CONTROL OF A WHEELED MOBILE ROBOT FOLLOWING A SET TRAJECTORY

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УПРАВЛЕНИЕ НА КОЛЕСЕН МОБИЛЕН РОБОТ ПРИ СЛЕДЕНЕ НА ЗАДАДЕНА ТРАЕКТОРИЯ

Abstract: *In the article, the problems of stability during movement of a four-wheeled mobile robot along a set trajectory are investigated. The object of the study is a wheeled mobile robot with front turning and rear driving wheels according to the Ackerman scheme. A simulation was made based on a bicycle mathematical model, in which the longitudinal and transverse stability are calculated. The simulation shows that the accelerations do not exceed the limit values guaranteeing the stability of the robot during the movement along the set trajectory.*

***Keywords**: Control; Wheeled Mobile Robot; Trajectory Tracking.*

Introduction

Today, the design of mobile wheeled robots is becoming an increasingly detailed process, due to the experience gained over the years and the increased use of this type of technique. These robots are used both in indoor work [4], [6] and as autonomous vehicles.

Autonomous mobile wheeled mechanisms are of particular interest. They solve various scientific tasks and problems such as: determining the trajectory of movement [1], [3], [7], following such a trajectory [2], [4]. As well as questions regarding the sustainability and stability of such robots.

According to D'alembert's principle [5] – the principle of kinetostatics – considering an inertial force allows us to consider the robot as a body on which a balanced system of forces acts. Thus, for a given motion law, we can determine the support reactions and thus obtain values for the model parameters at which the robot will remain stable – there will be no overturning relative to the motion surface.

In the present paper, we extend the well-known [2], [3] models, with calculation of longitudinal and transverse stability of wheeled mobile robots, containing geometric and mass design parameters. Driving, resistance, inertial forces and support reactions are included in the model, i.e. active forces, support reactions and inertial forces are included.

The article is organized as follows: in the second part, we have placed an implementation and description of the mathematical model of the robot, and its position in the lateral direction, including relative to a set trajectory, is determined by three coordinate systems. We propose a way in which the smoothness of trajectory following can be controlled. In the third part through numerical experiments we investigate the accelerations to which the robot is subjected and the smoothness of the trajectory described by it, while it is in the mode of following a set trajectory. In the last part, we draw conclusions about the performance of the model.

Methods and Materials

It is necessary to select one or more points of the robot structure, for which we will aim to follow the robot's motion trajectory $\boldsymbol{S}$, as close as possible (Figure 1 and 2).

Let point $\boldsymbol{A}$ be the support of a front wheel (we assume that the contact of the wheels with the road is a point, not a spot), point $\boldsymbol{B}$ be the support of a rear wheel. Let the steering wheel be one - the front one. Let the turning angle of the front wheel be θ_A and let there be a restriction on the angle by construction etc. considerations $-\theta_{Amax}$.

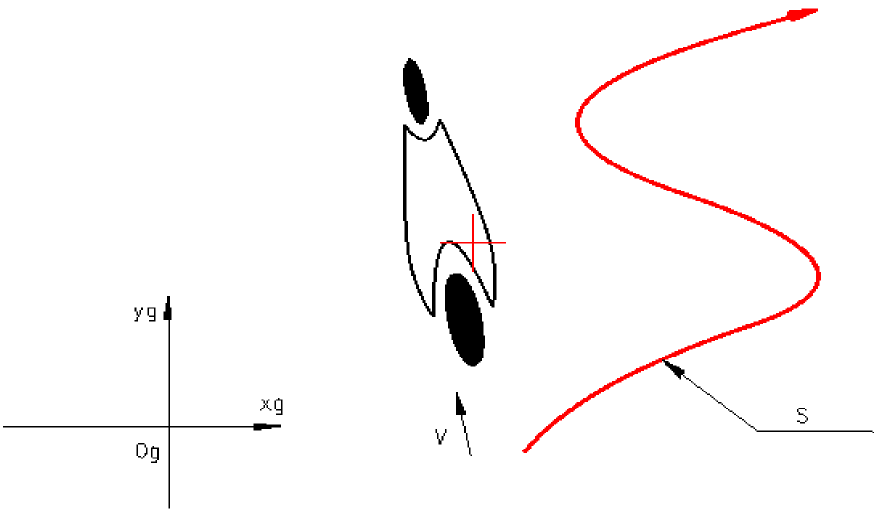

Figure 1. Random position of the robot (bicycle mode) and the set trajectory in the horizontal plane of movement

Using a circle with radius r_A and center point $\boldsymbol{A}$, we find the closest distance from point $\boldsymbol{A}$ to the trajectory $\boldsymbol{S}$. We denote the common point by A_{ps}. We introduce the angles:

- θ_{A1}, which is closed between the steering wheel direction and $\underline{AA_{ps}}$;
- θ_{A2}, which is closed between the x-axis of the robot body-related coordinate system and $\underline{AA_{ps}}$.

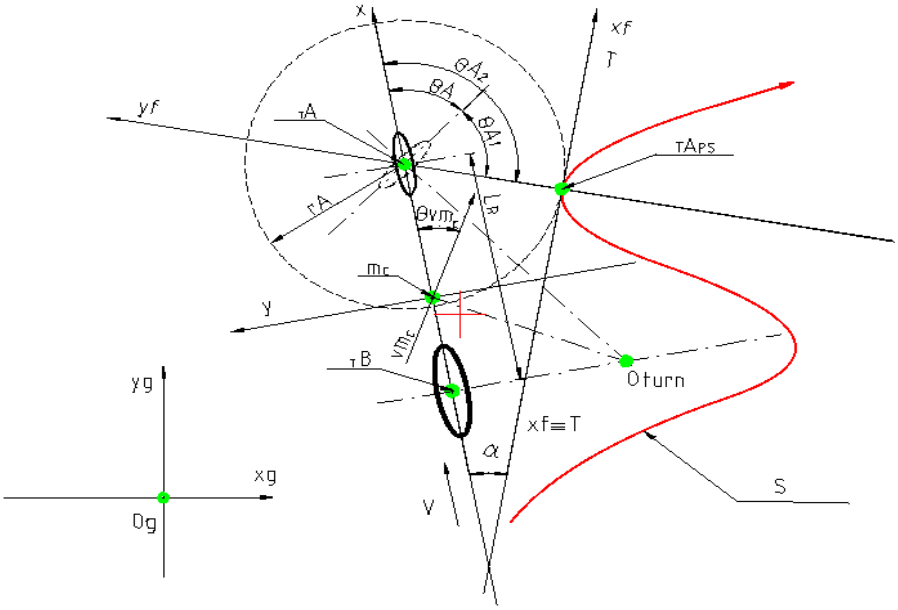

Figure 2. The robot, the set trajectory and the three coordinate systems

Angle θ_{A1} gives the orientation of the steering wheel relative to the tangent at the closest point of the trajectory ***S*** (closest to point ***A***, i.e. $\underline{AA_{ps}}$).

Angle θ_{A2} gives the orientation of the x-axis of the associated coordinate system (the x-axis is contained in the longitudinal plane of symmetry of the robot, coinciding with the plane $m_c xz$ of the robot-associated coordinate system $m_c xyz$, where in this case the center of mass of the robot is chosen as the center of the coordinate system) relative to the same tangent.

Let us denote the tangent by ***T***. We aim at $\underline{AA_{ps}} \to 0$, i.e. we choose point ***A*** to follow trajectory ***S*** as close as possible. It is possible to select another point of the robot to follow the trajectory, for example m_c or ***B***.

One clarification: in Bicycle mode, point ***A*** and point ***B*** are the contacts of the front and rear wheels with the road; in the case of a four-wheel scheme, these points are the geometric center of the front and rear axles.

The trajectory ***S*** is defined in the global coordinate system $O_g x_g y_g z_g$; the tangent ***T*** - too. We assume that the trajectory ***S*** is located in the plane $O_g x_g y_g$ and the set of points on the planes $O_g x_g y_g$ and $m_c xy$ is the same (in case of uneven terrain, the search for the closest distance between point ***A*** and the trajectory will be instead of a circle – with a sphere, with center point ***A***).

Thus:

(1)

$$\underline{AA_{ps}} = f(V; \theta_{A1}; \theta_{A2}; S; x_{gA}; y_{gA})$$

We introduce a degree of closeness of the robot to the trajectory ***S*** by the following inequality:

(2)

$$\underline{AA_{ps}} \geq kL_R$$

Where $\boldsymbol{k}$ is a coefficient taking into account the nature of the terrain, maneuverability of the structure, etc. If the inequality is satisfied, the robot is assumed to be far enough to advance to the trajectory by the shortest path, i.e. $\underline{AA_{ps}} \in m_c x$.

If:

(3)

$$\underline{AA_{ps}} < kL_R$$

as the trajectory approaches, angle $\boldsymbol{\alpha}$, the angle between the $m_c x$ axis and the tangent $\boldsymbol{T}$ should become more and more oblique, and when $A \equiv A_{ps}$, $\boldsymbol{\alpha}$ will assume a value of 0.

We will introduce a floating coordinate system $A_{ps} x_f y_f$, where the center is the floating point A_{ps}; $x_f \equiv T$; $\underline{AA_{ps}} \in y_f$.

We introduce a coefficient q of attraction to the trajectory:

(4)

$$q = f(V_A; \theta_{A1}; A_{yf}); q > 1$$

We define a dependency between some angles:

(5)

$$\text{For } \alpha \in [0; \pi), \theta_A = -\alpha . q;$$
$$\text{if } |-\alpha|. q > \theta_{Amax}, \theta_A := -\theta_{Amax}.$$
$$\text{For } \alpha \in [-\pi; 0), \theta_A = -\alpha . q;$$
$$\text{if } |-\alpha|. q > \theta_{Amax}, \ \theta_A := +\theta_{Amax}$$

We determine the direction of movement and the speed of point m_c when turning. We consider the case in which the travel speed of the leading wheel is constant, i.e. $V_B = const.$

Then:

(6)

$$V_A = \frac{V_B}{coscos\, \theta_A}, cos\, cos\, \theta_A \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$$

if there are no other restrictions.

We determine the center of rotation in a turn at a given θ_A. This is the intersection of the front and rear wheel axes: O_{turn}.

We pass axis vm_c through point m_c, which is perpendicular to $\underline{m_c O_{turn}}$ and it gives us the direction of velocity V_{m_c} of m_c.

We define an angle $\theta_{V_{m_c}}$. Then:

(7)

$$V_{m_c} = \frac{V_B}{\cos\cos\theta_{V_{m_c}}}$$

We determine the projections of V_{m_c} on the axes of the connected coordinate system $m_c xy$:

(8)

$$V_{m_c}x = V_{m_c}\cos\cos\theta_{vm_c}\,;\; V_{m_c}y = V_{m_c}\sin\sin\theta_{vm_c}$$

At $\theta_A = const$, i.e. the robot moves in a circle with center O_{tyrn}, only centrifugal acceleration and centrifugal force are present. For the center of mass, at a given peripheral velocity, we have a centrifugal force which is:

(9)

$$P_c = mr(\omega_{O_{turn}})^2,$$

where $\boldsymbol{m}$ is the mass of the robot centered at m_c, $r = \underline{m_c O_{turn}}$ and

(10)

$$\omega_{O_{turn}} = \frac{V_{m_c}}{\underline{m_c O_{turn}}}$$

If $V_B \neq const$ and/or $\theta_\mathrm{A} \neq const$, then $V_{m_c}x \neq const$, $V_{m_c}y \neq const$, respectively $\dot{V}_{m_c}x \neq 0$; $\dot{V}_{m_c}y \neq 0$. Then for the tangential and transverse acceleration we have:

(11)

$$a_{tan} = \dot{V}_{m_c}x;\; a_{lat} = r\left(\omega_{O_{turn}}\right)^2 + \dot{V}_{m_c}y$$

The maximum values for longitudinal and transverse acceleration are determined by the methods described in [5], which guarantee the stability of the robot in roll and pitch.

Experiments and Results

We calculate the necessary torques so that the mobile robot moves along the reference trajectory:

$$xr = 1.1 + 0.7\sin(2\pi/30);$$
$$yr = 0.9 + 0.7\sin(4\pi/30).$$

In the simulation, the torques of the robot are calculated using the inverse dynamic model, and the motion trajectory is plotted in Figure 3.

The parameters of the robot are mass ***m=0.25 kg***, ***J=0.01 kgm²*** →

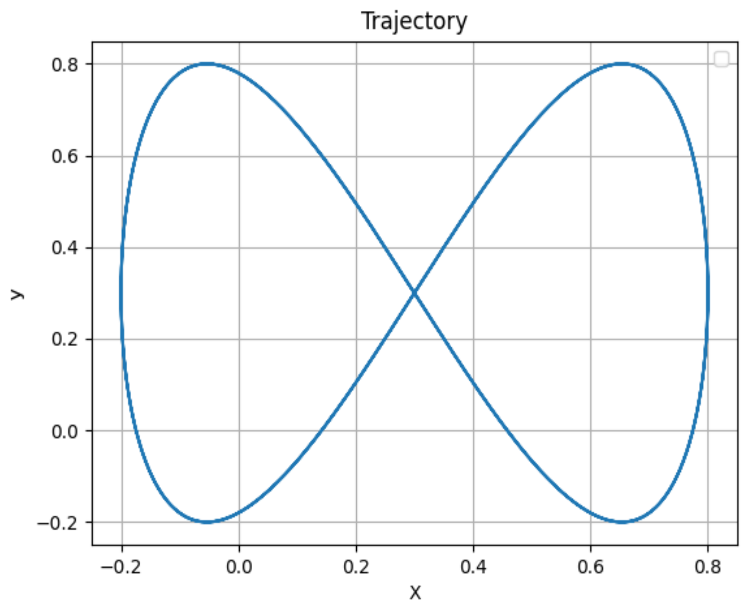

Figure 3. Траектория на движение на робота

Figure 4 and Figure 5 show the transverse and longitudinal acceleration of three robots with different wheel lengths and radii using the inverse dynamic model, and plotting the motion trajectory in Figure 3. With the parameters of the robot selected in this way [5], the transverse and longitudinal stability of the robot are guaranteed.

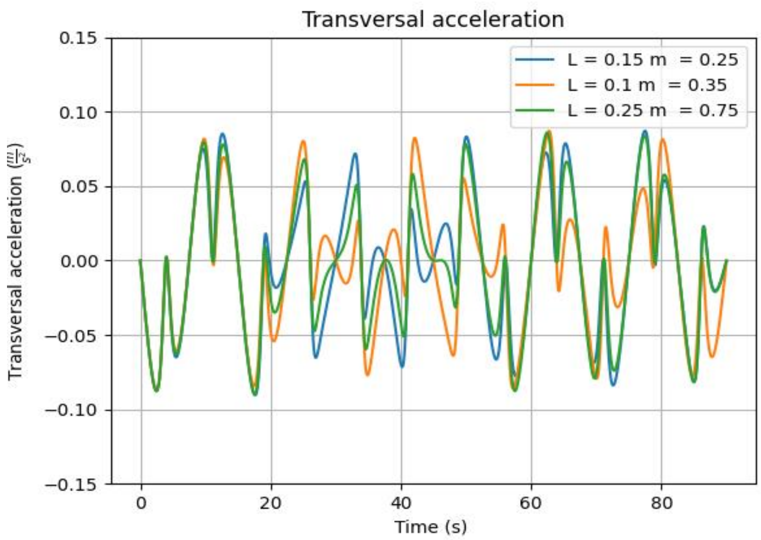

Figure 4. Transverse acceleration of three robots with different wheel lengths and radii

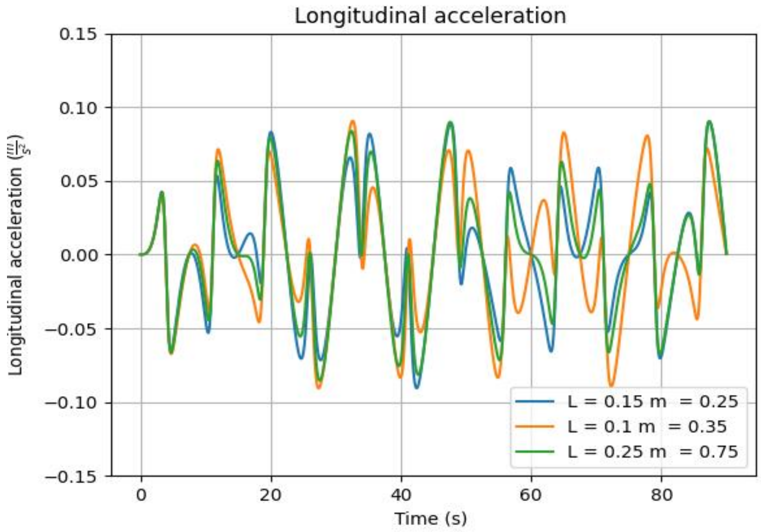

Fig. 5 Longitudinal acceleration of three robots with different wheel lengths and radii

Conclusion

The problems of stable control of a four-wheeled robot with front turning wheels according to the Ackerman scheme are investigated. A model of this type of robot was built based on the two-wheeled model based on the principles of kinetostatics. The longitudinal and transverse stability of wheeled mobile robots, depending on their geometric proportions, as well as the forces acting on them, have been added to the model. The influence of the length of the robot and the size of its wheels on its longitudinal and transverse stability was studied. Numerical simulation shows that the modeled robots have parameters where the longitudinal and transverse stability is guaranteed.

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Резюме: *В статията са изследвани проблемите на устойчивост при движение на четири-колесен мобилен робот по зададена траектория. Обект на изследването е колесен мобилен робот с предни завиващи и задни задвижващи колела по схема Акерман. Направена е симулация, базирана на bicycle математичен модел, в която се изчисляват надлъжната и напречната устойчивост. От симулацията се вижда че ускоренията не надвишават пределните стойности гарантиращи устойчивостта на робота по време на движението по зададената траектория.*

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