Privatization Policies by National and Regional Governments*

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Abstract

In order to analyze the privatization policies undertaken by the national and regional governments, I consider a horizontal differentiation model with price competition in which a country consists of two regions of different sizes. I show that public-sector intervention by either the national or regional government is essential for achieving the social optimum. The preferences of consumers and firms about privatization policy are completely opposite: consumers prefer a regional public-sector intervention, while firms prefer a national public-sector intervention. Finally, I find that the preferences of the two regions about market structures are also opposite: the least populated region prefers the private duopoly, while the most populated region prefers a government intervention in the market.

Keywords: Horizontal Differentiation; National and Regional Governments; Mixed Duopoly; Region Size; Partial Privatization

JEL classification: H42, L13, L32, L33, R59

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1 Introduction

In the eighties the British government began a wave of privatization that was echoed across the world in the following years. Bortolotti et al. (2003) empirically show that privatization takes place typically in wealthy democracies with high public debt, but endowed with deep and liquid stock markets.[1] They show that legal institutions are also important, in the sense that the law should provide protection for private investment. However, at the end of 2000, governments retained control of two-thirds of privatized firms (Bortolotti and Faccio (2009)). In civil law countries, governments tend to retain large ownership positions, whereas in common law countries they typically use golden shares. To put it another way, governments hold more influence over privatized firms in countries with proportional electoral rules and with a centralized system of political authority (Bortolotti and Faccio (2009)). Thus, in the current recession, which affects to most of the countries in the world, a new wave of privatization across the world can be expected, in which the local governments in each country may play an important role. Therefore, it is interesting to analyze strategic interaction at the privatization stage between local governments and the government of the nation to which they belong, where the local government is concerned about the welfare in the region and the national government for the welfare of the country.

In this paper I develop a model that analyzes strategic interaction between different tiers of government and its consequences on the markets, without neglecting a welfare analysis. Given the presence of firms owned by national and regional governments in many industries, the insights behind the paper can be incorporated into the analysis of different industries, e.g. the airport sector (Albalate et al. (2014) and Matsumura and Matsushima (2012)), the broadcasting market (Bel and Domènech (2009) and González-Maestre and Martínez-Sánchez (2014, 2015)), hospital markets (Brekke et al. (2008), Aiura and Sanjo (2010) and Aiura (2013)), the university system (De Fraja and Valbonesi (2012) and Cremer and Maldonado (2013)), and the development of public facilities (Takahashi (2004)) among others. For instances, the european broadcasting markets are characterized for the existence of national and regional public firms (Bel and Domènech (2009)). In particular, there are three national channels and twenty one regional in Italy, three national channels and thirteen regional in France, two national channels and thirteen regional in United Kingdom, two national channels and seventeen regional in Spain ... (Bel and Domènech (2009)).

The question of whether it is advisable to privatize public firms has been widely analyzed previously. In a seminal paper that assumes quantity competition and homogenous goods, De Fraja and Delbono (1989) show that the existence of a public firm is socially desirable if the number of firms is low enough. However, I consider horizontal product differentiation as in Cremer et al. (1991). They show that it is only when the total number of firms is two or at least six that a mixed oligopoly with one public firm is socially preferable to a private oligopoly. Following Matsumura (1998), I allow partial privatization, so a partially public-owned firm takes both profits and welfare into consideration. Matsumura (1998) analyzes a duopoly model with quantity competition and finds that neither full privatization nor full

[1] This result is also supported by Albalate et al. (2014), who find that it is more likely for airports to be privatized in countries with higher public debt.

nationalization is optimal. As in my model, recent research has focused on analyzing privatization policies in horizontal product differentiation models with price competition. For instance, Kumar and Saha (2008) show that unless public ownership exceeds a critical level, maximal differentiation continues to hold and social welfare does not improve with public ownership. Sanjo (2009) analyses simultaneous price choice and sequential price choice and shows that the degree of privatization of a publicly owned firm influences social welfare in a mixed duopoly market. From this last paper, Martínez-Sánchez (2011) shows that in the location game, in which firms set prices simultaneously, social welfare depends on the degree of privatization and is only maximized if the partially privatized firm is a fully publicly owned firm.

Previous papers have considered that public firms are owned by national governments that are concerned for the social welfare of their countries. Thus, they omit firms owned by regional governments that are concerned only for the social welfare of their regions. However, such firms are present in many sectors. My model incorporates regional public firms and analyzes the strategic interaction between them and the government of the country to which they belong. A recent paper by Tomaru and Nakamura (2012) studies a mixed oligopoly model with quantity competition in a country comprised of two regions. They find that when the national and regional governments independently consider whether to privatize their respective public firms, only the state-owned public firm should be privatized. A paper with features similar to mine is Inoue et al. (2009), in which a regional public firm competes against a private firm. They allow firms to choose their location and prices, but they do not allow partial privatization of the regional public firm. Finally, they show the existence of two types of equilibrium: i) the regional public and private firms locate in different regions; and ii) both firms locate in the same region.

My paper is related to the literature on the privatization policies of national governments in an international context. Bárcena-Ruiz and Garzón (2005a, 2005b) analyze strategic interaction between governments when they decide whether to privatize their publicly-owned firms. They consider two equal countries that form a single market in which there is free trade and firms produce a homogeneous good and decide its output level.[2] Bárcena-Ruiz and Garzón (2005a) includes the existence of a supra-national authority, whose goal is to maximize social welfare in both countries. They show that if the supra-national authority decides whether or not to privatize public firms, the aggregate politically weighted welfare is no less than if the decision is taken by the governments.

A feature of the aforementioned papers that consider horizontal differentiation is that consumers are uniformly distributed across a linear segment. However, following Gabszewicz and Wauthy (2012), I break with this assumption here.[3] These authors have developed a new variant of the Hotelling model in which the segment where consumers are distributed is divided into two, so there are two "natural" markets with different sizes. In a private duopoly, they show that equilibrium prices increase whenever the disparity of consumers between regions decreases. Moreover, as in Gabszewicz and Wauthy (2012), I assume that firms are located at the two endpoints of the unit interval $[0, 1]$, which is an equilibrium when

[2] Bárcena-Ruiz and Garzón (2005a, 2005b) do not allow partial privatization by governments.

[3] Calvó-Armengol and Zenou (2002) and Benassi et al. (2006) develop models in which consumers are not uniformly distributed.

private firms are not allowed to exceed the boundaries and the disparity of consumers is low enough (Guo and Lai (2013)). However, this assumption is not an equilibrium in a mixed duopoly in which consumers are uniformly distributed (Cremer et al. (1991) and Sanjo (2009)).

In short, I analyze the privatization policies undertaken by national and regional governments, under which they have the option to partially privatize firms. To that end I consider a horizontal differentiation model with price competition in which a country is divided into two regions with different sizes. We show that public intervention by either the national or regional government is essential for achieving the social optimum, because a private duopoly does not achieve the social optimum. On the other hand, the preferences of consumers and firms about market structures are completely opposite: consumers prefer the intervention regional governments while firms prefer the intervention of the national government. From a regional viewpoint, I find that the preferences of the two regions about market structures are also completely opposite: the least populated region prefers the private duopoly, while the most populated region prefers a (national and regional) government intervention in the market. In addition, both regions are indifferent to the public intervention by the national or regional government.

The paper is organized as follows. Section 2 describes the model formally. Sections 3, 4 and 5 consider the regional, the national and the private duopoly, respectively. A duopoly in which a national publicly-owned firm competes against a regional publicly-owned firm is analyzed in Section 6. Section 7 provides a social welfare analysis. Section 8 concludes.

2 The Model

A unit mass of consumers is distributed in the $[0, 1]$ interval, which represents the location of the consumers in a country which is divided into two regions: $[0, 1/2]$, which is referred to here as Region 0, and $[1/2, 1]$ which is referred to as Region 1. It is assumed that the regions have different population densities (or relative sizes). In particular, with no loss of generality, the density of Region 0 is $\mu \in (0, 1/2)$ and that of Region 1 is $1 - \mu$, as in Gabszewicz and Wauthy (2012), which means that most consumers are located in Region 1. There are two firms that sell a single product and are located at the two endpoints of the unit interval $[0, 1]$: one is located at zero and indexed by 0 and the other is located at one and indexed by 1. Notice that firm 0 is located in Region 0 and firm 1 is located in Region 1. It is assumed that each consumer can buy at most one unit of the product. Thus, the utility of the consumer located at x is:

$$U(x) = \begin{cases} v - x - p_0 & \text{if he/she buys from firm 0,} \\ v - (1 - x) - p_1 & \text{if he/she buys from firm 1,} \end{cases} \quad (1)$$

where v represents the consumer's utility obtained from buying from his/her location, p_i represents the price of the product $i = 0, 1$, and x and $1 - x$ represent the disutility from not buying from his/her location if he/she buys from firm 0 and firm 1, respectively. Thus, consumers located in $[0, 1/2]$ prefer to buy the product offered by firm 0 than that offered by firm 1 at equal prices, and conversely consumers located in $[1/2, 1]$ prefer product 1 to product 0 at equal prices. Since $\mu \in (0, 1/2)$, the majority of consumers prefer to buy from firm 1 than from firm 0 at equal prices. It is assumed that v is high enough for all consumers to buy at least from one firm. From the utility function (1), it is possible to find the consumer who is indifferent between buying from firm 0 and firm 1, which is given by:

$$\widehat{x}\left(p_0,p_1\right)=\frac{1}{2}+\frac{p_1-p_0}{2}. \tag{2}$$

In order to obtain the demand functions of the two firms, two cases must be analyzed: (i) when the indifferent consumer is located in Region 0, $\widehat{x}\left(p_0,p_1\right)<1/2$, and (ii) when he/she is located in Region 1, $\widehat{x}\left(p_0,p_1\right)>1/2$. Thus, if $p_0>p_1$, then $\widehat{x}\left(p_0,p_1\right)<1/2$, which means that there are some consumers in Region 0 that buy products from the firm located in Region 1. In particular, $[\widehat{x}\left(p_0,p_1\right),1/2]$. On the other hand, if $p_0<p_1$, then $\widehat{x}\left(p_0,p_1\right)>1/2$, which means that there are some consumers in Region 1 that buy from firm 0. In particular, $[1/2,\widehat{x}\left(p_0,p_1\right)]$. Thus, taking into account that the densities of the regions are different, the demand functions are:

$$D_0\left(p_0,p_1\right)=\begin{cases}\frac{\mu+(1-\mu)(p_1-p_0)}{2} & \text{if } p_0\leq p_1\\ \frac{\mu(1+p_1-p_0)}{2} & \text{if } p_0\geq p_1\end{cases} \tag{3}$$

$$D_1\left(p_0,p_1\right)=\begin{cases}\frac{(1-\mu)(1-p_1+p_0)}{2} & \text{if } p_0\leq p_1\\ \frac{1-\mu-\mu(p_1-p_0)}{2} & \text{if } p_0\geq p_1\end{cases} \tag{4}$$

It is assumed that the fixed cost of developing a product and the marginal cost are zero. Thus, the profit function of each firm is:

$$\pi_0\left(p_0,p_1\right)=\begin{cases}p_0\frac{\mu+(1-\mu)(p_1-p_0)}{2} & \text{if } p_0\leq p_1\\ p_0\frac{\mu(1+p_1-p_0)}{2} & \text{if } p_0\geq p_1\end{cases} \tag{5}$$

$$\pi_1\left(p_0,p_1\right)=\begin{cases}p_1\frac{(1-\mu)(1-p_1+p_0)}{2} & \text{if } p_0\leq p_1\\ p_1\frac{1-\mu-\mu(p_1-p_0)}{2} & \text{if } p_0\geq p_1\end{cases} \tag{6}$$

A third kind of agent consider here is comprised of regional governments that maximize regional social welfare. The social welfare of Region i is defined as the sum of the profit of the firm and the surplus of the consumers located in Region i, i.e. $W_i=\pi_i+CS_i$ $i=0,1$, where

$$CS_0=\begin{cases}\mu\int_0^{\frac{1}{2}}\left(v-x-p_0\right)dx & \text{if } p_0\leq p_1\\ \mu\int_0^{\widehat{x}}\left(v-x-p_0\right)dx+\mu\int_{\widehat{x}}^{\frac{1}{2}}\left(v-(1-x)-p_1\right)dx & \text{if } p_0\geq p_1.\end{cases} \tag{7}$$

$$CS_1=\begin{cases}(1-\mu)\left(\int_{\frac{1}{2}}^{\widehat{x}}\left(v-x-p_0\right)dx+\int_{\widehat{x}}^{1}\left(v-(1-x)-p_1\right)dx\right) & \text{if } p_0\leq p_1\\ (1-\mu)\int_{\frac{1}{2}}^{1}\left(v-(1-x)-p_1\right)dx & \text{if } p_0\geq p_1.\end{cases} \tag{8}$$

In the model proposed here regional governments may become shareholders of the firms located in their regions. Thus, the aims of these firms depend on the composition of their boards because regional government representatives advocate maximizing regional social welfare and private shareholders advocate a profit motive. Following Matsumura (1998), it is assumed here that the board of a partially publicly-owned regional firm seeks to maximize a well-balanced mean between its profit and regional social welfare, where the weights are the stakes held in it by the regional government and by private owners. The government of Region i is considered to own a stake of $\alpha_i \in [0,1]$ and the private owners $1-\alpha_i$ of the partially publicly-owned firm located in Region i, so the objective function of partially publicly-owned firms is $\Pi_i = \alpha_i W_i + (1-\alpha_i)\pi_i \; i=0,1$. Notice that full privatization is achieved when $\alpha_i = 0$. The timing of the game is as follows:

i) in order to maximize regional social welfare, each regional government simultaneously chooses its share in the partially publicly-owned firm located in its region, $\alpha_i \; i=0,1$; and,

ii) finally, firms simultaneously set prices.

In the following section, this game is solved by backward induction.

3 Regional Duopoly

From the profit functions of firm 0 and firm 1, (5) and (6), and the consumer surpluses of regions 0 and 1, (7) and (8), the expressions for social welfare in each region are found, i.e.:

$$W_0 = \pi_0 + CS_0 = \begin{cases} \frac{4v\mu-\mu+4(1-\mu)p_0(p_1-p_0)}{8} & \text{if } p_0 \le p_1 \\ \frac{\mu\left(4v-1+2\left(p_1^2-p_0^2\right)\right)}{8} & \text{if } p_0 \ge p_1 \end{cases} \tag{9}$$

$$W_1 = \pi_1 + CS_1 = \begin{cases} \frac{(1-\mu)\left(4v-2\left(p_1^2-p_0^2\right)-1\right)}{8} & \text{if } p_0 \le p_1 \\ \frac{(1-\mu)(4v-1)+4\mu p_1(p_0-p_1)}{8} & \text{if } p_0 \ge p_1, \end{cases} \tag{10}$$

Taking into account theses expressions, I maximize the objective functions $\Pi_0 = \alpha_0 W_0 + (1-\alpha_0)\pi_0$ (with respect to p_0) and $\Pi_1 = \alpha_1 W_1 + (1-\alpha_1)\pi_1$ (with respect to p_1), and obtain the firms' reaction functions, which are:

$$p_0(p_1) = \begin{cases} \varphi_0^l(p_1) = \frac{(1-\alpha_0)\mu}{2(1-\mu)} + \frac{p_1}{2} & \text{if } p_0 \le p_1 \\ \varphi_0^h(p_1) = (p_1+1)\frac{1-\alpha_0}{2-\alpha_0} & \text{if } p_0 \ge p_1, \end{cases} \tag{11}$$

$$p_1(p_0) = \begin{cases} \varphi_1^h(p_0) = (p_0+1)\frac{1-\alpha_1}{2-\alpha_1} & \text{if } p_0 \le p_1 \\ \varphi_1^l(p_0) = \frac{(1-\alpha_1)(1-\mu)}{2\mu} + \frac{p_0}{2} & \text{if } p_0 \ge p_1. \end{cases} \tag{12}$$

Since $\mu < 1/2$ and $\alpha_i \in [0,1] \; i=0,1$, we have that relationship (13) holds. This means that $\varphi_0^l(p_1) < \varphi_0^h(p_1)$ and $\varphi_1^l(p_0) > \varphi_1^h(p_0)$, which implies that firm 1's reaction function is continuous and kinked, while firm 0's reaction function is discontinuos at $p_1 = \widetilde{p}_1$ as can be seen in Figure 1.[4] $\widetilde{p}_1$ is the price at which firm 0 is indifferent between setting a high price and setting a low price. In particular, $\widetilde{p}_1$

[4] $\widetilde{p}_1 = \frac{1-\alpha_0}{4\mu+(1-\mu)\alpha_0-2}\left(\mu\alpha_0 - \sqrt{\frac{2(2-\alpha_0)\mu(1-2\mu)^2}{1-\mu}}\right)$.

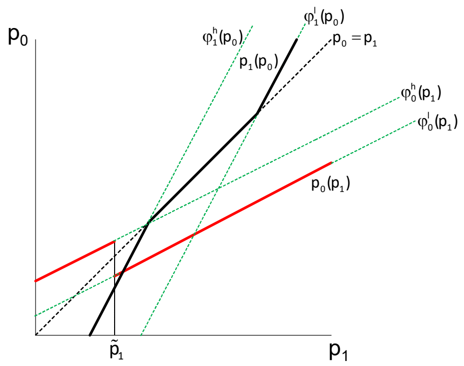

Figure 1: Regional Duopoly.

is the price that solves the following equality $\Pi_0\left(\varphi_0^l\left(p_1\right), p_1\right) = \Pi_0\left(\varphi_0^h\left(p_1\right), p_1\right)$. Thus, the existence of a pure strategy equilibrium is not guaranteed.

$$\frac{\mu}{2\left(1-\mu\right)} < \frac{1}{2} \leq \frac{1}{2-\alpha_i} \leq 1 < \frac{1-\mu}{\mu} \; i = 0, 1. \tag{13}$$

From the intersection of the price reaction function of regional publicly-owned firms, the prices at this stage of the game are found, i.e.:

$$p_0\left(\alpha_0, \alpha_1\right) = \begin{cases} \frac{\mu+1-\alpha_1-\mu\alpha_0(2-\alpha_1)}{(1-\mu)(3-\alpha_1)} & \text{if } p_0 \leq p_1 \\ \frac{(1-\alpha_0)(1+\mu-\alpha_1+\mu\alpha_1)}{\mu(3-\alpha_0)} & \text{if } p_0 \geq p_1 \end{cases} ; p_1\left(\alpha_0, \alpha_1\right) = \begin{cases} \frac{(1-\alpha_1)(2-\mu-\mu\alpha_0)}{(3-\alpha_1)(1-\mu)} & \text{if } p_0 \leq p_1 \\ \frac{(2-\alpha_0)(1-\alpha_1(1-\mu))-\mu}{\mu(3-\alpha_0)} & \text{if } p_0 \geq p_1 \end{cases} \tag{14}$$

Each regional government now maximizes the social welfare of its region, so that by substituting firms' prices (14) in regional social welfare functions (9) and (10), and maximizing these functions with respect to α_0 and α_1 , respectively, the regional governments' reaction functions of the share in the partially publicly-owned firms are found:

$$\alpha_0\left(\alpha_1\right) = \begin{cases} \frac{5\mu-1+\alpha_1(2-4\mu-\alpha_1(1-\mu))}{2\mu(2-\alpha_1)} & \text{if } p_0 \leq p_1 \\ \frac{3\mu}{2\mu-(1-\mu)\alpha_1+1} & \text{if } p_0 \geq p_1 \end{cases} ; \alpha_1\left(\alpha_0\right) = \begin{cases} \frac{3(1-\mu)}{3-2\mu-\mu\alpha_0} & \text{if } p_0 \leq p_1 \\ \frac{4-5\mu+\alpha_0(4\mu-\mu\alpha_0-2)}{2(2-\alpha_0)(1-\mu)} & \text{if } p_0 \geq p_1 \end{cases}$$

From the intersection of the above reaction functions and taking into account that $\alpha_i \in [0, 1]$, the Subgame Perfect Equilibrium (SPE) of this game is obtained, in which each regional government fully owns the firm that is located at its region. As a result, both regional publicly-owned firms set the same price, so the indifferent consumer is located at the border between the two regions, $\widehat{x}^R = 1/2$. Thus, consumers buy the product from the firm located in their region. These results are summarized in Proposition 1.

Proposition 1 *In the SPE of the Regional Duopoly, regional governments fully control firms* ($\alpha_0^R = \alpha_1^R = 1$) *and the prices, indifferent consumer, consumer surplus and social welfare are:*

$$p_0^R = p_1^R = 0; \widehat{x}^R = \frac{1}{2}; CS_0^R = W_0^R = \mu W^R; CS_1^R = W_1^R = \left(1-\mu\right) W^R; CS^R = W^R = \frac{4v-1}{8}. \tag{15}$$

An interesting result is that both firms set the same price at the SPE, although they face markets with different sizes. What is more they set a price of zero and thus do not obtain any revenue. Thus, the regional social welfare coincides with the regional consumer surplus for each region. Moreover, the regional social welfare depends on the population density of the region so that $W_0^R < W_1^R$ since $\mu \in (0, 1/2)$. However, the national social welfare does not depend on the relative sizes of the regions.

Given that firms cannot discriminate on prices according to the location of consumers, the best way of maximizing the regional social welfare is to set the lowest price (which is zero) and maximize the regional consumer surplus.

4 National Duopoly

I now consider the intervention of the national government as a shareholder of a publicly-owned firm. It is concerned for the social welfare of the country, which is defined as the sum of the social welfare of both regions, $W = W_0 + W_1$.[5] Taking into account the regional social welfare functions (9) and (10), the expression of national social welfare is found to be the following:

$$W = \begin{cases} \frac{4v-1-2(1-\mu)(p_0-p_1)^2}{8} & \text{if } p_0 \leq p_1 \\ \frac{4v-1-2\mu(p_0-p_1)^2}{8} & \text{if } p_0 \geq p_1 \end{cases} \tag{16}$$

In this section the regional governments do not participate in the game. Thus, the partially national publicly-owned firm competes against a private firm that maximizes its profit. The aims of the partially publicly-owned firm depend on the composition of its board because the national government's representatives advocate maximizing social welfare and private shareholders advocate a profit motive. It is assumed here that the board of the partially publicly-owned firm seeks to maximize a well-balanced mean between the national social welfare and its profit, where the weights are the stakes held by the government and private owners. I consider that the national government owns a stake of $\alpha \in [0, 1]$ of the partially publicly-owned firm and private owners $1 - \alpha$, so the objective function of the partially publicly-owned firm is $\Pi_i = \alpha W + (1 - \alpha)\pi_i$ $i = 0, 1$. Notice that $1 - \alpha$ measures the degree of privatization, so full privatization is achieved when $\alpha = 0$.

The timing of the game is as follows: (i) in order to maximize social welfare, the national government chooses the firm in which it is to take a stake; (ii) next, it decides how big a stake to take in that firm, α; and (iii) finally, firms simultaneously set prices.

In the following subsections, the subgames in which the national government partially owns firm 0 (subgame 0) and firm 1 (subgame 1) are solved.

4.1 Subgame 0

In this subgame, the national government takes a partial stake in firm 0, so firm 0 maximizes the objective function $\Pi_0(p_0, p_1) = \alpha W + (1 - \alpha)\pi_0$, which is given by:

[5] An equivalent definition of national social welfare is: the sum of firms' profits and the consumer surplus, $W = \pi_0 + \pi_1 + CS$, where $CS = CS_0 + CS_1$.

$$\Pi_0(p_0,p_1) = \begin{cases} \alpha\frac{4v-1-2(1-\mu)(p_0-p_1)^2}{8} + (1-\alpha)\,p_0\frac{\mu+(1-\mu)(p_1-p_0)}{2} & \text{if } p_0 \le p_1 \\ \alpha\frac{4v-1-2\mu(p_0-p_1)^2}{8} + (1-\alpha)\,p_0\mu\frac{1+p_1-p_0}{2} & \text{if } p_0 \ge p_1. \end{cases} \tag{17}$$

Next I solve the third stage, in which firms simultaneously choose prices. Therefore, by maximizing the function (17) with respect to p_0, the reaction function of the partially publicly-owned firm 0 is obtained, i.e.:

$$p_0(p_1) = \begin{cases} \frac{(1-\alpha)\mu}{(2-\alpha)(1-\mu)} + \frac{p_1}{2-\alpha} & \text{if } p_0 \le p_1 \\ \frac{1-\alpha}{2-\alpha} + \frac{p_1}{2-\alpha} & \text{if } p_0 \ge p_1 \end{cases} \tag{18}$$

Firm 1 is a private firm that aims to maximize the profit function (6). Thus, the reaction function of this firm is:

$$p_1(p_0) = \begin{cases} \frac{1}{2} + \frac{p_0}{2} & \text{if } p_0 \le p_1 \\ \frac{1-\mu}{2\mu} + \frac{p_0}{2} & \text{if } p_0 \ge p_1. \end{cases} \tag{19}$$

Although firm 0's reaction function is discontinuos at $\widehat{p}_1 = \sqrt{\mu/(1-\mu)}$, both reaction functions intersect.[6] From that interception, I obtain the prices at this stage, which are:

$$p_0(\alpha) = \frac{1+\mu-2\alpha\mu}{(1-\mu)(3-2\alpha)}; p_1(\alpha) = \frac{2-\alpha-\mu}{(1-\mu)(3-2\alpha)}.$$

Notice that $p_0 \le p_1$. By substituting the above prices in the social welfare function (16), it can be evaluated at the second stage of this subgame. Thus, the expression of the social welfare is:

$$W(\alpha) = \frac{4v(3-2\alpha)^2(1-\mu) + 16\alpha - 6\alpha^2 - 8\mu^2(1+\alpha^2) - 4\alpha\mu(7-4\mu-3\alpha) + 17\mu - 11}{8(3-2\alpha)^2(1-\mu)} \tag{20}$$

In order to maximize the social welfare function (20), the government decides to fully control firm 0, so that $\alpha^0 = 1$, as can be seen in Proposition 2:

Proposition 2 *In subgame 0 of the National Duopoly, the national government fully controls firm 0.*

4.2 Subgame 1

The national government now takes a stake in firm 1, so that the objective function of firm 1 is $\Pi_1 = \alpha W + (1-\alpha)\pi_1$. As in the previous subsection, the stage in which firms simultaneously choose prices is solved first. By maximizing the function Π_1 with respect to p_1, I obtain the reaction function of the partially publicly-owned firm 1, which is:

$$p_1(p_0) = \begin{cases} \frac{1-\alpha}{2-\alpha} + \frac{p_0}{2-\alpha} & \text{if } p_0 \le p_1 \\ \frac{(1-\mu)(1-\alpha)}{\mu(2-\alpha)} + \frac{p_0}{2-\alpha} & \text{if } p_0 \ge p_1 \end{cases} \tag{21}$$

Firm 0 is private, so it maximizes the profit function (5) and its reaction function is:

$$p_0(p_1) = \begin{cases} \frac{\mu}{2(1-\mu)} + \frac{p_1}{2} & \text{if } p_0 \le p_1 \\ \frac{1}{2} + \frac{p_1}{2} & \text{if } p_0 \ge p_1. \end{cases} \tag{22}$$

[6] $\widehat{p}_1$ is the price at which firm 0 is indifferent between setting a high price and setting a low price. In particular, $\widehat{p}_1$ is the price that solves the following equality $\Pi_0\left(\phi_0^l(p_1), p_1\right) = \Pi_0\left(\phi_0^h(p_1), p_1\right)$.

Given that firm 0's reaction function is discontinuous at $p_1 = \sqrt{\mu/(1-\mu)}$, it is possible that the two reaction functions may not intercept.[7] In particular, if $\alpha > \overline{\alpha} = 1 - \frac{1}{2}\sqrt{\frac{\mu}{1-\mu}}$, the two reaction functions do not intersect. Otherwise, if $\alpha \leq \overline{\alpha}$, they intercept and the prices are:[8]

$$p_0 = \frac{1-\alpha+\mu}{(1-\mu)(3-2\alpha)}; p_1 = \frac{2-\mu-2\alpha(1-\mu)}{(1-\mu)(3-2\alpha)}. \tag{23}$$

Notice that $p_0 \leq p_1$. By substituting the previous prices (23) in the social welfare function (16), it is evaluated at the second stage, which coincides with that obtained in previous section, that is (20). Therefore, taking into account that the social welfare function (20) is increasing when $\alpha < 1$, there is no solution when $\alpha > \overline{\alpha}$, and $\overline{\alpha} < 1$ (since $\mu \in (0, 1/2)$), I find that the national government partially privatize firm 1, as shown in the following proposition:

Proposition 3 *In subgame 1 of the National Duopoly, the national government takes a stake in firm 1 at $\alpha^1 = \overline{\alpha}$ level.*

4.3 Government: firm 0 or 1

Next, I look at which firm the government must take a stake in from a social point of view. The above subsections show that the government fully controls firm 0 or partially controls firm 1. Notice that when the government decides its stake in each firm, it faces the same objective function in both subgames. In particular, it faces the function (20). Given that the social welfare function (20) is increasing when $\alpha < 1$, and $\alpha^1 < \alpha^0 = 1$, the government takes a stake in firm 0. Therefore, the government fully controls firm 0 in the SPE of the complete game, which is shown in Proposition 4.

Proposition 4 *In the SPE for National Duopoly, the government fully controls firm 0, so that the prices, indifferent consumer, profits, consumer surplus and welfare are:*

$$\begin{aligned} p_0^N &= p_1^N = p^N \in \left[1, \frac{1-\mu}{\mu}\right]; \widehat{x}^N = \frac{1}{2}; \pi_0^N = \frac{\mu p^N}{2}; \pi_1^N = \frac{(1-\mu)p^N}{2}; CS_0^N = \mu CS^N; \\ CS_1^N &= (1-\mu)CS^N; W_0^N = \mu W^N; W_1^N = (1-\mu)W^N; CS^N = \frac{4v-4p^N-1}{8}; W^N = \frac{4v-1}{8}. \end{aligned} \tag{24}$$

Given that the government fully controls firm 0 ($\alpha^0 = 1$), both firms set the same price as can be seen in Figure 2. In contrast to the regional duopoly model, there is a multiplicity of equilibrium prices, although a unique value of welfare is obtained. Thus, both firms get positive profits, unlike the regional duopoly model. This last result is due to the existence of a rival private firm whose profit is included in the objective function of the publicly-owned firm, while in the regional duopoly both firms are owned by regional governments that are not concerned about the rival firm's profit. A strange result is that in the national duopoly both firms set the same price at the SPE, although they have different objective functions.

[7] Notice that $\sqrt{\mu/(1-\mu)}$ is the price that solves $\pi_0\left(\phi_0^l(p_1), p_1\right) = \pi_0\left(\phi_0^h(p_1), p_1\right)$.

[8] Notice that $p_1 = \frac{2-\mu-2\alpha(1-\mu)}{(1-\mu)(3-2\alpha)} \geq \sqrt{\frac{\mu}{1-\mu}}$ if and only if $\alpha \leq \overline{\alpha} = 1 - \frac{1}{2}\sqrt{\frac{\mu}{1-\mu}}$.

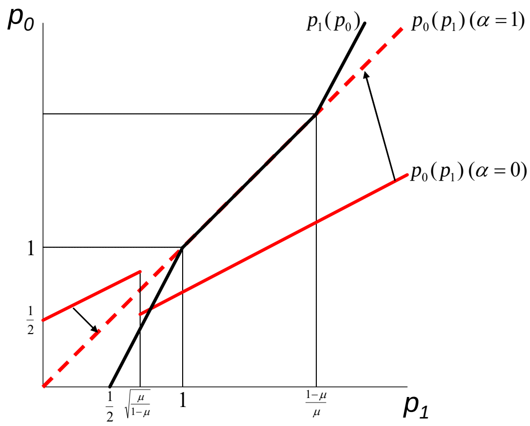

Figure 2: National Duopoly.

Notice that the national producer surplus, defined as $\pi^N = \pi_0^N + \pi_1^N$, does not depend on μ, although it does depend positively on prices. However, the profit of each firm is a fraction of the producer surplus, which is equal to the population density of the region in which it is located. Since $\mu \in (0, 1/2)$, I find that $\pi_0^N < \pi_1^N$. On the consumers side a similar result is observed. In particular, the national consumer surplus does not depend on μ and the consumer surplus of each region is a fraction of the national consumer surplus, which is equal to the population density of the region in which consumers are located. Since $\mu \in (0, 1/2)$, $CS_0^N < CS_1^N$. Thus, the regional social welfare depends on μ so that $W_0^N < W_1^N$, and the national social welfare does not depend on the relative sizes of the regions (μ).

5 Private Duopoly

We now consider the private duopoly in which the aim of both firms is to maximize their profits. Given that the market in Region 1 is bigger, the competition between firms focuses on consumers in Region 1. Thus, the firm located in Region 0 sets a lower price than firm 1 in order to sell its product in Region 1. This contrasts with the result that firms set the same price obtained when they are publicly-owned, regardless of whether they are national or regional. The private duopoly is analyzed by Gabszewicz and Wauthy (2012), and their result is reproduced here as follows:

Result 1 (Gabszewicz and Wauthy (2012)) *Assume $\mu \in (0, 1/2)$, there is a unique Nash equilibrium given by*

$$p_0^P = \frac{1+\mu}{3(1-\mu)} \text{ and } p_1^P = \frac{2-\mu}{3(1-\mu)}.$$

From Gabszewicz and Wauthy's result, the equilibrium values of the indifferent consumer, firms' profits, consumer surplus and social welfare at both national and regional level can be found. Given that $p_0^P < p_1^P$, I find that the indifferent consumer is located in Region 1, i.e. $\widehat{x}^P > 1/2$, which means that some consumers in Region 1 buy from firm 0. These additional results are shown in Proposition 5.

Proposition 5 *In the SPE of the Private Duopoly, the indifferent consumer, firms' profits, consumer surplus and social welfare are:*

$$\begin{aligned}
\widehat{x}^P &= \frac{4-5\mu}{6(1-\mu)}; \pi_0^P = \frac{(\mu+1)^2}{18(1-\mu)}; \pi_1^P = \frac{(2-\mu)^2}{18(1-\mu)}; CS_0^P = \frac{\mu(12v(1-\mu)-\mu-7)}{24(1-\mu)};\\
CS_1^P &= \frac{36v(1-\mu)^2+\mu(46-13\mu)-31}{72(1-\mu)}; CS^P = \frac{36v(1-\mu)+25\mu-16\mu^2-31}{72(1-\mu)};\\
W_0^P &= \frac{36v\mu(1-\mu)-13\mu+\mu^2+4}{72(1-\mu)}; W_1^P = \frac{12v(1-\mu)^2+10\mu-3\mu^2-5}{24(1-\mu)};\\
W^P &= \frac{36v(1-\mu)+17\mu-8\mu^2-11}{72(1-\mu)}.
\end{aligned}$$

Firm 0 sets an aggressive price strategy because there are fewer consumers near its location. To maximize profits, it therefore tries to sell to consumers located in Region 1, although they face a higher transport cost if they buy from firm 0. On the other hand, firm 1 has more consumers, whose transport cost is higher when they buy from the rival firm 0. Thus, it can set a higher price without losing many consumers, so that $\pi_0^P < \pi_1^P$ for $\mu \in (0, 1/2)$. The stronger competition for Region 1's consumers and larger size of Region 1 cause $CS_0^P < CS_1^P$. Therefore, $W_0^P < W_1^P$. These last results coincide with those of the previous sections.

6 Regional-National Duopoly

In this section, I consider the case in which a national publicly-owned firm competes against a regional publicly-owned firm. There are two possible cases: i) firm 0 is regional partially publicly-owned and firm 1 is national partially publicly-owned; and, ii) firm 0 is national partially publicly-owned and firm 1 is regional partially publicly-owned. In the first case, there is no SPE. Thus, I do not analyze it (see Martínez-Sánchez (2014) where this analysis can be found). Therefore, I focus on the case in which the national partially publicly-owned firm 0 competes against the regional partially publicly-owned firm 1.

The government of Region 1 takes a stake in firm 1, so firm 1's aim is to maximize the objective function $\Pi_1 = \alpha_1 W_1 + (1-\alpha_1)\pi_1$, and the national government takes a stake in firm 0, so firm 0's aim is to maximize the objective function $\Pi_0 = \alpha W + (1-\alpha)\pi_0$. The timing of the game is: i) the national and regional governments simultaneously choose their stakes in firms 0 and 1, respectively; and finally ii) the two firms simultaneously set prices. This game is solved by backward induction.

As can be seen in the national duopoly, firm 0's reaction function is (18), and in the regional duopoly firm 1's reaction function is (12). From the interception of these reaction functions, the following price levels are obtained:

$$p_0 = \begin{cases} \frac{\mu-2\alpha\mu-\alpha_1+\alpha\mu\alpha_1+1}{(3-2\alpha-\alpha_1+\alpha\alpha_1)(1-\mu)} & \text{if } \alpha_1 \le \frac{1-2\mu}{1-\mu} \\ \frac{\mu-2\alpha\mu+1-\alpha_1+\mu\alpha_1}{\mu(3-2\alpha)} & \text{if } \alpha_1 \ge \frac{1-2\mu}{1-\mu} \end{cases}; p_1 = \begin{cases} \frac{(1-\alpha_1)(2-\alpha-\mu)}{(3-2\alpha-\alpha_1+\alpha\alpha_1)(1-\mu)} & \text{if } \alpha_1 \le \frac{1-2\mu}{1-\mu} \\ \frac{(1-\alpha_1+\mu\alpha_1)(2-\alpha)-\mu}{\mu(3-2\alpha)} & \text{if } \alpha_1 \ge \frac{1-2\mu}{1-\mu} \end{cases}$$

By incorporating these prices into the welfare functions (10) and (16), the social welfare of Region 1

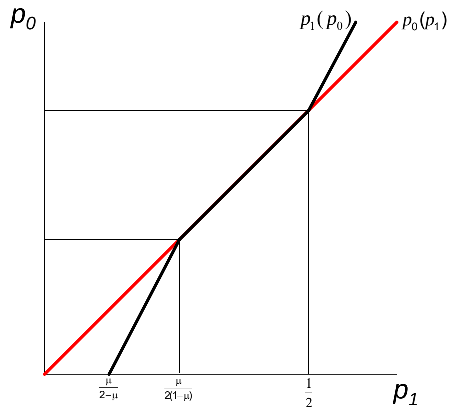

Figure 3: Regional National Duopoly.

and of the country evaluated at the first stage of this game is obtained, i.e.:

$$W_1 = \begin{cases} \frac{(1-\mu)(4v-1)}{8} - \frac{2(3-\alpha-3\alpha_1-2\alpha\mu+\alpha\alpha_1+\mu\alpha_1+\alpha\mu\alpha_1)(1-2\mu-\alpha_1+\mu\alpha_1)(1-\alpha)}{8(1-\mu)(-2\alpha-\alpha_1+\alpha\alpha_1+3)^2} & \text{if } \alpha_1 \leq \frac{1-2\mu}{1-\mu} \\ \frac{(1-\mu)(4v-1)\mu(2\alpha-3)^2-4(1-\alpha)(1-2\mu-\alpha_1+\mu\alpha_1)(\alpha\alpha_1-\mu-2\alpha_1-\alpha+2\mu\alpha_1-\alpha\mu\alpha_1+2)}{8\mu(3-2\alpha)^2} & \text{if } \alpha_1 \geq \frac{1-2\mu}{1-\mu} \end{cases}$$

$$W = \begin{cases} \frac{4v-1}{8} + \frac{2(1-\alpha)^2(1-2\mu-\alpha_1+\mu\alpha_1)^2}{8(\mu-1)(3-2\alpha-\alpha_1+\alpha\alpha_1)^2} & \text{if } \alpha_1 \leq \frac{1-2\mu}{1-\mu} \\ \frac{4v-1}{8} - \frac{2(1-\alpha)^2(1-2\mu-\alpha_1+\mu\alpha_1)^2}{8\mu(3-2\alpha)^2} & \text{if } \alpha_1 \geq \frac{1-2\mu}{1-\mu} \end{cases}$$

The national government chooses the α that maximizes W, and the government of Region 1 chooses the α_1 that maximizes W_1. Thus, the reaction functions of the two governments are:

$$\alpha(\alpha_1) = 1; \alpha_1(\alpha) = \begin{cases} \frac{3-\alpha-3\mu}{3-\alpha-2\mu} & \text{if } \alpha_1 \leq \frac{1-2\mu}{1-\mu} \\ \frac{4-2\alpha-5\mu+2\alpha\mu}{4-2\alpha-4\mu+2\alpha\mu} & \text{if } \alpha_1 \geq \frac{1-2\mu}{1-\mu} \end{cases}$$

As can be seen in Proposition 6, the government of Region 1 takes a partial stake in firm 1 and the national government fully controls firm 0.[9] This result contrasts with that obtained in previous literature by Tomaru and Nakamura (2012), who obtain that when the national and regional governments independently consider whether to privatize their respective public firms, only the state-owned public firm should be privatized.[10] Moreover, as can be seen in Figure 3, I find that there is a multiplicity of equilibrium prices, as in the national duopoly.

Proposition 6 *In the SPE of the Regional-National Duopoly, the national government fully controls firm 0 and the government of Region 1 decides to take a partial stake in firm 1, so the governments' stakes,*

[9] Notice that $\alpha_1^{RN} = \frac{2-3\mu}{2(1-\mu)} > \frac{1-2\mu}{1-\mu}$.

[10] Tomaru and Nakamura (2012) develop a mixed oligopoly model with quantity competition, in which firms produce a homogenous good and governments do not consider the option of partial privatization.

prices, the indifferent consumer, profits, consumer surplus and social welfare are:[11]

$$\begin{aligned}
\alpha^{RN} &= 1; \alpha_1^{RN} = \frac{2-3\mu}{2(1-\mu)}; p_0^{RN} = p_1^{RN} = p^{RN} \in \left[\frac{\mu}{2(1-\mu)}, \frac{1}{2}\right]; \widehat{x}^{RN} = \frac{1}{2}; \pi_0^{RN} = \frac{\mu p^{RN}}{2}; \\
\pi_1^{RN} &= \frac{(1-\mu)\, p^{RN}}{2}; CS_0^{RN} = \frac{\mu\left(4v-4p^{RN}-1\right)}{8}; CS_1^{RN} = \frac{(1-\mu)\left(4v-4p^{RN}-1\right)}{8}; \\
CS^{RN} &= \frac{4v-4p^{RN}-1}{8}; W_0^{RN} = \mu\frac{4v-1}{8}; W_1^{RN} = (1-\mu)\frac{4v-1}{8}; W^{RN} = \frac{4v-1}{8}
\end{aligned}$$

The government of Region 1 partially privatizes firm 1 but it does not set a different price than firm 0. This is because of the full control of firm 0 by the national government, which tries to avoid the welfare loss generated by consumers' disutility from not buying from the nearest firm. Thus, in equilibrium, both firms set the same prices and the maximum social welfare level is achieved, as in the national and the regional duopolies.

The government of Region 1 increases its stake in firm 1 as the relative size of Region 1 increases. The intuition behind this is that a bigger Region 1 reduces the incentive of private firm 1 to compete for new consumers, but increases the incentives of private firm 0. These firms' reactions entail that some consumers of Region 1 can buy from firm 0, which generates a social loss in Region 1 because it increases the transport cost of consumers when they buy from the firm furthest away. Thus, when the relative size of Region 1 increases, the government of Region 1 must take a bigger stake in firm 1 to avoid increasing the transport cost of consumers located in that region.[12]

7 Social Welfare Analysis

In the private duopoly, firms set different prices so that consumers' choice depends on their location and on prices. This implies a social loss because some consumers do not buy from the nearer firm, although they do buy the cheapest product. In particular, in the private duopoly some consumers in Region 1 buy from firm 0, which implies higher transport costs for consumers. Moreover, there are more consumers in Region 1. These facts explain why it is found that the privatization of publicly-owned firms is not socially optimal, as can be concluded from the relationship (25). This result holds regardless of whether the publicly-owned firms are owned by the national or the regional governments. Thus, it can be concluded that (national or regional) public intervention is essential for the social optimum to be achieved.

$$W^P < W^N = W^{RN} = W^R. \tag{25}$$

When a national or regional government takes a stake in a firm, both firms set the same price and avoid the welfare loss generated by the high transport costs for consumers in the private duopoly. However, although social welfare is the same in these market structures, the causes are different. In the regional duopoly both firms set a price of zero, which maximizes the consumer surplus and social welfare, although the two regional firms do not obtain profits. On the other hand, in the national duopoly firms set a higher

[11] Given that $\alpha^{RN} = 1$, there is no discontinuity in $p_0\,(p_1)$. Thus, the two price reaction functions always intercept.

[12] $\frac{\partial \alpha_1^{RN}}{\partial \mu} = -\frac{1}{2(\mu-1)^2} < 0$.

price than in other market structures, as can be see in (26). It is because prices are strategic complement and the national publicly-owned firm is located in the least populated region, so it has lower incentives to set lower prices and to attract more consumers of Region 1, which increases the welfare loss generated by the high transport costs. These high prices imply that the consumer surplus is lowest in the national duopoly, while the producer surplus ($\pi = \pi_0 + \pi_1$) is highest, as can be seen in the relationships (27) and (28). From the viewpoint of consumers and producers, the market structure in which the national partially publicly-owned firm 0 competes against the regional partially publicly-owned firm 1 represents a intermediate case between the national and regional duopolies.

$$p^R = 0 < p_0^P \leq p^{RN} < p_1^P < 1 \leq p^N \text{ if } \mu \leq \frac{1}{5} \qquad (26)$$

$$p^R = 0 < p^{RN} \leq p_0^P < p_1^P < 1 \leq p^N \text{ if } \frac{1}{5} \leq \mu$$

$$CS^N < CS^P < CS^{RN} < CS^R \qquad (27)$$

$$\pi^R < \pi^{RN} < \pi^P < \pi^N \qquad (28)$$

I now evaluate social welfare from a regional perspective. From (29) we observe that Region 0 prefers the private duopoly, while Region 1 does not. This result arises because firm 0 sets a low enough price in the private duopoly to obtain high demand (some consumers located in Region 1 buy from firm 0) and the transport cost of Region 0's consumers does no change with respect to other market structures. Thus, Region 0 prefers the private duopoly. However, in Region 1 consumers bear a higher transport cost and firm 1's profit is not too high, so Region 1 prefers (national and regional) government intervention in the market.

$$W_0^R = W_0^{RN} = W_0^N < W_0^P \qquad (29)$$

$$W_1^P < W_1^R = W_1^{RN} = W_1^N \qquad (30)$$

Since firms set the same price when a national or regional government takes a stake in a firm, it is avoided the welfare loss generated by the high transport costs for consumers in the private duopoly. Therefore, like the national viewpoint, both regions are indifferent to the public intervention by the national or regional government, as can be see from the relationships (29) and (30).

8 Conclusions

In this paper I analyze the privatization policies implemented by national and regional governments by considering a horizontal differentiation model with price competition in which a country is divided into two regions. Following Gabszewicz and Wauthy (2012), I assume that those regions are of different sizes.

My analysis shows that public intervention by either the national or regional government is essential for achieving the social optimum, because the private duopoly increases the consumers' disutility from not buying from the nearest firm. Thus, there are several ways to attain the social optimum: i) the firm located in the less populated region, firm 0, is fully controlled by the national government while the firm located in more populated region, firm 1, is private (national duopoly); ii) each firm is owned by the government of the region where it is located (regional duopoly); and iii) firm 0 is owned by the national government while firm 1 is partially owned by the government of Region 1 (regional-national duopoly). An underlying overall conclusion of this paper is that the relative size of the regions is an important feature in the design of the privatization policies implemented by national and regional governments.

An interesting result is that the preferences of consumers and firms about market structures are completely opposite. In particular, the national duopoly is the best outcome for firms but the worst for consumers, while the regional duopoly is the best outcome for consumers and the worst outcome for firms. From a regional viewpoint, the least populated region prefers the private duopoly, while most populated region prefers a (national and regional) government intervention in the market. In addition, both regions are indifferent to the public intervention by the national or regional government.

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