

A NON-LINEAR OPTIMISATION APPROACH TO IN-PIT HAUL ROAD DESIGN

Dimitar Kaykov¹, Danail Terziyski², Kremena Arsova-Borisova³, Milan Stajić⁴

¹University of Mining and Geology “St. Ivan Rilski”, 1700 Sofia; E-mail: dimitar.kaykov@mgu.bg

²University of Mining and Geology “St. Ivan Rilski”, 1700 Sofia; E-mail: d.terziyski@mgu.bg

³University of Mining and Geology “St. Ivan Rilski”, 1700 Sofia; E-mail: kremena.arsova@mgu.bg

⁴University of Belgrade, Technical Faculty in Bor, 19210 Bor; E-mail: mstajic@tfbor.bg.ac.rs

Corresponding author: dimitar.kaykov@mgu.bg

ABSTRACT. Truck haul costs, as one of the predominant operational costs for mining and quarrying operations, are known to be heavily dependent on the design parameters of haul roads. Furthermore, in-pit haul road design parameters determine the pit limits and therefore, the potential feasibility of the mining operation. Thus, when in search of an optimal solution in terms of in-pit haul roads, one should primarily consider the location of the in-pit haul road, its design features and the deriving operational costs regarding extraction and haul costs. A suitable objective function in this case may be the undiscounted profit for the ultimate pit design. However, for each considered scenario, truck and excavator operational costs can be calculated using simulation techniques for better accuracy. Furthermore, finding an optimal solution requires the execution of a reliable and efficient algorithm, depending on the shape of the objective function. Hence, a non-linear optimisation approach was proposed in this paper for solving the in-pit haul road optimisation problem, based on a simulation of the materials allocation, which was used for calculating the objective function. Design parameters were assumed to be predetermined, while the only variable used for finding an optimal solution was the location of the in-pit haul road inside the pit contour. In addition, two 1-D algorithms were compared for finding the optimal solution (Search with accelerated step size and 1-D Simplex method). Furthermore, two regression models are proposed (Multiple Linear Regression – MLR and Non-Linear Regression - NLR), which could identify the more feasible region for the in-pit haul road location and reduce the number of iterations required for convergence.

Key words: in-pit haul road, open-pit mine, design parameters, non-linear optimisation, simulation

Introduction

Creating an efficient haul road is one of the most important aspects of open-pit mine design. According to different sources, haul costs can comprise a significant portion of the total operational costs, ranging from 49% to 70% (Mohutsiwa and Musingwini, 2015; Nancel-Penard et al., 2019). The design of the haul road can also impact the pit's shape, potentially leading to the excavation of more material or limiting access to certain volumes. Various optimisation techniques have been previously used in pit design problems that aim to maximise discounted or undiscounted revenue for the ultimate pit contour. However, some parameters are not always accounted for in these solutions, which would lead to certain assumptions that simplify the actual problem. Hence, one should treat such methods as guidelines rather than robust techniques.

The haul road design problem in the field of open-pit mining can be divided into three separate subproblems: 1) optimising ex-pit haul road location and design features simultaneously with the earth allocation problem, 2) optimising in-pit haul roads (ramps) location and design features in addition to the ultimate pit design optimisation problem, and 3) optimising haul road construction methods and maintenance practices. Regarding overall costs, ramp construction costs are significantly less compared to haul costs. Therefore, ex-pit and in-pit haul roads location and design tend to be more significant for the project's evaluation and hence, they tend to be the focus of many researchers and optimisation problems.

The full extent of the ex-pit haul road optimisation problem deals with the required investment costs for road construction and the expected operational and maintenance costs (Akay et al., 2013). This problem can provide solutions that estimate the full amount of costs for the earthwork allocation problem depending on the provided iteration in terms of the haul road location and design. Usually, these problems are solved

separately as part of a larger problem so that computations can be performed more efficiently.

By comparison, the in-pit haul road optimisation problem deals not only with investment, operational and maintenance costs, but also with the revenue of the pit design. The in-pit haul road location is responsible for changes in the pit limits, which leads to the additional extraction (or potential losses) of certain amounts of ore and waste volumes. Therefore, the in-pit haul road optimisation problem is regarded by a growing number of authors as part of the pit optimisation problem (Morales - Varela, et al., 2017; Nancel - Pernard et al., 2019; Yarmuch et al., 2020).

However, the established paradigm in pit design optimisation does not fully recognise this approach yet. To some extent, actual pit designs rely primarily on subjective decisions as they do not consider the full spectrum of design features or criteria, on which the ultimate design is based. This can be attributed to two major reasons – 1) actual pit design can be a time-consuming task and 2) there are a lot of design features that require to be structured in a uniform feature space. Nowadays, automated pit design functions in pit design software make one's job easier in terms of designing an open-pit scenario.

However, this still requires a certain amount of manual work. In addition, manual design may lead to improved pit design and ore extraction, compared to simple contours offset automation. Nonetheless, automated pit design has provided a way to increase the development speed of pit design projects and the consideration of additional scenarios. Furthermore, it can be utilised as a tool for a preliminary exploration of the design parameters feature space to obtain a rational design solution. This is the main motivation why this paper focuses primarily on the in-pit haul road optimisation subproblem, based on automated design tools.

In-pit haul road optimisation problem

Optimisation criteria

Several critical factors must be taken into consideration during the design phase of an in-pit haul road regardless of its lifespan – low amount of ore losses, maximum economic value, low amount of environmental impact, etc. All of these factors are related and must be considered as a whole in the design of haul roads for both in-pit and ex-pit conditions.

Maximum economic value criteria

The first criterion is related to maximum economic value for the ultimate pit design, which requires careful consideration of access to valuable sections of the ore deposit while minimising waste volumes to reduce unnecessary mining costs. Secondly, it is essential to minimise haul costs throughout the life of mine which requires minimal overall haul distances. Furthermore, this also leads to the minimisation of maintenance costs. It is generally established that longer-life haul roads are preferred over shorter ones as they reduce overall road construction and operating costs (Atkinson, 1992). Moreover, the number of access points to the pit also affects haul costs. This could lead to better flexibility and less waiting times for trucks, although in some cases, more access points may not always be cost-effective (Hustrulid et al., 2013). In addition, crusher and waste dump locations can affect the feasibility of the operation (Hustrulid et al., 2013; Paricheh and Osanloo, 2016; Liu and Pourrahimian, 2020). However, such a complex optimisation problem that simultaneously optimises in-pit haul road location and design parameters, in addition to determining optimal waste dump locations and crusher locations are rarely considered to the full extent. The most common iteration of this problem is solved by treating waste dump and crusher locations as predefined, which puts the main focus on haul road design. The alternative approach is also valid – assuming a fixed in-pit haul road location and searching for an optimal location for crushers and waste dumps. It is crucial that the overall haul distance should be minimised, so that the operational costs during the life of the mine can also be minimal.

Nonetheless, in-pit haul road design may lead to unfavourable results in terms of increased operational or maintenance costs, but they may be redeemed by the incremental revenue from the additional extracted ore volumes due to changes in the pit design. Additionally, overall costs can also be reduced when the amount of waste required to be mined is decreased due to pit design changes. This is a good argument why in-pit haul road design and pit design optimisation problems must be considered simultaneously. Therefore, the criterion which considers this trade-off should be the profit obtained from the mining operation. However, as better suited it may be, the question remains whether this profit should be considered as a discounted or an undiscounted value. Hence, the problem can be solved in a different context, depending on whether pushback sequencing or annual scheduling is involved. Indeed, block extraction sequence is a crucial factor, especially when commodity prices fluctuate in an unpredictable manner. The use of discounted cash flows is known to lead to different outcomes compared to using undiscounted cash flows, based on the Caccetta-Hill theorem (Saleki et al., 2019). However, applying the discounted cashflow approach would require a detailed mine schedule and therefore, this would increase the complexity of the in-pit haul

road design problem, which could be unfavourable for bigger deposits. Thus, such complexity is usually reduced because of these reasons and different aspects of the overall problem are solved separately. Nonetheless, advances in mining software and open-pit optimisation problems (ultimate pit optimisation, scheduling problems, etc.) may soon provide integrated solutions.

Environmental criteria

The environmental impact of the haul road design is also recognised as a valid optimisation criterion, as this is crucial for the public reputation of mining operations. This involves reducing noise levels, dust generation, and CO₂ emissions, as well as exploring ways to utilise waste rock materials (Keckojevic and Komljenovic, 2010; Terziyski and Kaykov, 2022). However, in many cases, these criteria are rarely used as primary ones and alternatively, they are used as constraints for all considered solutions. One reason is that they can correlate with operational costs (e.g., CO₂ emissions). Nonetheless, they can also be used in multi-criteria optimisation problems to provide a better insight into the reduced environmental impact of the mining operation.

Constraints

Each solution belonging to the set of all considered solutions must meet legal, environmental, social and technological constraints, as well as safety limitations in terms of equipment exploitation, pit design parameters and haul road design parameters.

Operational safety constraints

Operational safety is one of the most important aspects of in-pit haul road design. Haul road width and road basis should be considered regarding safe passage for mining equipment. Additionally, it is crucial to ensure that sight distance is significantly higher than truck stopping distances. Thompson et al. (2019) have developed an interactive approach to haul road design that considers both the design and operational phases of the mine. The authors have established that the use of more formal design methods can significantly reduce the risk of accidents.

Slope stability constraints

Slope stability is also a crucial factor in terms of in-pit haul road design and ultimate pit design. It is essential to avoid design parameters that could compromise slope stability. Smaller slope angle values, wider bench widths and smaller bench heights are all valid ways of ensuring slope safety (Aleksandrova, 2008; Aleksandrova and Trifonova, 2012). However, it is established that this leads to decreased cash flows. It is known that a small decrease in the overall pit slope angle may lead to a substantial amount of waste extraction costs, which may not always be feasible (Koprev and Aleksandrova, 2022). Additional switchbacks may be a solution for slightly improved slope stability, while at the same time, they may also decrease haul road length. However, it should be noted that they tend to slow traffic and cause greater tire wear and maintenance costs (Hustrulid et al., 2013). Regardless, each considered solution should be accepted, only if the lowest pit slope factor of safety is above a desired (or required) threshold value.

Material output constraints

Material output is another key consideration in terms of haul road design, and the road width should be designed to minimise traffic congestion and reduce the overall waiting time for mining equipment (Koprev, 2015). However, excess road width also leads to a decrease in the overall pit slope angle, which impacts the undiscounted profit of the mine. The intuition behind these relations is that it is necessary to strike a balance between productivity, profitability and safety.

Environmental constraints

In most cases, complex environmental aspects are used as constraints for the optimisation problem when certain thresholds are established for each environmental aspect, which leads to accepting or neglecting a design alternative. This is especially related to waste dumping problems regarding foundation, area requirements, etc. (Pavlov et al., 2015; Ivanova et al., 2016). Additionally, waste volumes constraints could also be imposed on the design solution, where a certain amount of waste type should not exceed a certain threshold (Panayotova et al., 2013).

Ore blending constraints

Additional constraints to the optimisation problem may also consider ore blending targets in terms of metal contents or deleterious elements. Depending on the extracted ore and the established target value of metal content, such constraints could be used as a way of extending this optimisation problem to the scheduling aspect of the mining operation. However, for this case study, this is not considered, but it is a valid way of bringing the problem into a more generalised form.

Pre-established optimisation techniques

In general, designing an efficient haul road involves selecting reliable optimisation criteria, as well as a reliable and efficient optimisation algorithm so that the solutions converge to a global optimum with respect to all constraints. In many cases reaching a global optimum may not always be feasible in terms of computational and manual work time, due to the vast number of alternative design scenarios. This leads to the adoption of certain assumptions, certain stopping criteria, decision variable precision or a combination of these assumptions. Moreover, depending on the type of deposit, as well as the stage of the life of mine planning, the optimisation problem can be defined in numerous ways, which furthermore can be additionally adjusted to the uniqueness of the mining operation itself. This can be a positive aspect, as it can lead to valid assumptions which reduce the number of design features used for branching different design scenarios. So far, there is no generalised solution to the in-pit haul road optimisation problem. However, certain prominent methods have been established for different pre-defined conditions. Two of the most prominent optimisation problems follow a discrete optimisation approach for determining the location of the in-pit haul road.

Pit contour adjustments based on the economic block model

The block modelling approach is one of the most powerful and successfully implemented methods for solving mining problems, regarding the optimisation of the pit contour,

establishing pushbacks and scheduling mining operations, when a vast set of technologically feasible solutions exist. Each solution provided by solving the ultimate pit optimisation problem is initially in the discrete space of the block model. Hence, Morales-Varela and co-authors (2017) proposed that adjustments to the ultimate pit solution should be made, taking into account the location where the in-pit haul road is located (Figure 1).

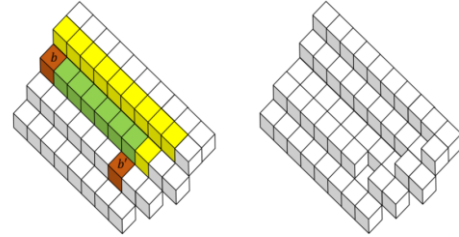


Fig. 1. Ramp set, blocks b and b' selected as accesses: green blocks are in the path, yellow blocks have to be removed due to precedence constraints; (left) Resulting profile after ramp construction (right) (Nancel-Penard et al., 2019)

The objective function of this algorithm is to maximise the economic value of all extracted blocks from the block model (including the blocks representing the pit ramps). The optimisation technique was applied for multiple scenarios for different starting point locations, as well as clockwise and counter-clockwise directions of the ramp.

Ramp design based on shortest path optimisation

The shortest path optimisation problem is one of the most well-known optimisation problem classes in the field of Operations Research. Indeed, it is related to a wide range of practical problems, including optimising haul road design and pit ramps locations in open-pit mining. Yarmuch et al. (2020) have proposed a new approach to solve the in-pit haul road optimisation problem by treating different road segments as part of a graph. This method is based on minimising the sum of extraction costs, haul costs, as well as costs, accounting for changes in road direction. The authors have classified the problem as a binary linear programming one (BLP). For solving the problem, the authors have applied an adaptation of the Depth-first search algorithm – the Mutually exclusive greedy adaptive path (MEGAP) algorithm, which reportedly yields better results than conventional commercial Mixed-Integer Programming solvers (Yarmuch et al., 2020). An example of the solution, of the problem is shown in Figure 2.

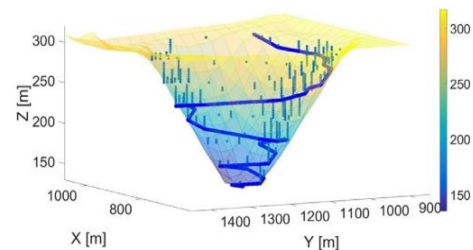


Fig. 2. Illustration of in-pit ramps from solving BLP method (Yarmuch et al., 2020)

The authors have also included a local search method for a partial exploration of all possible paths in the constructed graph, which aims to improve the solution by pruning paths

consisting of poor values. In addition, the time complexity of the proposed method for finding a design solution is linear.

Discussion of possible optimisation methods, assumptions and limitations

Both approaches are based on discrete optimisation techniques, which are very powerful methods for automating calculations and the evaluation of each solution candidate. Furthermore, the iterative approach based on investigating different in-pit haul road location candidates is crucial for solving the generalised in-pit haul road problem.

However, a limitation of both approaches is that the in-pit haul road width is limited to block size. This may not always be the case for actual ramps design and further subdivision of different blocks may lead to skewing ore grades for the subdivided blocks. Nonetheless, the position of the set of blocks along the pit contour can adjust the haul road slope grade and width of the ramp stepwise, which could serve as a guideline for pit design at a final stage. Furthermore, in terms of both described approaches, they both deal only with a single pit contour, rather than a set of pushbacks. The multi-stage pushback approach is indeed a crucial part of the problem as it could provide a better insight into the evolving stages of the in-pit haul road locations and design. However, advances in this field of the problem are yet to be seen.

Based on this short review, it can be pointed out that the generalised form of the in-pit haul road optimisation problem should take into account the following key aspects:

- Actual pit design features;
- Actual road design features;
- Deposit block model;
- Robust cost model;
- Pushbacks sequencing;
- Materials flow scheduling;
- Occupational safety and traffic rules for mining equipment;
- Overall slope safety constraints;
- Environmental constraints and/or criteria;
- Site-specific constraints;
- Potential stochastic implementation of one or more of the above aspects.

Indeed, all these parameters are related to the overall problem of pit design optimisation. Therefore, the haul road optimisation problem should be regarded as a part of pit design optimisation, considering latest advances in computational power and newly developed algorithms.

A non-linear approach to the in-pit haul road optimisation problem

Most currently developed algorithms used for ultimate pit design optimisation are primarily based on solving actual design problems in a discrete space. It is known that this assumption yields an error of about 5% in terms of tonnage calculations, which can be considered negligible (Poniewierski, 2017). However, actual design features are continuous, which would mean that most optimisation solutions provide only a guideline for actual design. Yet, the problem for optimising the final actual design remains open. Therefore, to “close the gap” between discrete optimisation solutions and the continuous nature of actual design features, a different class of optimisation methods should also be investigated. A linear optimisation approach is utilised for determining an optimal underground shaft location (Stoyanchev, 2014). However, the

assumption made for this problem is that the terrain surface is approximated by a best-fit plane, using the Ordinal least squares method. Linearisation is a good option for cases when the terrain surface is relatively simple. However, for complex terrain and deposit morphology a different approach is required. A good starting point would be the use of non-linear optimisation methods. So far, no prior studies have been found, which treat the in-pit haul road problem as a non-linear one. Hence, this study aims to introduce this perspective to the pool of possible solutions for the in-pit haul road problem, as well as to illustrate some key aspects of the implementation of this approach, which would be useful for future interpretation and generalisation of the problem.

Utilised methodology and assumptions

For this case study, a quarry dealing with basalt extraction of approximately 100000 m³/a is taken as an example. The main assumption for this iteration of the in-pit haul road design problem is that the haul road follows a spiral along the pit shell. Therefore, no potential switchbacks were considered for this analysis. Furthermore, the assumed design features for this case study include:

- In-pit haul road slope grade: 10%;
- In-pit haul road width: 10 m;
- Flat road distance between road segments: 20 m.

The desired Factor of Safety (1.6) was achieved for the overall pit slope with a maximum bench height of 15 m, a slope angle of 60° and a berm width of 10 m. The remaining ore volume for the deposit is estimated to be extracted with 3 more benches.

The ex-pit haul road was predefined as it led from the pit's exit to the stockpile. In order to take into consideration how haul road design affects operational costs, RPM Global's software HAULSIM was utilised for all performed simulations (<https://rpmglobal.com/product/haulsim/>).

Haul road design can affect operational costs in two major ways: 1) haul distance affects travel time and fuel consumption; 2) travel time affects the proportionality of truck cycle times in terms of their productive work state. In addition to travel time, certain traffic rules along the designed haul road may also lead to varying time utilisation, e. g. time of truck queues at the loading point, or wait times at “bottleneck” road sectors. Therefore, to obtain a more robust estimate of operational costs the simulation was utilised. One should take note that HAULSIM's best field of application is for short-term planning. However, as scheduling was not considered for this particular case study, haul distances were assumed to reach the centroid for each bench's toe polygon. For this particular quarry, the final pushback is mined bench-by-bench. Therefore, these assumptions, as basic as they may be, coincide with the vertical advance of mining for the remaining life of mine for the considered quarry. Therefore, the objective function can be formulated as:

$$Q = \sum_{i=1}^n R_i - EC_i - HC_i - const \quad (1)$$

where Q is the objective function, representing the total mining costs, EUR; R_i – revenue of the extracted commodity for bench i , EUR; EC_i – excavation costs for bench i , EUR; HC_i – hauling costs for bench i , EUR; i – index number of the extracted bench; n – total number of extracted benches.

For this particular objective function, the sum of drilling, blasting, crushing, sieving costs, as well as general costs are regarded as constant, regardless of the in-pit haul road location and design features. Indeed, drilling and blasting costs and their resulting rock fragments distribution influence excavation and hauling costs. However, for this case study blasting efficiency is regarded as constant for all haul road design scenarios, so that the effect of haul road location is investigated independently.

The assumed parameters regarding the mining equipment include:

- Type of excavator: Backhoe excavator;
- Bucket size: 2 m³;
- Number of excavators: 1;
- Haul truck type: dumper truck;
- Haul truck payload: 22.68 t;
- Number of haul trucks: 2.

The assumed working conditions are relatively basic, but they provide an easy way to validate the optimisation result against practical expertise. Furthermore, this initial iteration of the problem can provide key insights into certain dependencies, which cannot be established only by practical experience. Hence, this example aims to evaluate how the non-linear approach could provide guidance for a basic example and determine its feasibility for more complex applications.

Established 1-D optimisation methods

Initially, the most prominent 1-D optimisation methods were reviewed and evaluated in terms of their applicability for the current optimisation problem. Results from a similar previous case study indicate that the initial assumption for the objective function could be unimodal and exhibit a behaviour that can be approximated by a polynomial or a spline (Terziyski and Kaykov, 2022). However, this was not the case, which led to the review of different non-linear optimisation methods.

In general, 1-D optimisation methods can be divided into three main classes - Elimination methods, Interpolation methods and Direct root finding methods (Rao, 2009). Indeed, each category has a great number of optimisation techniques. As requirements for the ones considered, it was initially assumed that methods which are thought to be robust and have a fast convergence rate would be examined for this case study. Hence, a short description of the initially considered methods is provided in this paper.

Elimination methods

The most used 1-D optimisation methods, which were initially considered for this case study include Exhaustive search, Search with accelerated step size, Dichotomous search, Fibonacci search, Golden section search (Stoyanov, 1993; Rao, 2009).

The Exhaustive Search approach is relatively simple and involves calculating the objective function at predetermined regular intervals for the decision variable. This technique is suitable when the interval for the objective function is finite, which was exactly the case for this problem. This approach may require a large amount of computational work and in many cases is not preferred. However, as a simultaneous search method, each calculation does not require prior knowledge on

where the most feasible region for the optimum value lies (Stoyanov, 1993; Rao, 2009).

The Search with an accelerated step size follows a similar approach, however, the step size is reduced after bracketing the most feasible region from the objective function. This process can be repeated until the desired precision for the decision variables is reached. As an Unrestricted search method, the Search with an accelerated step size is primarily applicable in cases when the range of the decision variable, where the optimum solution lies, is not necessarily known (Stoyanov, 1993; Rao, 2009).

The Fibonacci method is a heuristic optimisation technique that uses the Fibonacci sequence to find the optimum value of a function, even if the function is not continuous. In the Fibonacci method, the optimisation problem is reduced to a sequence of one-dimensional problems. It should be pointed out that the most important limitation of the method include that the objective function should be unimodal for the interval of uncertainty (Stoyanov, 1993; Rao, 2009).

The Golden-section search method is also based on the Golden ratio, a mathematical constant that has been used in art, architecture, and mathematics for centuries. The Golden section method is similar to the Fibonacci method, except that in the Fibonacci method the total number of experiments to be conducted has to be specified before beginning the calculation, whereas this is not required in the Golden section method. For this method one starts with the assumption that a large number of experiments would be conducted. However, the total number of experiments can be decided during the computation. One of the main advantages of this method is that it always retains the golden ratio between the segments of the decision variable, which ensures a fast convergence rate. Furthermore, it is robust and easy to implement, making it a popular choice for optimisation problems where the function is smooth and unimodal (Stoyanov, 1993; Rao, 2009).

Interpolation methods

Interpolation methods were originally developed as 1-D search methods within multivariable optimisation techniques. They are considered to be generally more efficient than Fibonacci-type approaches (Stoyanov, 1993; Rao, 2009). The most prominent Interpolation methods include Quadratic interpolation and Cubic Interpolation.

Quadratic interpolation is a numerical optimisation method used to find the minimum or maximum of a unimodal function, which is a function that has a single local minimum or maximum. The method approximates the function using a quadratic polynomial and then finds the minimum or maximum of the quadratic polynomial. Quadratic interpolation is a relatively efficient optimisation method that can converge to the minimum or maximum of the function in just a few iterations. However, it can be sensitive to the choice of the three interior points and may not converge if the function is not smooth or if it has multiple local minima or maxima (Stoyanov, 1993; Rao, 2009).

Cubic interpolation is another numerical optimisation method for unimodal functions. The method approximates the function using a cubic polynomial and then finds the minimum or maximum of the cubic polynomial. Cubic interpolation is a relatively efficient optimisation method that can converge to the minimum or maximum of the function in just a few iterations. It requires the evaluation of the function at four interior points. In general, it is considered to be more robust and accurate than

quadratic interpolation, especially for functions that are not smooth or have multiple local minima or maxima (Stoyanov, 1993; Rao, 2009).

Direct root methods

Newton's optimisation method, also known as the Newton-Raphson method, is used to find the optimum of a function by iteratively improving an initial estimate. It uses the second derivative of the function to compute a quadratic approximation of the function and then finds the minimum or maximum of the quadratic approximation. Newton's optimisation method is relatively efficient and can converge to the minimum or maximum of the function in just a few iterations, especially for functions that are smooth and well-behaved. However, the solution may not converge, if the initial estimate is not close enough to the minimum or maximum of the function or if the objective function is not well-behaved, such as having a flat or steep section or having multiple local minima and maxima (Stoyanov, 1993; Rao, 2009).

The Secant optimisation method is a numerical optimisation method which also relies on iteratively improving an initial estimate of the optimum. Unlike Newton's method and its variants, the Secant method does not require the evaluation of the second or higher derivatives of the function. However, it may not converge, if the initial estimates are not close enough to the minimum or maximum of the function or if the function is multimodal (Rao, 2009).

The Bisection optimisation method is a simple numerical optimisation method used to find the root of a function, which is also equivalent to finding the minimum or maximum of a function. The method is based on the intermediate value theorem, which states that if a continuous function changes sign over an interval, then it must have at least one root in that interval. The Bisection method is a simple and robust optimisation method that guarantees convergence to a root of the function, provided that the function is continuous, and changes sign over the interval. However, the method is relatively slow and requires a large number of iterations to converge, especially if the interval is large or the function has multiple roots. Additionally, the method does not provide information about the nature of the root, such as whether it is a minimum or maximum of the function (Rao, 2009).

Direct search methods

Apart from the mentioned above classes of non-linear methods, the non-gradient methods, often known as direct search methods, utilise solely the objective function values and do not rely on the partial derivatives of the function to determine the minimum. A type of a direct search method is the **Simplex method**, which is also considered as a very powerful optimisation technique. A regular simplex is a geometric shape created by a collection of $n+1$ equidistant points in an n -dimensional space. For instance, in two dimensions, a regular simplex is referred to as a triangle, while in three dimensions, it is called a tetrahedron. The basic concept of the Simplex method is that the objective function is calculated at each simplex point and the worse-off solution is the vertex which is moved in the opposite direction. Following this approach, the simplex gradually moves to the location of an optimal solution (Rao, 2009). The method is capable of optimising higher dimensional problems, including simpler

problems in 1-D with a single decision variable. In addition, it has several modifications when choosing a starting point for the simplex. The first one uses one of the limits of the scanned interval. Initially the simplex method starts with a bigger step size, which is reduced 4 times when an unsuccessful attempt is made. This is considered to be a good way for achieving a fast convergence rate (Stoyanov, 1993). The second modification utilises a random starting point and increases the step size until it localises the most feasible region. After that, the step size is reduced. This approach is suitable for problems which are unconstrained in terms of the decision variable. The third variant is the generalised simplex method, also known as the Nelder-Mead method. The step size in this method can be stretched or reduced, which is especially useful when the constraints for the decision variable are not defined. Regardless of which modification is used, the main heuristic rule of successfully implementing the method is that the starting point should be close to the presupposed optimum location to reduce the number of iterations.

Discussion and choice of method

Regarding this case study, the objective function can be evaluated only after the design phase and the simulation are over. Therefore, the calculation of the objective function cannot be considered as a relatively easy task. Simultaneous search could be a good way to reduce the overall computational time for this case study. However, the application of the Exhaustive search approach can yield a large number of design scenarios and simulations for comparison, which is not favourable in the cases of a small mining site, let be for a big mining operation. Furthermore, in cases when the interpolating polynomial is not representative of the variation of the function being minimised, the Fibonacci or Golden section methods are a more favourable choice. However, in some problems it might prove to be more efficient to combine several techniques. For example, the unrestricted search with an accelerated step size can be used to bracket the minimum and then the Fibonacci, the Golden section method or the Simplex method can be used to find the optimum point.

However, it can be observed that most mentioned non-linear methods work well with unimodal functions, as their primary assumption of unimodality determines the following iteration. When dealing with a multimodal function, each method's behaviour is unpredictable in terms of convergence rate and solution optimality. Furthermore, methods based on derivative calculations can be problematic in cases of non-differentiable objective functions. In either case, if a function is known to be piecewise or multimodal, the range of the function can be subdivided into multiple parts and the function could be treated as a unimodal function in each part. However, determining different intervals can be challenging and time consuming for the task at hand as for this basic iteration of the in-pit haul optimisation problem there was no prior information about the nature of the evaluated function. Hence, the Accelerated step size approach is assumed to potentially be the best choice for an initial assessment of the objective function. In addition, the Simplex method was also selected for evaluating its performance in an initial interval of uncertainty, where the optimal value is expected to lie. The motivation behind the Simplex method is because of its versatility in higher dimensional problems and its potential scalability for future work. Cubic and Quadratic interpolation could be

extended to higher dimensions, however, they are not commonly used for this purpose. Therefore, the two Interpolation methods were disregarded for this case study at this point due to their limited use for higher dimensional optimisation. Direct root finding methods were also disregarded at this point, as they were considered not suitable in case the objective function proves to be multimodal.

Results

Based on an initial step with a size of 100 m, the pit contour bottom line was divided into sections by 7 points, which provided an initial overview of the evaluated function (Figure 3).

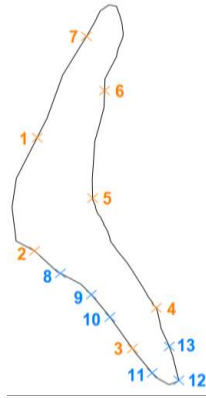


Fig. 3. In-pit haul road design starting points (bottom-up approach) for Accelerated step size search method

For each of the starting points, shown in Figure 3, three simulations were performed, for each of the 3 remaining benches in all considered cases. The costs were then summed for each scenario and were subtracted by the revenue of the mining operation. This yielded the results of the assumed objective function, shown in Figure 4.

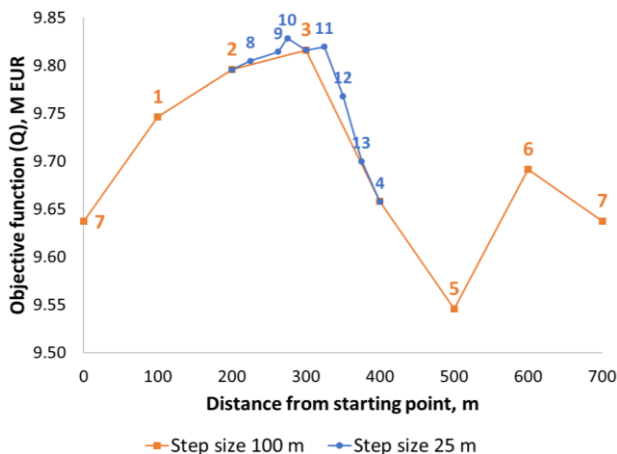


Fig. 4. Search with accelerated step size optimisation results

As it can be observed, the function indeed proved to be multimodal. Furthermore, the objective function is also assumed to be piecewise. Although it is not easily observable, the reason behind this assumption is due to the incremental volumes added or subtracted from the pit contour, which depend on the terrain interpolation model. For mining CAD software is usually a triangulated irregular network. Each new starting point for the in-pit haul road can also lead to changes

in the pit design shape, which are also not smooth. The piecewise nature of the function is primarily observable when two consecutive designs are made on both sides of a random point from the pit bottom line. Hence, the objective function is non-differentiable. For this reason, the actual shape of objective function is hard to be determined, which leads to the use of the conventional spline interpolation, primarily for visual interpretation. In terms of the interpolation's predictive capabilities, it should be carefully considered whether to use interpolated values, however, this was not in the scope of this case study.

The initial step size of 100 m was reduced to 25 m for the two intervals around point 3. The additional points (8 to 13) validated the initial conclusion that the objective function is multimodal, as an additional local optimum appeared, one of which was assumed to be the solution of the problem (point 10).

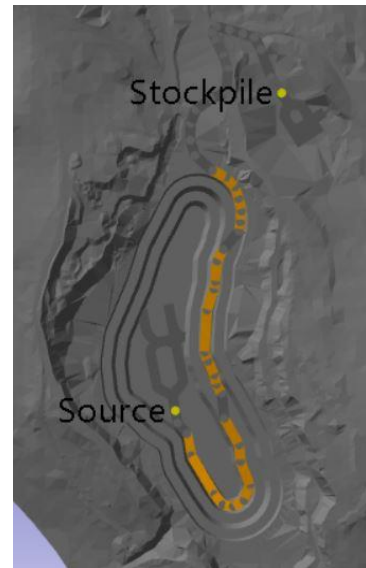


Fig. 5. Visual representation of candidate problem solution point 10 and the assumed traffic rules

Furthermore, the function continued to exhibit indicators of continuous but non-differentiable behaviour. In addition, these features of the objective function are expected to be present in a higher dimensional variation of the problem, when additional degrees of freedom regarding the haul road design are considered (i.e., haul road slope, width, direction, etc.). Nonetheless, given that the desired precision level is 25 m for localising the in-pit haul road starting point, an optimal solution in the set of scanned solutions can be reached within 13 iterations. All cases for each step size can be computed simultaneously, however sequential steps require an evaluation of the most feasible interval for scanning with the reduced step size. Hence, this approach is suitable for automation and parallelisation. A visual representation of the optimal solution can be seen on Figure 5.

Additionally, the 1-D Simplex method was applied for this problem as a fast-converging algorithm. Given that the objective function is multimodal, it should be taken into consideration that the Simplex method would require multiple starting points, as it is known to easily fall in locally optimum results. Indeed, this was the case for this study. Due to its relatively good convergence rate, the starting point of the Simplex method search was assumed to be where the smallest

haul road length is located (distance from starting point: 275 m). Assuming that the Simplex method starts from the same point as the Accelerated step search method, with an initial step size of 100 m, the method yields a locally optimum result in 6 iterations. However, when the lowest known transport distance is assumed as a starting point, as well as a clockwise search direction, the method reaches an improved solution, once again in 6 iterations, as shown in Figure 6.

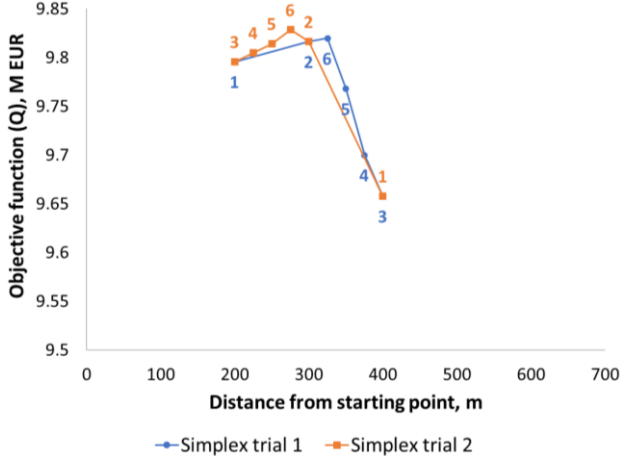


Fig. 6. 1-D Simplex method optimisation results

Hence, the 1-D Simplex method also proves to be a viable option for similar optimisation problems, however, it requires some initial guidance of the sector, where the global optimum is expected to lie. In addition, the search direction should also be carefully considered, as this could lead to reaching a locally optimum solution.

Possibilities for applying regression models in pit design optimisation

The data acquired from different design features and their respective objective function values can be used for building regression models, which could be further utilised as an indicator for faster convergence in each sequential iteration. In terms of this case study, a direct regression model between the in-pit haul road end point location and the result from the objective function may not be possible to be established. However, there is a correlation between the volume of extractable ore for each pit design ($r = 0.49$), the haul distance ($r = -0.82$) with the objective function values from Formula 1. Theoretically, if a given profit (within the model's range) is desired by the pit designer, the regression model can be used for an initial estimation of the expected range of the in-pit haul distance for the ideal design. And vice versa, when the designer meets a certain haul distance, he can estimate the range of the expected value for the objective function (Figure 7).

In a previous case study, a regression approach was proposed for estimating confidence intervals for the supposed in-pit haul road length, based on the predetermined undiscounted profit from the interpolation model (Terziyski and Kaykov, 2022). However, in this particular case study, the regression model, based on the initial 7 candidates, is not satisfactory and cannot be used as a reference for finding a feasible region along the pit bottom polyline, due to the short in-pit haul road distances. With a confidence level of 95% the

prediction interval for the expected haul distance ranges between ± 369.6 up to ± 438 m (Figure 7).

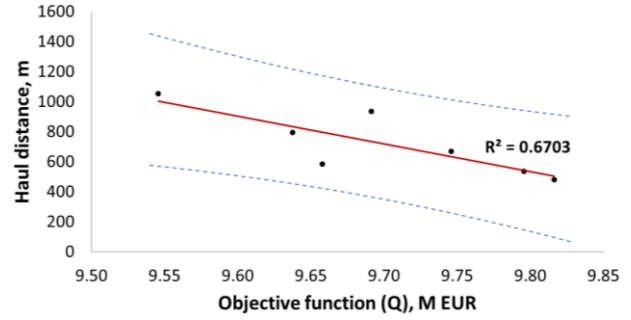


Fig. 7. Linear regression model for Objective function result and Haul distance

Therefore, when seeking a higher value for the objective function one cannot place accurately the direction and position of the next iteration, solely on the prediction confidence interval. As it can be observed, the interval is very wide, and therefore, in this case, it cannot be used for practical purposes, although the relationship between the objective function values and the haul distance is statistically significant ($p = 0.024$). Hence, a higher dimensional model is proposed for this case study, which utilises a multiple linear regression model (MLR) and includes the extracted commodity mass for the design candidate. The proposed MLR model uses the following expression:

$$Q = -371.70 L + 4.08 V_{com} + 583\,310 \quad (2)$$

where Q is the objective function value, obtained from the regression model, EUR; L – the Haul distance, m; V_{com} – is the Extractable commodity tonnage, t.

The model is considered to be a good initial estimation of the dependence between the parameters obtained from redesigning the pit contour, as it follows the intuition that lower haul distance and higher extracted commodity volumes lead to higher profit. However, it has some limitations and drawbacks, which need to be addressed. For instance, the Durbin-Watson statistic is $d = 2.2602$. The upper and lower bounds are assumed to be $d_L = 0.47$ and $d_U = 1.9$ for $\alpha = 0.05$. Hence, the value falls in the interval $(d_U, 4 - d_U)$, which means that the test is inconclusive in terms of residuals autocorrelation. Furthermore, the p-value for the model's intercept is 0.82, which means that it is not robustly estimated. Nonetheless, the p-values for the haul distance ($p = 0.003$) and commodity tonnage ($p = 0.017$) coefficients are good estimates of the true relation. Moreover, the p-value for the regression model is estimated to be $p = 0.005$, which can be considered satisfactory.

Another alternative model implements a quadratic relationship between the haul distance and the objective function:

$$Q = -0.756 L^2 + 782 L + 5.020 V_{com} - 1\,984\,012 \quad (3)$$

Once more, the coefficient for Haul distance and Extractable commodity tonnage are a good approximation of the actual ones, while the intercept should be regarded with caution due to the low number of observations. A reasonably good explanation for the behaviour of the NLR model is that

the haul distance can influence the objective function not only in terms of its physical length but also in terms of road curvature, traffic rules and therefore truck hours. This would explain the higher order of the model in terms of the Haul length variable. A visual representation of both models can be seen on Figure 8.

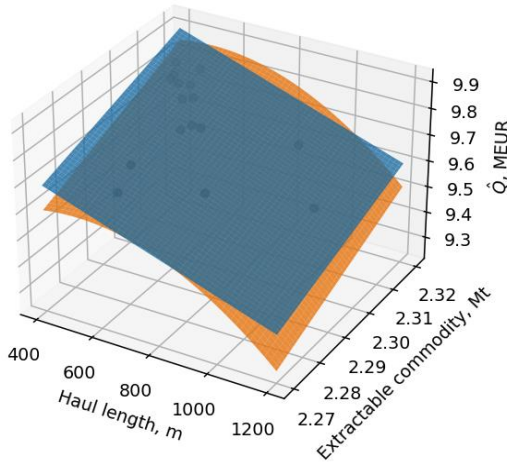


Fig. 8. Surface plots for MLR (blue) and NLR (orange) based on initial step size results

Furthermore, in terms of both models, no significant relationship between the haul distance and the extracted commodity tonnage for the pit design was established ($r = 0.03$). Therefore, the linear independence of Haul distance and the extractable commodity tonnage for each design candidate and the inherent stochasticity of the problem may lead to counter-intuitive results. For example, pit designs with lower haul distances can lead to lower expected profit due to a reduced extracted commodity tonnage in the designed pit contour.

Regardless, of their drawbacks, both models provide good initial estimates of the objective function and can be used as a guideline for narrowing the search limits for an optimal solution. Overall prediction metrics for MLR and NLR models can be seen in Table 1. Their performance was evaluated via the conventional Regression Coefficient (R^2), Mean average error (MAE), Root mean square error (RMSE).

Table 1. Prediction metrics for MLR and NLR models, based on initial step size results

Prediction metric	MLR	NLR
R^2	0.9317	0.9835
MAE, EUR	19 371.27	9 853.56
RMSE, EUR	23 328.51	11 359.26
Sample size	7	7

Apart from the results from the Accelerated step size search method and the 1-D Simplex method, additional simulations were performed in order to acquire more data for validating both models. The regression models were validated against the objective function values from the actual design and simulations. Performance metrics for the validation of both models can be seen in Table 2.

Table 2. Prediction metrics for MLR and NLR models, based on additional simulations results

Prediction metric	MLR	NLR
R^2	0.9533	0.9114
MAE, EUR	12 913.75	9 913.23
RMSE, EUR	16 018.82	13 339.36
Sample size	9	9

A representation of the objective function results, obtained from both regression models for all considered design candidates, can be seen in Figure 9.

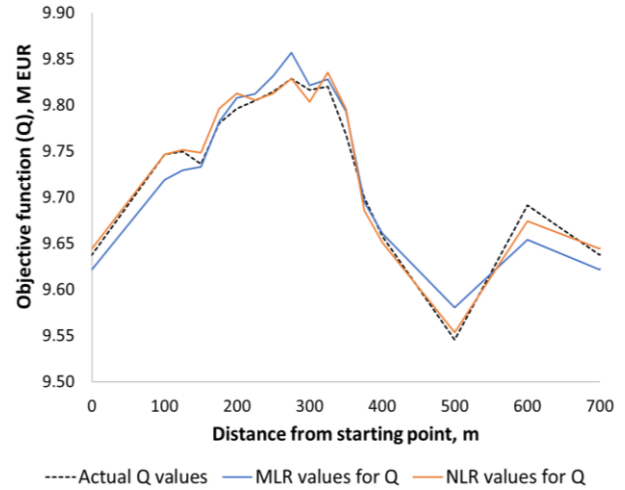


Fig. 9. Objective function estimation comparison for MLR and NLR models

Indeed, both models proved to have excellent accuracy compared to the actual values of the objective function. The MLR model tends to follow the curves of the actual objective function, while at the same time its results tend to be offset from the actual ones, due to the initially estimated intercept value. In contrast, the NLR model's deviation from the actual results is smaller but the model is less accurate for predicting the objective function's curvature. Nonetheless, both models prove to be good estimates and therefore suitable for narrowing down the search region for future iterations, especially when better accuracy is required.

Discussion

In order to use a regression model for guidance, certain iterations could be parallelly computed. In contrast, all other elimination methods, interpolation methods and direct root finding methods are sequential search methods and require prior information regarding results from subsequent experiments. In addition, when considering a higher dimensional optimisation approach for the in-pit haul road optimisation problem, all current findings for the 1-D approach should be taken into account. For instance, this case study does not prove, but rather demonstrates that the objective function is expected to behave in a piecewise manner and that it is non-differentiable. Furthermore, the objective function is expected to be multimodal in the general case when the resolution of the objective function is increased. Hence, a good step forward for future research is the implementation of global optimisation techniques, in addition to parallelising

computations. Additionally, the acquired data could be used for building more robust regression models, should better precision of the solution be required.

Conclusions

In-pit and ex-pit haul road construction are based on similar design parameters and design principles. The slope angle and height of the haul roads depend on the physical and mechanical properties of the rockmass, while the road width is determined by the width of the trucks being used and the work shift output of the pit. In addition, maintaining one or two lanes may be attributed to certain design requirements allowing trucks to allocate materials with minimal waiting times. To establish a well-structured decision, one should take into account all these design parameters when optimising a haul road and pit design. In addition, each optimisation problem should aim to find a solution that is both efficient and safe. Furthermore, understanding the technological constraints involved in road construction is crucial for achieving this goal. Additionally, a good understanding of the specific work conditions of the mining operation could lead to assumptions which could simplify the model, as a generalised solution to the in-pit haul road design problem is not yet introduced.

This case study proposes the use of non-linear optimisation approach for estimating the most efficient location of the in-pit haul road. The two methods used include the Search with an accelerated step size and the 1-D Simplex method. Both assumed a stopping criterion of reaching a precision of 25 m for the ramp's location. Additionally, both methods converged to the best solution for the assumed precision. However, the Simplex method proved to be sensitive to its starting point, which may lead to converging to a local optimum. The Search with an accelerated step size method required 13 iterations, while the 1-D Simplex methods required only 6 iterations, when the starting points is near the supposed global optimum. Furthermore, two regression models were proposed for guidance on locating the most feasible region in the search space for sequential iterations, given that a better precision of the ramp's location is required. The MLR model yielded a better result in terms of following the objective functions curvature ($R^2 = 0.9533$, MAE = 12 913.75 EUR, RMSE = 16 018.82 EUR), while the NLR reached more accurate results in terms of the objective function's values ($R^2 = 0.9114$, MAE = 9 913.23 EUR, RMSE = 13 339.36 EUR).

Future work is planned to continue the investigation of applying non-linear optimisation methods in more complex conditions, including solving the problem in a higher dimensional space, when adding more degrees of freedom such as road width and slope grade. Additionally, other potential aspect of future work is to include switchbacks to the haul road design and solving a multi-criteria optimisation problem with an environmental focus. Regardless, it should be pointed out that future non-linear optimisation methods should consider that the objective function is periodic and could prove to be non-differentiable and multimodal. Furthermore, the shape of the objective function strongly depends on the nature and morphology of the deposit, the topographical features, the pit design features, as well as the operational costs and selling prices of the extracted commodity. Last but not least, a parallelisation of the computation of different design scenarios is also considered to be promising when dealing with a higher dimensional iteration of the in-pit haul road design problem.

References

- Akay, A. E., Aruga, K., Bettinger, P. & Sessions, J. (2013). Using optimisation techniques in designing forest roads and road networks. *Bartın Orman Fakültesi Dergisi*, 15 (1), (pp. 49-62). Retrieved from <https://dergipark.org.tr/en/pub/barofd/issue/3390/46709>
- Aleksandrova E. & Trifonova M. Otsenka ustoychivostta na otkosi s otchitane na riska. Nauchni trudove na Sayuza na uchenite v Bulgaria - Plovdiv, Seria B. Estetstveni i humanitarni nauki, tom HV, Nauchna sesia "Tehnika i tehnologii, estetstveni i humanitarni nauki" 25-26th October 2012, Dom na uchenite, (pp. 107-111), ISSN 1311 - 9192, (in Bulgarian),
- Aleksandrova, E. (2008). Vazmozhnosti za tehnologichno upravlenie na ustoychivostta na otkosite. Nauchna konferentsia s mezhdunarodno uchastie "Naukata v usloviyata na globalizatsia", tom III, chast II, 1-2 oktomvri, 2008, (pp. 337 – 341), Izdatelstvo "Daga-JK", Kardzhali, ISBN 978-954-9634-33-4, (in Bulgarian)
- Atkinson, T. (1992). *Design and layout of haul roads*, in *SME Mining Engineering Handbook* (2nd ed.), Society for Mining, Metallurgy, and Exploration, Inc., Littleton, Colorado, USA., (pp. 1334-1342), ISBN 978-0873351003
- Hustrulid, W., Kuchta, M. & Martin, R. (2013). *Open pit mine planning and design*. (3rd Edition), CRC, UK, ISBN 978-1466575127
- Ivanova, V., Karacholov, P., Malinova, D., Pavlov, P. & Balev, V. (2016). Geotekhnicheski izsledvania na glinite ot dolnia horizont na vatreshnoto nasipite na rudnik "Troyanovo-1", klon na "Mini Maritsa Iztok" EAD vav vrazka s vtorichnoto mu izzemvane. VII International Geomechanics Conference, 1 juli 2016, Varna, (pp. 34-44), ISSN 1314-6467 (in Bulgarian).
- Kecojevic, V. & Komljenovic, D. (2010). Haul truck fuel consumption and CO₂ emission under various engine load conditions. *Mining engineering*, 62(12), 44-48.
- Koprev, I. (2015). *Rudnichni patishta*. Sofia, Avangard Prima, Sofia, ISBN 978-619-160-529-3 (in Bulgarian)
- Koprev, I., Aleksandrova, E. (2022). *Otkrit dobiv na polezni izkopaemi*. Sofia, Izdatelska kashta "Sv. Ivan Rilski", ISBN 978-954-353-459-3, (in Bulgarian)
- Liu, D. & Pourrahimian Y. (2021). A framework for open-pit mine production scheduling under semi-mobile in-pit crushing and conveying systems with the high-angle conveyor, *Mining*, 1(1). 59-79. <https://doi.org/10.3390/mining1010005>
- Mohutsiwa, M. & Musingwini, C. (2015). Parametric estimation of capital costs for establishing a coal mine: South Africa case study. *Journal of the Southern African Institute of Mining and Metallurgy*, 115(8), 789-797. <http://dx.doi.org/10.17159/2411-9717/2015/V115N8A17>
- Morales-Varela, N., Nancel-Penard, P. & Parra, A. An integer linear programming model for optimising open pit ramp design, 38th APCOM proceedings, Session 11, May 2017, Denver, Colorado, USA, (pp. 9-16)
- Nancel-Penard, P., Parra, A., Morales-Varela, M., Díaz, C. & Widzyk- Capehart, E. (2019). Value-optimal design of ramps in open pit mining, *Archives of Mining Sciences*, 64(2), 399-413. DOI: 10.24425/ams.2019.128691

- Panayotova, M., Vlasseva, E., Aleksandrova, E. & Bratkova, S. (2013). *Vazdeystvie na dobiva i prerabotvaneto na polezni izkopaemi varhu okolnata sreda*. Sofia, Izdatelska kashta "Sv. Ivan Rilski", ISBN 978-954-353-213-1, (in Bulgarian)
- Paricheh, M. & Osanloo, M.. Determination of the optimum in-pit crusher location in open-pit mining under production and operating cost uncertainties, *Proceedings of the 6th International Conference on Computer Applications in the Minerals Industries*, 3-4 October 2016, Istanbul, Turkey (pp. 1–7).
- Pavlov, P., Banov, M. & Totev, L. (2015). Environmental consequences of copper mining. 8. Freibergeb Geotechnik-Kolloquium 2015. Technische Universitaet – Bergakademie Freiberg, 18 Juni 2015, Germany, (pp.177-182), ISBN 978-3-86012-517-5
- Poniewierski, J. (2017). *Pseudoflow explained. A discussion of Deswik pseudoflow pit optimisation in comparison to Whittle LG pit optimisation*. Deswik Mining Consultants Pty Ltd. <https://www.deswik.com/wp-content/uploads/2017/05/Pseudoflow-Explained.pdf>
- Rao S. S. (2009). *Engineering optimisation. Theory and practice*. John Wiley & Sons, Inc. DOI:10.1002/9780470549124
- Saleki, M., Kakaie, R. & Ataei, M. (2019). Mathematical relationship between ultimate pit limits generated by discounted and undiscounted block value maximisation in open pit mining. *Journal of Sustainable Mining*. <https://doi.org/10.1016/j.jsm.2019.03.003>
- Stoyanchev, G. (2014). *Podzemen vagledobiv*. IK "Dvorets na kulturata", Pazardzhik, ISBN: 978-954-8944-26-7 (in Bulgarian).
- Stoyanov, S. (1993). *Optimizatsia na tehnologichni protsesi*. Izdatelstvo Tehnika, Sofia. ISBN 954-03-0256-0 (in Bulgarian).
- Terziyski, D. & Kaykov, D. (2022). Haul road design optimisation approach following technological constrains in open pit mining. *Sustainable Extraction and Processing of Raw Materials*, 3(3), 78–88. <https://doi.org/10.58903/c16182123>
- Thompson, R., Peroni, R., & Visser, A. (2018). *Mining haul roads: theory and practice* (1st ed.). CRC Press. <https://doi.org/10.1201/9780429491474>
- Yarmuch, J. L., Brazil, M., Rubinstein, H., & Thomas, D. A. (2020). Optimum ramp design in open pit mines. *Computers & Operations Research*. 115, 104739 <https://doi.org/10.1016/j.cor.2019.06.013>
- <https://rpmglobal.com/product/haulsim/> (Last accessed on 28th April 2023)