

Deterministic Sampling on the Torus for Bivariate Circular Estimation

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Abstract—Many modern approaches to nonlinear filtering employ sample-based density approximations. These approximations are generated via random (Monte Carlo methods) or deterministic sampling (say, the UKF). The advantages of deterministic techniques are their reproducibility and that they require fewer samples. While the UKF is designed for real vector spaces, we present an approach for deterministic sampling applicable to two-dimensional periodic manifolds. This approach employs five weighted samples and matches trigonometric moments and a circular-circular correlation coefficient.

I. INTRODUCTION

Estimation of angles and other periodic quantities is a widespread problem in many areas. When more than one angle is to be considered, it becomes necessary to take the dependence between the angles into account. While a single angle can be represented by a value on the unit circle, two or more angles can be seen as a value on the torus or hypertorus, respectively. In this paper, we focus on the two-dimensional case, i.e., on bivariate circular problems.

Bivariate circular estimation is of great importance in a number of applications in the fields of aerospace, signal processing, and robotics. Consider for example the heading of two aircraft in proximity. Even though the headings of two separate aircraft may seem independent at first glance, they are affected by common noise (e.g., the wind or variations in the Earth’s magnetic field influencing the navigation of both aircraft) and may thus be correlated. Another example is the orientation of a humanoid robot’s head and its torso, which can be moved independently but are subject to correlated disturbances that affect the entire robot. Other examples include the phase of two received signals as well as bearings-only measurements obtained from two different locations that are affected by the common environment.

There has been some research on probability distributions on the torus in the field of directional statistics [16]. Probability distributions on the torus have to be distinguished from distributions on the unit (hyper)sphere (see, e.g., [3]) because they consider a different underlying topology. The torus is useful when considering multiple quantities, where each of them is periodic on its own, whereas the sphere is useful when considering unit vectors or angles that correspond to (hyper)spherical coordinates. On the torus, the bivariate von Mises [15, Sec. 2.4], [18] and the bivariate wrapped normal distributions [6], [4] have been considered in literature. Some further investigations of these densities can be found in [17], [13]. In the past years, we have published a recursive filtering algorithm based on the bivariate wrapped normal distribution [8]. However, this algorithm is limited to very simple system and measurement models, because there was no easy way to propagate the uncertainty on the torus through an arbitrary nonlinear function.

To address this deficiency, a technique called *deterministic*

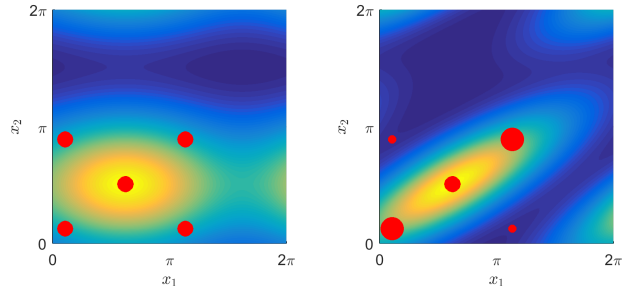


Figure 1: Uncorrelated (left) and correlated (right) bivariate wrapped normal distributions with the samples obtained using the proposed sampling scheme. The samples are depicted as red dots and their weight is indicated by their size. Note that both x_1 and x_2 are 2π -periodic.

sampling can be employed. Deterministic sampling is an approach used by various nonlinear filters on vector spaces such as the unscented Kalman filter (UKF) [7], the Smart Sampling Kalman filter (S^2 KF) [19], the cubature Kalman filter [1], [5], and others. The key advantage of deterministic sampling compared with stochastic sampling is that a much smaller number of carefully chosen samples can closely represent a probability density. We have previously published several papers about deterministic sampling on periodic manifolds, e.g., the unit circle [9], [10], the hypersphere [3], and the group of rigid body motions in the plane $SE(2)$ [2].

In this paper, we propose a novel deterministic sampling scheme for bivariate circular densities (see Fig. 1). For this purpose, we generalize ideas based on trigonometric moment matching that were previously used in the circular case [9]. However, it is not straightforward to ensure that not only the uncertainty in each dimension but also the circular correlation between the dimensions is accurately represented. To solve this problem, we use an intelligent choice of the sample weights to match a circular correlation coefficient [4]. Thus, we obtain a deterministic sample set by computing both suitable sample positions and sample weights.

II. PREREQUISITES

The Bivariate Wrapped Normal (BWN) distribution has the probability density function (pdf)

$$BWN(\underline{x}; \underline{\mu}, \mathbf{C}) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \mathcal{N}(\underline{x} + 2\pi[j, k]^T; \underline{\mu}, \mathbf{C}) ,$$

where $\underline{x} = [x_1, x_2]^T \in [0, 2\pi)^2$, $\underline{\mu} \in [0, 2\pi)^2$, and $\mathbf{C} \in \mathbb{R}^{2 \times 2}$ symmetric positive definite. A bivariate (wrapped) Dirac mixture on the torus $[0, 2\pi)^2$ is given by

$$\sum_{j=1}^L w_j \delta(\beta_j - \underline{x}_j) , \quad (1)$$

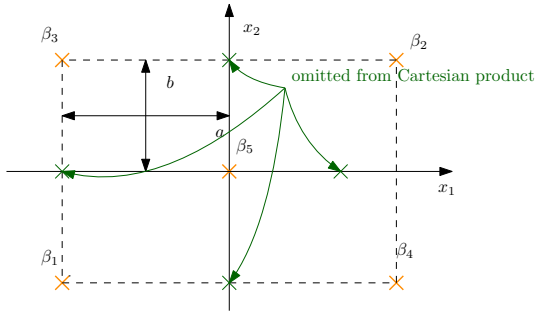


Figure 2: Idea of the proposed sampling scheme.

where $w_1, \dots, w_L > 0$ are weights with $\sum_{j=1}^L w_j = 1$ and $\underline{\beta}_1, \dots, \underline{\beta}_L \in [0, 2\pi]^d$ are the positions of the Dirac components.

We define the trigonometric moments for the bivariate case as the componentwise trigonometric moments (see [8])

$$\underline{m}_n = \begin{bmatrix} m_{n,1} \\ m_{n,2} \end{bmatrix} = \begin{bmatrix} \mathbb{E}(\exp(inx_1)) \\ \mathbb{E}(\exp(inx_2)) \end{bmatrix} \in \mathbb{C}^2. \quad (2)$$

It is possible to obtain the circular mean of each dimension as the complex argument of $m_{1,1}$ and $m_{1,2}$, respectively. The trigonometric moments can be calculated in closed form for the BWN distribution [8, Lemma 1].

In order to quantify the correlation between two circular random variables x_1 and x_2 , a number of different correlation coefficients has been proposed. In the following, we use the correlation coefficient by Jammalamadaka and Sarma [4], which is given by

$$r_c = \frac{\mathbb{E}(\sin(x_1 - \mu_1) \sin(x_2 - \mu_2))}{\mathbb{E}(\sin^2(x_1 - \mu_1)) \mathbb{E}(\sin^2(x_2 - \mu_2))},$$

where μ_1 and μ_2 are the circular means in each dimension. A closed-form solution for the BWN distribution can be found in [4, eq. (3.3)], [8, Lemma 2].

Remark 1. *The following sampling scheme is not limited to the BWN distribution but can be applied to any toroidal distribution, where the trigonometric moments and Jammalamadaka's correlation coefficient can be computed either analytically or numerically, e.g., various versions of the bivariate von Mises distribution [17]. A more thorough discussion of such distributions can be found in [18], [17], [14].*

III. DETERMINISTIC SAMPLING

In this section, we show how to deterministically sample from a bivariate circular density with known first trigonometric moment \underline{m}_1 and known circular correlation coefficient r_c . To simplify the derivations, we consider the special case where $\mu_1 = \mu_2 = 0$. In this case, all entries of the first trigonometric moment \underline{m}_1 are real numbers. Later, we can shift the locations by μ to obtain the samples for the general case.

The samples are chosen symmetrically around $[0, 0]^T$ with positions

$$\begin{aligned} \beta_1 &= [\beta_{1,1}, \beta_{1,2}]^T = [-a, -b]^T, \\ \beta_2 &= [\beta_{2,1}, \beta_{2,2}]^T = [a, b]^T, \\ \beta_3 &= [\beta_{3,1}, \beta_{3,2}]^T = [-a, b]^T, \\ \beta_4 &= [\beta_{4,1}, \beta_{4,2}]^T = [a, -b]^T, \\ \beta_5 &= [\beta_{5,1}, \beta_{5,2}]^T = [0, 0]^T, \end{aligned}$$

where $a, b \in (0, \pi)$ and weights $w_1 = w_2 > 0$, $w_3 = w_4 > 0$, and $w_5 = 1 - 2w_1 - 2w_3 > 0$ (see Fig. 2). This choice is motivated by using the Cartesian product of samples at $-a, 0, a$ and $-b, 0, b$ in each dimension similar to [9] and setting four of the weights to zero, which effectively reduces the number of samples from nine to five¹. The weights are chosen such that the Dirac mixture is point symmetric with respect to the origin. Thus, it holds that the imaginary part of the first trigonometric moment is always 0, i.e., the circular mean $\mu_1 = \mu_2 = 0$ is preserved. The idea is that for densities with positive correlation, the weights $w_1 = w_2$ are large and the weights $w_3 = w_4$ are small, whereas for densities with negative correlation the opposite is true. For uncorrelated densities, the weights w_1, w_2, w_3, w_4 are all identical.

A. Obtain a and b from the First Trigonometric Moment

For convenience, we define the abbreviation $\tilde{w} = w_1 + w_3$. The first component of \underline{m}_1 is given by the real part of (2)

$$\begin{aligned} m_{1,1} &= \sum_{n=1}^5 w_n \cos(\beta_{n,1}) \\ &= w_1 \cos(-a) + w_1 \cos(a) \\ &\quad + w_3 \cos(-a) + w_3 \cos(a) + w_5 \cos(0) \\ &= 2w_1 \cos(a) + 2w_3 \cos(a) + w_5 \\ &= 2\tilde{w} \cos(a) + w_5, \end{aligned}$$

and the second component of \underline{m}_1 is analogously given by

$$m_{1,2} = \sum_{n=1}^5 w_n \cos(\beta_{n,2}) = 2\tilde{w} \cos(b) + w_5.$$

For known weights, we can easily solve these equations for a and b , which results in

$$a = \arccos\left(\frac{m_{1,1} - w_5}{2\tilde{w}}\right), \quad b = \arccos\left(\frac{m_{1,2} - w_5}{2\tilde{w}}\right).$$

Notice that these equations only depend on \tilde{w} but not on the relation between the weights w_1 and w_3 , which determines the correlation. Furthermore, we can obtain $w_5 = 1 - 2\tilde{w}$, i.e., only \tilde{w} needs to be chosen. Hence, we get

$$\begin{aligned} a &= \arccos\left(\frac{m_{1,1} - 1 + 2\tilde{w}}{2\tilde{w}}\right) = \arccos\left(\frac{m_{1,1} - 1}{2\tilde{w}} + 1\right), \\ b &= \arccos\left(\frac{m_{1,2} - 1 + 2\tilde{w}}{2\tilde{w}}\right) = \arccos\left(\frac{m_{1,2} - 1}{2\tilde{w}} + 1\right). \end{aligned}$$

We need to ensure that real-valued solutions for a and b exist, i.e., the argument of $\arccos(\cdot)$ must be in $[-1, 1]$. We consider the equation for a and obtain

$$\frac{m_{1,1} - 1}{2\tilde{w}} + 1 < 1 \Leftrightarrow \frac{m_{1,1} - 1}{2\tilde{w}} < 0 \Leftrightarrow m_{1,1} < 1,$$

which always holds because $m_{1,1} \in [0, 1)$. Furthermore,

$$\begin{aligned} \frac{m_{1,1} - 1}{2\tilde{w}} + 1 > -1 &\Leftrightarrow \frac{m_{1,1} - 1}{2\tilde{w}} > -2 \\ \Leftrightarrow m_{1,1} - 1 > -4\tilde{w} &\Leftrightarrow \frac{1 - m_{1,1}}{4} < \tilde{w}, \end{aligned}$$

which gives a lower bound for \tilde{w} . We observe that because of $m_{1,1} \in [0, 1)$, any $\tilde{w} > 1/4$ fulfills this condition. The same results are obtained by considering the equation for b .

¹Unlike the UKF [7], the proposed approach uses samples on the diagonals rather than samples on the axes, which allows adjusting the correlation by varying the weights. By doing so, we avoid the need for rotating the samples to match the correlation, which is nontrivial on the torus.

B. Choose Weights w_1 and w_3 to Match Correlation

Jammalamadaka's correlation coefficient of the Dirac mixture (1) is given by

$$r_c = \frac{\sum_{n=1}^5 w_n \sin(\beta_{n,1}) \sin(\beta_{n,2})}{\sqrt{\left(\sum_{n=1}^5 w_n \sin^2(\beta_{n,1})\right) \left(\sum_{n=1}^5 w_n \sin^2(\beta_{n,2})\right)}}. \quad (3)$$

We can derive the term in the numerator according to

$$\begin{aligned} & \sum_{n=1}^5 w_n \sin(\beta_{n,1}) \sin(\beta_{n,2}) \\ = & w_1 \sin(-a) \sin(-b) + w_1 \sin(a) \sin(b) + w_3 \sin(-a) \sin(b) \\ & + w_3 \sin(a) \sin(-b) + w_5 \sin(0) \sin(0) \\ = & 2w_1 \sin(a) \sin(b) - 2w_3 \sin(a) \sin(b) \\ = & 2(w_1 - w_3) \sin(a) \sin(b) \\ = & 2(\tilde{w} - 2w_3) \sin(a) \sin(b). \end{aligned}$$

The terms in the denominator are given by

$$\begin{aligned} & \sum_{n=1}^5 w_n \sin^2(\beta_{n,1}) \\ = & w_1 \sin^2(-a) + w_1 \sin^2(a) \\ & + w_3 \sin^2(-a) + w_3 \sin^2(a) + w_5 \sin^2(0) \\ = & 2w_1 \sin^2(a) + 2w_3 \sin^2(a) \\ = & 2\tilde{w} \sin^2(a), \end{aligned}$$

and analogously

$$\sum_{n=1}^5 w_n \sin^2(\beta_{n,2}) = 2\tilde{w} \sin^2(b).$$

Now, we can solve (3) for w_3 assuming a fixed \tilde{w}

$$\begin{aligned} r_c &= \frac{2(\tilde{w} - 2w_3) \sin(a) \sin(b)}{\sqrt{4\tilde{w}^2 \sin^2(a) \sin^2(b)}} \\ \Leftrightarrow r_c \cdot 2|\tilde{w} \sin(a) \sin(b)| &= 2(\tilde{w} - 2w_3) \sin(a) \sin(b) \\ \Leftrightarrow r_c \cdot |\tilde{w}| &= \tilde{w} - 2w_3 \\ \Leftrightarrow w_3 &= (\tilde{w} - r_c |\tilde{w}|) / 2, \end{aligned}$$

where we use $\sin(a) > 0, \sin(b) > 0$. The value $w_1 = \tilde{w} - w_3$ can then be calculated as well. To obtain a valid Dirac mixture, we need to ensure that $w_1, w_3 \geq 0$.

From $w_3 \geq 0$, we get

$$\begin{aligned} & (\tilde{w} - r_c \cdot |\tilde{w}|) / 2 \geq 0 \\ \Leftrightarrow \tilde{w} &\geq r_c \cdot |\tilde{w}| \quad \Leftrightarrow \text{sign } \tilde{w} \geq r_c \end{aligned}$$

and from $w_1 \geq 0$, we obtain

$$\begin{aligned} & \tilde{w} - (\tilde{w} - r_c \cdot |\tilde{w}|) / 2 \geq 0 \\ \Leftrightarrow 2\tilde{w} - &(\tilde{w} - r_c \cdot |\tilde{w}|) \geq 0 \\ \Leftrightarrow \tilde{w} &\geq -r_c \cdot |\tilde{w}| \geq 0 \\ \Leftrightarrow \text{sign } \tilde{w} &\geq -r_c. \end{aligned}$$

Thus, we just need to ensure that $\tilde{w} \geq 0$.

We suggest the choice $w_5 = \frac{1}{5}$, which implies $\tilde{w} = \frac{2}{5}$ (i.e., $\tilde{w} > \frac{1}{4}$ also holds). This choice has the advantage that the Dirac mixture has uniform weights in the case of uncorrelated random variables, i.e., $r_c = 0$.

Pseudocode of the resulting procedure is given in Algorithm 1. It can be seen that the algorithm does not require any numerical methods. It is very easy to implement and fast to execute.

Algorithm 1: Deterministic Sampling

Input: first trigonometric moment m_1 ,

Jammalamadaka's correlation coefficient r_c

Output: Dirac positions β_1, \dots, β_5 and weights

w_1, \dots, w_5

/* Reduce to the case with circular mean zero */

$\underline{\mu} \leftarrow [\text{atan2}(\Im m_{1,1}, \Re m_{1,1}), \text{atan2}(\Im m_{1,2}, \Re m_{1,2})]^T$; */

$\underline{m}_1 \leftarrow [|m_{1,1}|, |m_{1,2}|]^T$;

/* Compute Dirac weights */

$\tilde{w} \leftarrow \frac{2}{5}$;

$w_5 \leftarrow 1 - 2\tilde{w}$;

$w_3 \leftarrow \frac{1}{2}(\tilde{w} - r_c |\tilde{w}|)$;

$w_1 \leftarrow \tilde{w} - w_3$;

/* Compute Dirac locations */

$a \leftarrow \arccos((m_{1,1} - 1)/(2\tilde{w}) + 1)$;

$b \leftarrow \arccos((m_{1,2} - 1)/(2\tilde{w}) + 1)$;

$\beta_1 \leftarrow \underline{\mu} + [-a, -b]^T$;

$\beta_2 \leftarrow \underline{\mu} + [a, b]^T$;

$\beta_3 \leftarrow \underline{\mu} + [-a, b]^T$;

$\beta_4 \leftarrow \underline{\mu} + [a, -b]^T$;

$\beta_5 \leftarrow \underline{\mu} + [0, 0]^T$;

return $(\beta_1, \dots, \beta_5, w_1, \dots, w_5)$;

IV. EVALUATION

To evaluate the proposed deterministic sampling scheme, we apply it to the problem of propagating a bivariate wrapped normal density through a nonlinear function. This function is given by

$$f_c(\underline{x}) = (\underline{x} + c \cdot [\cos(x_2), \sin(x_1)]^T) \bmod 2\pi,$$

where $c \in \mathbb{R}$ is a parameter that influences the nonlinearity. The evaluation consists of the following steps:

1) Sample from

$$\text{BWN} \left(\underline{x}; \underline{\mu} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \right).$$

2) Propagate all samples through $f_c(\cdot)$.

3) Estimate parameters of a BWN density from propagated samples using the Mixed-MLE method proposed in [13].

4) Compare $\underline{\mu}$ and \mathbf{C} of the resulting BWN density to the ground truth.

The ground truth is obtained by performing the step 1–3 with 10 000 Monte Carlo samples. In the evaluation, we compare the proposed sampling scheme to the UKF samples and to 50 Monte Carlo samples (averaged over 100 runs). We use the parameters $\sigma_1 = 0.8$, $\sigma_2 = 1.3$ and different values of ρ .

The error measure for $\underline{\mu}$ is given by the mean angular error across both dimensions

$$\frac{1}{2} \sum_{k=1}^2 \min(|\mu_k - \mu_k^{true}|, 2\pi - |\mu_k - \mu_k^{true}|).$$

For the matrix \mathbf{C} , we use the mean absolute error given by $\frac{1}{4} \|\mathbf{C} - \mathbf{C}^{true}\|_1$.

The results are shown in Figure 3. It can be seen that the proposed method significantly outperforms the UKF and the Monte Carlo solution with 50 samples even though it only uses five samples.

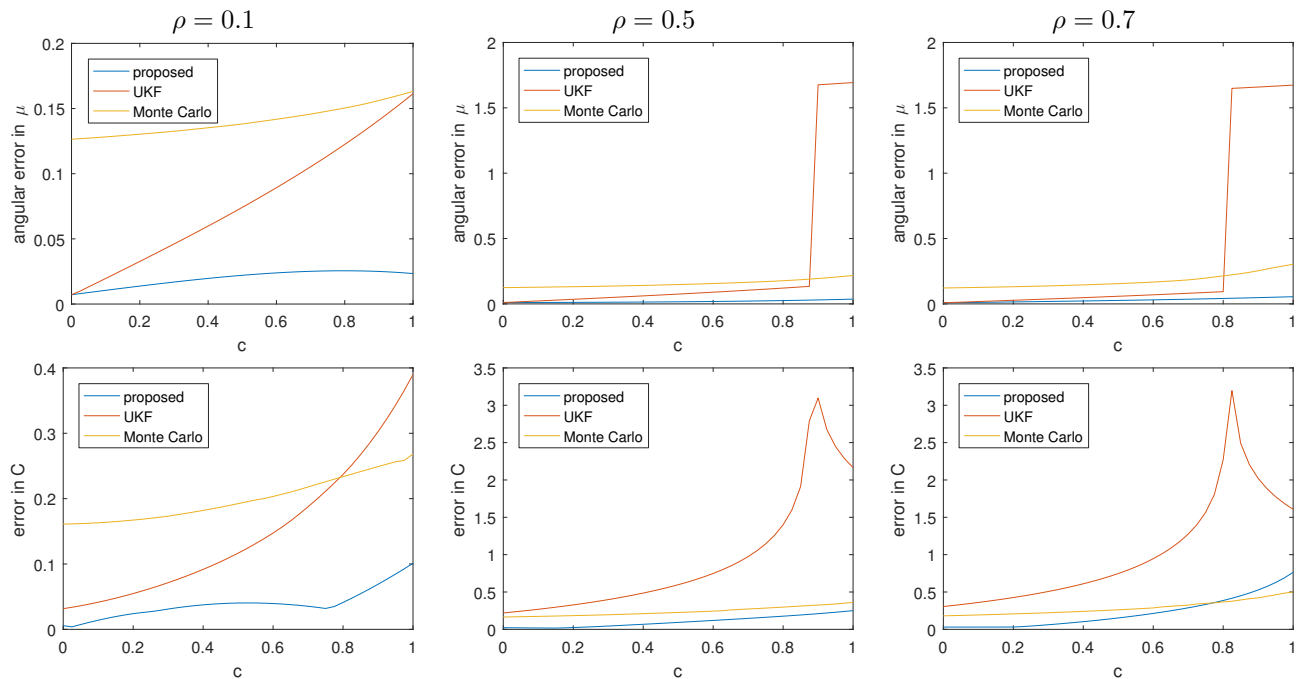


Figure 3: Evaluation results for different values of the correlation parameter ρ .

V. CONCLUSION

In this paper, we have presented a novel deterministic sampling algorithm for bivariate circular distributions. The proposed algorithm preserves the first trigonometric moment as well as Jammalamadaka's circular correlation coefficient. It is easy to implement and can be calculated very efficiently.

In the future, we plan to develop nonlinear recursive Bayesian filtering algorithms based on the proposed sampling scheme. The resulting filter will constitute a bivariate generalization of the circular filtering algorithms presented in [11]. Furthermore, a thorough comparison to the toroidal versions of the EKF, UKF, and the particle filter will be performed.

An implementation of the proposed method can be found in `libDirectional` [12], a MATLAB library for directional statistics and directional estimation.

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BIOGRAPHIES

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Gerhard Kurz received his diploma in computer science from the Karlsruhe Institute of Technology (KIT), Germany, in 2012. He obtained his Ph.D. in 2015 at the Intelligent Sensor-Actuator-Systems Laboratory, Karlsruhe Institute of Technology (KIT), Germany. His research interests include directional filtering, nonlinear estimation, and medical data fusion. He has authored multiple award-winning publications on these topics.



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