Recursive LMMSE Centralized Fusion with Compressed Multi-Radar Measurements

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***Abstract*—For multi-sensor centralized fusion with linear measurements, simply stacking all measurements up and then applying the Kalman filter at the fusion center can give the optimal estimation performance. This optimal performance is independent of how the measurements from different sensors are stacked up. However, for centralized fusion with multiple radar measurements under the recursive LMMSE filtering framework, the performance really matters as to how to stack the measurements from different radars. In [1], we have shown that centralized fusion with stacked recombined multi-radar measurements outperforms the one with stacked original measurements under the recursive LMMSE filtering framework. In this paper, we further develop a new multi-radar centralized fusion approach by compressing all measurements first and then applying the recursive LMMSE filter with single radar measurements at the fusion center. Numerical examples show that the new centralized fusion with compressed measurements has better estimation accuracy and smaller noncredibility than the ones with stacked measurements.**

I. Introduction

In tracking applications, target motion is usually modeled in Cartesian coordinates while the radar measurements are available in polar or spherical coordinates. It is obvious target tracking with radar measurements is a nonlinear filtering problem due to the nonlinear relationship between the radar measurements and the target motion state, whether the target motion model is linear or nonlinear. Many existing nonlinear filters are available for this problem, e.g. extended Kalman filter (EKF), unscented filter (UF) [2], cubature Kalman filter (CKF) [3], and particle filter (PF) [4]. However, since the nonlinear relationship between radar measurements and target motion state is known explicitly, specifically designed nonlinear filters may perform better.

The converted measurement Kalman filter [5]–[8] is such a popular specifically designed filtering approach. It converts the measurement model from polar/spherical coordinates into Cartesian coordinates so that the converted model is pseudo linear. Then the standard Kalman filter (KF) can be applied because the standard KF can only be applied to linear models. However, the converted measurement errors have the following mismatch with the assumptions of the standard KF: 1) the converted measurement errors are dependent on state; 2) the converted measurement noise sequence is not white; 3) the covariances are estimated conditioned on the measurement or state [1], [9], [10]. Many ways have been proposed to debias the converted measurements by using additive debiasing [5], multiplicative debiasing [6], and decorrelated converted measurements [7], [8]. Extension of debiased conversion to the case when range rate measurements [6], [11] are also available. In contrast, a recursive LMMSE filter was proposed in [10] using the converted measurements directly in Cartesian coordinates. This recursive LMMSE filter is free of the above three mismatches of the converted measurement Kalman filter.

For multi-sensor centralized fusion [12] with linear measurements, optimal estimation performance can be achieved by simply stacking all measurements up and then applying the Kalman filter at the fusion center. This optimal performance stays the same no matter how the measurements from different sensors are stacked up. However, for centralized fusion with multiple radar measurements under the recursive LMMSE filtering framework, the performance depends on how the measurements from different radars are stacked up. It is shown in [1] that centralized fusion with stacked recombined multi-radar measurements outperforms the one with stacked original measurements under the recursive LMMSE filtering framework. In this paper, we propose a new multi-radar centralized fusion approach under the recursive LMMSE filtering framework. The key idea of this new approach is to compress the measurements from all radars at the fusion center first and then apply the recursive LMMSE filter to the single radar measurements. Due to the reduction of uncertainty and nonlinearity associated with the compressed single radar measurements, the new centralized fusion with compressed measurements outperforms the ones with stacked measurements in terms of both estimation accuracy and credibility.

The rest of the paper is organized as follows. Section II formulates the problem. In Section III, we summarize recursive LMMSE centralized fusion with stacked original measurements and with recombined measurements. New recursive LMMSE centralized fusion with compressed measurements is presented In Section IV. Section V provides experimental results. Section VI gives conclusions.

II. Problem Formulation

Consider the following typical linear target motion model in Cartesian coordinates

$$\mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{G}_{k-1}\boldsymbol{\omega}_{k-1}, \quad k = 1, 2, \cdots \tag{1}$$

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where $\mathbf{x}_k = [x_k, \dot{x}_k, y_k, \dot{y}_k, z_k, \dot{z}_k]'$ is the target motion state at time k. $\langle \boldsymbol{\omega}_k \rangle$ is the process noise sequence which is white Gaussian and zero-mean with covariance $E[\boldsymbol{\omega}_k \boldsymbol{\omega}_k'] = \mathbf{Q}_k$. It is assumed that $\mathbf{x}_0 \sim \mathcal{N}(\bar{\mathbf{x}}_0, \mathbf{P}_0)$ and $\text{cov}(\mathbf{x}_0, \boldsymbol{\omega}_k) = 0$.

In this paper, we assume that M radars are mounted at the origin of the Cartesian coordinates and the measurements of M radars are mutually independent. Thus, in spherical coordinates, the measurements of the i-th radar at time k are range r_k^i, bearing b_k^i, and elevation e_k^i. Assume that they are generated as

$$\begin{bmatrix} r_k^i \\ b_k^i \\ e_k^i \end{bmatrix} = \begin{bmatrix} r_k \\ b_k \\ e_k \end{bmatrix} + \begin{bmatrix} \tilde{r}_k^i \\ \tilde{b}_k^i \\ \tilde{e}_k^i \end{bmatrix} . \; i = 1, \cdots, M \tag{2}$$

where

$$\begin{aligned} r_k &= (x_k^2 + y_k^2 + z_k^2)^{1/2} \\ b_k &= \tan^{-1}(y_k / x_k) \\ e_k &= \tan^{-1}(z_k / (x_k^2 + y_k^2)^{1/2}) \end{aligned}$$

and $\langle \tilde{r}_k^i \rangle, \langle \tilde{b}_k^i \rangle, \langle \tilde{e}_k^i \rangle$ are the corresponding measurement noise sequences. It is further assumed that measurement noise sequences, process noise sequences and $\mathbf{x}_0$ are mutually independent. Each of the measurement noise sequences is assumed as a zero-mean white Gaussian sequence, and $\sigma_r^i, \sigma_b^i, \sigma_b^i$ are the corresponding standard deviations.

For comparison purpose, we summarize two existing centralized fusion approaches with multi-radar measurements under the recursive LMMSE filtering framework next.

III. Summary of Two Existing Approaches

Recursive LMMSE centralized fusion with stacked original measurements and with stacked recombined measurements are summarized in this section.

A. Centralized Fusion with Stacked Original Measurements

The basic idea of centralized fusion with stacked original measurements is to stack the measurements from M radars first and then use the stacked measurements for estimation. First, the original measurements need to be converted from the spherical coordinates to the Cartesian coordinates as

$$\begin{aligned} x_k^i &= r_k^i \cos e_k^i \cos b_k^i \\ y_k^i &= r_k^i \cos e_k^i \sin b_k^i \\ z_k^i &= r_k^i \sin e_k^i \end{aligned}$$

Define

$$\begin{aligned} \mathbf{z}_k^i &= [x_k^i, y_k^i, z_k^i]' \\ \lambda_1^i &= E[\cos \tilde{b}_k^i] = e^{-(\sigma_b^i)^2/2} \\ \mu_1^i &= E[\cos \tilde{e}_k^i] = e^{-(\sigma_e^i)^2/2} \\ \boldsymbol{\Omega}_i &= \text{diag}(\lambda_1^i \mu_1^i, \lambda_1^i \mu_1^i, \mu_1^i) \end{aligned}$$

Next we stack the measurements from M radars up and apply recursive LMMESE filtering [1].

Let $\mathbf{z}_k = [(\mathbf{z}_k^1)', (\mathbf{z}_k^2)', \cdots, (\mathbf{z}_k^M)']'$, $\mathbf{Z}^k = \{\mathbf{z}_j\}_{j=1}^k$. Given $\hat{\mathbf{x}}_{k-1|k-1} = E^*[\mathbf{x}_{k-1} | \mathbf{Z}^{k-1}]$ and $\mathbf{P}_{k-1|k-1} = \text{MSE}(\hat{\mathbf{x}}_{k-1|k-1} | \mathbf{Z}^{k-1})$, then one cycle of centralized fusion with stacked original measurements is

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_{k-1} \hat{\mathbf{x}}_{k-1|k-1}$$

$$\begin{aligned} \mathbf{P}_{k|k-1} &= \mathbf{F}_{k-1} \mathbf{P}_{k-1|k-1} \mathbf{F}_{k-1}' + \mathbf{G}_{k-1} \mathbf{Q}_{k-1} \mathbf{G}_{k-1}' \\ \hat{\mathbf{z}}_{k|k-1} &= [(\hat{\mathbf{z}}_{k|k-1}^1)', \cdots, (\hat{\mathbf{z}}_{k|k-1}^M)']' \\ \mathbf{C}_{k|k-1} &= [\mathbf{C}_{k|k-1}^1, \cdots, \mathbf{C}_{k|k-1}^M] \\ \mathbf{S}_k &= \begin{bmatrix} \mathbf{S}_k^1 & \mathbf{S}_k^{1,2} & \dots & \mathbf{S}_k^{1,M} \\ \mathbf{S}_k^{2,1} & \mathbf{S}_k^2 & \dots & \mathbf{S}_k^{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}_k^{M,1} & \mathbf{S}_k^{M,2} & \dots & \mathbf{S}_k^M \end{bmatrix} \\ \mathbf{S}_k^{i,j} &= \boldsymbol{\Omega}_i \mathbf{P}_{k|k-1}([1,3,5],[1,3,5]) \boldsymbol{\Omega}_j', \\ & \quad i \neq j, \; i,j = 1, \cdots, M \end{aligned}$$

$$\begin{aligned} & \mathbf{P}_{k|k-1}([1,3,5],[1,3,5]) \\ &= \begin{bmatrix} \mathbf{P}_{k|k-1}(1,1) & \mathbf{P}_{k|k-1}(1,3) & \mathbf{P}_{k|k-1}(1,5) \\ \mathbf{P}_{k|k-1}(3,1) & \mathbf{P}_{k|k-1}(3,3) & \mathbf{P}_{k|k-1}(3,5) \\ \mathbf{P}_{k|k-1}(5,1) & \mathbf{P}_{k|k-1}(5,3) & \mathbf{P}_{k|k-1}(5,5) \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + \mathbf{C}_{k|k-1} \mathbf{S}_k^{-1} (\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}) \\ \mathbf{P}_{k|k} &= \mathbf{P}_{k|k-1} - \mathbf{C}_{k|k-1} \mathbf{S}_k^{-1} \mathbf{C}_{k|k-1}' \end{aligned}$$

where $\hat{\mathbf{z}}_{k|k-1}^i$, $\mathbf{C}_{k|k-1}^i$ and $\mathbf{S}_k^i$ can be referred to the recursive LMMSE filter in [10].

B. Centralized Fusion with Recombined Measurements

The "recombination" in [1] means to reshuffle the measurements from all radars according to the measurement noise covariances dimension by dimension. And the core idea of recursive LMMSE centralized fusion with recombined radar measurements is that measurements from the j-th "virtual sensor" have the j-th smallest standard deviations of range, bearing, and elevation. Because of this, the performance of centralized fusion with recombined stacked measurements is better than the one with stacked measurements. This approach is summarized as follows.

First, we order the standard deviations of all M radars at the fusion center

$$\begin{aligned} \sigma_r^{(1)} &\le \sigma_r^{(2)} \le \cdots \le \sigma_r^{(M)} \\ \sigma_b^{(1)} &\le \sigma_b^{(2)} \le \cdots \le \sigma_b^{(M)} \\ \sigma_e^{(1)} &\le \sigma_e^{(2)} \le \cdots \le \sigma_e^{(M)} \end{aligned}$$

Suppose that $\sigma_r^{(i)} = \sigma_r^{l_i}$, $\sigma_b^{(i)} = \sigma_b^{m_i}$, $\sigma_e^{(i)} = \sigma_e^{n_i}$, $\lambda_h^{(i)} = \lambda_h^{m_i}$, and $\mu_h^{(i)} = \mu_h^{n_i}$, where $l_i, m_i, n_i = 1, 2, \cdots, M$ and $h = 1, 2, 3$. Then the i-th recombined converted measurements can be obtained as

$$\begin{aligned} x_k^{(i)} &= r_k^{l_i} \cos e_k^{n_i} \cos b_k^{m_i} \\ y_k^{(i)} &= r_k^{l_i} \cos e_k^{n_i} \sin b_k^{m_i} \\ z_k^{(i)} &= r_k^{l_i} \sin e_k^{n_i} \end{aligned}$$

Correspondingly, the i-th recombined measurements $\mathbf{z}_k^{(i)}$ and the recombined stacked measurements $\mathbf{z}_k^r$ are

$$\begin{aligned} \mathbf{z}_k^{(i)} &= [x_k^{(i)}, y_k^{(i)}, z_k^{(i)}]' \\ \mathbf{z}_k^r &= [(\mathbf{z}_k^{(1)})', (\mathbf{z}_k^{(2)})', \cdots, (\mathbf{z}_k^{(M)})']' \end{aligned}$$

Given $\hat{\mathbf{x}}^r_{k-1|k-1} = E^*[\mathbf{x}_{k-1}|\mathbf{Z}^{k-1,r}]$ and $\mathbf{P}^r_{k-1|k-1} = \text{MSE}(\hat{\mathbf{x}}_{k-1|k-1}|\mathbf{Z}^{k-1,r})$, where $\mathbf{Z}^{k,r} = \{\mathbf{z}^r_j\}^k_{j=1}$, then one cycle of centralized fusion with recombined measurements is

$$\begin{aligned}
\hat{\mathbf{x}}^r_{k|k-1} &= \mathbf{F}_{k-1}\hat{\mathbf{x}}^r_{k-1|k-1} \\
\mathbf{P}^r_{k|k-1} &= \mathbf{F}_{k-1}\mathbf{P}^r_{k-1|k-1}\mathbf{F}'_{k-1} + \mathbf{G}_{k-1}\mathbf{Q}_{k-1}\mathbf{G}'_{k-1} \\
\hat{\mathbf{z}}^r_{k|k-1} &= [(\hat{\mathbf{z}}^{(1)}_{k|k-1})', \cdots, (\hat{\mathbf{z}}^{(M)}_{k|k-1})']' \\
\mathbf{C}^r_{k|k-1} &= [\mathbf{C}^{(1)}_{k|k-1}, \cdots, \mathbf{C}^{(M)}_{k|k-1}] \\
\mathbf{S}^r_k &= \begin{bmatrix} \mathbf{S}^{(1)}_k & \mathbf{S}^{(1),(2)}_k & \dots & \mathbf{S}^{(1),(M)}_k \\ \mathbf{S}^{(2),(1)}_k & \mathbf{S}^{(2)}_k & \dots & \mathbf{S}^{(2),(M)}_k \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}^{(M),(1)}_k & \mathbf{S}^{(M),(2)}_k & \dots & \mathbf{S}^{(M)}_k \end{bmatrix} \\
\mathbf{S}^{(i),(j)}_k &= \mathbf{\Omega}_{(i)}\mathbf{P}^r_{k|k-1}([1,3,5],[1,3,5])\mathbf{\Omega}'_{(j)}, \\
&\quad i \neq j,\ i,j = 1, \cdots, M \\
\mathbf{\Omega}_{(i)} &= \text{diag}(\lambda^{(i)}_1\mu^{(i)}_1, \lambda^{(i)}_1\mu^{(i)}_1, \mu^{(i)}_1) \\
\hat{\mathbf{x}}^r_{k|k} &= \hat{\mathbf{x}}^r_{k|k-1} + \mathbf{C}^r_{k|k-1}(\mathbf{S}^r_k)^{-1}(\mathbf{z}^r_k - \hat{\mathbf{z}}^r_{k|k-1}) \\
\mathbf{P}^r_{k|k} &= \mathbf{P}^r_{k|k-1} - \mathbf{C}^r_{k|k-1}(\mathbf{S}^r_k)^{-1}(\mathbf{C}^r_{k|k-1})'
\end{aligned}$$

From [1], we know that, for centralized fusion with multi-radar measurements under the recursive LMMSE filtering framework, the performance really matters as to how to stack the measurements from different radars. Also, centralized fusion with stacked recombined multi-radar measurements outperforms the one with stacked original measurements.

IV. New Recursive LMMSE Centralized Fusion with Compressed Measurements

The two existing approaches in the above section stack all measurements in two different ways first and then apply the recursive LMMSE filtering framework. Next we present a new recursive centralized fusion approach. The key idea of this new approach is to compress all measurements from different radars into a pseudo single radar measurement first and then apply the recursive LMMSE filtering with this single radar measurement.

For recursive LMMSE centralized fusion with compressed measurements, first and foremost, we need to compress the range measurements, bearing measurements, and elevation measurements from all radars, respectively.

Define

$$\begin{aligned}
\mathbf{r}^s_k &= [r^1_k, r^2_k, \cdots, r^M_k]' \\
\mathbf{b}^s_k &= [b^1_k, b^2_k, \cdots, b^M_k]' \\
\mathbf{e}^s_k &= [e^1_k, e^2_k, \cdots, e^M_k]' \\
\mathbf{H}^r_k = \mathbf{H}^b_k = \mathbf{H}^e_k &= [\underbrace{1, 1, \cdots, 1}_{M}]' \\
\tilde{\mathbf{r}}^s_k &= [\tilde{r}^1_k, \tilde{r}^2_k, \cdots, \tilde{r}^M_k]' \\
\tilde{\mathbf{b}}^s_k &= [\tilde{b}^1_k, \tilde{b}^2_k, \cdots, \tilde{b}^M_k]' \\
\tilde{\mathbf{e}}^s_k &= [\tilde{e}^1_k, \tilde{e}^2_k, \cdots, \tilde{e}^M_k]'
\end{aligned}$$

where $\mathbf{r}^s_k, \mathbf{b}^s_k, \mathbf{e}^s_k$ are the stacked measurements of range, bearing and elevation, and $\tilde{\mathbf{r}}^s_k, \tilde{\mathbf{b}}^s_k, \tilde{\mathbf{e}}^s_k$ are their measurement noises.

From Eq. (2), the stacked measurement equation at the fusion center can be written as

$$\begin{aligned}
\mathbf{r}^s_k &= \mathbf{H}^r_k r_k + \tilde{\mathbf{r}}^s_k \\
\mathbf{b}^s_k &= \mathbf{H}^b_k b_k + \tilde{\mathbf{b}}^s_k \\
\mathbf{e}^s_k &= \mathbf{H}^e_k e_k + \tilde{\mathbf{e}}^s_k
\end{aligned}$$

Since $\langle \tilde{r}^i_k \rangle$, $\langle \tilde{b}^i_k \rangle$ and $\langle \tilde{e}^i_k \rangle$ are white Gaussian sequences and mutually independent, the covariances of $\tilde{\mathbf{r}}^s_k$, $\tilde{\mathbf{b}}^s_k$, $\tilde{\mathbf{e}}^s_k$ are

$$\begin{aligned}
\mathbf{R}^r_k &= \text{cov}(\tilde{\mathbf{r}}^s_k) = \text{diag}((\sigma^1_r)^2, (\sigma^2_r)^2, \cdots, (\sigma^M_r)^2) \\
\mathbf{R}^b_k &= \text{cov}(\tilde{\mathbf{b}}^s_k) = \text{diag}((\sigma^1_b)^2, (\sigma^2_b)^2, \cdots, (\sigma^M_b)^2) \\
\mathbf{R}^e_k &= \text{cov}(\tilde{\mathbf{e}}^s_k) = \text{diag}((\sigma^1_e)^2, (\sigma^2_e)^2, \cdots, (\sigma^M_e)^2)
\end{aligned}$$

Then the optimal WLS estimates [13] of r_k, b_k, e_k and their corresponding MSE matrices are

$$\begin{aligned}
\hat{r}^{\text{OWLS}}_k &= ((\mathbf{H}^r_k)'(\mathbf{R}^r_k)^{-1}(\mathbf{H}^r_k))^{-1}(\mathbf{H}^r_k)'(\mathbf{R}^r_k)^{-1}\mathbf{r}^s_k \\
P^r_k &= ((\mathbf{H}^r_k)'(\mathbf{R}^r_k)^{-1}(\mathbf{H}^r_k))^{-1} \\
\hat{b}^{\text{OWLS}}_k &= ((\mathbf{H}^b_k)'(\mathbf{R}^b_k)^{-1}(\mathbf{H}^b_k))^{-1}(\mathbf{H}^b_k)'(\mathbf{R}^b_k)^{-1}\mathbf{b}^s_k \\
P^b_k &= ((\mathbf{H}^b_k)'(\mathbf{R}^b_k)^{-1}(\mathbf{H}^b_k))^{-1} \\
\hat{e}^{\text{OWLS}}_k &= ((\mathbf{H}^e_k)'(\mathbf{R}^e_k)^{-1}(\mathbf{H}^e_k))^{-1}(\mathbf{H}^e_k)'(\mathbf{R}^e_k)^{-1}\mathbf{e}^s_k \\
P^e_k &= ((\mathbf{H}^e_k)'(\mathbf{R}^e_k)^{-1}(\mathbf{H}^e_k))^{-1}
\end{aligned}$$

Thus we have the following compressed measurement equation at the fusion center

$$\begin{aligned}
r^c_k &= H^{r,c}_k r_k + \tilde{r}^c_k \\
b^c_k &= H^{b,c}_k b_k + \tilde{b}^c_k \\
e^c_k &= H^{e,c}_k e_k + \tilde{e}^c_k
\end{aligned}$$

where

$$\begin{aligned}
r^c_k &= \hat{r}^{\text{OWLS}}_k \\
b^c_k &= \hat{b}^{\text{OWLS}}_k \\
e^c_k &= \hat{e}^{\text{OWLS}}_k \\
H^{r,c}_k &= H^{b,c}_k = H^{e,c}_k = 1
\end{aligned}$$

And the covariances of $\tilde{r}^c_k, \tilde{b}^c_k, \tilde{e}^c_k$ are

$$\begin{aligned}
R^{r,c}_k &= \text{cov}(\tilde{r}^c_k) = P^r_k \\
R^{b,c}_k &= \text{cov}(\tilde{b}^c_k) = P^b_k \\
R^{e,c}_k &= \text{cov}(\tilde{e}^c_k) = P^e_k
\end{aligned}$$

We can see that the compressed measurement $[r^c_k, b^c_k, e^c_k]'$ at time k is a 3×1 vector, while the stacked original measurement at time k is a $3M \times 1$ vector. It is obvious that the compressed measurement has smaller nonlinearity than the stacked original measurement with respect to the system state. In addition, covariances of compressed measurements are smaller than the ones of the measurements of each radar, whether recombined or not. Thus recursive LMMSE centralized fusion with compressed measurements is guaranteed to perform better than the two existing centralized fusion approaches using stacked measurements.

Next, we apply the LMMSE filtering framework at the fusion center. By treating the compressed measurement equation as the pseudo measurement equation of a single radar, we have the following converted measurements from the spherical coordinates into the Cartesian coordinates

$$x^c_k = r^c_k \cos e^c_k \cos b^c_k$$

$$y_k^c = r_k^c \cos e_k^c \sin b_k^c$$
$$z_k^c = r_k^c \sin e_k^c$$

Define

$$\begin{aligned}
\mathbf{z}_k^c &= [x_k^c, y_k^c, z_k^c]' \\
\lambda_1^c &= E[\cos \tilde{b}_k^c] = e^{-R_k^{b,c}/2} \\
\lambda_2^c &= E[\cos^2 \tilde{b}_k^c] = (1 + e^{-2R_k^{b,c}})/2 \\
\lambda_3^c &= E[\sin^2 \tilde{b}_k^c] = (1 - e^{-2R_k^{b,c}})/2 \\
\mu_1^c &= E[\cos \tilde{e}_k^c] = e^{-R_k^{e,c}/2} \\
\mu_2^c &= E[\cos^2 \tilde{e}_k^c] = (1 + e^{-2R_k^{e,c}})/2 \\
\mu_3^c &= E[\sin^2 \tilde{e}_k^c] = (1 - e^{-2R_k^{e,c}})/2 \\
\mathbf{\Omega}_c &= \text{diag}(\lambda_1^c \mu_1^c, \lambda_1^c \mu_1^c, \mu_1^c)
\end{aligned}$$

Given $\hat{\mathbf{x}}_{k-1|k-1}^c$ and $\mathbf{P}_{k-1|k-1}^c$, one cycle of centralized fusion with compressed multi-radar measurements can be summarized as

$$\begin{aligned}
\hat{\mathbf{x}}_{k|k-1}^c &= \mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1|k-1}^c \\
\mathbf{P}_{k|k-1}^c &= \mathbf{F}_{k-1}\mathbf{P}_{k-1|k-1}^c\mathbf{F}_{k-1}' + \mathbf{G}_{k-1}\mathbf{Q}_{k-1}\mathbf{G}_{k-1}' \\
\hat{\mathbf{z}}_{k|k-1}^c &= \mathbf{\Omega}_c[\hat{\mathbf{x}}_{k|k-1}^c(1), \hat{\mathbf{x}}_{k|k-1}^c(3), \hat{\mathbf{x}}_{k|k-1}^c(5)]' \\
\bar{d}^c &= ([\hat{\mathbf{x}}_{k|k-1}^c(1)]^2 + [\hat{\mathbf{x}}_{k|k-1}^c(3)]^2 + [\hat{\mathbf{x}}_{k|k-1}^c(5)]^2)^{1/2} \\
\bar{d}_1^c &= ([\hat{\mathbf{x}}_{k|k-1}^c(1)]^2 + [\hat{\mathbf{x}}_{k|k-1}^c(3)]^2)^{1/2} \\
\alpha_k^c &= \mu_2^c R_k^{r,c}/(\bar{d}^c)^2 + \mu_3^c[\hat{\mathbf{x}}_{k|k-1}^c(5)]^2/(\bar{d}_1^c)^2 \\
&\quad + \mu_3^c R_k^{r,c}[\hat{\mathbf{x}}_{k|k-1}^c(5)]^2/(\bar{d}^c\bar{d}_1^c)^2 \\
\boldsymbol{\beta}_k^c(1) &= [\lambda_2^c\mu_2^c - (\lambda_1^c\mu_1^c)^2][\hat{\mathbf{x}}_{k|k-1}^c(1)]^2 \\
&\quad + \lambda_3^c\mu_2^c[\hat{\mathbf{x}}_{k|k-1}^c(3)]^2 \\
\boldsymbol{\beta}_k^c(2) &= [\lambda_2^c\mu_2^c - (\lambda_1^c\mu_1^c)^2][\hat{\mathbf{x}}_{k|k-1}^c(3)]^2 \\
&\quad + \lambda_3^c\mu_2^c[\hat{\mathbf{x}}_{k|k-1}^c(1)]^2 \\
\boldsymbol{\beta}_k^c(3) &= [\mu_2^c - (\mu_1^c)^2][\hat{\mathbf{x}}_{k|k-1}^c(5)]^2 + \mu_3^c(\bar{d}_1^c)^2 \\
\boldsymbol{\beta}_k^c(4) &= [\mu_2^c(\lambda_2^c - \lambda_3^c) - (\lambda_1^c\mu_1^c)^2]\hat{\mathbf{x}}_{k|k-1}^c(1)\hat{\mathbf{x}}_{k|k-1}^c(3) \\
\boldsymbol{\beta}_k^c(5) &= [\lambda_1^c(\mu_2^c - \mu_3^c) - \lambda_1^c(\mu_1^c)^2]\hat{\mathbf{x}}_{k|k-1}^c(5) \\
\mathbf{S}_k^c(1,1) &\approx \lambda_2^c\mu_2^c\mathbf{P}_{k|k-1}^c(1,1) + \lambda_3^c\mu_2^c\mathbf{P}_{k|k-1}^c(3,3) \\
&\quad + \alpha_k^c(\lambda_2^c[\hat{\mathbf{x}}_{k|k-1}^c(1)]^2 + \lambda_3^c[\hat{\mathbf{x}}_{k|k-1}^c(3)]^2) + \boldsymbol{\beta}_k^c(1) \\
\mathbf{S}_k^c(2,2) &\approx \lambda_2^c\mu_2^c\mathbf{P}_{k|k-1}^c(3,3) + \lambda_3^c\mu_2^c\mathbf{P}_{k|k-1}^c(1,1) \\
&\quad + \alpha_k^c(\lambda_3^c[\hat{\mathbf{x}}_{k|k-1}^c(1)]^2 + \lambda_2^c[\hat{\mathbf{x}}_{k|k-1}^c(3)]^2) + \boldsymbol{\beta}_k^c(2) \\
\mathbf{S}_k^c(3,3) &\approx \mu_2^c\mathbf{P}_{k|k-1}^c(5,5) + \mu_3^c[\mathbf{P}_{k|k-1}^c(1,1) + \mathbf{P}_{k|k-1}^c(3,3) \\
&\quad + \mu_2^c R_k^{r,c}[\hat{\mathbf{x}}_{k|k-1}^c(5)]^2/(\bar{d}^c)^2 \\
&\quad + \mu_3^c R_k^{r,c}(\bar{d}_1^c)^2/(\bar{d}^c)^2 + \boldsymbol{\beta}_k^c(3) \\
\mathbf{S}_k^c(1,2) &= \mathbf{S}_k^c(2,1) \approx (\lambda_2^c - \lambda_3^c)[\mu_2^c\mathbf{P}_{k|k-1}^c(1,3) \\
&\quad + \alpha_k^c\hat{\mathbf{x}}_{k|k-1}^c(1)\hat{\mathbf{x}}_{k|k-1}^c(3)] + \boldsymbol{\beta}_k^c(4) \\
\mathbf{S}_k^c(1,3) &= \mathbf{S}_k^c(3,1) \approx \lambda_1^c(\mu_2^c - \mu_3^c)[\mathbf{P}_{k|k-1}^c(1,5) \\
&\quad + R_k^{r,c}\hat{\mathbf{x}}_{k|k-1}^c(1)\hat{\mathbf{x}}_{k|k-1}^c(5)/(\bar{d}^c)^2] + \boldsymbol{\beta}_k^c(5)\hat{\mathbf{x}}_{k|k-1}^c(1) \\
\mathbf{S}_k^c(2,3) &= \mathbf{S}_k^c(3,2) \approx \lambda_1^c(\mu_2^c - \mu_3^c)[\mathbf{P}_{k|k-1}^c(3,5) \\
&\quad + R_k^{r,c}\hat{\mathbf{x}}_{k|k-1}^c(3)\hat{\mathbf{x}}_{k|k-1}^c(5)/(\bar{d}^c)^2] + \boldsymbol{\beta}_k^c(5)\hat{\mathbf{x}}_{k|k-1}^c(3) \\
\mathbf{S}_k^c &= [\mathbf{S}_k^c(m,n)]_{m,n=1}^3 \\
\mathbf{C}_{k|k-1}^c &= [\mathbf{P}_{k|k-1}^c(:,1), \mathbf{P}_{k|k-1}^c(:,3), \mathbf{P}_{k|k-1}^c(:,5)]\mathbf{\Omega}_c' \\
\hat{\mathbf{x}}_{k|k}^c &= \hat{\mathbf{x}}_{k|k-1}^c + \mathbf{C}_{k|k-1}^c(\mathbf{S}_k^c)^{-1}(\mathbf{z}_k^c - \hat{\mathbf{z}}_{k|k-1}^c) \\
\mathbf{P}_{k|k}^c &= \mathbf{P}_{k|k-1}^c - \mathbf{C}_{k|k-1}^c(\mathbf{S}_k^c)^{-1}(\mathbf{C}_{k|k-1}^c)'
\end{aligned}$$

V. Experimental Results and Discussions

In this section, we compare the performance of recursive LMMSE multi-radar centralized fusion with stacked original measurements (**CF-o**), with stacked recombined measurements (**CF-r**), and with compressed measurement (**CF-c**) to verify that **CF-c** performs better than the other two.

For Eq. (1), we use the discrete-time NCV kinematic model for target motion. The involved parameters are set as

$$\begin{aligned}
\mathbf{F}_k &= \text{diag}(\mathbf{F}, \mathbf{F}, \mathbf{F}) \\
\mathbf{G}_k &= \text{diag}(\mathbf{G}, \mathbf{G}, \mathbf{G}) \\
\mathbf{F} &= \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \\
\mathbf{G} &= [T^2/2, T^2/2]', \ T = 1\text{s} \\
\boldsymbol{\omega}_k &\sim \mathcal{N}([0,0,0]', \mathbf{Q_k}) \\
\mathbf{Q}_k &= \text{diag}((0.01\text{m/s}^2)^2, (0.01\text{m/s}^2)^2, (0.01\text{m/s}^2)^2) \\
\bar{\mathbf{x}}_0 &= E(\mathbf{x}_0) \\
&= [10\text{km}, 100\text{m/s}, 1\text{km}, 100\text{m/s}, 10\text{km}, 100\text{m/s}]' \\
\mathbf{P}_0 &= \text{cov}(\bar{\mathbf{x}}_0) \\
&= \text{diag}(10^6\text{m}^2, 20^2\text{m}^2/\text{s}^2, 10^6\text{m}^2, 20^2\text{m}^2/\text{s}^2, \\
&\qquad 10^6\text{m}^2, 20^2\text{m}^2/\text{s}^2)
\end{aligned}$$

In the following numerical examples, we assume that there are 3 radars mounted at the origin of the Cartesian coordinates, and $\sigma_r^i, \sigma_b^i, \sigma_e^i, \ i = 1,2,3$ are the standard deviations of $\tilde{r}_k^i, \tilde{b}_k^i, \tilde{e}_k^i$. We use root mean-squared (RMS) position and velocity errors to evaluate the filter's accuracy [13], and noncredibility index (NCI) [14] and inclination indicator (II) [14] to evaluate the filter's credibility.

A total of 1,000 Monte Carlo runs are performed. And we initialize all the filters as follows.

$$\hat{\mathbf{x}}_{0|0} = \bar{\mathbf{x}}_0, \ \mathbf{P}_{0|0} = \mathbf{P}_0$$

Two cases are considered to compare the performances of the three approaches. In Case 1,

$$\begin{aligned}
\sigma_r^1 &= 10\text{m}, \sigma_b^1 = 10\text{mrad}, \sigma_e^1 = 80\text{mrad} \\
\sigma_r^2 &= 100\text{m}, \sigma_b^2 = 1\text{mrad}, \sigma_e^2 = 10\text{mrad} \\
\sigma_r^3 &= 400\text{m}, \sigma_b^3 = 80\text{mrad}, \sigma_e^3 = 1\text{mrad}
\end{aligned}$$

Figs. 1-4 show the performance in **Case 1**. From all these four figures, we can see that, in terms of estimation accuracy, centralized fusion with compressed measurements is better than the one with recombined measurements obviously and the one with recombined measurements is much better than the one with stacked original measurements, whether it is about position or velocity. In terms of filter credibility, after 60 steps, the noncredibility of centralized fusion with compressed measurements is slightly smaller than the one with recombined measurements, and also smaller than the one with stacked original measurements obviously.

In Case 2,

$$\begin{aligned}
\sigma_r^1 &= 40\text{m}, \sigma_b^1 = 30\text{mrad}, \sigma_e^1 = 15\text{mrad} \\
\sigma_r^2 &= 20\text{m}, \sigma_b^2 = 15\text{mrad}, \sigma_e^2 = 20\text{mrad} \\
\sigma_r^3 &= 10\text{m}, \sigma_b^3 = 20\text{mrad}, \sigma_e^3 = 30\text{mrad}
\end{aligned}$$

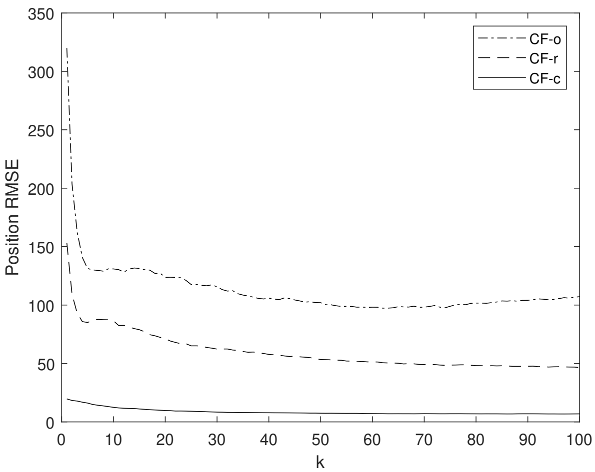

Fig. 1: Position RMSE (Case 1)

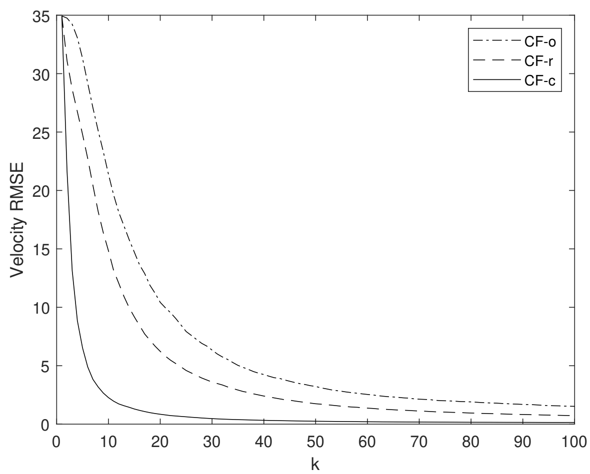

Fig. 2: Velocity RMSE (Case 1)

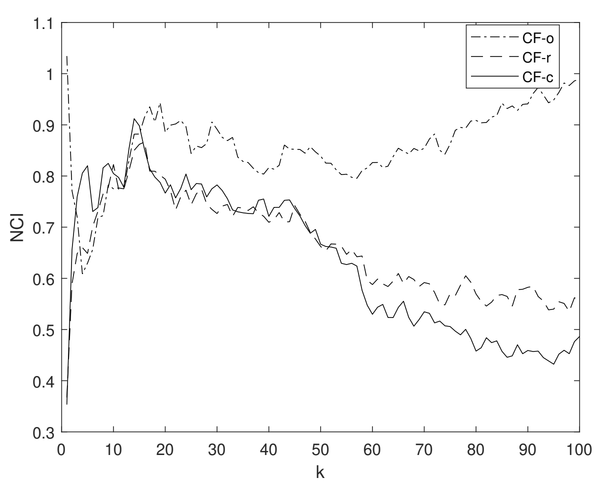

Fig. 3: NCI (Case 1)

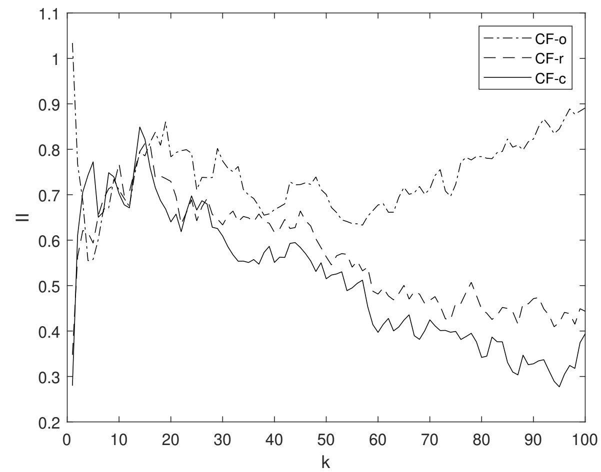

Fig. 4: II (Case 1)

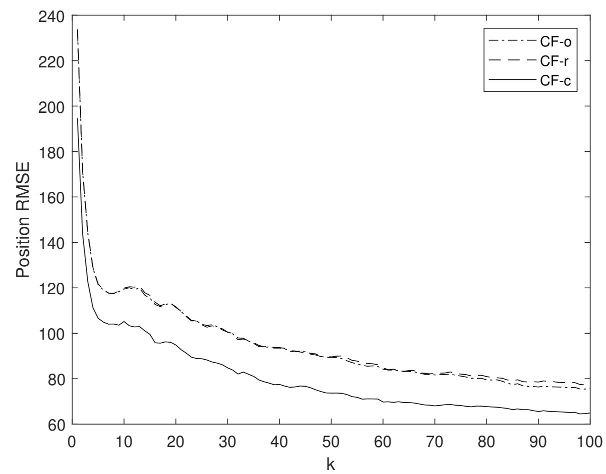

Fig. 5: Position RMSE (Case 2)

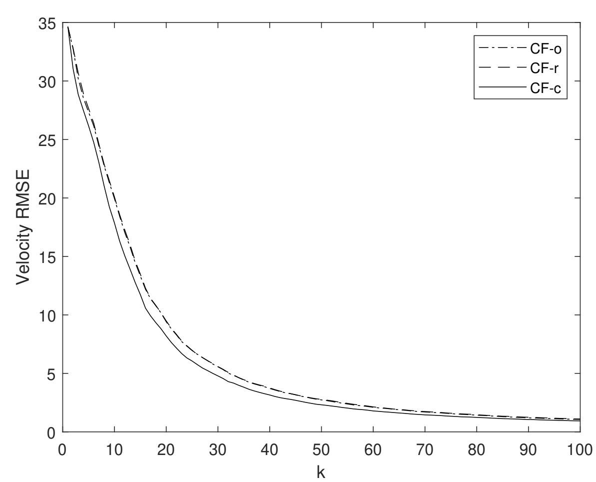

Fig. 6: Velocity RMSE (Case 2)

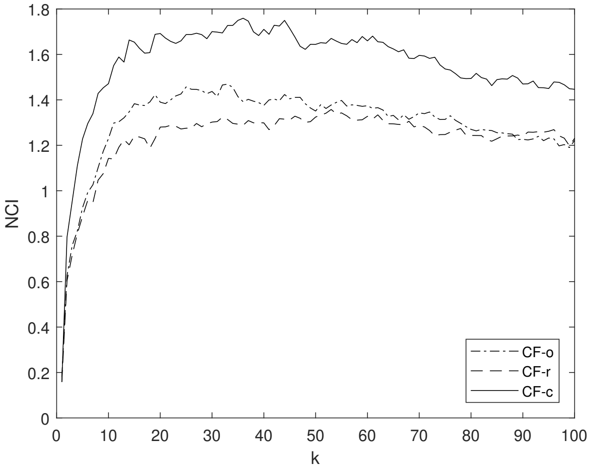

Fig. 7: NCI (Case 2)

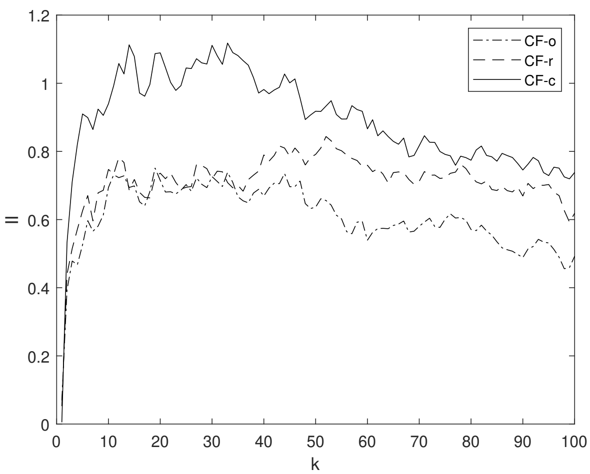

Fig. 8: II (Case 2)

Figs. 5-8 show the performance in **Case 2**. From all these four figures, we can see that, in terms of estimation accuracy, centralized fusion with recombined measurements is similar to the one with stacked original measurements, but the centralized fusion with compressed measurements is much better than the one with recombined measurements and with stacked original measurements in position. And in velocity, after 80 steps, these three approaches perform very closely. For the filter credibility, these three approaches are all credible.

In summary, the above numerical examples show that new centralized fusion with compressed measurements provides better estimation accuracy and smaller noncredibility under the recursive LMMSE framework. The compressed measurement at time k is a 3×1 vector, while the stacked measurement at time k is a 9×1 vector. Thus, nonlinearity of the compressed measurement is reduced compared with that of the stacked measurement with respect to the system state. Also, covariances of compressed measurements are reduced compared with the ones of the measurements of each radar, whether recombined or not. This explains why recursive LMMSE centralized fusion with compressed measurements performs better than the two existing centralized fusion approaches using stacked measurements.

VI. Conclusion

For centralized fusion with multi-radar measurements under the recursive LMMSE filtering framework, we propose a new centralized fusion approach. In this new approach, we compress the measurements from all radars at the fusion center first and then apply the recursive LMMSE filter with single radar measurements. Comparing the new approach to centralized fusion using both stacked original and recombined measurements under the recursive LMMSE filtering framework, it is found that new approach can provide better estimation accuracy and smaller noncredibility. That is mainly because the new approach uses more accurate radar measurements with smaller nonlinearity.

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