This is a peer-reviewed, accepted author manuscript of the following research article: Ahmed, M, Hamad, MS, Abdel-Khalik, AS, Hamdan, E, Ahmed, S & Elmalhy, NA 2023, 'Improved utilization for "smart parking systems" based on paging technique', IEEE Transactions on Intelligent Transportation Systems. https://doi.org/10.1109/TITS.2023.3323097

Improved Utilization for "Smart Parking Systems" based on Paging Technique

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***Abstract*—Considering the rapid urbanization and the road congestion, the development of smart parking solutions becomes more crucial, especially in terms of economic interests. Thanks to IoT-connectivity and the cloud-integrated platforms, drivers can easily find a vacant parking lot with smart parking services. This paper intervenes in the profit of parking management systems. The paper proposes a new technique "paging technique" which increases the utilization factor of parking slots. The proposed method takes advantage of the idle time that exists between two successful parking services in the same slot. Besides, it investigates the possibility of using the idle times from different parking slots to provide a continuous parking time for an additional car. The paging technique is optimally implemented using mixed-integer linear programming that maximizes the utilization factor for the parking slots with minimum car transitions. Moreover, a data model for the parking management system has been constructed while considering the three major customers, namely, regular, prepaid, and walk-in customers. The difference between fixed and dynamic pricing for parking has been investigated. The technique has been validated using GAMS optimization software and hardware using DSP with Coin-or branch and cut solver (CBC) under real-life conditions. The statistical results prove that the revenue for the proposed parking system has increased significantly. Finally, a comparative analysis is performed, benchmarking our proposed method against recent competing algorithms in real world applications to demonstrate its superiority.**

***Index Terms*—smart parking services, parking utilization, smart cities, IoT, Coin-or branch and cut MIP solver**

I. INTRODUCTION

The concentrated emergence of vehicle fleets in the transportation sector poses unprecedented challenges for urban management, such as road congestion, pollution, and a lack of parking slots. On the other hand, smart parking systems are emerging to provide solutions for urban mobility. This is aided by the rapid development of vehicle IoT and sensor technologies, which enables the transfer of real-time data regarding parking availability, traffic, and road conditions [1]. The authors in [2] highlight the need for innovative approaches in addressing parking problems, particularly in the context of connected and automated technologies. The authors cover various topics such as autonomous vehicles, shared parking mechanisms, dynamic pricing, and parking market interactions. While the papers offer valuable insights, further research is needed in parking planning, operation, and management, as well as modeling and analyzing the economic impact of smart parking systems. The fully automated parking system comprises three essential platforms: path planning, statistical reporting, and parking management. Literature is consistently striving to develop the objectives of these platforms to create a fully automated parking system.

The objectives of the path planning platform include: (i) allocating the most effective route between the user and parking spaces in order to reduce the cruising time and (ii) minimizing pollution and congestion caused by cruising vehicles. The majority of drivers are willing to travel non-optimal distances in order to get a parking space close to their destinations According to [3], the observed mean non-optimal cruising distance is 2.75 times the observed mean optimal one. Moreover, during peak visiting hours, a number of vehicles were unable to obtain a parking slot in the area of interest, resulting in excessive CO_2 emissions. In the meantime, [4] has presented an eco-parking system, which is an innovative green parking solution that employs IoT sensors and a CO_2 emissions-based parking space allocation algorithm to reduce the cruising time of more polluting vehicles. Moreover, between 9 and 56% of traffic, according to the findings of [5], was cruising for parking; hence reducing cruising will therefore significantly decrease congestion. To this end, [5, 6] developed an IoT-based platform to reduce cruising time. In addition, this objective has been accomplished by assigning the most efficient route between the user and the parking spot based on real-time traffic conditions, as described in [7]. Moreover, the authors in [8] propose a smart parking system based on matching theory, addressing traffic congestion, and meeting the preferences of drivers and parking managers. It introduces a prior reservation system and a dynamic parking fee design. However, the effectiveness of this method heavily relies on accurate predictions of waiting times and parking statuses, which can be challenging to achieve in real-world scenarios.

The statistical reporting platform provides the client with reports and data regarding the parking lots, whether they are on-street or off-street, the time frame of each available parking slot, and the pricing, whether it is dynamic or fixed. The main

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objective for this platform is to minimize the parking cost for the users [9]. In addition, the administration can distribute resources in the most efficient manner possible thanks to this platform’s feedback reports. To this end, [10] proposed a parking system that reserved the ideal parking spot based on the driver's cost function, which combined proximity to the destination and the parking fee. Beside minimizing the parking costs, the authors in [11] utilized mixed-integer linear programming to balance parking demand between different parking lots. One key observation is that the drivers prefer the cheapest parking spaces. Therefore, they all head to the on-street parking lots, causing road congestion, and leaving the off-street with great availability due to their high cost. To address this problem, the authors in [12, 13] investigated the efficacy of certain parking pricing strategies in achieving a balance between demand and supply and between on-street and off-street parking lots in urban areas. Furthermore, the authors in [14] propose a demand-driven dynamic parking pricing strategy to regulate parking demand and ensure the utilization of parking facilities in business districts. The strategy is based on a multi-agent based on-street parking simulation that explores the effects of time-varying parking prices on parking demand. The results show that the strategy can effectively regulate the distribution of parking demand and reduce parking and traffic problems. However, it is not effective in all cases. For example, if the price change is too high, it could lead to drivers cruising for parking for longer periods of time. Alternative approaches depend on Auctions. The authors in [15] propose two new auction mechanisms for shared parking. These mechanisms are truthful, individually rational, and budget balanced. They can achieve asymptotic efficiency as the number of participants increases. However, they may be more complex to implement. Moreover, the author in [16] proposes a new auction mechanism for shared parking. The proposed mechanism is fair, recurrent, and efficient. It takes into account the priority attributes of participants and can be used repeatedly over time. However, it is also complex to implement.

The parking management platform is responsible for regulating, monitoring, and increasing the revenues of urban parking facilities. From the government perspective, the parking area is considered one of the most important income-generating assets as long as it is managed efficiently. In [17], the parking coverage is surveyed, which is the ratio of parking area to land area. The parking coverage accounts for 31% of land use in most cities, like Sydney, San Francisco, and even more, 81% in Los Angeles and 76% in Melbourne, while in the most populated cities, one can find New York (18%), London (16%), and Tokyo (7%). These parking areas should be utilized efficiently to accommodate the rapid growth in the number of vehicles and maximize the income from these parking areas. There are many techniques used to increase the revenue from parking areas. For instance, [18] provided a multi-layer architecture for smart parking system and proposed a higher pricing as the parking requests increase. This approach maximizes the revenue for the parking authority; however, it does not provide fairness between customers and does not consider the stochasticity of the parking demands. In [19], the issue of stochasticity has been tackled, as the Poisson process is used to model the parking requests. Besides, this technique maximizes the revenue through a dynamic pricing scheme that varies according to the arrival demand and the number of vacant parking spaces. However, the technique used in [19] is complex and has a high computational cost. Another approach to increase the capacity and revenue of the parking authority is to rent out the private parking spaces during the periods when the owners are not using their parking spaces. Such an approach is mentioned in [20-22], where the parking authority receives information about the private parking spaces, and then manages them optimally with the public parking spaces in order to increase the net revenue, but at the cost of added complexity and a heavy computational burden. Moreover, the authors in [23] propose a new method for improving the reliability of on-street parking information (OSPI) systems. The proposed method uses parking events (PEs) to develop dynamic features that can make the system more adaptive to changes that impact on-street parking availability. The authors also develop a parking behavior change detection (PBCD) model to trigger potential parking map updates, despite having simpler but more enhanced and adaptive features. One potential drawback of the proposed OSPI system is the reliance on vehicles parked-in and parked-out events for gathering data. While these events provide valuable information, their accuracy and availability may be subject to certain limitations. Factors such as sensor malfunctions, incomplete data, or unreported parking events could affect the reliability and completeness of the system's information. Table 1 summarizes the main platforms of a fully automated parking system and their objectives.

On the other hand, certain approaches employ a hybrid combination that addresses various objectives from different platforms. One such approach is discussed in [24] which proposes two methods to tackle two objectives: minimizing parking expenses and balancing parking demand among multiple parking lots. The first method is a matching game

Table 1: The platforms of a fully automated parking system and their objectives.

Fully automated parking system Platforms	Objectives
Statistical Reporting Platform	- Providing the client with reports and data regarding the available parking lots. - Minimizing the parking cost for the clients.
Path planning Platform	- Allocating the most efficient route between the user and the desired parking lot in order to (i) reduce cruising time, (ii) minimize pollution and congestion caused by cruising vehicles.
The parking management platform	- Increasing the revenue for the garage by employing one of the following techniques: (i) increasing the parking spaces by optimally renting the private parking slots, (ii) using different tariff for on street and off-street parking, and (iii) utilizing the dynamic pricing which charges high prices during the peak parking demand periods.

approach, which outperforms the greedy approach by 8.5% in terms of parking utilization. The second method is an alternating direction method of multipliers (ADMM)-based algorithm, which produces performance gains up to 27.5% compared with the matching game approach. However, matching game approach may not be optimal in all cases, and the ADMM-based algorithm may be computationally expensive for large-scale problems. Alternative approaches incorporate intelligent forecasting behavior, as exemplified by the methods proposed in [25] which proposes a parking behavior forecast for smart parking allocation algorithm. This approach predicts driver behavior and estimated parking traffic in the near future, which helps to better match parking demands and the resource of available parking grids. However, this approach requires accurate information about parking demand and costs. If this information is not accurate, then the algorithm may not be able to allocate parking spaces in the most effective way possible. Moreover, the authors in [26] propose a stochastic dynamic parking management model that can simultaneously minimize the total travelers' costs and maximize the parking agency's revenue. This model proposes a promising solution to the problem of parking management in congested areas, however, if the model is not properly calibrated, it could lead to inaccurate results.

Out of the various prior objectives, this article intervenes in parking management platform that maximizes the revenue from the parking spaces. It is quite clear from the discussion above that the revenue has been increased using; (i) dynamic pricing, which charges high prices during the peak parking demand periods, (ii) increasing the parking spaces by optimally renting the private parking slots, and (iii) using different tariffs for on-street and off-street parking. However, this article investigates the utilization factor of parking spaces. It proposes a new technique (the paging technique), which serves as an add-on to the parking management platform to further increase the revenue without compromising the objectives of other platforms. This technique uses the idle time between two successful parking services in the same parking space and investigates the possibility of using the idle times from different parking slots to provide a continuous parking time for an additional car. Therefore, the total revenue is increased through parking more vehicles in the same parking spaces.

The key contributions for the paper are concluded as follows,

- Proposing a new technique (paging technique) which serves as an add-on to the parking management platform to further increase the utilization factor of the parking spaces.
- Investigating the difference between the fixed pricing and dynamic pricing using paging technique.
- Validating the proposed technique using digital signal processor (DSP).

This paper is organized as follows: Section II covers the optimization problem formulation. The overall system structure is described in Section III. Section IV presents the statistical results using GAMS and the DSP. Section V concludes the paper.

II. Problem Formulation

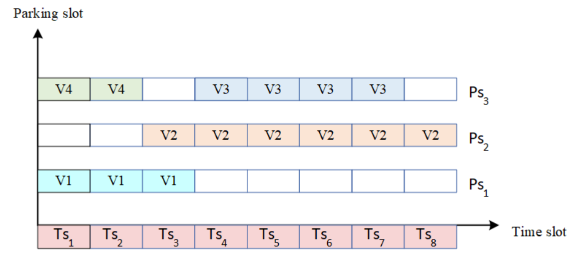

Fig. 1. The traditional allocation of the vehicles in parking slots.

This section introduces the formulation of the optimization problem with the required constraints associated with the controller, that will be implemented in the parking lot. Fig. 1 illustrates the traditional allocation of the vehicles in the parking slots and describes the basic idea of the proposed method. For simplicity, three parking slots (Ps) and eight time slots (Ts) are assumed. As shown in Fig. 1, first the vehicle 1 requires a parking service from Ts_1 to Ts_3 in the Ps_1, then vehicle 2 reserves a parking service from Ts_3 to Ts_8 in the Ps_2, then vehicle 3 reserves a parking service from Ts_4 to Ts_7 in the Ps_3 and vehicle 4 reserves the service from Ts_1 to Ts_2 and will be located in the Ps_3 instead of Ps_2 to prevent the overlapping between vehicles 2 and 4. The overlapping may occur if vehicle 4 left the Ps later than the predetermined departure time and vehicle 2 arrives on time or much earlier. Now, if another vehicle (vehicle 5) requires a parking service from Ts_1 to Ts_8, the traditional techniques will fail to park the car, because there is no parking slot free during all this period. However, by using the paging technique, that uses the idle time for each parking slot. The vehicle 5 can be allocated in the Ps_2 in the first two time slots, then allocated in Ps_3 in third time slot and finally allocated in Ps_1 from Ts_4 to Ts_8. This technique will park more vehicles in the same number of parking slots and increase the revenue of the parking management system. It is worth mentioning that, as the number of parking and time slots increases, the number of idle times increases, and the paging technique becomes more efficient. However, the vehicles' transition over the parking slots increases, as illustrated by (V5) that makes 3 transitions over the three parking slots. Therefore, the paging technique is implemented by an optimization problem that optimally allocates the vehicles over the parking slots with minimum vehicle transition and late departure penalties to prevent the overlapping of vehicles.

A. Objective function and constraints formulation

The coordination problem for the vehicles in the parking lot is targeted to maximize parking revenue. The controller employs a new technique called “paging technique”, which optimally allocates the vehicles to take the advantage of the idle time that exists in the same slot. Therefore, the revenue is increased by parking more vehicles in the same number of parking slots.

For simplicity, the decision variables are visualized as a 3D array x_{kji}, as shown in Fig.2. The first dimension (k) represents the number of vehicles (V) that require a parking service, the second dimension (j) represents the number of parking slots (Ps) in the parking lot, and the third dimension (i) represents the number of time slots (Ts), at which the parking lot is

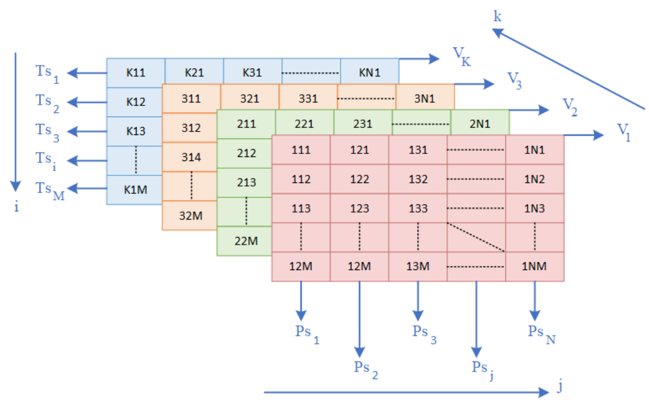

Fig. 2. The array of decision variables.

available for parking. These decision variables are modelled as binary variables (0: if the vehicle is not parked and 1: if the vehicle is parked).

The objective of the optimization problem is to maximize the utilization factor for the parking slot with the least number of vehicle transitions, as described in (1).

$$\underset{x}{\text{Max}}\, F_{obj} = \max_{x} \sum_{k=1}^{K} \sum_{j=1}^{N} \prod_{i=a(k)}^{i=d(k)} x_{kji} \tag{1}$$

$$\sum_{i=1}^{M} x_{kji} \leq 1 \qquad \forall k \in K, j \in N \tag{2}$$

$$\sum_{k=1}^{K} x_{kji} \leq 1 \ \forall i \in M, j \in N \tag{3}$$

$$\sum_{j=1}^{N} \sum_{i=a(k)}^{i=d(k)} x_{kji} = d(k) - a(k) \qquad \forall k \in K \tag{4}$$

$$\sum_{j=1}^{N} \sum_{i=0}^{i=a(k)-1} x_{kji} = 0 \qquad \forall k \in K \tag{5}$$

$$\sum_{j=1}^{N} \sum_{i=d(k)+1}^{i=M} x_{kji} = 0 \qquad \forall k \in K \tag{6}$$

where a_k and d_k are the arriving time and departure time for vehicle k, respectively, N is the total number of parking slots, K is the total number of vehicles that require parking services, while M is the total number of the time slots.

As shown in (1), the objective function depends on the product of the decision variables for each vehicle from its arrival time slot to its departure time slot. This product will be repeated for each parking slot. If the vehicle parks in the same parking slot for all of its parking time, the product value will be 1, and if it moves to another parking slot in any time slot during its parking time, the product will be zero. Therefore, maximizing the product value will optimally minimum transitions of vehicles over the parking slots and maximize the possible numbers of parked vehicles.

The constraint illustrated in (2) prevents the vehicle from being in more than one parking slot at the same time slot. Moreover, the constraint illustrated in (3) prevents more than one vehicle from being in the same parking slot at the same time slot. The constraints (4, 5 and 6) ensure that each vehicle will only park at the predetermined time slots.

B. Objective function linearization

As shown in (1), the objective function is highly nonlinear and represents a computational burden for the existing hardware platform. Therefore, linearizing this objective function allows for simple linear programming methods, which dramatically reduce the complexity and the computation time.

Suppose we have s binary variables x_i, then, the product of these variables can be linearized by introducing a new binary variable z that represents the value of this product and model it by introducing the following constraints;

$$z \leq x_i \quad \forall\, i \in [1,2,\dots,s] \tag{7}$$

$$z \geq \sum_{i=1}^{i=s} x_i - (s-1) \tag{8}$$

By applying this technique to all the parking slots and vehicles, the objective function in (1) is reformulated as follows,

$$\max_{z} F_{obj,linearized} = \max_{z} \sum_{k=1}^{K} \sum_{j=1}^{N} z_{jk} \tag{9}$$

$$z_{jk} \leq x_{ijk} \quad \Leftrightarrow a(k) \leq i \leq d(k), \ \ \forall k \in K, j \in N \tag{10}$$

$$z_{jk} \geq \sum_{i=d(k)}^{i=end} x_{ijk} - [d(k) - a(k)], \forall k \in K, j \in N \tag{11}$$

This new linearized formulation can easily be addressed using any algorithm that solves the linear problem, namely, simplex, dual-simplex, CPLEX, or the interior point method. Moreover, this formulation does not require a highly specific platform. Therefore, the initial cost of providing the existing parking lots with the required controllers (hardware platforms) will be significantly reduced compared to the original nonlinear representation.

C. Comparison Between Proposed Technique and Other Resource Assignment Problems

Resource assignment and schedule-related problems are responsible for allocating different jobs to limited resources across time, such as job shop problems (JSP), gate assignment at airports, customer support centers and parking assignment problem. Nonetheless, each problem has unique characteristics that might drastically alter the optimization problem nature. The answers to the following questions describe the nature of the optimization problem: i) Should the objective be accomplished within a certain time frame, or can it be extended beyond the time frame? , ii) should each job start at a specific time or can it be dynamically shifted within the time frame? iii) similarly, should the end time of each job occur at a specified time or it can be dynamically adjusted within the time frame, and whether or not is it constrained by a deadline? iv) is the scheduling day-ahead or real-time? , v) are the arrival and departure times of each task random or predetermined? Based on the answers to the previous questions, Table 2 compares the proposed parking problem to the other resource assignment problems.

Regarding the job shop problem, as shown in Table 2, the total number of jobs and machines is predetermined before executing the optimization problem; hence this is not a real-time problem. Each job for job shop problem has a dynamic starting time, but it must be completed before the deadline; therefore, the starting and ending times of each job may be dynamic. This is not the case with the parking problem, where the arrival and departure times of each vehicle are not dynamic (it is fixed and predetermined by the client). In addition, unlike the parking problem, there is no stochastic behavior in the arrival, departure, or number of jobs.

Table 2: The difference between the proposed problem and the other resource assignment problems.

Job shop problem	Objective function	Minimizing the total length of the schedule (that is, when all the jobs have finished processing)
	Start time of the job	Dynamic
	End time of the job	Dynamic
	Time scheduled	Day-ahead
	deterministic nature of the system parameters	Deterministic programming problem
Gate assignment at airports	Objective function	Minimizing the number of ungated flights and the total walking distances
	Start time of the job	Fixed
	End time of the job	Fixed
	Time scheduled	Day-ahead
	deterministic nature of the system parameters	Deterministic programming problem
Inquiry handling at customer support centers	Objective function	Minimizing the cost of providing service and minimize the customer wait
	Start time of the job	Dynamic
	End time of the job	Dynamic
	Time scheduled	Real-time
	deterministic nature of the system parameters	Stochastic programming problem
Proposed parking problem	Objective function	Maximizing the number of parked vehicles and minimize their transitions
	Start time of the job	Fixed
	End time of the job	Fixed
	Time scheduled	Real-time
	deterministic nature of the system parameters	Stochastic programming problem

For the gate assignment at airports, the arrival and departure, number of flights are predetermined; therefore, there is no stochastic behavior in this problem unlike the parking problem. Moreover, the total number of flights and gates are predetermined before executing the optimization problem; hence this is not a real-time problem and can be performed as a day-ahead. Finally, for the gate assignment problem, the concept of making a transition for the same flight between gates is not accepted. If no gate is available when they arrive, they will be ungated and must wait outside the gates for a certain amount of time until one becomes available.

For the customer support center problems, each job start time is defined based on when each resource becomes available; hence, the start and end times of each job may be changeable, and it is not the case with the parking problem. Also, the concept of making a transition for the same customer between service offices is not acceptable.

III. The Overall System Structure

This section describes the overall system structure and the data model at which the parking controller is validated.

A. Parking lot

The typical parking lot consists of one or more blocks. Each block contains many floors that are further subdivided into multiple wings. Each wing helps the vehicle driver to correctly orient and remember their parking area. Moreover, each wing is subdivided into parking spaces that are uniquely numbered. According to the standards, some of these spaces are reserved for handicapped people. Nowadays, some spaces are fitted by an electric charger to charge the recent electric vehicles.

There are two main systems of parking, namely, on-street parking and off-street parking. On-street parking usually refers to the available parking slots on the street (on the side of a road). This type of parking lot is usually managed and controlled by the government. On the other hand, off-street parking refers to the available slots within a garage or an enclosed parking lot. These parking lots can be owned by the government, privately owned or municipality. Sometimes the customers are allowed to park for a limited amount of time or during certain hours of the day, especially in on-street parking. Therefore, the parking lots are also categorized based on two zones, namely, short term parking spaces (available for up to 4 hours) and long-term parking spaces (available for over 4 hours). Fig. 3 describes the parking lot structure for off-street parking, which is used in this study to validate the system. The parking lot consists of one

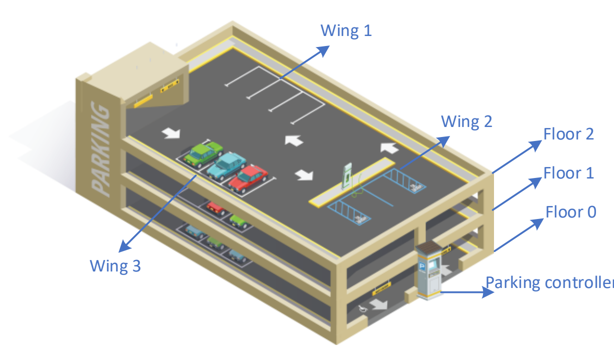

Fig. 3. The parking lot with gated parking controller.

block with 3 floors. Each floor consists of 3 wings, namely, wing 1 (which consists of three parking spaces), wing 3 (which consists of three parking spaces), and wing 2 (which consists of one parking space). The second-floor wing 2 is fitted with a parking space with an electric charger, and floor zero is fitted with a parking space for handicapped people. The total number of parking slots is 24. Moreover, access and control for the parking lot can be managed by gated parking controller, as shown in Fig. 3.

B. Customer types

The customers who use the car park can be classified into 3 categories: namely, regular customers, prepaid customers, and walk-in customers. The regular customer, who has paid for a monthly or annual pass, may reserve the same designated slots for specific periods every day, which may correspond to an extra cost. The prepaid customer represents the customer who booked the slot remotely through the online service, which is controlled by the parking management system. This type of customer usually uses the slots for a limited time (couple of hours). Moreover, they must park in the reserved slots, and late departure penalties are charged for the customers who do not leave the reserved slot after their allocated time. It is worth mentioning that some management systems have a fixed minimum reservation time, for instance, the customers have to reserve the slot for two hours even if they only need one or a fraction of an hour. In the paper, each day is divided into time slots with a 30-minute time window. Therefore, the minimum reservation time in this study is 30 minutes. The "wake-in" customer is the one who does not have a pass or reserved slot in advance. Therefore, based on the slot's availability, the slot can be assigned.

C. Parking tariff

There are two approaches to determining the parking price, namely, the fixed pricing and dynamic pricing. In the fixed pricing strategy, the drivers pay a fixed tariff regardless of the total parking period or whether demand for parking is high or low. However, the dynamic time-of-use (ToU) pricing can make the best use of the capacity of the parking facility, and the corresponding revenue is further increased. In the case of dynamic pricing, customers are often more willing to pay a higher price for parking spaces at peak times and expect to pay a lower price during off-peak periods. The proposed method applies dynamic pricing to walk-in and prepaid customers. Fig. 4 illustrates the histogram for the numbers of prepaid and walk-in customers during a working day. The data set is derived from a parking lot in Istanbul [27]. Then, these data are fitted using a

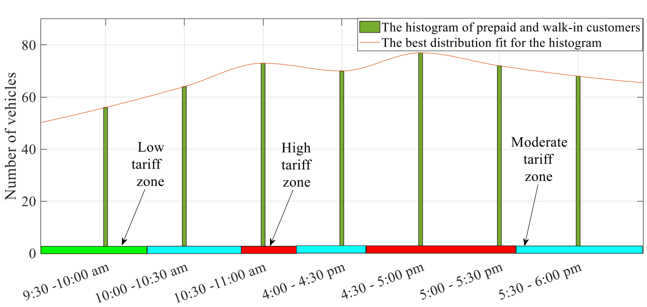

Fig. 4. Number of off-street parking at working day [13].

nonparametric kernel-smoothing distribution using MATLAB, as shown in Fig. 4. Then, the dynamic real time price for the day is set according to the highest demand. The system has 3 different categories for the tariffs, namely, the high tariff zone (red colored), the moderate tariff zone (blue colored), and the low tariff zone (green colored) in Fig. 4.

D. Arrival and departure patterns

Vehicle arrival and departure patterns are considered one of the most important parameters while constructing a realistic model of a parking lot. Therefore, the statistical model for the diversity of vehicles entering the parking lot is constructed based on statistical data and random arrival/departure times. In [27], 22-weekday statistical data for the parking lot is used to model the statical arrival/departure of the vehicles. The study in [27] found that the Weibull distribution, exponential distribution, and lognormal distribution can be used to model the arrival time. However, among these distributions, the two-parameter Weibull distribution gives reasonable performance for efficiently validating the controller. The two-parameter probability density function for the Weibull distribution is illustrated in (12).

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \tag{12}$$

where α and β represent the shape and the scale parameters, respectively. Moreover, the most appropriate values for these parameters are found to be 0.9831 and 16.8.

On the other hand, the performance of the common probability density functions used to fit the departure times is not found to be as reasonable as that of the arrival times. Therefore, it is more efficient to employ kernel density estimation (KDE). The KDE is a nonparametric estimation for the probability density function of a random variable. This random variable is chosen to be the parking duration instead of the departure times to provide a better results [27]. The KDE is given by (13) and is used to estimate the probability density function of the parking duration.

$$g(x) = \frac{1}{Nh}\sum_{i=1}^{N} K\left(\frac{x-x_i}{h}\right) \tag{13}$$

where N is the sample size, K is the kernel smoothing function, h is the bandwidth of the kernel estimator and x_i is the sample data points. In this paper, the Gaussian kernel is used as a smoothing function and h is chosen to be 1. These parameters are defined by training the estimator on the realistic data set, which is given in [27]. Now, the arrival times are generated using the Weibull distribution in (12), the parking times are generated using the KDE in (13), and the departure times can be given using (14).

$$t_{departure} = t_{arrival} + t_{parking\_duration} \tag{14}$$

In addition, the arrival rate and parking time models are recalibrated using another practical recording of data from Melbourne parking spaces [28]. A total of 750 random recordings from the 13.5M are used to calibrate the distributions. The Weibull, Normal, Lognormal, and Exponential distributions are shortlisted from the typical distributions that can be used for fitting the arrival rate. Then, with a sample size of 750, the distribution fitting toolbox in MATLAB is used to determine the optimal distribution parameters. Fig. 5 illustrates the empirical cumulative distribution function (CDF) for the arrival rate along with the CDFs of the optimally shortlisted distributions. From Fig. 5, it can be deduced that the Weibull distribution provides the most accurate CDF for the arrival rate, whereas the exponential distribution gives the most imprecise CDF.

In addition, the one-sample Kolmogorov-Smirnov (KN) test is performed to validate the distributions that have been shortlisted. The null hypothesis assumes that the data is given from the shortlisted distributions, against the alternative which does not come from such a distribution. The test result is true if the test rejects the null hypothesis at the 5% significance level, or false otherwise. The results are shown in Table 3. The Weibull distribution yields a false result, indicating that the null hypothesis cannot be rejected, and the sample is modelled using the Weibull distribution at the 5% significance level. Whilst the other distributions exhibit true results, the null hypothesis is therefore rejected.

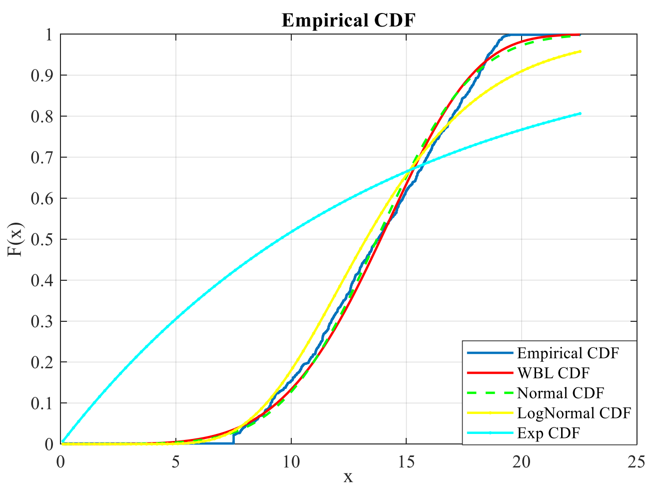

Fig. 5. The empirical CDF for the arrival rate in addition to the CDFs of the optimally shortlisted distributions.

Table 3: One-sample Kolmogorov-Smirnov (KN) test for the shortlisted distributions to test arrival rate.

Distribution	Weibull	Normal	Lognormal	Exponential
KN-test	False	True	True	True

The kernel density estimation (KDE), Weibull, Normal, Lognormal, and Exponential distributions are shortlisted from the typical distributions that can be used for fitting the stay duration. Then, the distribution fitting toolbox in MATLAB is used to determine the optimal distribution parameters. Fig. 6 illustrates the empirical CDF for the stay duration along with CDFs of the optimally shortlisted distributions. From the graph, it can be deduced that the KDE distribution provides the most accurate CDF for the stay duration, whilst the normal distribution gives the most imprecise CDF.

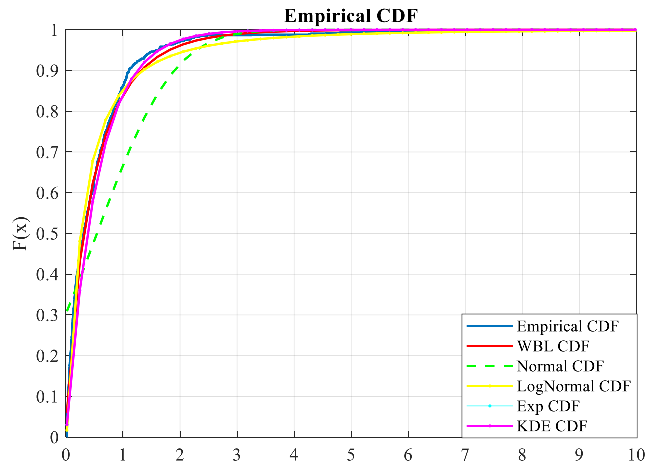

Fig. 6. The empirical CDF for the stay duration in addition to the CDFs of the optimally shortlisted distributions.

Moreover, KN test is performed to validate the distributions that have been shortlisted. The null and alternative hypotheses are expressed in the same manner as the arrival rate case. The results are shown in Table 4. The KDE yields a false result, indicating that the null hypothesis cannot be rejected, and the sample is modelled using the KDE at the 5% significance level. Whilst the other distributions exhibit true results, the null hypothesis is therefore rejected.

Table 4: One-sample Kolmogorov-Smirnov (KN) test for the shortlisted distributions to test stay duration.

Distribution	KDE	Weibull	Normal	Lognormal	Exponential
KN-test	False	True	True	True	True

This stochastic model is used to generate 84 random arrival/departure patterns, which are listed in TABLE A1 in the Appendix and used to validate the parking controller.

E. Workflow diagram of the parking lot management system

The flow diagram of the proposed paging method is described in Fig. 7. First, the driver sends a reservation request for the parking lot. The paging technique receives the request and addresses the linear programming problem in (9). Then, if there is no vacant parking space at the time requested by the driver, the algorithm asks the driver to choose another parking lot. However, if there is a continuous-time vacant space, the algorithm sends the parking confirmation to the driver. On the other hand, if there is a discontinuous-time vacant space, which indicates that the vehicle will be transmitted from one parking slot to another during the parking time, then the algorithm sends the car entry slot and car exit slot to the driver and waits (<5 minutes) to be confirmed by the driver. Finally, the algorithm updates the operating diagram for the operator. This operating diagram includes information about the vehicle that will be moved (from or to which slots and when will it be moved).

IV. Results and discussion

This section describes the effects of the proposed controller on maximizing the revenue of the parking lots using general algebraic modeling system (GAMS) optimizer package. Moreover, it tests the validity of implementing the proposed

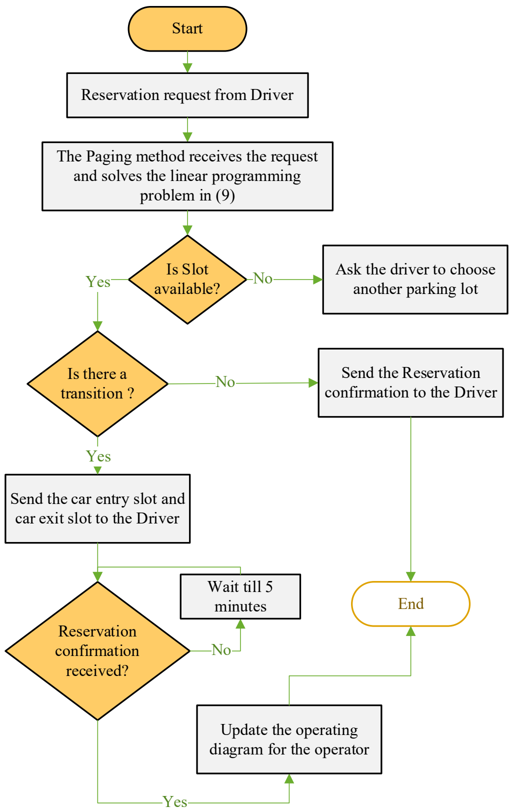

Fig. 7. The workflow diagram of the proposed method.

algorithms on the available hardware in the lab based on Control Hardware in the Loop (CHiL) using OPAL-RT and DSP. Additionally, a benchmark analysis was conducted to compare the proposed method to other recent algorithms.

A. Simulation results:

The off-street parking lot given in Fig. 3 is used to validate the proposed parking controller. The mixed-integer linear programming optimization problem for the controller has been implemented in the GAMS modelling environment. The coin-or branch and cut (CBC) solver is used to address this mixed-integer optimization problem. Furthermore, the validation considers the stochastic arrival/departure of vehicles, which are listed in TABLE A1 in the Appendix with different types of customers, namely, regular, pre-paid, and walk-in customers. The results describe the difference between the coordinated allocation of vehicles using the proposed method (paging technique) and the random allocation on the revenue of the parking lot

. In random allocation, the vehicle is located in any free slot suitable with the vehicle’s parking time, then this process continues till all parking slots are filled. Fig. 8 describes the random allocation of vehicles in the parking lot and shows the occupation of the parking lot for 24 parking slots (y-axis) over the day (x-axis) in the case of a 30-minute time slot. It is worth mentioning that slot-23 is fitted with an electric charger and used only to park the electric vehicles, for instance, Vehicles 85 and 86 which are indicated by the shaded orange area in Fig. 8. Moreover, the slot-24 is used by only the handicapped people, for instance, Vehicle-87 which indicated by the shaded red area in Fig. 8. The random allocation can locate only 70 vehicles out of 87. The vehicles with numbers (53, 63-66, 70-75, 77-79, 81-83) have not been allocated with this technique, as shown in Fig. 8. The idle times for the parking lot are 214 slots. The utilization factor is calculated as follows;

$$U_f = \frac{Ts_{occupied}}{Ts_{total}} \tag{15}$$

where U_f is the utilization factor, $Ts_{occupied}$ and Ts_{total} are the total number of time slots which are occupied and the overall time slots, respectively. Therefore, the utilization factor is found to be 938/1152 = 81.4 %.

On the other hand, the coordinated allocation of vehicles using mixed integer linear programming is shown in Fig. 9. The vehicles that located in the parking lots have been dramatically increased, as the proposed method can locate 84 out of 87 vehicles. The vehicles with numbers (53, 66, 82) have not been allocated with the proposed technique, as shown in Fig. 9. The idle time for the parking lot is 84 slots, which is dramatically decreased by approximately 61% compared to the random allocation method. Moreover, the utilization factor dramatically increased to (1041/1152) = 90.4 % compared to the random allocation.

One key observation that there are three vehicle transition between the parking slots, namely, vehicle-51is moved from slot-22 to slot-19 at time slot-48, vehicle-25 is moved from slot-9 to slot-11 at time slot-8, and vehicle-49 is moved from slot-8 to slot-6 at time slot-6. These transitions are illustrated by the shaded blue area in Fig. 9.

B. Cost analysis:

This subsection discusses the difference between the fixed tariff and dynamic tariff on the overall revenue of a parking lot.

For cost analysis, the price list for the West End-long stay car park is considered. The West End car park is a well-known off-street parking in Bristol [29]. This price list is applied to the proposed parking lot in Section III, and the revenue is calculated. In the fixed pricing strategy, the price is considered as 3.5£/car. There are 70 vehicles that have been allocated to parking spaces using the random allocation, so the total revenue per day will be 245£ per day. However, this revenue is increased to 245£ per day by using day. However, this revenue is increased to 294£ per day by using the proposed method, because the proposed method can park 14 more vehicles.

Moreover, the overall revenue has dramatically increased using the dynamic pricing strategy. As shown in Fig. 4, the demand for parking is high from 8:00 AM to 6:00 PM, therefore, dynamic pricing will be applied during this period, and the traffic will be stated as a time dependent instead of being constant [29]. Furthermore, during this period the demand is further increased in the periods 10:00 AM to 12:00 PM and 4:00 PM to 6:00 PM, so the traffic is further increased. Fig. 10 describes the dynamic price strategy over the day (based on Fig. 4). As shown in Fig. 10, during the period 8:00 AM to 6:00 PM (high demand), the price is dynamic and equal to 1.5£/h. However, during the periods 10:00 AM to 12:00 PM and 4:00 PM to 6:00 PM (extremely high demand), the dynamic price is further increased to 1.6£/h.

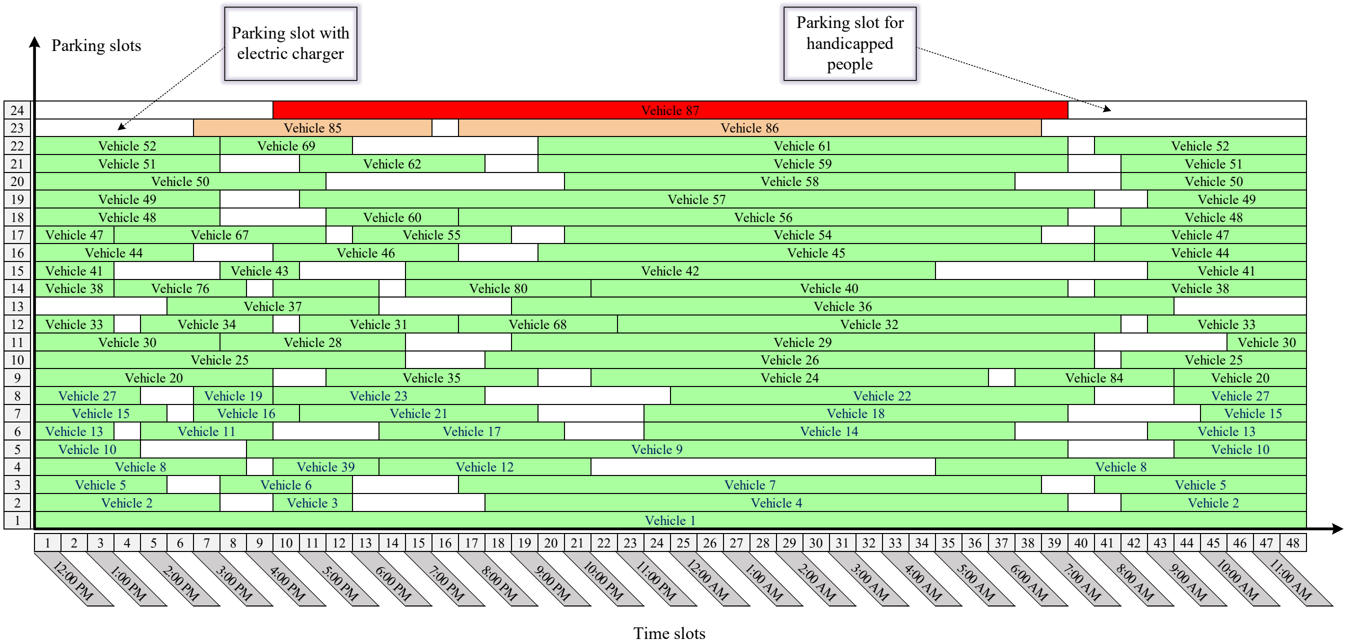

Fig. 8. The random allocation of vehicles in the parking lot.

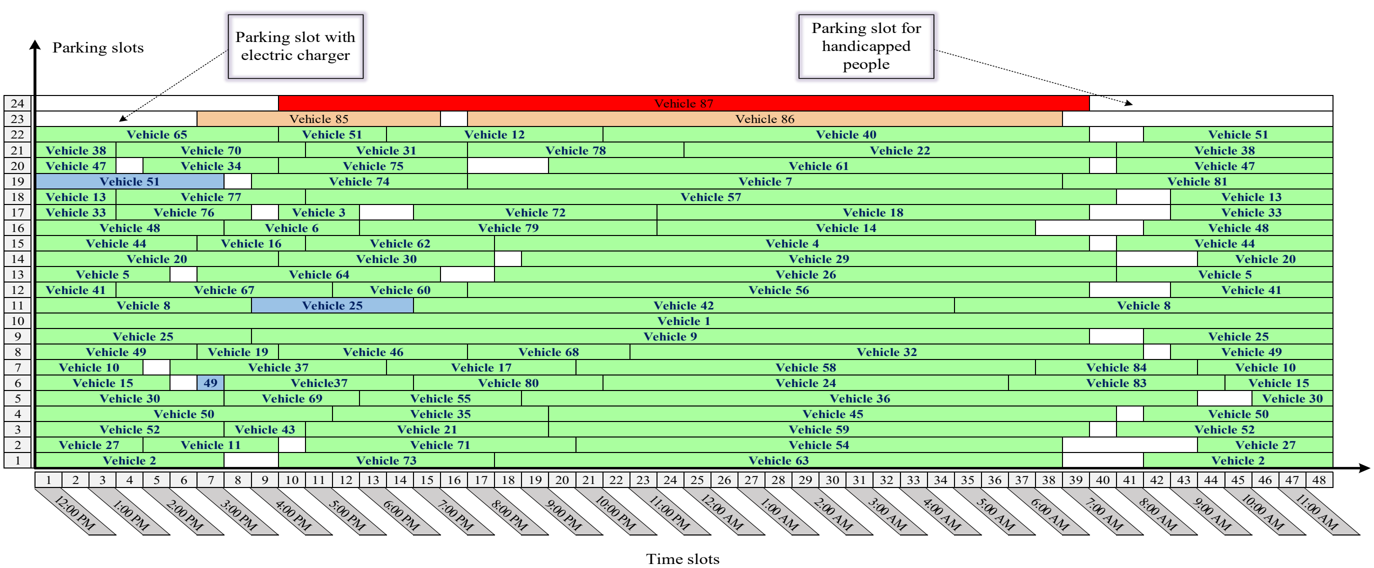

Fig. 9. The coordinated allocation of vehicles in the parking lot using the proposed method.

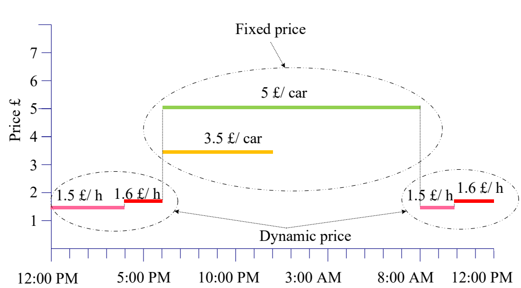

Fig. 10. The dynamic pricing strategy.

If the car parks in the evening from 6:00 PM to 12:00 midnight (low demand), the traffic will be fixed (time independent) and equal to 3.5£/car. If the car parks to overnight from 6:00 PM to 8:00 AM (park in the low demand but for long period), the traffic will be also fixed and equal to 5£/car. The total revenue for the random and coordinated allocation of vehicles in the parking lot is calculated using dynamic pricing. The total revenue for the proposed parking lot is approximately 437£ per day using the random allocation, however, this value is dramatically increased to approximately 525£ per day for the coordinated allocation. The prices are summarized in Table. 5

TABLE 5: The overall revenue for parking lot.

Pricing strategy	Random allocation	Coordinated allocation using the proposed method	Percentage increase
Fixed pricing	245 £/day	294 £/day	20%
Dynamic pricing	437 £/day	525 £/day	20.14%

Therefore, the annual revenue is increased by 31,328£ (20.14%) in the case of using the proposed method to allocate the vehicles in the parking lot as opposed to using random allocation. Moreover, the cost analysis is performed for 26 different days using 26 different scenarios of random arrivals and departures. The analysis is also accomplished on two different parking lots, namely, a parking lot with 24 parking spaces and a parking lot with 48 parking spaces. The statistical analysis is performed using the Wilcoxon signed rank test (non-parametric statistical test) at a 5% significance level. Table 6 shows the total revenue for the two parking lots for different 26 scenarios and also describes the results of the Wilcoxon test. The p-value tests the null hypothesis of zero medians at 5% significant level and the non-parametric signs (+, −, ≈) verified the superiority of the paging technique. The non-parametric signs ‘‘+’’, ‘‘−’’, and ‘‘≈’’ indicate that the enhancement of the paging technique is statistically superior to, inferior to, and similar to the random allocation, respectively. The latest rows of Table 6 display that the number of ‘‘+’’ is 50 out of 52, the number of ‘‘−’’ is 0 out of 52, and the number of ‘‘≈’’ is 2 out of 52. Therefore, the total revenue is significantly increased by the paging technique against the random allocation.

C. Controller hardware in the loop (CHiL):

The system is validated based on DSP and Control Hardware in the Loop (CHiL) using OPAL-RT, as shown in Fig. 11. The OPAL-RT platform is operating on 4 cores based on Intel Core Xeon processor at 3 GHz and RAM 2 × 8 GB. The system controller is uploaded on a 150 MHz DSP labelled as (TMS320F28335ZJZA). The optimization algorithm of the parking controller is to be validated; therefore, the parking controller algorithm is set up on the DSP, and the OPAL-RT platform is used only for supervisory control and data acquisition.

Also, the coin-or branch and cut (CBC) solver is used to solve the optimization problem for the parking controller. The CBC is an open-source mixed integer programming solver that is written in C++ and its libraries can be accessed at [30]. First, the optimization problem is solved by the DSP using the libraries from the CBC solver. Then, the OPAL-RT receives the optimal allocation for the vehicles from the DSP. The optimal fitness value from the DSP matches the fitness value given by the GAMS optimizer. Fig. 12 describes the graphical user interface for parking reservations, which is filled out by the driver. This parking request contains the data and time for the car entry, the data and time for the car exit, the name, phone number, and email of the driver, and if the customer requires a special slot with an electric charger or for a handicapped person. Consequently, this request will be analyzed by the paging technique to determine if there is a vacant space or not. On the other hand, Fig. 13 describes the operating diagram that will guide the operator. This diagram determines which vehicles will be transferred from which slot and to which slot and when will be exactly transferred. The alarm alerts the operator when it's time to move the car.

TABLE 6: Non-parametric sign test results.

	Parking lot with 24 parking spaces				Parking lot with 48 parking spaces		
	Rand.	Coord.			Rand.	Coord.	
Run #	Total revenue (£/day)		sign	Run #	Total revenue (£/day)		sign
1	437	525	+	27	920	1090	+
2	414	529	+	28	915	1065	+
3	436	533	+	29	891	1074	+
4	422	529	+	30	900	1084	+
5	413	521	+	31	892	1106	+
6	445	525	+	32	894	1106	+
7	440	515	+	33	922	1068	+
8	422	533	+	34	915	1107	+
9	426	510	+	35	901	1106	+
10	424	531	+	36	932	1082	+
11	405	511	+	37	891	1095	+
12	410	518	+	38	905	1068	+
13	419	528	+	39	903	1079	+
14	406	535	+	40	918	1102	+
15	438	505	+	41	920	1095	+
16	404	541	+	42	897	1106	+
17	406	534	+	43	907	1089	+
18	403	523	+	44	905	1062	+
19	406	521	+	45	913	1098	+
20	417	525	+	46	916	1104	+
21	411	515	+	47	918	1090	+
22	451	451	≈	48	900	1093	+
23	417	527	+	49	914	1093	+
24	404	536	+	50	913	1078	+
25	442	535	+	51	896	1089	+
26	448	448	≈	52	894	1069	+
$\sum +$	50						
$\sum -$	0						
$\sum \approx$	2						
p-value	8.27e-6						

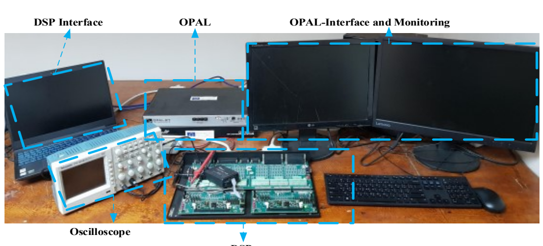

Fig. 11. Controller hardware in the loop.

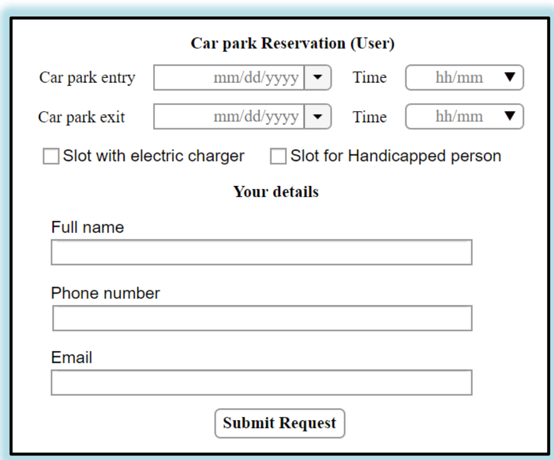

Fig. 12. The parking request.

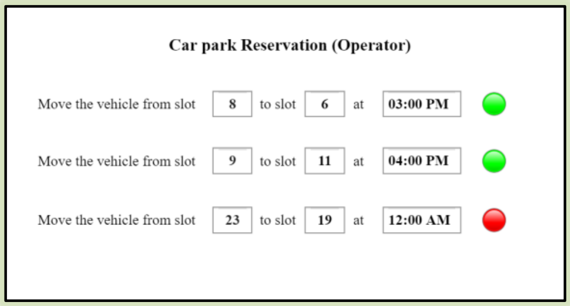

Fig. 13. The operating diagram for the operator.

D. Computational complexity

The computational time is the time required by the parking controller to finish its task. Using the optimization problem on the non-linear form is computationally complex. However, the linearized form of the problem dramatically reduces the computational time. As shown in Fig. 14, which shows the histogram generated from 100 runs for the linearized problem, the average execution time for the parking controller is approximately 2.5 minutes for a 30-minute time slot.

Moreover, in order to study how the computational time is affected as the problem size increases, many different scenarios for different parking sizes are performed on core I9, 16 G ram, and the computation time is recorded. For a parking lot with 100 parking spaces that can accommodate approximately 200 vehicles per day, the computational time is 3.6463 minutes, while for a parking lot with 150 parking spaces that can accommodate approximately 350 vehicles per day, the computational time is 7.8073 minutes. Nevertheless, the proposed method continues to be effective because the study assumes a 30-minute time span and all execution times fall inside this time period.

E. Benchmark

In this subsection, a comparative analysis is conducted between the proposed methodology and state-of-the-art methodologies found in [24, 25] besides the greedy algorithm stated in [26]. It is important to mention that the approach discussed in reference [24] strives to achieve a balanced distribution of parking demand across numerous parking lots. However, it relies on the availability of a substantial number of parking lots in comparison to the number of vehicles that required parking services. Unfortunately, this condition is not met in many countries, as there is a scarcity of parking lots relative to the number of cars. In light of this constraint, the approach has been modified by retaining the same objective function. However, instead of minimizing it to prevent overcrowding of vehicles in a single parking lot compared to others, the function has been opted for maximization to increase the utilization of the parking lots. The proposed comparison aims to demonstrate the superiority of the proposed method in efficiently managing vehicles within parking slots. Table 7 compares the performance of several algorithms for allocating parking lots to vehicles. The algorithms are evaluated on the following metrics: percentage of lost users, utilization factor, computational time, and accuracy. The proposed method outperforms the other algorithms on most of the metrics. It has the lowest percentage of lost users (3.44%), the highest utilization factor (90.4%), and relatively short computational time (1.99 minutes). The other algorithms such as WF-SPA and BF-SPA have a slightly lower computational time than the proposed method. However, they have a much higher percentage of lost users and a lower utilization factor, therefore, do not guarantee optimality. Moreover, the greedy algorithm is a simple and intuitive algorithm that allocates parking spots to vehicles in a first-come, first-served manner. It is not the most accurate algorithm, and it is not efficient with large dimension problems. The greedy algorithm has a lower percentage of lost users than WF-SPA and BF-SPA, but it has a higher percentage of lost users than the proposed method. The greedy algorithm also has a lower utilization factor than the proposed method. ADMM is accurate, but it has a much higher computational time. Overall, the proposed method is the best algorithm for allocating parking spots to vehicles. It has the best combination of accuracy, utilization factor, and computational time. It has the lowest percentage of lost users, which means that it is the best at ensuring that all vehicles are able to find a parking spot. It is a simple linear programming formulation that can guarantee optimal solutions. It is fast and efficient, and it can be efficiently scaled to large problem instances.

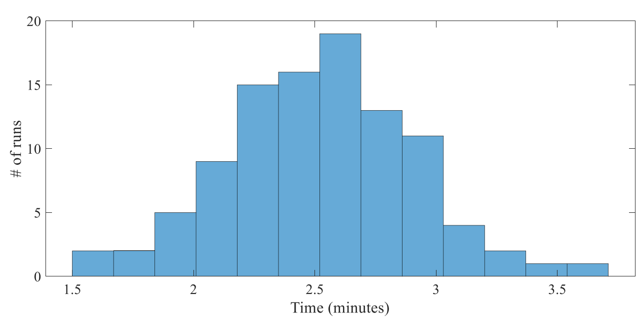

Fig. 14. Histogram for the parking controller execution time.

Table 7. Performance Comparison of Parking Assignment Problem Algorithms.

	WF-SPA	**BF-SPA**	**ADMM**	**Greedy Approach**	**Proposed method**
Percentage number of lost users	16.09%	16.09%	9.19%	4.6%	3.44%
Utilization Factor	85.93%	88.19%	90.01%	90.2%	90.4%
Computational time	31 sec	35 sec	2.6 mins	1.2 mins	1.9 mins
Accuracy	Can be sub-optimal	Can be sub-optimal	find optimal solutions	Can be sub-optimal with large problem	find optimal solutions

F. Proposed Method validation in real world applications with different pricing strategies:

A comparative analysis of revenue was conducted between our proposed model and the models mentioned in, demonstrating the superior performance of our model in [24-26], minimizing unoccupied parking spaces and maximizing revenue compared to the competing methodologies. To provide practical examples, three real-world scenarios were utilized. Firstly, Premium P1080 SAN Airport Parking in the United States implemented a fixed pricing strategy, with a constant parking fee of $8.99 per day regardless of the parking duration [31]. Secondly, Covent Garden Car Park in London employed a dynamic real-time pricing approach, where parking rates varied based on the duration of stay, ranging from £9.00 for up to 1 hour to £44.00 for up to 24 hours, with potential rate increases based on demand and the number of vehicles present in the parking lot [32]. During the study, we assumed that the parking rates would be increased by £0.5 if there were more than 25 vehicles in the parking lot. If there were more than 50 vehicles, the rates would be increased by £1. And if there were more than 75 vehicles, the rates would be increased by £1.5. Lastly, the West End long-stay car park in Bristol City, England, operated under a fixed-time, variable-rate pricing system. It offered different fees for various durations, such as £2.50 for 1 hour, £5.00 for 2 hours, and £18.00 for daytime parking (entry after 08:00 and exit by 18:00). Additionally, distinct rates were applicable for evening and overnight parking periods [29]. These practical examples serve to illustrate the diverse pricing strategies employed in different parking scenarios. Table 8 compares the revenue generated by different parking strategies: Fixed pricing, dynamic real-time pricing, and dynamic fixed-time with variable-rate pricing. The proposed method outperforms the other strategies, consistently generating the highest daily revenue. This demonstrates the superiority of the proposed method in maximizing revenue generation for parking services, regardless of the pricing method. Due to the primary focus of the proposed method on significantly increasing the utilization factor through the utilization of the paging algorithm, it consequently results in a substantial increase in revenue.

Table 8: Comparative Revenue Analysis Applied on real world application with different pricing methods.

	Premium P1080 SAN Airport Parking in the United States (Fixed Pricing)	**Covent Garden Car Park in London (Dynamic Real-Time Pricing)**	**West End long stay car park in Bristol City (Dynamic, Fixed-Time, Variable-Rate Pricing)**
WF-SPA	656.27 $/day	2858.5 £/day	1014.2 £/day
BF-SPA	665.26 $/day	2932.5 £/day	1026 £/day
ADMM	719.2 $/day	2988.5 £/day	1061.5 £/day
Greedy Approach	746.17 $/day	3014.5 £/day	1067.5 £/day
Proposed method	755.16 $/day	3078.5 £/day	1087.5 £/day

V. Conclusion

This paper has investigated the revenue for the parking authorities and proposed a new paging technique in order to maximize the parking revenues. The idea behind the proposed technique is to make the best use of parking spaces by exploiting their idle times. Consequently, the utilization factor for the parking spaces is maximized. First, the proposed technique was formulated as non-linear optimization problem, then it was reformulated as a mixed integer linear programming to reduce the computational time and complexity. The optimization problem was addressed using GAMS optimizer by CBC solver. Moreover, it was validated on a hardware platform using a DSP controller. The paper investigated the difference between fixed and dynamic pricing when employing the proposed paging technique. The results showed that the revenue has dramatically increased by 20% by using the proposed method instead of conventional random allocation. Moreover, using the random allocation, the parking management can only assign 70 vehicles to the 120 clients, resulting in a customer rejection percentage of 50/120 = 41.67%. However, by Using the proposed paging algorithm, the number of parked vehicles increases to 84 and the customer rejection rate decreases to 30%. Consequently, the proposed algorithm significantly increases the utilization rate of the parking spaces. As a future work, the authors will study addressing the large-scale parking problem with the most prevalent solvers for MIP, such as CPLEX, SCIP, CBC, and GLPK, and then evaluate which solver will be the most efficient for the large-scale scenario. Moreover, the authors will investigate the use of the most used meta-heuristic techniques, such as the Salp swarm algorithm (SSA), to handle large-scale parking problems. Since SSA outperforms other swarm-based algorithms, it has recently been employed in various applications. Consequently, research will be conducted by the authors to improve the exploration and exploitation for the SSA, especially for large-scale optimization problems. Finally, a comparative analysis is performed, benchmarking the proposed method against recent competing algorithms. The proposed method demonstrates superior performance compared to other algorithms in terms of lower percentage of lost users, higher utilization factor, reasonable computational time, and the ability to find optimal solutions in the parking assignment problem.

Appendix

The stochastic arrival/departure scenarios for vehicles in the parking lot are listed in TABLE A1.

TABLE A1

ARRIVAL/DEPARTURE SCENARIOS FOR DIFFERENT VEHICLES

Vehicle #	Arrival Time	Departure Time	Vehicle #	Arrival Time	Departure Time
1	12:00 PM	11:30 AM	45	9:30 PM	7:30 AM
2	8:30 AM	3:00 PM	46	4:30 PM	7:30 PM
3	4:30 PM	5:30 PM	47	8:00 AM	1:00 PM
4	8:30 PM	7:00 AM	48	8:30 AM	3:00 PM
5	8:00 AM	2:00 PM	49	9:00 AM	3:00 PM
6	3:30 PM	5:30 PM	50	8:30 AM	5:00 PM
7	8:00 PM	6:30 AM	51	8:30 AM	3:00 PM
8	5:00 AM	3:30 PM	52	8:00 AM	3:00 PM
9	4:00 PM	7:00 AM	53	8:30 AM	4:00 PM
10	9:30 AM	1:30 PM	54	10:00PM	6:30 AM
11	2:00 PM	4:00 PM	55	6:00 PM	8:30 PM

12	6:30 PM	10:00 PM	56	8:00 PM	7:00 AM
13	9:00 AM	1:00 PM	57	5:00 PM	7:30 AM
14	11:30 PM	6:00 AM	58	10:00 PM	6:00 AM
15	10:00 AM	2:00 PM	59	9:30 PM	7:00 AM
16	3:00 PM	4:30 PM	60	5:30 PM	7:30 PM
17	6:30 PM	9:30 PM	61	9:30 PM	7:00 AM
18	11:30 PM	7:00 AM	62	5:00 PM	8:00 PM
19	3:00 PM	4:00 PM	63	8:30 PM	6:30 AM
20	9:30 AM	4:00 PM	64	3:00 PM	7:00 PM
21	5:00 PM	9:00 PM	65	12:00 PM	4:00 PM
22	12:00 AM	7:30 AM	66	12:00 PM	4:00 PM
23	4:30 PM	8:00 PM	67	1:30 PM	5:00 PM
24	10:30 PM	5:30 AM	68	8:00 PM	10:30 PM
25	8:30 AM	6:30 PM	69	3:30 PM	5:30 PM
26	8:30 PM	7:30 AM	70	1:30 PM	4:30 PM
27	9:30 AM	1:30 PM	71	5:00 PM	9:30 PM
28	3:30 PM	6:30 PM	72	7:00 PM	11:00 PM
29	9:00 PM	7:30 AM	73	4:30 PM	8:00 PM
30	10:30 AM	3:00 PM	74	4:00 PM	7:30 PM
31	5:00 PM	7:30 PM	75	4:30 PM	7:30 PM
32	11:00 PM	8:00 AM	76	1:30 PM	3:30 PM
33	9:00 AM	1:00 PM	77	1:30 PM	4:30 PM
34	2:00 PM	4:00 PM	78	8:00 PM	11:30 PM
35	5:30 PM	9:00 PM	79	6:00 PM	11:00 PM
36	9:00 PM	9:00 AM	80	7:00 PM	10:00 PM
37	2:30 PM	6:00 PM	81	7:00 AM	11:30 AM
38	8:00 AM	1:00 PM	82	5:30 AM	10:00 AM
39	4:30 PM	6:00 PM	83	6:00 AM	9:30 AM
40	10:30 PM	7:00 AM	84	6:30 AM	9:00 AM
41	9:00 AM	1:00 PM	85	3:00 PM	7:00 PM
42	7:00 PM	4:30 AM	86	8:00 PM	6:30 AM
43	3:30 PM	4:30 PM	87	4:30 PM	7:00 AM
44	8:00 AM	2:30 PM			
	Handicapped case		Electric vehicle		Regular customer
	Prepaid and walk-in customers				

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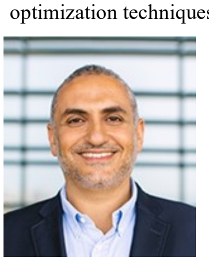

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