened. After discussing the generalizable idea, the team constructed the learning trajectory using a collaborative asset-based perspective (Celedón-Pattichis et al., 2018). The Hillview team had a clear vision of the instructional goals for students. They understood how rich math tasks served as a vehicle to get students to generalize the concept. The Hillview team built learning trajectories with the knowledge of the generalizable idea. They also understood where each student entered into the concept. Curriculum designers and district leaders could provide resources to teachers that clearly outline the learning trajectory pathway (Barrett et al., 2019; Sztajn et al., 2019). Providing only an outline of the learning trajectory usually does not provide enough support for teachers. Learning trajectory documents also need to provide information on concepts that may be especially hard for students to grasp. Teachers may need to pause on a concept or backtrack to resume forward momentum on the learning trajectory. Without guidance or caution, teachers may become focused on the small skill and not know how to regain focus on the big generalizable skill. In other words, learning trajectories are not always a clear linear progression, and learning trajectory documents need to reflect how the small daily goal nests inside the large generalizable idea.

Learning Trajectories at Three Peaks

The Three Peaks team struggled to develop a common understanding of the big idea for students to generalize because the work focused on small skills. With IEP goals as a focus, planned lessons focused on a small skill or concept rather than a large generalizable idea. With the primary focus on the daily lesson and the small concept, learning trajectories that included a larger idea never fully developed. At Three Peaks, the teachers were not able to construct a learning trajectory because they did not have similar instructional goals for students. Megan wanted narrow goals for students based on IEP goals, and Sally wanted bigger goals focused on grade-level standards. With differing views on the goals, that they each deemed important, a collaborative learning trajectory constructed by both teachers never became actualized. Megan and Sally were able to collaborate when the grain size was small. Megan and Sally discussed the daily lesson and revised the math task that allowed for more solution strategies. The revised task with more solutions provided access for students with mathematics difficulty, but this work did not help these students to connect and generalize what the different solutions had in common. If the curriculum materials showed how the small goals that were important to Megan fit inside of the large goals important to Sally, the Three Peaks team might have had the support needed to move forward collaboratively with the construction of a learning trajectory. This suggests that curriculum materials that provide a clear learning trajectory pathway support collaboration between special education teachers and general education teachers. This also suggests that curriculum materials that do not provide a learning trajectory can be a barrier for teacher collaboration.

Learning Trajectories at Oakview

The Oakview team struggled to create learning trajectories because the EngageNY (New York State Education Department, 2012) materials made thinking about a large math concept difficult for teachers. The Oakview team needed to redo the EngageNY (New York State Education Department, 2012) lesson progressions to allow for a large generalizable idea. With support from the math interventionists, the general education teachers created the learning trajectory on paper. However, after lesson observation and data analysis, it was clear that even though the general education teachers helped to create the learning trajectory, they fully did not understand how to implement the lessons they designed. Instead of enacting the newly created lessons, the enacted lessons came from the Engage NY materials (New York State Education Department, 2012). A barrier in thinking about lesson planning with a larger grain size came from the materials the teachers used in planning (Remillard, 1999; 2000; Remillard & Bryans, 2004).

These findings have implications for designers of professional development and curricula. Teachers continued to plan lessons in the ways they were familiar with thinking about lesson planning (Remillard, 1999; 2000; Remillard & Bryans, 2004; Sztajn et al., 2019). If teachers tended to use narrowly focused materials and thought in terms of small daily goals, then the planned lessons tended to have a limited scope. Teacher leaders, curriculum designers, and professional development designers need to expose teachers to a variety of materials to aid in lesson design. These materials need to include explicit information about learning trajectories. Professional development designers and instructional coaches need to provide support to teachers in learning how to create a learning trajectory if the curriculum materials are lacking in this area. When teachers understand how the small daily goal builds to the large generalizable idea, then teachers are more prepared to help students reach the primary instructional outcome. Without this support, teachers will continue to plan in the ways they are familiar with and may never have an opportunity to learn to plan with a learning trajectory in mind.

Five Practices and Lesson Planning

I found that when teachers used the structure of the *Five Practices* (Smith & Stein, 2011, 2018) in anticipating solution strategies, then teachers were more likely to understand the importance of selecting and sequencing student solution strategies with an intentional goal for the connect and generalize portion of the math lesson. This is consistent with other research on the use of the *Five Practices* (Sztajn et al., 2019). When teachers did the work of anticipating solution strategies, they were more likely to understand the math concept on a deeper level (Sztajn et al., 2019). Therefore, teachers were more likely to highlight a big idea during the lesson summary or the connect and generalize portion of the lesson. When teachers learned about various solution strategies from CGI (Carpenter et al., 2015) and were encouraged to anticipate solution strategies (Smith & Stein, 2011, 2018), they naturally thought about a direct modeling solution strategy, a counting-on solution strategy, and more advanced solution strategies that used relational thinking (Carpenter et al., 2015). When teachers planned using a cognitively guided approach, (Carpenter et al., 2015; Empson & Levi, 2011) towards solution strategies, they generated at least four different solution pathways together as a team. Teachers also indicated which solutions were more concrete and provided access to all students and which solution strategies were more abstract and would require students to reason and connect solution strategies to each other and to the larger mathematical idea. In the sections below, I summarize how the teachers worked with the *Five Practices* (Smith & Stein, 2011, 2018) within each of the schools, and then I discuss implications from these findings.

Five Practices at Hillview

During the PD sessions, the Hillview team moved fluidly from discussing a rich math task into solving for different solution strategies. As the Hillview team worked on solutions for the task individually, they transitioned to discussing the various solutions they discovered. With little to no coaching support, they moved into discussing which solution strategy to highlight first and which solution strategy to highlight second and so forth. Then the Hillview team discussed how the solutions connected to each other. During the collaboration meetings, Holly prompted Chris and Maggie to think about questions to facilitate student connections to the various solution strategies. The Hillview team understood the importance between anticipating various solution strategies and selecting and sequencing those strategies, so mathematical ideas build on each other. The Hillview team understood the *Five Practices* (Smith & Stein, 2011) of anticipate, monitor, select, sequence, and connect solution strategies led to deeper mathematical learning for their students. However, the Hillview team had a more difficult time with learning how to develop mathematical discourse during the final connect and generalize portion of math workshop in which students do the heavy cognitive lifting.

Five Practices at Oakview

When the Oakview team learned about the *Five Practices* (Smith & Stein, 2011), they were skilled at anticipating a variety of solution strategies. With the support of the math interventionists, the general education teachers described and labeled the solutions from concrete to abstract. For the Oakview team, the planning and thinking about the *Five Practices* (Smith & Stein, 2011) stopped at this point. The Oakview team did not enact the lessons they planned, and they did not have the same experience as the other teams in understanding how the approach in planning with the *Five Practices* (Smith & Stein, 2011) in mind led students to generalize a big math concept. The Oakview team did the work of anticipating solution strategies (Smith & Stein, 2011) during the collaboration meetings but did not reap the benefit of seeing how this approach could shift student's understanding of big math concepts. The routine and comfort of the scripted EngageNY (New York State Education Department, 2012) materials got in the way of the Oakview general education teachers experiencing how the *Five Practices* (Smith & Stein, 2011) support student learning. More specifically, the Oakview team missed a learning opportunity of observing how a series of complete L-E-S math lessons (Lappan et al., 2007; Schroyer & Fitzgerald, 1986) could make a difference for their students.

Five Practices at Three Peaks

The Three Peaks team was strong with revising math tasks for a variety of solution strategies. As Megan and Sally reframed math tasks that allowed for more correct solutions and solution pathways, they also solved the tasks as students. Sally and Megan recorded the various ways they anticipated students would solve the task and then rank ordered the solutions from the most concrete to most abstract. The work Megan and Sally did during the collaboration meetings set them up for a powerful connect and generalize portion of the math lesson. Megan and Sally rarely discussed the big generalizable math idea. When Sally brought this up with Megan, Megan reverted to IEP goals and kept the conversation limited to various ways students could solve the task. Megan was resistant to discuss the generalizable idea. The link to her special education background and the use of behaviorist learning theory in special education lesson planning helps to explain her approach to instruction.

Implications of the *Five Practices*

District leaders and coaches need to provide PD that allows teachers to co-construct knowledge around anticipating solution strategies. When teachers anticipate solution strategies, they can then select and sequence solution strategies to engage students in a meaningful lesson summary. PD opportunity should engage teachers in at least a single round of solving a rich math task as a student would, generating possible solution pathways, and then selecting and sequencing the solution strategies. My findings suggest teachers need multiple opportunities to become comfortable and accustom to this work. My findings also suggest that curriculum designers need to find a balance between designing materials that are supportive of building teacher capacity in researched-based instructional approaches and not telling too much information. Powerful teacher learning occurs when teachers generate solution pathways on their own rather than reading about potential solution pathways in curriculum materials. When teachers plan with their students in mind, teachers are more likely to generate the same solution strategies as their students. Although curriculum designers should not do all the work with providing all possible solution pathways in resource materials, the opposite is true with examples of lesson summary enactment.

Lesson Summary Enactment

I found that even though teachers had a supportive structure with the *Five Practices* in lesson planning (Smith & Stein, 2011, 2018), teachers needed extensive support to approximate the lesson summary. Out of the nine teachers who participated in this study, only Holly was well skilled at lesson summary enactment. This need for extensive support has implications for teachers and coaches, as discussed in the context of these teachers.

Holly

Holly had 30 years of teaching experience and worked for a not-for-profit math education consulting company in which she provided PD to districts on L-E-S (Lappan et al., 2007; Schroyer & Fitzgerald, 1986) lesson structure and rich math tasks. Part of the PD that Holly provided in her prior job was on researched-based instructional approaches to lesson summary enactment. Holly's extensive knowledge and experience with math education and instruction helped her be an expert at lesson summary enactment. The field of math education cannot wait for teachers to gain an equivalent number of years or types of experiences similar to Holly to become proficient at lesson summary enactment. Curriculum and PD designers need to provide ways that can actually accelerate teacher learning in this area. Researchers need to discover the most efficient and effective ways to build this teacher knowledge, and then districts need to invest in these methods. Holly and Sally both had specialized learning opportunities throughout their careers, but the types of learning experiences make a difference for teachers.

Sally

Sally was well versed in the ideas of L-E-S lesson structure (Lappan et al., 2007; Schroyer & Fitzgerald, 1986) and could articulate the importance of lesson summary (Smith & Stein, 2011, 2018) during the collaboration meetings. However, when it was time for the lesson summary, Sally struggled with facilitating a discussion in which students generated the big idea. Lesson summary enactment from Sally often included an IRE pattern (Cazden, 2001), telling students of important math content, and lacked discourse moves that required students to make sense of each other's thinking (Smith & Stein, 2011, 2018). Even with the PD, provided resources on lesson summary enactment, and coaching at the time of need, Sally struggled to make a shift in her lesson summary enactment away from mostly vertical discourse and teacher telling of ideas. The duration of the PD was not long enough to observe an instructional shift in Sally's approach to lesson summary enactment. Even though Sally struggled to facilitate a powerful lesson summary discussion, she believed in the fundamental importance of an L-E-S lesson structure (Lappan et al., 2007; Schroyer & Fitzgerald, 1986) and understood the purpose of a mathematical discussion at the end of the lesson. Curriculum designers could provide resources that facilitate teacher learning with lesson summary enactment. Resources could help teachers monitor for horizontal discourse and provide tips on how to increase student discourse on solution strategy connections. My findings suggest resource materials that provide extra support on lesson summary enactment can help teachers develop these skills. Resource materials need to provide for a wide range of teacher knowledge as Cheryl was still developing this knowledge at the end of the PD.

Cheryl

Cheryl also received coaching at the time of need around lesson summary enactment. At the time of the PD, Cheryl was not sure of the importance of the lesson summary. Her first attempts at lesson summary enactment included a review of student work time and compliments on working quietly and independently to students. During the PD, Cheryl was provided a copy of the *Five Practices* for Orchestrating a Powerful Math Discussion (Smith & Stein, 2011), and an article on principles of good math instruction (Protheroe, 2007). Cheryl was coached after I observed the lesson summary to help her reflect on how to improve the lesson summary enactment. With coaching in the moment of need, (Aguilar, 2013; Sweeney, 2011) with provided research-based resources, (Carpenter et al., 2015; Smith & Stein, 2011; Protheroe, 2007) and with 12 weeks of math PD, Cheryl made little to no change to her lesson structure or lesson summary during the duration of the study. As designers of professional development, it takes time and multiple lesson summary engagement opportunities to shift teacher practice.

These findings suggest that mathematics teacher educators and curriculum designers need to provide multiple examples of lesson summary discourse for teachers to discuss and analyze. This kind of professional development needs to be central to math education learning. Teachers need rich experiences in understanding the key moves in orchestrating powerful mathematics discussions. Likewise, teachers also need time working with examples in which the lesson summary lost focus and power. Teachers need to practice instructional moves to pull a lesson summary back on track should it degrade to an IRE pattern (Cazden, 2001), or lack an instructional focus. Some curriculum materials include lesson discourse content, but these kind of curricular materials need to become more prevalent in today's elementary classroom (Lappan et al., 2007; Russell et al., 2017).

As teacher leaders and designers of professional development, we need to find a more efficient and effective way at turning the lesson summary novice teacher into a teacher who is highly skilled with lesson summary enactment. Teachers might have developed or improved understandings about lesson summary enactment during the 12-week PD. The PD and coaching at the time of need were not long enough to learn about, implement, and evidence change in practice through the data I collected. Alternatively, perhaps the PD experiences did not provide the type or quality of experiences for teachers to shift their instructional practice. Current research (Loucks-Horsley et al., 2010) indicates that 12, weekly, 90-minute PD meetings focused on student data and teacher instructional practice should provide ample opportunity to shift instructional practice, yet my study indicated that longer periods are needed for teachers to shift instructional practice and/or to observe shifts in practice. One way to provide the needed time is for districts and schools to provide regular and systematic PD in combination with research-based coaching for teachers. This PD needs to build across time and include ideas on rich math tasks, (Carpenter et al., 2015; Boaler et al., 2018a, 2018b, 2018c; Flynn, 2017; San Giovani, 2016), principles of CGI, (Carpenter et al., 1999, 2015, Carpenter et al., 1989; Carpenter, Franke, & Levi, 2003; Empson & Levi, 2011; Fennema et al., 1996; Franke, Kazemi, & Battey, 2007) and the *Five Practices* (Smith & Stein, 2011, 2018). I have worked in my district for 17 years prior to the 2016-2017 school year. During this time, there has never been ongoing math PD provided at the district level. Any math PD occurred at the building level, and as a result, the PD lacked coherence with the many other district-directed instructional initiatives. For student learning to improve, my district needs to make ongoing, in-depth, research-based, math PD a priority, and it seems that other districts might consider this need also. Without coherent and purposeful PD, teachers are likely to lack support to improve lesson summary enactment and lesson summary discourse.

Discourse in the Lesson Summary

I found that only one of the teachers in my study effectively used the discourse moves in the *Five Practices* (Smith & Stein, 2011) and required students to engage in horizontal student-to-student discourse. Providing PD on how to enact a lesson summary, and even the frequent occurrence of an actual lesson summary itself is not enough to increase the rigor or demand in student mathematical thinking if the teacher does all the talking (Cazden, 2001).

Holly used discourse moves of holding students accountable for the thinking and reasoning of others and knew the critical moment of when to revoice student thinking. Holly pushed student thinking during the lesson summary until a student made a claim or began to articulate the big generalizable idea. As soon as student thinking emerged in this way, Holly leveraged this thinking and made instructional moves to hold all students accountable for this thinking. Holly remained focused on students articulating the big idea, and as soon as she reached that goal, she leveraged the student’s thinking for the benefit of other students in the class.

Sally was the other participant that had a higher occurrence of an enacted lesson summary, but she lost sight on the focus of the discussion and reduced the discourse to an IRE pattern (Cazden, 2001). To support teacher thinking and planning for facilitating discourse during the final connect and generalize portion of the lesson, strong curriculum materials are needed. Curriculum materials could prompt teachers with potential questions, discourse moves, and strategies to help students articulate a large generalizable claim and hold other students accountable for this thinking. Although examples of curriculum programs providing support for discourse moves exist (e.g., Investigations 3, Russell et al., 2017) these curriculum resource materials are limited.

Teachers and students both might need time to develop practices for facilitating and participating in classroom discourse during the lesson summary. Students at first may only indicate the correct solution with a minimal description of the solution pathway. Teachers can develop facilitation practices and discourse moves to prompt students to describe how solution pathways connect to each other during the lesson summary. Eventually, these discourse patterns can become classroom norms (Cai, 2010; Cazden, 2001; Herbel-Eisenmann & Breyfogle, 2005; Smith & Stein, 2011, 2018). Teachers can employ advanced discourse moves that go beyond students connecting solution pathways. These advanced discourse moves can lead students to make claims about the base 10 number system and how the four operations function. This high level of mathematical justification is one of the five strands of mathematical proficiency known as adaptive reasoning (National Academies Press, 2001).

For this study, it is not clear whether the resources provided during the PD, the length of the PD, and/or the PD process was insufficient. However, the findings indicate that the participants needed more support to enact instructional discourse moves to facilitate student thinking at a higher level. Discovering PD instructional approaches that are most effective in advancing teacher discourse moves during lesson summary enactment is a topic worth pursuing in further research. The desired goal is that teachers effectively learn to hold students accountable for mathematical thinking and discourse throughout a lesson.

Changes in Instructional Leadership

I found that changes in instructional leadership played a significant role in how well a PLC or individual teachers were able to shift instructional practice. The Hillview team was the most successful at building their co-teaching relationship and calibrating around student needs and goals. Even though the Hillview teachers were new to Hillview, unchanged collaboration goals, and student learning outcomes, by the school leadership remained. The same principal and instructional coach worked together over the past 10 years at Hillview. The consistency in school leadership and maintained objectives in professional learning allowed the team to become cohesive quickly, without mixed messages on curriculum, instructional priorities, or instructional methods.

The Oakview teachers had a continual change in instructional leadership. Consistent with other research, instructional leadership changes can create isolation and make shifts in instructional practice difficult (Jametz, 2002). In the five years prior to 2016-2017, the Oakview teachers had three different principals, each with their own unique focus for professional learning. As teachers at Oakview expected to pick up divergent instructional approaches with each changing principal, the teachers became more reliant on the use of the EngageNY (New York State Education Department, 2012) curriculum. The constant changes in instructional leadership, changes in professional development learning, and a scripted math curriculum were barriers for the teachers at Oakview. Even with the PD, the teachers did not have a strong enough level of safety to take a risk and enact lessons outside of the EngageNY (New York State Education Department, 2012) curriculum.

The Three Peaks teachers' experiences seemed somewhere in the middle between the two extremes of the Hillview and Oakview leadership contexts. Even though the instructional leadership at Three Peaks had been consistent, no provided PD occurred in the ten years prior to the 2016-2017 school year on research-based math instructional practice or the fundamentals of effective intervention. With a lack of research-based math PD experiences, the staff at Three Peaks used any mathematical resources they could find, including searching websites from Teachers Pay Teachers (https://www.teacherspayteachers.com) and Pinterest (https://www.pinterest.com). Using any math resource as curriculum results in student learning goals focused only on the daily objective or the small-based IEP skill (Sztajn & Wilson, 2019). With limited PD on researched-based instructional practice, a narrow focus of student goals and student work developed (Loucks-Horsley et al., 2010). With the instructional leadership at Three Peaks unaware of research-based math, instructional practice, teachers were focused on daily lessons with a narrow focus. The narrow focus on student goals and student work became a barrier for the team at Three Peaks.

Future research might examine how schools and districts can maintain instructional leadership. Future research could also explore how to maintain instructional focus when the leadership changes or if a school experiences substantial changes in a single year. Future research could examine the following questions:

- How can schools adjust for constant changing staff despite in-depth learning that has potentially occurred in the past?

- How can the focused learning from prior years remain when a school has changes in instructional leadership and changes in instructional priority?
- How can a school hold onto collective learning from professional development over time?
- How does a school compensate for changes to staff attrition and hiring and still maintain the PD learning from years prior?

Using the data from this research project, I would like to extend cross-case analysis between the three school contexts. I am especially interested in the role of the facilitator within each context and other factors that influenced or inhibited instructional shifts at the different locations. I am also interested in continuing to examine Oakview on a deeper level. Even though Smarter Balance math scores were down overall across the district for the 2018-2019 school year, Oakview's scores for this year increased. Principal C has had a consistent PD instructional focus on the Launch-Explore-Summary lesson structure for the past three years (Lappan et al., 2006). It would be interesting to study the teachers again at Oakview to see if the continued instructional focus helped them navigate away from the direct instructional approach used in the EngageNY curriculum materials (New York State Education Department, 2012). I am curious to discover how PD and curriculum materials help teachers to implement new instructional approaches and if these new instructional approaches actually result in higher Smarter Balance test scores. If a link could be demonstrated through research, would this result in a new approach towards PD across the district?

One system that districts could implement would be continued PD for new teachers, or teachers new to the district regarding researched-based math instructional practices. If districts had online documents and self-guided professional development videos, outlining expected instructional practices, then teachers could participate in this PD on their own schedule. Teachers would have a resource to access when unsure of how to implement an instructional practice. This self-guided professional development could include links to research-based articles, videos demonstrating how to implement L-E-S based lessons, and videos demonstrating how to make important instructional moves during the summary portion of the lesson. Providing a plethora of excellent lesson summary discussions, along with coaching cycles, could help teachers know the desired expectation and outcome of a powerful lesson summary.

Districts and curriculum designers need to provide examples of annotated lesson summaries in which teachers can observe highlighted, powerful discourse moves. Utilizing videos and transcripts that demonstrate lesson summaries in which the teacher was able to recover from an IRE pattern (Cazden, 2001), or how the teacher regained instructional focus after a lapse in coherent student thinking would help teachers understand even a weak lesson summary can regain power. Provided face-to-face and online PD would allow every teacher from the experienced, expert teacher in lesson summary, to the new, beginning teacher, to grow in his or her instructional practice. Most districts do not provide a systemic way for schools to maintain professional development learning despite changes in leadership, changes in staff, or changes in instructional focus from year to year (Borasi & Fonzi, 2002; Desimone, 2009; Desimone et al., 2002; Garet et al., 2001; Murry, 2014). This kind of district investment and focus can maintain the lift from provided PD and of district initiatives.

Conclusion

The unique combination of a collaborative team and unchanged instructional priorities at one location made a difference for how well that team helped students with mathematics difficulty. The Hillview team was the most successful with this endeavor, and it seemed that this success was due to teachers developing and strengthening their co-teaching relationships. In addition, the teachers shared the same ideals about math instruction and received consistent support from the math interventionist and special education teacher. Hillview also had unchanging instructional leadership and consistent instructional priorities.

Other teams were not as successful in helping students with mathematics difficulty, and it seemed that the curriculum materials or IEP goals served as barriers to lesson planning. We need to continue to study teachers enacting lesson summaries and utilizing discourse moves that require students to connect and generalize mathematical ideas. Clearly, in my study, teachers struggled the most with lesson summary enactment, and for teachers to improve in this area, they need rich models and plenty of guidance on how to implement a meaningful lesson summary. Curriculum designers, districts, and schools need systems and structures for developing highly proficient teachers in lesson summary enactment, and lesson summary discourse moves to accelerate learning for students with mathematics difficulty (Lewis, 2014).

In conclusion, my study advanced the knowledge base on how special education teachers, general education teachers, and/or math interventionists collaborated to support students with mathematics difficulty. My study furthered the field of math education and professional development to understand ways teachers can make instructional shifts and apply research-based instructional practices. Helping students address and resolve mathematics difficulty is a worthy and important goal. This study endeavored to highlight ways math educators and teacher leaders can support students and teachers in this complex task.

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APPENDICES

APPENDIX A:

Codebook

Code	Code Description
Collaboration	•Any conversation teachers have together with shared decision making towards a common goal (Friend and Cook, 1995) •Should focus around the teaching learning cycle •And benefit students
Calibration RQ #1	Becoming aligned in thinking around an idea or concept. Could include student need, Roles and Responsibilities, designing math instruction, etc.
Co-teaching RQ #1	The act of teaching together in the same physical space Could include: •Consultation •Lead and support •Teaching in Tandem (Friend et al., 2010; Wilson & Blednick, 2011)
Roles and Responsibilities RQ #1	Discourse around work load, expectations of each other, defining or putting parameters around the job of the general education or special education teacher (Cook & Friend, 1995; Friend, 2007; Scruggs, Mastropieri, & McDuffie, 2007)
Designing math instruction RQ #2	The act of planning for math instruction in which curricular resources, CCSSM documents, district scope and sequence documents may or may not be used
Teaching Learning Cycle RQ #2	Discourse around assessment, planning for instruction, implementing instruction, and collecting new assessment data (Jones, 2008)
Assessment RQ #2	Mentioning or designing formative or summative assessment in terms of using this to inform instruction
Differentiation RQ #2	Taking into account different student needs and planning for variety of instructional approaches in either presentation, process, product, or assessment (Tomlinson, 2014)
Adapting materials RQ #3	The act of taking what was planned use for the Gen Ed student and recreating or changing to allow the Sp Ed student access to the learning (Wilson & Blednick, 2011)
Students in special education instructional needs RQ #2 and #3	Using some form of student data to drive perceived instructional needs
SDI RQ #2	Any specially designed instructional strategy
IEPs	Use or mention of the student's Individualized Education Plan

RQ #2	
Intervention RQ #2	Use of assessment data to drive designing and/or implementing supplementary lessons intended for a targeted group of students
Concept/Skill transfer RQ #2	Looking for or examining if ideas are generalizing from one context to another
Student Ownership RQ #1	Mentioning students in terms of belonging to either the Gen Ed teacher or the Sp Ed teacher (Cook & Friend, 1995; Friend, 2007; Scruggs, Mastropieri, & McDuffie, 2007)
Resistant or Persistent Practices RQ #2 and #3	When a teacher has knowledge that an instructional practice is not beneficial for students but continues to use that instructional strategy anyway (i.e. using a timed addition and subtraction test to build fluency)
CGI RQ #3	Discourse between the special education teacher and the general education teacher in using CGI framework
Five Practices RQ #3	Teacher discourse in planning to use any form of Five practices for a mathematical discussion
+/- Benefits or Limitations RQ 1, 2 & 3	Could be combined with any other code to express feeling about an idea or concept

APPENDIX B:

Timeline of Data Collection and Analysis Activities

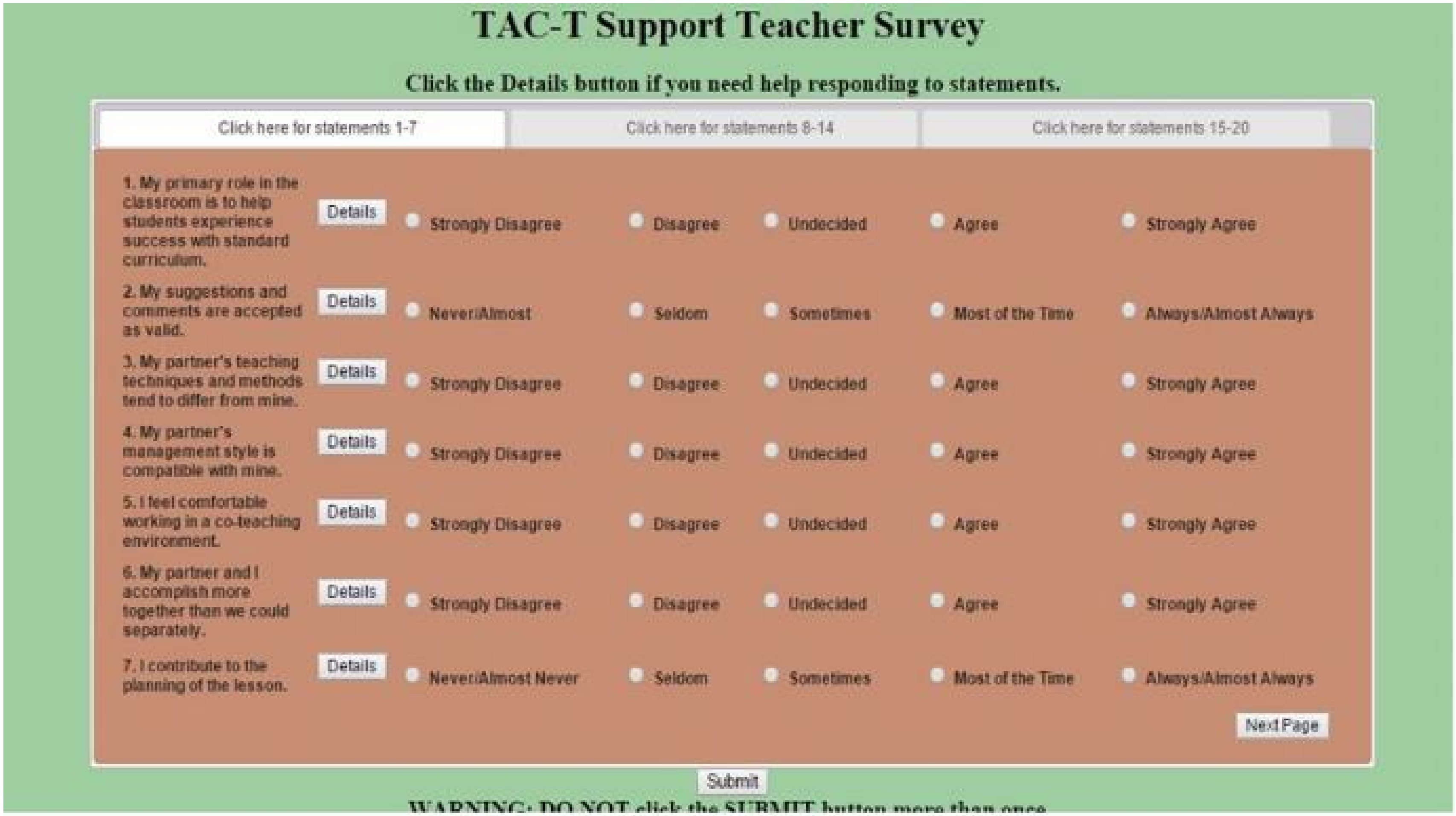
Data Collection and Analysis Activity	March				April				May				June				August-Nov.			
	w1	w2	w3	w4	w1	w2	w3	w4	w1	w2	w3	w4	w1	w2	w3	w4	A	S	O	N
Administer co-teaching survey RQ #1																				
Compile co-teaching survey responses RQ #1																				
Create digital system to for PD documents RQ #3																				
Conduct pre PD individual interviews RQ # 1, 2, & 3																				
Conduct pre PD partnership interviews RQ # 1, 2, & 3																				
Observe and video record PLC meetings RQ # 2																				
Use observation tool to collect data from watching PLC video RQ # 2																				
Analyze observation data and generate research memo from observations # 1 & 2																				
Create questions for collaboration meetings RQ # 2																				
Attend and video record collaboration meetings RQ # 2																				
Attend and observe collaboration meetings RQ #2																				
Facilitate PD RQ #3																				
Observe and video record Co-taught lessons RQ # 2 & 3																				

Complete observation tool for lesson observations RQ # 2																				
Collect student work samples on paper RQ # 3																				
Copy artifacts teachers bring to the PD RQ # 3																				
Conduct post PD individual interviews RQ # 1, 2, & 3																				
Conduct post PD partnership interviews RQ # 1, 2, & 3																				
Analyze Observation notes gathered from PD RQ # 3																				
Transcribe Pre and Post individual and partnership interviews RQ # 1, 2, & 3																				
Analyze Pre and post individual and partnership interviews using HR RQ# 1, 2, & 3																				
Analyze lesson observation tools and generate research memo RQ #2																				
Generate Secondary data analysis in HR by analyzing first round data collected RQ# 1, 2, & 3																				
Create claims and evidence table using all data sources																				
Write and finalize results, discussion, conclusion, and implication sections																				

APPENDIX C:

Teachers Analyzing Co-Teaching Survey

Questions 1 – 7



TAC-T Support Teacher Survey

Click the Details button if you need help responding to statements.

Click here for statements 1-7 | Click here for statements 8-14 | Click here for statements 15-20

1. My primary role in the classroom is to help students experience success with standard curriculum.	Details	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
2. My suggestions and comments are accepted as valid.	Details	Never/Almost	Seldom	Sometimes	Most of the Time	Always/Almost Always
3. My partner's teaching techniques and methods tend to differ from mine.	Details	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
4. My partner's management style is compatible with mine.	Details	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
5. I feel comfortable working in a co-teaching environment.	Details	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
6. My partner and I accomplish more together than we could separately.	Details	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
7. I contribute to the planning of the lesson.	Details	Never/Almost Never	Seldom	Sometimes	Most of the Time	Always/Almost Always

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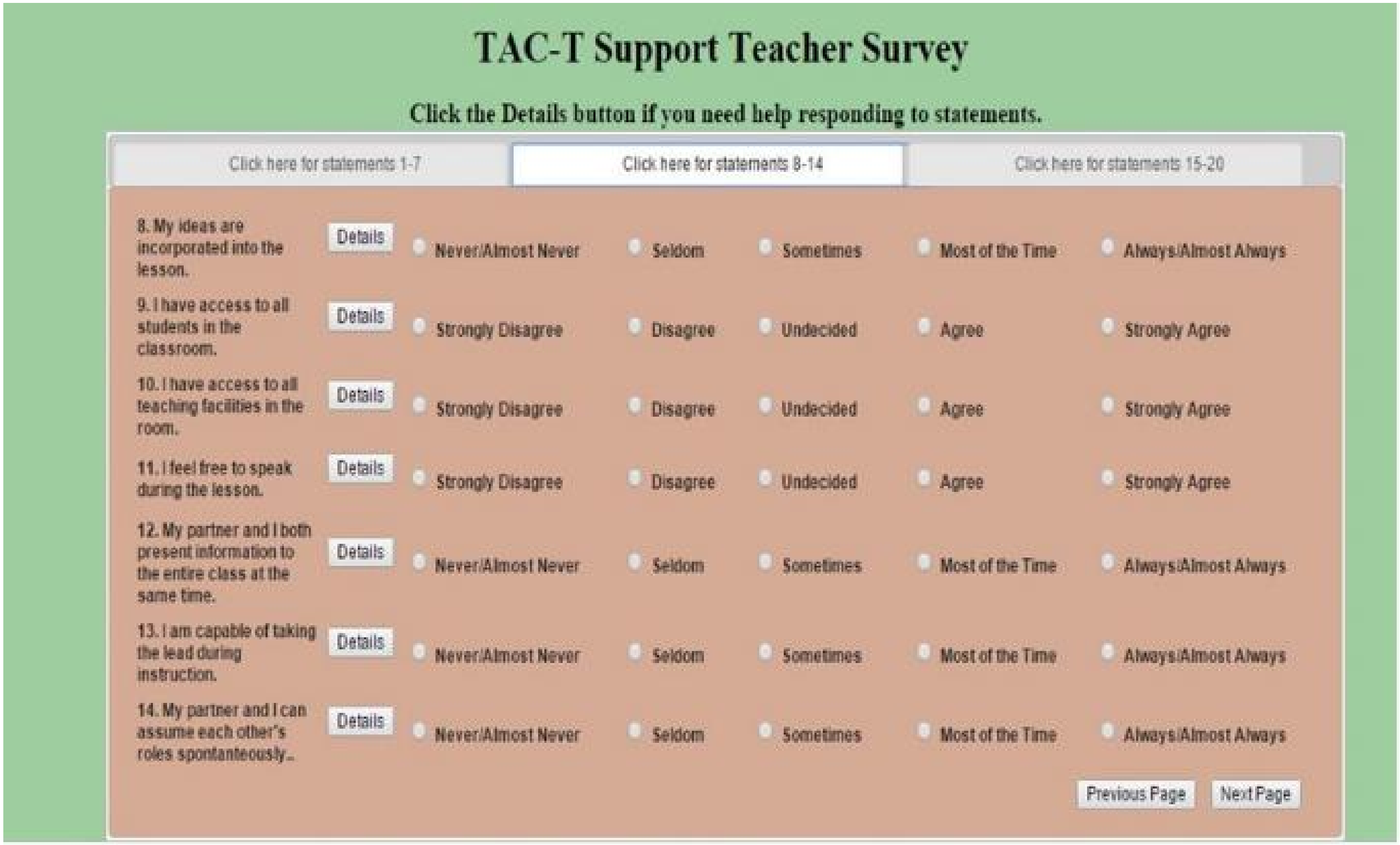
Submit

WARNING: DO NOT click the SUBMIT button more than once

Teachers Analyzing Co-Teaching Survey

Questions 8 – 14



TAC-T Support Teacher Survey

Click the Details button if you need help responding to statements.

Click here for statements 1-7 | Click here for statements 8-14 | Click here for statements 15-20

8. My ideas are incorporated into the lesson.	Details	Never/Almost Never	Seldom	Sometimes	Most of the Time	Always/Almost Always
9. I have access to all students in the classroom.	Details	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
10. I have access to all teaching facilities in the room.	Details	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
11. I feel free to speak during the lesson.	Details	Strongly Disagree	Disagree	Undecided	Agree	Strongly Agree
12. My partner and I both present information to the entire class at the same time.	Details	Never/Almost Never	Seldom	Sometimes	Most of the Time	Always/Almost Always
13. I am capable of taking the lead during instruction.	Details	Never/Almost Never	Seldom	Sometimes	Most of the Time	Always/Almost Always
14. My partner and I can assume each other's roles spontanteously..	Details	Never/Almost Never	Seldom	Sometimes	Most of the Time	Always/Almost Always

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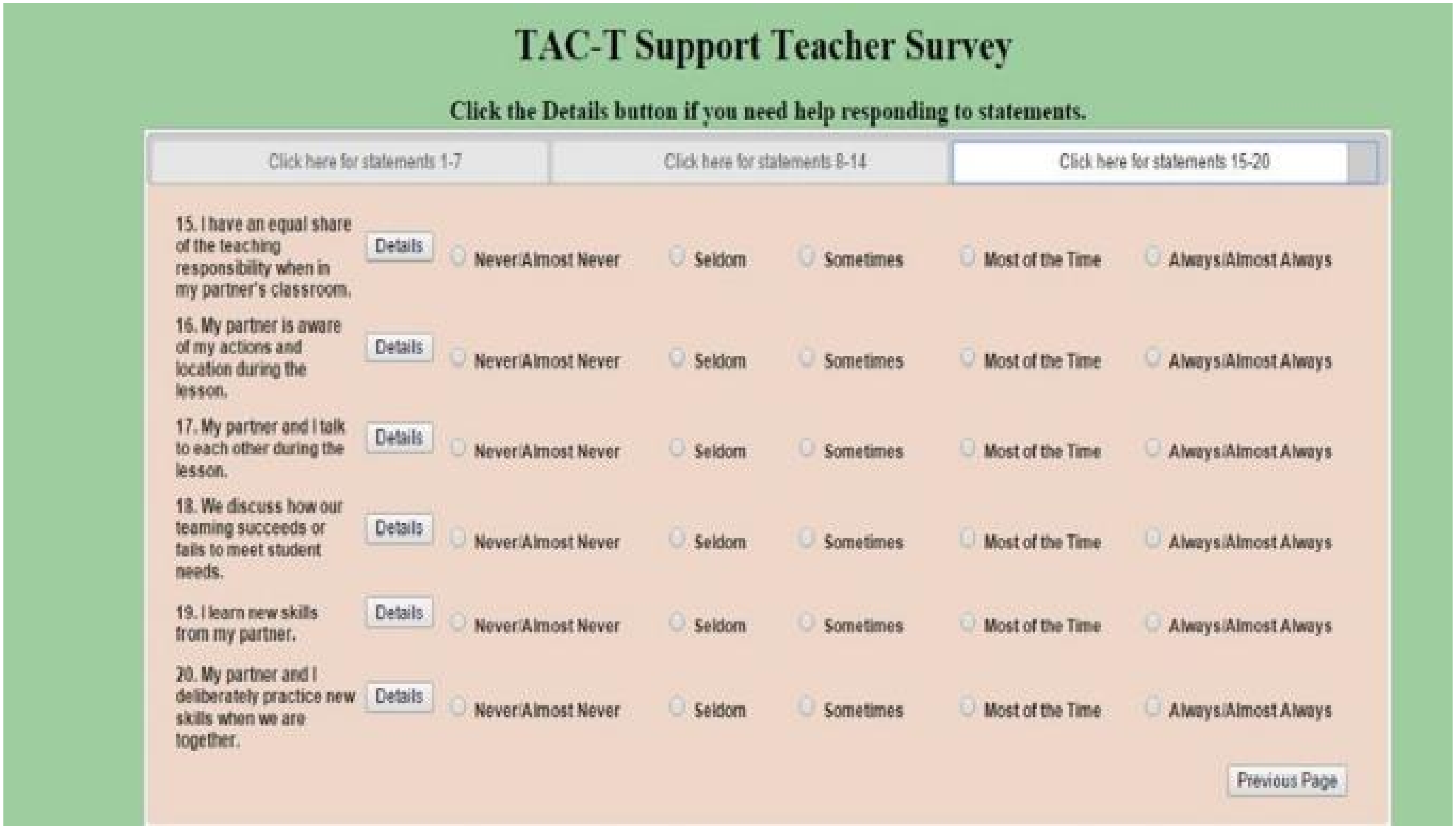
Teachers Analyzing Co-Teaching Survey

Questions 15 – 20



TAC-T Support Teacher Survey

Click the Details button if you need help responding to statements.

Click here for statements 1-7 | Click here for statements 8-14 | Click here for statements 15-20

Statement						
15. I have an equal share of the teaching responsibility when in my partner's classroom.	Details	Never/Almost Never	Seldom	Sometimes	Most of the Time	Always/Almost Always
16. My partner is aware of my actions and location during the lesson.	Details	Never/Almost Never	Seldom	Sometimes	Most of the Time	Always/Almost Always
17. My partner and I talk to each other during the lesson.	Details	Never/Almost Never	Seldom	Sometimes	Most of the Time	Always/Almost Always
18. We discuss how our teaming succeeds or fails to meet student needs.	Details	Never/Almost Never	Seldom	Sometimes	Most of the Time	Always/Almost Always
19. I learn new skills from my partner.	Details	Never/Almost Never	Seldom	Sometimes	Most of the Time	Always/Almost Always
20. My partner and I deliberately practice new skills when we are together.	Details	Never/Almost Never	Seldom	Sometimes	Most of the Time	Always/Almost Always

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APPENDIX D:

Behavior Analysis Co-Teaching Report

T-1

Teachers Analyzing Co-Teaching (TAC-T) Report 1: Behavior Analysis

Data Collection Date =	10/24/2014	Classroom Teacher =	Phyllis Madison
School =	Wy'east Middle School	Support Teacher =	Phyllis Madison

TAC-T analysis indicates that the members of this team MAY DISAGREE in their perception of the teaming relationship.

TAC-T analysis indicates that improvement is needed in 18 of the 20 co-teaching behaviors.

Info.	Behaviors	Team Teaching Behavior	Emerging	Developing	Established
?	1	My partner's role in the classroom is to help students experience success with standard curriculum.			X
?	2	Support teacher's ideas accepted as valid.		X	
?	3	Teachers have divergent approaches to instruction.		X	
?	4	Teachers have compatible approaches to management.		X	
?	5	Teachers feel comfortable with the co-teaching model.	X		
?	6	Teachers consider the co-teaching model to be effective.	X		
?	7	There is evidence of joint planning.	X		
?	8	Support teacher's ideas incorporated into lesson.	X		
?	9	Both teachers have access to all students in the class.			X
?	10	Both teachers have access to all teaching facilities in the classroom.		X	
?	11	Teachers both have verbal access to lesson.		X	
?	12	Both teachers teach to whole group simultaneously.	X		
?	13	Both teachers capable of sharing leadership role.		X	
?	14	Both teachers capable of total role release.	X		
?	15	Teachers share the instructional responsibilities during the lesson.	X		
?	16	Teachers keep track of each other during the lesson.	X		
?	17	Teachers conference during the lesson.	X		
?	18	Teachers evaluate the effect of teaming on instruction and students.	X		

APPENDIX E:

Co-Teaching Action Improvement Plan Report

Report 2

TEACHERS ANALYZING CO-TEACHING (TAC-T) Report 2: Improven

For: Phyllis Madison and Phyllis Madison

Based on the responses provided by you and your partner, and to encourage success, the TA(

targeted for improvement in the order suggested below.

Info.	Behavior	Target Co-teaching Behavior
		************** WORK ON THIS/THESE BEHAVIOR(S) FIRST ************
?	10	Both teachers have access to all teaching facilities in the classroom.
?	3	Teachers have divergent approaches to instruction.
?	2	Support teacher's ideas accepted as valid.
?	11	Teachers both have verbal access to lesson.
		************** WORK ON THIS/THESE BEHAVIOR(S) NEXT ************
?	13	Both teachers capable of sharing leadership role.
		************** WORK ON THIS/THESE BEHAVIOR(S) NEXT ************
?	19	There is evidence of exchange of professional skills.
		************** WORK ON THIS/THESE BEHAVIOR(S) NEXT ************
?	4	Teachers have compatible approaches to management.
		************** WORK ON THIS/THESE BEHAVIOR(S) NEXT ************
?	20	Teachers use co-teaching as an opportunity to practice new skills.
?	18	Teachers evaluate the effect of teaming on instruction and students.
?	7	There is evidence of joint planning.

APPENDIX F:

Individual Semi-Structured Interview Protocol Prior to PD

1. Describe any PD you have participated in that helped you teach math. Why was that PD helpful? RQ # 3
2. What you do think are important elements of math instruction? RQ #3
3. Describe how your administrator has supporting co-teaching in your building. Give a brief overview of how inclusion and co-teaching have looked or been enacted in your building in the past four years. RQ # 1
4. Please describe your relationship with your co-teaching partner. RQ #1
5. What do you see as the benefits or limitations of working with another teacher? RQ #1
6. If a student has been identified as having a learning disability, what does that mean for instruction you plan? RQ #2
7. What specific strategies have you learned for teaching mathematics to students with learning disabilities? How did you learn those strategies? In what ways have these strategies been effective (or not effective) in helping students approach grade-level standards? RQ # 2 & 3
8. What do you see as challenges or difficulties in teaching math to students with identified learning disabilities? RQ #2
9. What types of specialized instruction do you think students with learning disabilities need for effectively learning mathematics? RQ #2
10. How would you describe effective mathematics instruction? RQ #3
11. What do you think are the most important responsibilities/roles of the general educator teaching in a co-taught classroom? (Probe for top three, if needed) RQ #1

12. What do you think are the most important responsibilities/roles of the special education teacher in collaborating with the general education teacher? (Probe for top three, if needed) RQ #1
13. Describe inclusion and co-teaching at your school. RQ #1
14. What are your experiences with co-teaching math?
15. What do you see as the benefits of inclusion and co-teaching? Probe: Please tell me more about inclusive mathematics instruction for students with a learning disability. RQ #1
16. Please describe in detail how you plan for a typical mathematics lesson Probe: Tell me how you planned for the last math lesson you taught. Please describe the topic/context and how you go through the planning process? RQ # 3
17. What are your views and experiences collaborating with your general education or special education co-teaching partner? RQ #1
18. How, if at all, have your instructional methods changed since teaching in an inclusive co-teaching classroom? RQ #1
19. Is there anything else you would like to share with me about co-teaching in an inclusive math classroom?

APPENDIX G:

Individual Semi-Structured Interview Protocol Post PD

1. Please tell me about what you currently think about co-teaching. RQ # 1
2. What were some of the initial conversations you and the special education/general education teacher had when you first discovered that you would be co-teaching together? RQ #2
3. How have the conversations changed over time to support student learning? RQ #2
4. How did those first conversations evolve into a way of planning or preparing for the co-taught math lesson? RQ #2
5. How would you describe the structure of any planning meetings you had with your partner for preparing for a co-taught lesson? RQ #2
6. How has student data been used to plan for instruction? Probe: Tell me about how student data informs your instruction. RQ #2
7. How would you describe the collaboration meetings with your co-taught partner? How do you see these meetings impacting student learning? RQ #2
8. Describe how the general education and special education teacher have worked together during a co-taught lesson? Probe – what do you see as successes and challenges. RQ #1
9. Please describe how the PD you participated in impacted your co-teaching relationship. RQ #3
10. Please describe how the PD you participated in supported you with planning for and implementing math instruction. RQ #3

11. How has your participation in the PD has impacted your PLC meetings and the shared instructional practice of the general education and special education teacher during co-taught lessons? RQ #3
12. What do you think might be some benefits of co-teaching between general education and special education teachers? RQ #1
13. What do you think might be some limitations of co-teaching between general education and special education teachers? RQ #1
14. Is there anything else you would like to share with me about special education and general education teachers working together that we haven't already discussed?

APPENDIX H:

Partnership Semi-Structured Interview Protocol Prior to PD

1. Describe some of your first conversations together when you knew you would be co-teaching? How did you develop your co-teaching relationship? RQ #1
2. Please describe how you would define the role of your co-teaching partner. RQ #1
3. Think of a time when you thought collaboration was at its best. What made that collaborative experience so effective or beneficial? RQ #1
4. How would you like to improve your co-taught math practice? RQ #1
5. Describe how you decide on the needs of your students. RQ #2
6. How do you discuss differentiation with your co-taught partner? RQ #2
7. Describe how you plan together for differentiated lessons? RQ #2
8. In your view, what instructional approaches are needed for students working a year or more below grade level? RQ #2
9. Please describe how, together, you use student work in planning for instruction. RQ #2
10. What are you hoping to learn through participating in the PD? RQ #3
11. How has PD helped support instruction for you in the past? RQ #3

Is there anything else either one of you would like to share about collaboration or co-teaching?

APPENDIX I:

Partnership Semi-Structured Interview Protocol Post PD

1. How did the PD you participated in support math instruction in the co-taught classroom? RQ # 3
2. Describe how your conversations around collaboration and co-teaching changed as a result of the PD. RQ #3
3. What do you perceive that has specifically developed within your co-teaching relationship to enable for more effective math collaboration? RQ #1
4. Think of a special education student you both work with who is at least a year behind. How did you collaborate together to provide access to core content for this particular student? Probe: What resources did you use to help with lesson planning or lesson implementation? How do you determine if the student in special education is able to access grade-level math content? RQ #2
5. How will you continue to collaborate and support each other even though the PD and collaboration meetings are coming to a conclusion? RQ #1
6. Describe how your collaborative conversations have changed. RQ #1
7. What do you think is the next step in your co-teaching relationship? RQ # 1
8. If you were to get a new co-teaching partner for next school year, please explain what you would do first to develop your co-teaching relationship. Describe why this is the most important issue/idea/concept to address. RQ #1
9. What do you perceive is important that administration understand/change/or add to foster or develop co-teaching at your building? RQ #1
10. How has co-designed math instruction supported students in special education? RQ #2

11. How do you ensure the students in special education aren't instructionally left behind?

    RQ #2

APPENDIX J:

Collecting Observational Data during a PLC Meeting

Teacher name and position	Analyzing student data (RQ #3)	Designing instruction (RQ #2)	Creating assessments (RQ #3)	Using CGI principles (RQ #2 & #3)	U I Pra (R
Protected					
Protected					

APPENDIX K:

Problem Type and Solution Strategy Matrix

11 Different Addition and Subtraction Problem Types

Easiest --> Hardest

Simple ---------- Student Solution Strategies ----------→Complex

	Join Result unknown	Join Change unknown	Join Start Unknown	Separate Result unknown	Separate Change Unknown	Separate Start Unknown	Part-Part Whole Whole Unknown	Part Part Whole Part unknown	Compare Difference Unknown	Compare Quantity Unknown	Compare Referent Unknown
Direct Modeling											
Counting On											
Incrementing											
Compensation											
Derived Facts											

APPENDIX L:

Data Display Merging Student Work Sample Data and Observational Data

Student Work Sample Data	Observational Data
How did the student enter the task?	Analyzing student data
What are the student's strengths and next steps? How will you use this work to inform your instruction?	Designing instruction
How can this work inform formative or end of unit assessments?	Creating assessments
What solution strategy did the student use?	CGI principles
How will the teacher advance student mathematical thinking?	Five Practices
What additional supports could be put in place to allow students with math difficulty more access to mathematic content?	Evidence of planning with students with math difficulty (receiving math intervention or is special education) in mind

APPENDIX M:

Student Support Tool

Student Support Tool	Student Interests and Strengths	Math Conceptual Understanding Needs	Math Procedural Fluency Needs	Ways Co-Teachers can Support Math Content
Student A				
Student B				

APPENDIX N:

Reframe a Math Task Tool

Use the Following Criteria to Analyze and Reframe the Selected Math Task

Goal: Change a math task into a high quality rich math task by including at least 5 of the 6 elements

Qualities of Rich Math Tasks

- Relevant, meaningful, engaging context
- Multiple solution pathways
- Multiple correct solutions
- Multiple entry points (Low floor – High ceiling)
- Students can generalize ideas around a big math concept
- Manipulatives will bring meaning to the problem

Common Issues and Ideas for Reframing a Task, so the Task is High-Quality

Issue: The context is not engaging enough

✓Try changing the context to be about yourself. When you give your student's insider knowledge, you will hook them

✓Pose a real-world question about the school or community

✓Show an image and ask students to generate the questions

Issue: I can only think of one solution pathway

✓ Solve the problem with your teammates and discuss solutions

✓ Solve using manipulatives, then a picture, then a chart

✓ Think about adding or removing a constraint

✓ If you add a constraint lower the number to keep the problem accessible

✓ If you reduce a constraint consider increasing the number to balance rigor

Issue: There is only one correct solution

- ✓ Change the question, so the answer is more open-ended
- ✓ Change the wording to allow for equivalent answers (142 as well as 14 tens and 2 ones)

Issue: I am unsure if the problem has multiple entry points

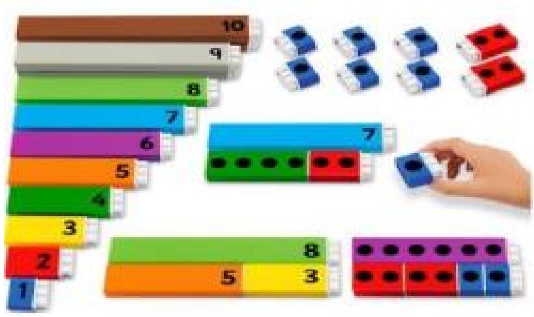
- ✓Can you solve the problem using manipulatives? Which ones? Try doing this.
- ✓Can you solve with drawing a picture?
- ✓Can you begin with a different constraint?
- ✓Can you make a chart or a table?
- ✓Can you find a shortcut in calculations?

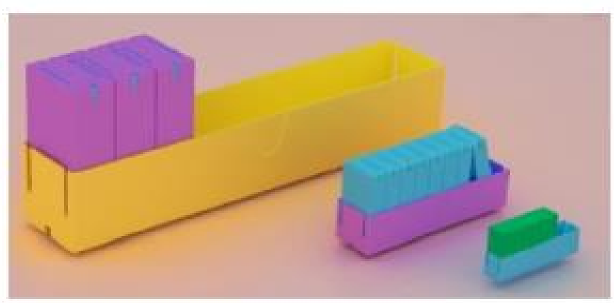
Issue: I am unsure of the idea students should generalize

- ✓Try looking at the progression documents to see where the learning is headed
- ✓Try asking yourself why it would be important for students to solve this task 5 times in a row
- ✓What is math property is at work in this task?
- ✓Try solving the problem yourself. Then reflect, how could the thinking you just did help you solve other tasks?

Issue: I am unsure of what manipulatives to use to help my students with the most mathematics difficulty

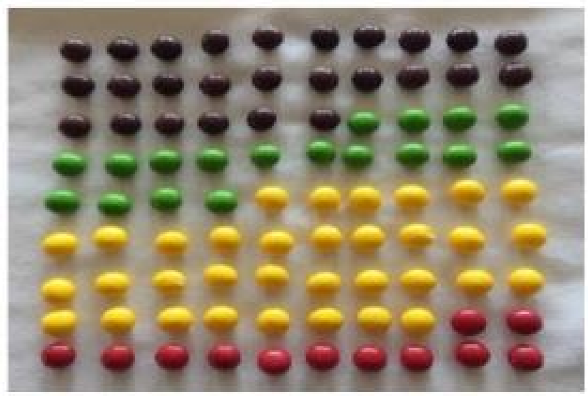
- ✓Remember to use our intervention manipulates to highlight certain concepts
  - oCuisenaire Rods – great for highlighting number relationships
  - oDigi Blocks® – conceptual manipulatives for base 10 number system
  - oNumber Balance Scale – perfect to highlight equivalency

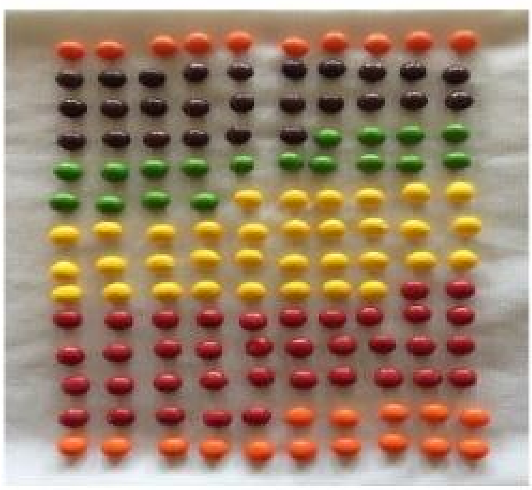




Issue: How do I shift my instructional practice away from explicit strategy instruction?

- ✓Create a context that will foster the strategy you would like to highlight.
    - Want to highlight a number line strategy? – Think about a context that is walking/biking/driving steps or miles.
    - Want to highlight using a hundreds chart? – Think about a context that would allow for rows of different colored items. Using an image is great for this.
    - Here are a few sample images:





- ✓Assume if students understand the problem and find the problem meaningful, they will develop their own solution strategies.
    - Be available to watch and listen to how students think about the problem and solve.
- ✓Want to highlight a skip counting strategy? – Think about a context in which there is a repeated action.