A decision theoretic framework for reliability-based optimal wind turbine selection

by

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Abstract

The problem of choosing the optimal wind turbine for a specific site is of special importance in the desing process of wind farm. Manifestly, the selection of the optimal wind turbine should depend on a certain criteria. In this paper, optimal wind turbine selection is studied in terms of the capacity factor of wind turbine generator and the Expected Energy not Supplied which is one of the most commonly used reliability indices for power systems. The latter one considers the load profile of the system and is suitable to compare different wind farm compositions while the former one completely ignores the load profile of the system. This paper presents general theoretical results

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that are helpful to compare performance of wind turbines and wind farms without data collection and further numerical assesment. In particular, the conditions on wind turbine characteristics and availability values of wind turbines are determined to compare wind turbines and wind farms in terms of the capacity factor and Expected Energy not Supplied.

Keywords. Capacity factor; Expected Energy not Supplied; Reliability; Stochastic dominance; Wind power.

Nomenclature

CF: Capacity factor

FOR: Forced-Outage-Rate

LOLP: Loss of Load Probability

EENS: Expected Energy not Supplied

SSD: Second order stochastic dominance

WT: Wind turbine

WF: Wind farm

1 Introduction

The amount of power generated by a wind turbine at a specific site depends on many factors such as wind speed conditions at the location, the characteristics of the wind turbine generator itself, particularly the cut-in, rated and cut-out wind speed parameters (Billinton and Chen (1999)). In a long term, the power produced by the wind turbine also depends on its availability/reliability. There are many different types of wind turbine models commercially available. Thus, it is desirable to choose a wind turbine which is most suitable for a particular site to obtain the maximum capacity benefit. To this end, various measures have been proposed and calculated based on wind speed distribution and wind turbine models.

Reliability of a wind power system, in a general sense, is a measure of the ability of the system to generate and supply electrical energy (Nemes and Munteanu (2010)). In the context of power systems, reliability has two main aspects: system adequacy aspect and system security aspect. The former is concerned with the existence of sufficient facilities within the system to satisfy the load demand while the latter one is the ability of the system to respond to the disturbances arising within the system. That is, the security aspect is related to the mechanical failure/operation of the system's components, i.e. wind turbines for the wind power system. A good reliability index should combine both aspects. Reliability based planning of wind power systems has been of great interest. The methods of power system reliability assessment have been reviewed by Wen et al. (2009). Lin et al. (2014) investigated the features of reliability models of wind power, reliability assessment algorithms and its applications in wind power related decision making problems. Mani et al. (2016) proposed a reliability model for wind turbines considering their subcomponent failure rates and downtimes. Čepin (2019) analyzed the replacement of nuclear power plant with wind power plants to compare both cases in terms of power system reliability. Devrim and Eryilmaz (2020) considered a hybrid system that consists of a specified number of wind turbines and solar modules, and evaluated the performance of the system using weighted-k-out-of-n system model.

Various reliability indices have been defined and studied to evaluate power systems. The most commonly used reliability indices are Loss of Load Probability (LOLP), Loss of Load Expectation (LOLE) and Expected Energy not Supplied (EENS). These indices have been widely used in reliability and performance evaluation, and optimization of power systems. Volkanovski et al. (2008) developed a method for the optimization of maintenance scheduling of generating units in power system by minimizing the LOLE. Volkanovski (2017) investigated the impact of the introduction of the wind generating units in the power system using LOLP. Beyza et al. (2020) evaluated the performance of interconnection lines by measuring their impacts on the main reliability indicators of interconnected power systems by using LOLP, LOLE and EENS indices. Beyza and Yusta (2021) analyzed the reliability and vulnerability of electrical networks to quantify systems' performance by increasing and decreasing renewable resources using LOLP, LOLE and EENS indices. LOLP, LOLE and EENS have also been used to select the most suitable wind turbine model. Fotuhi-Firuzabad and Dobakhshari (2009), Nemes and Munteanu (2010), Mohiley and Moharil (2013) discussed reliability-based selection of wind turbines for a specific wind farm by calculating power system reliability indices such as LOLP, LOLE and EENS.

In the existing literature, optimal wind turbine selection and performance comparison of different wind farm compositions have been investigated through numerical evaluations which are based on wind speed data observed at a particular location. The choice of optimal wind turbine is of special importance since the wind turbines that are used by a wind farm heavily effects the reliability of the entire wind power system. Thus, for two given wind turbine models that may have different characteristics and availability values, it is important to choose the better one in terms of performance and reliability. For example, a wind farm that has a smaller EENS value has a larger reliability. Therefore, it is important to obtain necessary conditions to compare two different wind farm compositions in terms of reliability which considers both system adequacy and security aspects.

This work is motivated by the purpose of obtaining general theoretical results that will enable us to compare performance of wind turbines and wind farms without wind speed data and further numerical assessment. In particular, we determine some conditions on wind turbine characteristics to compare the capacity factors of two wind turbine models and to stochastically compare the powers produced by the turbines. The conditions are also obtained to compare the powers produced by different wind farm compositions. By the help of these findings, we can compare the two wind farm compositions in terms of EENS.

The results and application-oriented findings of the present paper are reported in the following order. In Section 2, an overview of reliability modeling in the context of wind power systems is presented in order to explain the basic concepts. Section 3 is devoted to optimal wind turbine selection with respect to the capacity factor and the power produced by wind turbine. In Section 4, the comparison of different wind farm compositions is discussed. Finally, in Section 5, the theoretical results are illustrated with numerical examples.

2 Reliability Modeling

The development of a reliability model for a wind turbine (WT) generator requires consideration of three factors which affect the output of the wind turbine (Giorsetto and Utsurogi (1983)). The first of these factors is the random nature of the wind speed at the location which can be modeled by a proper probability distribution. The second factor is the functional relationship between wind speed and power output of the WT. The last one is the Forced-Outage-Rate (FOR) or equivalently the unreliability of the WT generator. Thus, the stochastic model that describes the long term performance, i.e. the output of the WT can be represented as

$$P_{WT} = g(V) \cdot X, \tag{1}$$

where V denotes wind speed random variable, X is a binary variable that represents the state of the WT generator such that $X = 1$ if the WT works and $X = 0$ if the WT has failed, $g : (0, \infty) \to [0, P_r]$ is a function that describes the relation between wind speed and power output of the WT, where P_r denotes the nominal power of the WT. The probability $P\{X = 0\}$ gives the FOR of the WT. Equivalently, the probability $p = P\{X = 1\} = 1 - FOR$ defines the reliability (availability) of the WT. Thus, the probability distribution of the WT output which is defined by (1) can be derived based on the following:

1. The probability distribution of the wind speed random variable, i.e. $F(v) = P\{V \leq v\}$.

2. The shape of the function g and WT characteristics.

3. The $FOR = 1 - p$ or equivalently the availability value p of the WT.

In the literature, numerous works have been devoted to determine the suitable probability distribution for the wind speed data obtained at selected locations. Various well-known probability distributions such as Weibull distribution, Birnbaum-Saunders distribution, and Gamma distribution have been found to be useful for modeling wind speed data (Pobočíková et al. (2017), Wais (2017), Mohammadia et al. (2017)). Various forms of the function g has been considered to define the relation between the wind speed and the WT power output (see, e.g. Diyoke (2019)). In the present paper, we consider the following function which defines a cubic relationship between the wind speed and the WT output. Such a function has been found to be accurate in various cases (see, e.g. Villanueva and Feijoo (2020)).

$$g(v) = \begin{cases} 0, & \text{if } v < v_{ci} \text{ or } v \geq v_{co} \\ \\ P_r \frac{(v^3 - v_{ci}^3)}{(v_r^3 - v_{ci}^3)}, & \text{if } v_{ci} \leq v < v_r \\ \\ P_r, & \text{if } v_r \leq v < v_{co}, \end{cases} \tag{2}$$

where the WT characteristics v_{ci}, v_r and v_{co} respectively denote the cut-in wind speed, rated wind speed and cut-out wind speed values. The cut-in wind speed v_{ci} is the speed at which the wind turbine starts producing power. If the wind speed is below the cut-in wind speed, then turbine cannot produce electricity. When the wind speed is between the cut-in wind speed v_{ci} and rated wind speed v_r, the wind turbine produces power and there is a cubic relationship between the wind speed and the power. If the wind speed is above the rated wind speed v_r and below the cut-out wind speed v_{co}, then the wind turbine produces its nominal power. When the wind speed is above the cut-out wind speed v_{co}, the turbine is stopped since it is in a danger of mechanical failure and hence no power is produced.

The determination of the FOR of the WT or equivalently the availability is also necessary for finding and evaluating the power output of the WT. A WT can be modeled as a series system consisting of specified number of components, e.g. electrical system, electronic control, yaw system, rotor blades. If each component of the WT is modeled with two mechanical states as complete failure and perfect functioning, then under Markov process based modeling, the availability of the ith component of the WT is

$$A_i(t) = \frac{\mu_i}{\mu_i + \lambda_i} + \frac{\lambda_i}{\mu_i + \lambda_i} \exp\left\{-(\lambda_i + \mu_i)t\right\}.$$

Thus, the long term availability of the ith component is

$$p_i = \Pr\left\{X_i = 1\right\} = \lim_{t \longrightarrow \infty} A_i(t) = \frac{\mu_i}{\mu_i + \lambda_i},$$

where λ_i and μ_i represent respectively the failure and repair rates of the ith component. Therefore, the overall availability of the WT can be computed from

$$p = \prod_{i=1}^{n} p_i,$$

where n is the number of components within the WT. The FOR can be calculated from $1 - p$. In the present paper, we are not interested in computing the value of p. The models for computing the reliability value p have been reviewed by Alhmoud and Wang (2018).

If the random variables V and X are independent, i.e. the availability (or FOR) is not affected by the wind speed, then the cumulative distribution function of the power output of the WT defined by (1) is

$$H(x) = \Pr\{P_{WT} \leq x\} = \begin{cases} 0, & \text{if } x < 0 \\ pQ_1(x) + 1 - p, & \text{if } 0 \leq x < P_r \\ 1, & \text{if } x \geq P_r, \end{cases} \quad (3)$$

where

$$Q_1(x) = 1 - F(v_{co}) + F\left(\left[\frac{x}{P_r}(v_r^3 - v_{ci}^3) + v_{ci}^3\right]^{\frac{1}{3}}\right)$$

(see, e.g. Eryilmaz and Devrim (2019)). The distribution of the power output of the WT has been obtained and evaluated by Kan et al. (2020) when there is dependence between wind speed and wind turbine availability. Define the following conditional WT availabilities:

$$\begin{aligned} r_1 &= P\{X = 1 \mid V < v_{ci}\} \\ r_2(x) &= P\left\{X = 1 \mid v_{ci} < V < \left[\frac{x}{P_r}(v_r^3 - v_{ci}^3) + v_{ci}^3\right]^{\frac{1}{3}}\right\}, \ x > 0 \\ r_3 &= P\{X = 1 \mid V > v_{co}\}. \end{aligned}$$

Let $H^d(x) = P\{P_{WT} \leq x\}$ denote the cumulative distribution function of the power produced by a single turbine when V and X are dependent. Then, for $0 \leq x < P_r$, Kan et al. (2020) derived the following expression:

$$\begin{aligned} H^d(x) &= 1 - p + r_1 F(v_{ci}) \\ &\quad + r_2^*(x)\left[F\left(\left[\frac{x}{P_r}(v_r^3 - v_{ci}^3) + v_{ci}^3\right]^{\frac{1}{3}}\right) - F(v_{ci})\right] \\ &\quad + r_3\left[1 - F(v_{co})\right], \end{aligned}$$

where $r_2^*(x) = r_2(x)$ if $x > 0$ and $r_2^*(x) = 0$ if $x = 0$. Clearly, $H^d(x) = 0$ if $x < 0$ and $H^d(x) = 1$ if $x \geq P_r$.

Throughout the present paper, the random variables V and X are assumed to be independent. For a wind farm consisting of n identical WTs each having rated power P_r and availability p, let $P_{WF} = g(V) \cdot S_n$ denote the wind farm output, where S_n denotes the total number of available turbines. Then, the cumulative distribution function of the random variable P_{WF} can be computed from

$$\Pr(P_{WF} \leq x) = \sum_{i=0}^{n} \binom{n}{i} p^i (1-p)^{n-i} Q_i(x), \tag{4}$$

where

$$Q_i(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1 - F(v_{co}) + F\left(\left[\frac{x}{i \cdot P_r} (v_r^3 - v_{ci}^3) + v_{ci}^3 \right]^{\frac{1}{3}} \right), & \text{if } 0 \leq x < i \cdot P_r \\ 1, & \text{if } x \geq i \cdot P_r, \end{cases}$$

with $Q_0(x) = 0$ if $x < 0$ and $Q_0(x) = 1$ if $x \geq 0$ (see, e.g. Eryilmaz and Devrim (2019)). Note that $Q_i(x)$ denotes the cumulative distribution function of the wind farm power output when there are exactly i working/available wind turbines. It should be noted that in (4), the wind speed is assumed to have a continuous probability distribution and no discretization is applied. In the literature, the cumulative distribution function of the wind farm output has also been calculated by discretizing the wind speed distribution (see, e.g. Giorsetto and Utsurogi (1983), Fotuhi-Firuzabad and Dobakhshari (2009), Li and Zio (2012)).

LOLP is defined to be the probability that the local load L exceeds the available generating capacity. For a wind farm, with output P_{WF}, it is defined by $LOLP = \Pr\{P_{WF} < L\}$. A wind farm that has a smaller $LOLP$ has a higher reliability in terms of providing the required energy demand. Another useful reliability index denoted as EENS, is the expected energy that will not be supplied when the local load L exceeds the available generation. It is defined by

$$EENS = E(\max(L - P_{WF}, 0)).$$

For a wind farm that consists of n WTs with rated powers P_r, the $EENS$ can be computed from the following equation (see, e.g. Eryilmaz et al. (2021)):

$$EENS = L - \int_{0}^{\min(L, nP_r)} P\{P_{WF} \geq u\}\, du. \qquad (5)$$

3 Optimal Wind Turbine Selection

The earliest method for selecting the optimal WT is based on WT capacity factor (Salameh and Safari (1992)). The capacity factor (CF) of a WT describes the gap between nominal and realistic power production of a WT. It is the ratio of the wind turbine's actual power output to its nominal power output. Mathematically,

$$CF = \frac{\mu_{WT}}{P_r}, \qquad (6)$$

where μ_{WT} denotes the mean power produced by the WT. For a continuous wind speed distribution $F(v) = P\{V \leq v\}$, it can be computed from the following equation

$$\mu_{WT} = E(P_{WT}) = p\left[P_r F(v_{co}) - \int_0^{P_r} F\left(\left[\frac{x}{P_r}(v_r^3 - v_{ci}^3) + v_{ci}^3\right]^{\frac{1}{3}}\right) dx\right], \qquad (7)$$

where p denotes the availability value of the WT (see, e.g. Eryilmaz et al. (2021)). As it is clear from (6) and (7), the computation of the capacity factor of a WT needs wind speed distribution, WT characteristics and WT availability (or FOR). The inclusion of the wind speed distribution in the equation clearly exhibits the dependence of the capacity factor on the location of the wind farm. That is, the turbines having same characteristics and availability values may have different capacity factors at different locations. Substituting (7) in (6) and then applying the transformation $\left[\frac{x}{P_r}(v_r^3 - v_{ci}^3) + v_{ci}^3\right]^{\frac{1}{3}} = u$, the capacity factor can be represented as

$$CF = p\left[F(v_{co}) - \frac{3}{v_r^3 - v_{ci}^3}\int_{v_{ci}}^{v_r} u^2 F(u) du\right]. \qquad (8)$$

Thus, the capacity factor of the WT is independent of the turbine's rated power. Mathematically, one can write

$$CF = CF(v_{ci}, v_r, v_{co}; p; F).$$

For the two wind turbine models having respective characteristics $(v_{ci,1}, v_{r,1}, v_{co,1})$ and $(v_{ci,2}, v_{r,2}, v_{co,2})$, and availability values p_1 and p_2, wind turbine 1 (WT1) is preferred to wind turbine 2 (WT2) with respect to the capacity factor at a location with distribution F if

$$CF_1(v_{ci,1}, v_{r,1}, v_{co,1}; p_1; F) \geq CF_2(v_{ci,2}, v_{r,2}, v_{co,2}; p_2; F), \tag{9}$$

where

$$CF_1(v_{ci,1}, v_{r,1}, v_{co,1}; p_1; F) = p_1 \left[F(v_{co,1}) - \frac{3}{v_{r,1}^3 - v_{ci,1}^3} \int_{v_{ci,1}}^{v_{r,1}} u^2 F(u) du \right],$$

and

$$CF_2(v_{ci,2}, v_{r,2}, v_{co,2}; p_2; F) = p_2 \left[F(v_{co,2}) - \frac{3}{v_{r,2}^3 - v_{ci,2}^3} \int_{v_{ci,2}}^{v_{r,2}} u^2 F(u) du \right].$$

In the literature, capacity factors of wind turbines have been numerically compared based on wind speed data obtained at a certain location. That is, to choose the optimal WT, the capacity factors are calculated for given turbine characteristics and wind speed distribution and the decision is made based on the numerical comparison of the capacity factors. In this paper, we aim to obtain the necessary conditions on $(v_{ci,1}, v_{r,1}, v_{co,1})$ and $(v_{ci,2}, v_{r,2}, v_{co,2})$ to have the relation (9). This enables us to compare the capacity factors of the two wind turbine models without any numerical calculation and wind speed data collection. The first result can be stated as follows.

Proposition 1. There exists $\xi \in [v_{ci}, v_r]$ such that

$$CF(v_{ci}, v_r, v_{co}; p; F) = p \left[F(v_{co}) - F(\xi) \right].$$

Proof. By the mean value theorem (see, e.g. Bao-lin (1997)), there exists a number $\xi \in [v_{ci}, v_r]$ such that

$$\int_{v_{ci}}^{v_r} u^2 F(u) du = F(\xi) \int_{v_{ci}}^{v_r} u^2 du = F(\xi) \left(\frac{v_r^3 - v_{ci}^3}{3} \right).$$

Thus, from (8), we have

$$\begin{aligned} CF(v_{ci}, v_r, v_{co}; p; F) &= p \left[F(v_{co}) - \frac{3}{v_r^3 - v_{ci}^3} F(\xi) \left(\frac{v_r^3 - v_{ci}^3}{3} \right) \right] \\ &= p \left[F(v_{co}) - F(\xi) \right]. \blacksquare \end{aligned}$$

For $x < y$, define a function

$$m(x, y) = \frac{1}{y^3 - x^3} \int_x^y u^2 F(u) du.$$

Then, from (8), the CF of the wind turbine can be represented as

$$CF(v_{ci}, v_r, v_{co}; p; F) = p \left[F(v_{co}) - 3m(v_{ci}, v_r) \right].$$

In the following, we investigate some properties of the function m which will be useful in our developments.

Proposition 2. (a) The function m is increasing in both x and y. (b) The function m uniquely determines F (i.e. each wind characteristics have its own unique function m).

Proof. From the definition we get

$$(y^3 - x^3) m(x, y) = \int_x^y u^2 F(u) du.$$

Then

$$(y^3 - x^3)\partial_1 m(x,y) - 3x^2 m(x,y) = -x^2 F(x)$$

and

$$\partial_1 m(x,y) = \frac{x^2}{y^3 - x^3}(3m(x,y) - F(x)).$$

Analogously, we get

$$\partial_2 m(x,y) = \frac{y^2}{y^3 - x^3}(F(y) - 3m(x,y)).$$

Moreover, we know that

$$m(x,y) = \frac{1}{y^3 - x^3}\int_x^y u^2 F(u)du \leq \frac{F(y)}{y^3 - x^3}\int_x^y u^2 du = \frac{F(y)}{3}$$

and

$$m(x,y) = \frac{1}{y^3 - x^3}\int_x^y u^2 F(u)du \geq \frac{F(x)}{y^3 - x^3}\int_x^y u^2 du = \frac{F(x)}{3}.$$

Hence both partial derivatives are non-negative.

The proof of part (b) is immediate from

$$(y^3 - x^3)\partial_1 m(x,y) - 3x^2 m(x,y) = -x^2 F(x).\blacksquare$$

In the following, we obtain necessary conditions to compare the CFs of two WTs.

Theorem 1. Let WT1 have characteristics $v_{ci,1}, v_{r,1}, v_{co,1}$ and WT2 have characteristics $v_{ci,2}, v_{r,2}, v_{co,2}$. The WT$i$ has availability value p_i $i = 1, 2$. Suppose $p_1 \geq p_2$. Then,

(a) Let $v_{co,1} = v_{co,2}$. If

$$m(v_{ci,1}, v_{r,1}) \leq m(v_{ci,2}, v_{r,2}), \tag{10}$$

then

$$CF_1(v_{ci,1}, v_{r,1}, v_{co,1}; p_1; F) \geq CF_2(v_{ci,2}, v_{r,2}, v_{co,2}; p_2; F).$$

(b) If $v_{ci,1} \leq v_{ci,2}, v_{r,1} \leq v_{r,2}$ and $v_{co,1} \geq v_{co,2}$, then

$$CF_1(v_{ci,1}, v_{r,1}, v_{co,1}; p_1; F) \geq CF_2(v_{ci,2}, v_{r,2}, v_{co,2}; p_2; F).$$

Proof. The proof of part (a) is immediate from

$$CF(v_{ci}, v_r, v_{co}; p; F) = p\left[F(v_{co}) - 3m(v_{ci}, v_r)\right].$$

The proof of part (b) follows from Proposition 2 (a). ■

Various comparisons can be made based on Theorem 1. For example, for the two wind turbines that have same rated and cut-out wind speed values, a wind turbine that has a smaller cut-in wind speed is better in terms of the capacity factor. Similarly, for the two wind turbines that have same cut-in and cut-out wind speed values, a wind turbine that has a smaller rated wind speed is better in terms of capacity factor.

The results presented in Theorem 1 can also be given in terms of the function defined by

$$\bar{m}(x, y) = \frac{1}{y^3 - x^3} \int_x^y u^2 \bar{F}(u) du,$$

where $\bar{F}(u) = 1 - F(u)$. For some wind speed distribution models it is easier to compute this function. As

$$m(x,y) = \frac{1}{3} - \bar{m}(x,y)$$

the properties of $\bar{m}$ are similar. Indeed, $\bar{m}$ is decreasing in both x and y and it uniquely determines F. (10) is equivalent to $\bar{m}(v_{ci,1}, v_{r,1}) \geq \bar{m}(v_{ci,2}, v_{r,2})$.

To illustrate the shape of the function m, we give an example when the wind speed distribution is Weibull with known parameter values. Weibull distribution is one of the most commonly used wind speed distributions (see, e.g. Wais (2017)). It has been shown to be suitable for modeling wind speed regime at many locations. In practice, the unknown parameters of the Weibull distribution are estimated mostly using the method of maximum likelihood based on daily average wind speed values measured at a certain location. In the present paper, we are not interested in estimating the parameters and hence the parameter values are chosen in such a way to obtain a reasonable mean wind speed.

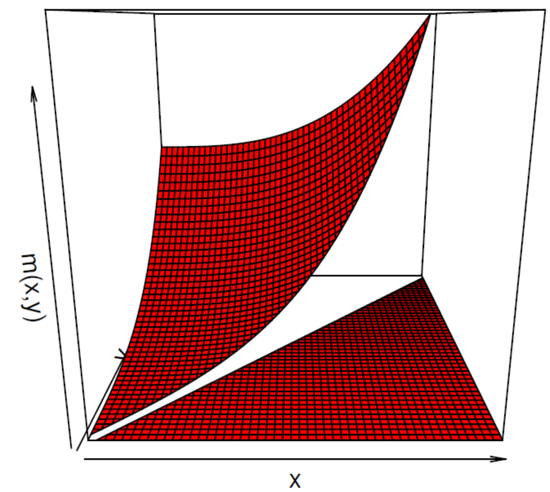
Let $F(u) = 1 - \exp(-(u/9)^3)$ for $u \geq 0$. Then

$$m(x,y) = \frac{1}{3} - \bar{m}(x,y) = \frac{1}{3}\frac{1}{y^3 - x^3}\int_x^y u^2 \exp(-(u/9)^3)du$$

for $0 \leq x < y$. Hence,

$$m(x,y) = \frac{1}{3} - \frac{1}{3}9^3\frac{\exp(-(x/9)^3) - \exp(-(y/9)^3)}{y^3 - x^3}$$

for $0 \leq x < y$. The plots for m are given in Figure 1.



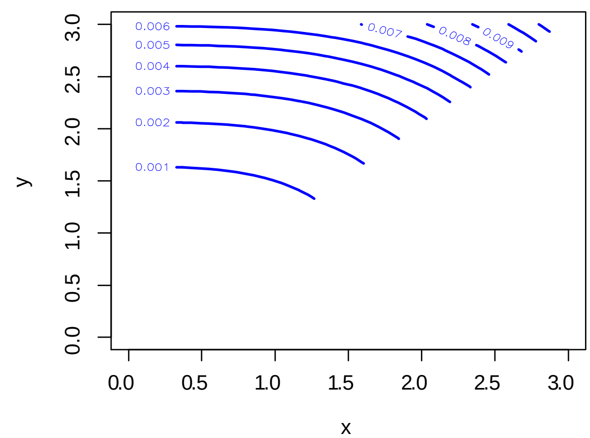


Figure 1.a. 3D-plot for the function m Figure 1.b. Level curves for the function m

3.1 Stochastic comparison of the powers of two wind turbines

Consider the wind turbines whose characteristics are presented in Table 1. Both turbines are assumed to have equal $FOR = 0.04$, i.e. availability $p = 0.96$. Assume that these turbines are installed at a location where the wind speed distribution follows a Weibull model with cumulative distribution function $F(v) = 1 - e^{-\left(\frac{v}{\alpha}\right)^\beta}$ with $\alpha = 9$ and $\beta = 2$. Because $v_{co,1} = v_{co,2}$ and the condition (10) is satisfied, we have $CF_1(v_{ci,1}, v_{r,1}, v_{co,1}; p; F) \geq CF_2(v_{ci,2}, v_{r,2}, v_{co,2}; p; F)$. Indeed, $CF_1(v_{ci,1}, v_{r,1}, v_{co,1}; p; F) = 0.3505$ and $CF_2(v_{ci,2}, v_{r,2}, v_{co,2}; p; F) = 0.2667$.

Table 1. Characteristics of two wind turbine models

	WT1 (1.5 s-GE)	WT2 (1.5 sle-GE)
v_{ci}	4 m/s	3.5 m/s
v_r	12 m/s	14 m/s
v_{co}	25 m/s	25 m/s
P_r	1.5 MW	1.5 MW

In Figure 2, we plot the cumulative distribution functions of the power produced by each WT, i.e. $H_1(x) = P\{P_{WT1} \leq x\}$ and $H_2(x) = P\{P_{WT2} \leq x\}$. Clearly, $H_1(x) \leq H_2(x)$ does not hold for all x. That is, we do not have the stochastic ordering relation $P_{WT1} \geq_{st} P_{WT2}$. Note that a random variable X is stochastically larger than the random variable Y (denoted by $X \geq_{st} Y$) if $P\{X \leq x\} \leq P\{Y \leq x\}$ for all x (see, e.g. Belzunce et al. (2016)). Manifestly, if $X \geq_{st} Y$ then $E(X) \geq E(Y)$. That is, if X is stochastically larger than Y, then the mean of X is greater than the mean of Y. This example demonstrates that although the WT1 has a larger capacity factor, the power produced by the WT1 is not necessarily larger than the power produced by the WT2. In Proposition 3, we obtain a necessary and sufficient condition to have $P_{WT1} \geq_{st} P_{WT2}$ for the two turbines having same cut-out wind speed values and different cut-in and rated wind speed values, nominal powers and FOR values.

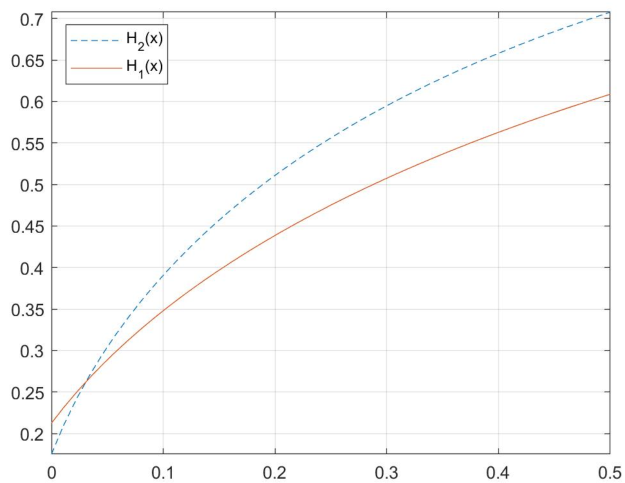


Figure 2. The cumulative distribution functions of

the power outputs of two WTs

Proposition 3. Let WT1 have characteristics $v_{ci,1}, v_{r,1}, v_{co,1}, P_{r,1}$ and WT2 have characteristics $v_{ci,2}, v_{r,2}, v_{co,2}, P_{r,2}$ with $v_{co,1} = v_{co,2} = v_{co}$. Then,

$$P_{WT}^1 \geq_{st} P_{WT}^2 \text{ iff } p_2 \left[F(v_{co}) - F(v_{x,2})\right] \leq p_1 \left[F(v_{co}) - F(v_{x,1})\right] \text{ for all } x,$$

where

$$v_{x,1} = \left[\frac{x}{P_{r,1}}(v_{r,1}^3 - v_{ci,1}^3) + v_{ci,1}^3\right], \quad v_{x,2} = \left[\frac{x}{P_{r,2}}(v_{r,2}^3 - v_{ci,2}^3) + v_{ci,2}^3\right].$$

Proof. Immediately follows from (3). ■

Theorem 2. Let $v_{co,1} = v_{co,2}$ and $P_{r,1} = P_{r,2}$, i.e. the turbines have the same cut-out wind speed values and nominal powers.

(a) If $p_1 \geq p_2$, $v_{ci,1} = v_{ci,2}$ and $v_{r,1} \leq v_{r,2}$, then $P_{WT1} \geq_{st} P_{WT2}$,

(b) If $p_1 = p_2$, $v_{ci,1} \leq v_{ci,2}$ and $v_{r,1}^3 - v_{ci,1}^3 \leq v_{r,2}^3 - v_{ci,2}^3$, then $P_{WT1} \geq_{st} P_{WT2}$.

Proof. Under the conditions of part (a), we clearly have $v_{x,1} \leq v_{x,2}$ which implies $F(v_{x,1}) \leq F(v_{x,2})$ because F is nondecreasing. Thus, since $p_1 \geq p_2$

$$p_1 \left[F(v_{co}) - F(v_{x,1})\right] \geq p_1 \left[F(v_{co}) - F(v_{x,2})\right] \geq p_2 \left[F(v_{co}) - F(v_{x,2})\right]$$

for all x, and hence the proof of part (a) follows from Proposition 3. The proof of part (b) can be established similarly.■

For an illustration, consider the wind turbine models given in Table 2. Assume that FOR values for both turbines are 0.04. Because $v_{ci,1} \leq v_{ci,2}$ and $v_{r,1}^3 - v_{ci,1}^3 \leq v_{r,2}^3 - v_{ci,2}^3$, we have $P_{WT1} \geq_{st} P_{WT2}$. That is, if the two turbines are installed at the same location, then the power produced by WT1 becomes larger than the power generated by WT2.

Table 2. Characteristics of two WT models

	WT1 (N70-1.5)	WT2 (1.5 sle-GE)
v_{ci}	3 m/s	3.5 m/s
v_r	13 m/s	14 m/s
v_{co}	25 m/s	25 m/s
P_r	1.5 MW	1.5 MW

4 Comparisons of wind farms

Consider two wind farm compositions which are denoted by WF1 and WF2. The WF1 consists of n_1 identical WTs having common availability p_1, and turbine characteristics $v_{ci,1}, v_{r,1}, v_{co,1}$ and $P_{r,1}$, and the WF2 consists of n_2 identical WTs having common availability p_2 and turbine characteristics $v_{ci,2}, v_{r,2}, v_{co,2}$ and $P_{r,2}$. Then, the power outputs of the wind farms can be represented respectively as

$$P_{WF1} = g_1(V) \cdot S^1_{n_1}, \tag{11}$$

and

$$P_{WF2} = g_2(V) \cdot S^2_{n_2}, \tag{12}$$

where $S^i_{n_i}, i = 1, 2$ follows a Binomial distribution with the probability mass function

$$P\left\{S^i_{n_i} = k\right\} = \binom{n_i}{k} p_i^k (1 - p_i)^{n_i - k},$$

for $k = 0, 1, ..., n_i$. The functions g_1 and g_2 have the form of (2) with respective turbine characteristics $(v_{ci,1}, v_{r,1}, v_{co,1}, P_{r,1})$ and $(v_{ci,2}, v_{r,2}, v_{co,2}, P_{r,2})$. Our aim is to compare the power outputs (11) and (12) with respect to the turbine characteristics.

Theorem 3. Let $v_{co,1} = v_{co,2}$ and $P_{r,1} = P_{r,2}$, i.e. the turbines within the wind farms have the same cut-out wind speed values and nominal powers. For $n_1 \geq n_2$, if the following three conditions hold true, then $P_{WF1} \geq_{st} P_{WF2}$.

(i) $(1 - p_1)^{n_1} \leq (1 - p_2)^{n_2}$.

(ii) $v_{ci,1} \leq v_{ci,2}$.

(iii) $v_{r,1}^3 - v_{ci,1}^3 \leq v_{r,2}^3 - v_{ci,2}^3$.

Proof. Because $\ln(x)$ is an increasing function of x, it is sufficient to show that $\ln P_{WF1} \geq_{st} \ln P_{WF2}$ under the listed conditions. Clearly,

$$\ln P_{WF1} = \ln g_1(V) + \ln S_{n_1}^1,$$

$$\ln P_{WF2} = \ln g_2(V) + \ln S_{n_2}^2.$$

For $n_1 \geq n_2$ and $(1-p_1)^{n_1} \leq (1-p_2)^{n_2}$, it is known that $S_{n_1}^1 \geq_{st} S_{n_2}^2$ (see, e.g. Klenke and Mattner (2010)). Because $\ln(x)$ is increasing in x, we have $\ln S_{n_1}^1 \geq_{st} \ln S_{n_2}^2$. On the other hand,

$$P\{g_1(V) \leq x\} = 1 - F(v_{co,1}) + F\left(\left[\frac{x}{P_{r,1}}(v_{r,1}^3 - v_{ci,1}^3) + v_{ci,1}^3\right]^{\frac{1}{3}}\right),$$

for $0 \leq x < P_{r,1}$, and

$$P\{g_2(V) \leq x\} = 1 - F(v_{co,2}) + F\left(\left[\frac{x}{P_{r,2}}(v_{r,2}^3 - v_{ci,2}^3) + v_{ci,2}^3\right]^{\frac{1}{3}}\right),$$

for $0 \leq x < P_{r,2}$. If (ii) and (iii) hold true, then $P\{g_1(V) \leq x\} \leq P\{g_2(V) \leq x\}$ for all x, which implies $g_1(V) \geq_{st} g_2(V)$ and hence $\ln g_1(V) \geq_{st} \ln g_2(V)$. Combining

$\ln S^1_{n_1} \geq_{st} \ln S^2_{n_2}$ and $\ln g_1(V) \geq_{st} \ln g_2(V)$ we obtain $\ln P_{WF1} \geq_{st} \ln P_{WF2}$. Thus, the proof is complete. ■

It should be noted that if there is a significant difference between n_1 and n_2, then n_1 and n_2 are of prior importance when comparing two WFs even if other characteristics of the WTs are not considered. However, in practice the difference between n_1 and n_2 is not large and the WT characteristics play important role in choosing the better WF composition. This is numerically shown in Section 5.

A random variable X second order stochastically dominates (SSD) another random variable Y $(X \geq_{SSD} Y)$ if

$$\int_0^t P\{X > x\}\, dx \geq \int_0^t P\{Y > y\}\, dy \text{ for all } t > 0$$

(see, e.g. Levy (1992)). It should be noted that if $X \geq_{st} Y$, then $X \geq_{SSD} Y$.

The wind farm that has a smaller $EENS$ value has a higher reliability. Therefore, $EENS$ can be used in optimal wind farm composition selection. Consider two wind farm compositions such that $n_1 P_{r,1} = n_2 P_{r,2}$. That is, the nominal powers of the two wind farms are assumed to be same. Clearly, if $P_{WF1} \geq_{SSD} P_{WF2}$, then

$$\begin{aligned} EENS_1 &= L - \int_0^{\min(L, n_1 P_{r,1})} P\{P_{WF1} \geq u\}\, du \\ &\leq L - \int_0^{\min(L, n_2 P_{r,2})} P\{P_{WF2} \geq u\}\, du = EENS_2. \end{aligned}$$

Based on the last statement, we establish the following definition.

Definition 1. A wind farm composition WF1 is said to be preferred to another wind farm composition WF2 with respect to $EENS$ if

$$P_{WF1} \geq_{SSD} P_{WF2},$$

where P_{WFi} denotes the power output of WFi, $i = 1, 2$.

Because usual stochastic ordering implies SSD ordering, we obtain the following Corollary as an immediate consequence of Theorem 3.

Corollary 1. Consider two wind farms WF1 and WF2. Assume that each WF consists of n WTs such that $v_{co,1} = v_{co,2}$, $P_{r,1} = P_{r,2}$ and $p_1 = p_2$. If $v_{ci,1} \leq v_{ci,2}$, and $v_{r,1}^3 - v_{ci,1}^3 \leq v_{r,2}^3 - v_{ci,2}^3$, then WF1 is preferred to WF2 with respect to $EENS$. That is, the WF1 has a smaller $EENS$ value.■

In reality, there might be more than two possible wind farm compositions and the main problem is the determination of the optimal one among all possible alternatives. In the following, we define the optimal wind farm in terms of $EENS$.

Definition 2. Let n_i denote the number of WTs used by WFi, and let $P_{r,i}$ be the rated power of the WT used by WFi, $i = 1, 2, ..., m$. Assume that

$$n_1 P_{r,1} = n_2 P_{r,2} = ... = n_m P_{r,m}.$$

A wind farm WF^* is said to be $EENS$-efficient if there is no feasible WF such that $P_{WF} \geq_{SSD} P_{WF^*}$.

Corollary 2. Consider m wind farms each having n WTs. Let $P_{r,1} = P_{r,2} = ... =$

$P_{r,m}$ and $p_1 = p_2 = ... = p_m$, where p_i denotes the reliability of the WTs in WFi, $i = 1, 2, ..., m$. Then, the wind farm WFk is $EENS$-efficient if

$$v_{ci,k} \leq \min(v_{ci,1}, ..., v_{ci,k-1}, v_{ci,k+1}, ..., v_{ci,m}),$$

and

$$v_{r,k}^3 - v_{ci,k}^3 \leq \min(v_{r,1}^3 - v_{ci,1}^3, ..., v_{r,k-1}^3 - v_{ci,k-1}^3, v_{r,k+1}^3 - v_{ci,k+1}^3, ..., v_{r,m}^3 - v_{ci,m}^3).$$

5 Discussion with numerical examples

With the findings presented in the previous sections, for given WT models and/or WF compositions, comparisons on capacity factors and $EENS$ values can be established with respect to turbine characteristics without any numerical assessment. In this section, we corroborate our theoretical findings with numerical results. To this end, in Table 3, we compute and present $EENS$ values for the wind farm compositions previously considered by Nemes and Munteanu (2010) when the wind speed distribution is Weibull with parameters $\alpha = 9$ and $\beta = 2$. Each WF consists of n identical WTs with $FOR = 0.04$ ($p = 0.96$). The nominal power of each WF is fixed as 30 MW. The CF of each WT model is also computed. As it is clear from Table 3, the WFs that consist of the WTs having same capacity factors may have different $EENS$ values.

Among all possible alternatives listed in Table 3, the wind farm that consists of $n = 20$ WT models of type 1.5xle-GE is $EENS$-efficient. Although we have computed

$EENS$ values to compare the WF compositions, some pairwise comparisons in terms of $EENS$ can be done without any calculation only using Corollary 1. Indeed, let WF1 consist of $n = 20$ WTs of type N70-1.5 ($v_{ci,1} = 3, v_{r,1} = 13$) and WF2 consist of $n = 20$ WTs of type 1.5sle-GE ($v_{ci,2} = 3.5$ and $v_{r,2} = 14$). Then, because $v_{ci,1} \leq v_{ci,2}$, and $v_{r,1}^3 - v_{ci,1}^3 \leq v_{r,2}^3 - v_{ci,2}^3$, WF1 is preferred to WF2 with respect to $EENS$. As it is observed from Table 3, a wind farm with a fewer number of WTs may have a smaller $EENS$ value due to the characteristics of the WTs used by the WF. Indeed, as it can be observed from Table 3, the wind farm that consists of $n_1 = 20$ WTs of 1.5sle-GE model has a larger $EENS$ than the wind farm with $n_2 = 15$ WTs of V90-2 model. Thus, the characteristics of the used wind turbines within a wind farm have a significant effect on wind farm performance.

From Theorem 1 (b), we can immediately state that the WT of type 1.5xle-GE has a larger CF than the WT of type 1.5s-GE. This is numerically demonstrated in Table 3.

Table 3. CF and $EENS$ values for a WF consisting of different types of WTs

($FOR = 0.04$ for each WT and $L = 10$ MW)

						Wind farm	
WT Model	P_r (MW)	v_{ci} (m/s)	v_r (m/s)	v_{co} (m/s)	CF	n	$EENS$
1.5xle-GE	1.5	3.5	11.5	20	0.3752	20	4.1094
1.5sle-GE	1.5	3.5	14	25	0.2667	20	5.1212
1.5s-GE	1.5	4	12	25	0.3505	20	4.3986
2.5xl-GE	2.5	3.5	12.5	25	0.3319	12	4.4802
V90-2	2	4	12	25	0.3505	15	4.3994
V90-3	3	3.5	15	25	0.2297	10	5.5359
N70-1.5	1.5	3	13	25	0.3128	20	4.5973
N80-2.5	2.5	3	13	25	0.3128	12	4.5991

6 Concluding Remarks

The research reported in this paper is the first attempt to compare wind turbines and wind farms without using wind speed data. The theoretical results have been established to compare the performances of the wind turbines and wind farm compositions with respect to the turbine characteristics. Specifically, necessary conditions have been obtained to compare the CFs of different WT models. The conditions have also been obtained to compare WF compositions in terms of stochastic ordering. As it has been pointed out, the second order stochastic dominance is closely related to the ordering of $EENS$ values of WF compositions. Some results obtained in the paper are wind speed distribution free and useful to select optimal WT model for a WF. Although we have chosen Weibull distribution to model wind speed in the numerical illustrations, some theoretical results hold true for an arbitrary choice of the wind speed distribution.

In the development of the results, the wind speed and wind turbine availability were assumed to be statistically independent. As a future work, analogous theoretical results could be obtained when there is a dependence between the wind speed and the wind turbine availability.

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