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Between Sentential and Model-Based Abductions
A Dialogical Approach

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Abstract

Most of the standard approaches consider abduction in terms of a backward reasoning, and miss some of its fundamental features. Overall they neglect its pragmatic dimension and the conjectural aspect of the conclusion. In this paper, we approach abduction in terms of strategic adjustment process in the context of dialogical logic. This sheds light on the use of conjectures in argumentative interactions. Although abductive dialogues are sometimes based upon sentential conjectures, they can also involve hypotheses about the context of argumentation itself. Indeed, the underlying logic of an argumentative interaction is not always settled since the beginning. In this context, abduction is not only concerned with the introduction of sentential hypotheses, but also with hypotheses concerning the structural rules governing the dialogue itself. We thus emphasize the instrumental dimension of abduction in dialogues.

1 Introduction

In dialogical logic, deductive validity is approached within a game between the proponent of a thesis and an opponent. The proponent's thesis is valid if he has a winning strategy; that is, if he is able to defend himself against every criticism of the opponent. In abductive dialogues, the proponent is allowed to introduce hypotheses as a basis for hypothetical plays. Even if deductive validity is not reached, a hypothetical winning strategy is displayed by the proponent despite a lack of concessions. Abductive dialogues are therefore understood in terms of strategic adjustment processes. Conjectures are set as a basis for new plays in which abduction exhibits its dialogical virtue. Although hypothetical plays might be understood as explanations of what is lacking for the validity of a thesis, dialogical logic highlights the instrumental aspect of abductive hypotheses. This sheds light on different kinds of hypotheses, namely sentential, formal and frame-based hypotheses, and subsequently different kinds of abduction, in which its pragmatic aspect is fundamental. Our understanding of abduction in terms of strategic adjustment process has its roots in the GW model of abduction (GW-m), following Gabbay and Woods [8]. In the GW-m, abduction is an inference triggered by an ignorance-problem. However, abduction does not consist in solving this problem, but rather in setting a hypothesis as a basis for new actions despite a persistent state of ignorance. Abduction is thus ignorance-preserving. This inference is pragmatic, agent-based, and goal-oriented. Although it does not fit with the standard of deductive validity, it has a cognitive virtue and should not be taken as a fallacy (see Woods [20]). This can be recognized by paying a peculiar attention to consequence-drawing; that is, the way real agents actually draw conclusions from premises. That is why dialogical logic, in which the consequence relation is somewhat dialectifed, constitutes a good candidate to study abduction.

In addition, different levels of rules can be clearly identified. The definitory rules, namely the "particle rules" and the "structural rules", say what is allowed in the course of a dialogue. They do not say how to play well, or how to win. This might be grasped by means of another level of rules, the strategic rules. By considering abduction in terms of strategic adjustment process, we meet Hintikka's [11] (p. 513) proposal on how to understand Peirce's claim that the legitimacy of abduction is based upon "altogether different principles" (CP 6.525).

By defending the instrumental dimension of abduction, we also agree with Hintikka's rejection of its identification with Inference to the Best Explanation (IBE).[1] Peirce himself speaks of abduction in terms of "the process of forming an explanatory hypothesis" (CP 5.171).[2] What does it mean? In the absence of a sufficient answer, this cannot clarify the nature of abduction: "Most of people who speak of 'inference to the best explanation' seem to imagine they know what explanation is. In reality, the nature of explanation is scarcely any clearer than the nature of abduction". ([11], p. 507) For example, Doven's entry in the Stanford Encyclopedia of Philosophy [6] is "exclusively concerned with abduction in the modern sense" (namely IBE), but the meaning of "explanation" is left unclear. First, explanation is not a clearer concept than abduction. Second, even if we agree on a concept of explanation, being explanatory is not sufficient for being abductive. Third, hypotheses introduced in the course of an abduction need not be explanatory. For example, Einstein's explanation of the perihelion movement of Mercury was not an IBE, even though an explanation was given within the General Theory of Relativity. That is, Einstein's explanation was rather an inference *from* an abductive (non-explanatory) hypothesis, rather an inference *to* the best explanation. Another example is the reconciliation of Maxwell's electromagnetic theory with Newtonian mechanics in the Special Theory of Relativity. This is a case of a wider new theory unifying

[1] Following Harman [10].
[2] CP refers to the Collected Papers [16].

earlier ones, but it does not explain the earlier ones. To put it in Hintikka's terms, "it would be ridiculous to say that Einstein's theory "explains" Maxwell's theory any more than it "explains" Newton's laws of motion". ([11], p. 510)

In what follows, we begin with the presentation of the GW-m (section 2). Then, we introduce standard deductive dialogical logic (section 3). And we add the rules to handle sentential abductive hypotheses (section 4.1), formal hypotheses (section 4.2) and frame-based hypotheses (section 4.3). We conclude with more general comments on how to understand abduction in dialogue (section 5).

2 GW Model of Abduction

In his wonderful book *Errors of Reasoning*, Woods [20] (pp. 364 ff.) carries on the definition of the GW-m of abduction, previously put forward by Gabbay & Woods [8]. Abduction is an inference triggered by an ignorance-problem. The relation between the premises and the conclusion is ignorance-preserving. It is pragmatically oriented since it provides the basis for new actions.

To begin, abduction is usually defined in reference to the well-known Peirce's schema (CP 5.189):

- The surprising fact C is observed.
- But if A were true, C would be a matter of course.
- Hence there is reason to suspect that A is true.

From the perspective of deductive reasoning, this scheme might be confused with the well-known fallacy of affirming the consequent. According to Woods [20] (p.135), a fallacy is not just an error. It is an error which is also attractive, universal and incorrigible. Woods coins the acronym EAUI (pronounced "Yowee") to refer to fallacies so conceived. Badness should also be added. But from the viewpoint of the cognitive economy, non-deductively valid arguments can have a cognitive virtue and be useful for various purposes.

In order to determine whether an argument is a fallacy or not, we first have to pay a peculiar attention to how the consequence-relation manifests itself in three non-equivalent ways:

- Consequence-having
- Consequence-spotting
- Consequence-drawing

According to Woods, consequence-having occurs in logical space and deals with entailment; between e.g. a statement A, or a set of statements Σ, and a statement B . Consequence-spotting is an epistemic achievement that occurs in the psychological space. It is knowing such an entailment relation. Consequence-drawing occurs in the inferential subspace of psychological space. Both are related. We cannot spot a consequence that there is not. And we cannot draw a consequence without having spotted it. However, it would be a mistake to reduce the two later relations to the former, as it is usually done in most of the standard approaches to deductive logic. In particular, correct inferences should not be reduced to deductive entailment. Indeed, in the case of abductive reasoning, the case might appear to be fuzzier. Let us illustrate the point with an example, taken from Peirce (CP 2.623):

1. All the beans from this bag are white.
2. These beans are white.
3. These beans are from this bag.

This argument does not compel with standard of deductive-entailment. The conclusion is only plausible. It is a hypothesis (to put it in Peirce's terms). Nonetheless, there could be good reasons to draw such a plausible conclusion from the two premises. However, consequence-drawing is not possible if there is no consequence-having. Is there any entailment-relation peculiar to abduction (or reasoning by hypotheses)? According to Woods [21] (pp. 148-9), there is no relation of logical consequence between the premises and the conclusion of an abductive reasoning. It would be more accurate to speak of a relation of conclusionality, an epistemic relation. Indeed, abduction is a response to an ignorance-problem. And, whereas deduction is truth-preserving and induction is likelihood-enhancing, the relation between the premises and the conclusion of abductive reasoning is ignorance-preserving. What does this mean?

Let Q be a question we cannot answer with our present knowledge and which acts as a cognitive irritant. Three situations are possible:

- Subduance: new knowledge removes ignorance (e.g., by discovering an empirical explanation),
- Surrender: we give up and do not look for an answer,
- Abduction: we set a hypothesis as a basis of new actions.

To put it in Woods's terms [20] (p. 368): "With subduance, the agent overcomes his ignorance. With surrender, his ignorance overcomes him. With abduction, his ignorance remains, but he is not overcome by it." Abduction is an agent-based and a goal-oriented inference, by means of which an agent draws conclusions in an ignorance-preserving way. Ignorance is not removed. The ignorance-problem is not solved. The conclusion of an abduction needs not be a true sentence or a new piece of knowledge. According to Woods [20] (p. 374), it needs not even be explanatory. So, what is specific to the relation between premises and conclusions in abduction? It is a relation in which a hypothesis is set as a reasoned basis for new actions, despite a persisting state of ignorance. As such, abduction has obviously a cognitive virtue from a pragmatic and an economic perspective (e.g. in relation to the agent's limitation of resources and abilities). Although consequence-drawing pertains to the psychological sphere, abduction does not assume any commitment to believe the conclusions that are drawn. Abduction "is a response that offers the agent a reasoned basis for new action in the presence of that ignorance" (Woods [20] (p. 368)).[3]

Let T be an agent's epistemic target at a specific time, K the agent's knowledge-base at that time, $K*$ an immediate successor-base of K, R an attainment relation for T (that is, $R(K,T)$ means that the knowledge-base K is sufficient to reach the target T), $\rightsquigarrow$ a symbol denoting the subjunctive conditional connective, for which no particular formal interpretation is assumed, and $K(H)$ the revision of K upon the addition of H. $C(H)$ denotes the conjecture of H and H^c its activation. Let $T!Q(\alpha)$ denotes the setting of T as an epistemic target with respect to an unanswered question Q to which, if known, α would be the answer. According to Woods [20] (p. 369), abduction has the following general structure.

1. $T!Q(\alpha)$
2. $\neg(R(K,T))$ [fact]
3. $\neg(R(K*,T))$ [fact]
4. $H \notin K$ [fact]
5. $H \notin K*$ [fact]
6. $\neg R(H,T)$ [fact]
7. $\neg R(K(H),T)$ [fact]
8. $H \rightsquigarrow R(K(H),T)$ [fact]
9. H meets further conditions $S_1, ..., S_n$ [fact]
10. Therefore, $C(H)$ [sub-conclusion, 1-7]
11. Therefore, H^c [conclusion,1-8]

Steps 2 and 3 state that the target $T!Q(\alpha)$ is not attained by means of the current knowledge or any immediate successor. Steps 3 and 4 says that the hypothesis H does not pertain to K or $K*$. Since H is only a hypothesis, and not a solution to the ignorance-problem, it does not relate to the cognitive target either (step 6), even in combination with the knowledge base (step 7). Thus, the hypothesis only relates subjunctively to the cognitive-target (step 8). This is how Gabbay and Woods understand Peirce's subjunctive in the second premise of the schema we previously mentioned (Peirce [16] (5.189)) and consequently how the "hence" should also be understood. That is, H is not assumed to be true or known, but that if it were true, it would relate to the target. Given

[3]This does not mean that the abducer "be wholly in the dark" [20](p. 371). For two kinds of ignorance depending on selective or creative abduction, see [4]; for a discusion between ignorance and knowledge enhancing, see [14] (ch. 3); for the ignorance preserving character of abduction (or ignorance mitigating), see [13] (chapter 2.1) and [14] (chapter 1.1).

certain conditions, to be specified, met by H (step 9), the hypothesis H can be conjectured (step 10). This is the first sub-conclusion. We write $C(H)$ to say that the hypothesis H is conjectured. Nevertheless, the abduction does not end in this step. Here, we face two possibilities: either we test the hypothesis, e.g. by empirical methods, and then three results are possible:

- the hypothesis is confirmed and we obtain a new piece of knowledge (included in a successor $K*$),
- the hypothesis is not confirmed, or invalidated, and we give it up,
- the hypothesis is not confirmed, but we maintain it anyway.

Or we directly use the hypothesis in a full abduction, in an ignorance-preservsing way (i.e. without confirmation) as a basis for new actions. The notation H^C keeps trace of the conjectural origin of the hypothesis (step 11). Following Woods [20] (p. 371) we will call an inference that ends at step 10 a *partial abduction*, and an inference continuing with step 11 a *full abduction*.[4]

3 Dialogical Logic

According to the GW-m, abduction is agent-centred. Thereby, abduction must be understood at the level of consequence-drawing. However, there cannot be consequence-drawing without consequence-having. In dialogical logic, validity is determined by the existence of a winning strategy for the proponent. Thus, the consequence-having is dialectified and the having-drawing distinction is somewhat broken.

More concretely, in dialogical logic, the proof process is approached through a game between the proponent of a thesis and the opponent, who challenges that thesis. The moves consist of challenges and defenses, that can be performed by means of two kinds of illocutionary acts: assertions and questions. Whereas deductive validity is defined in terms of existence of a winning strategy for the proponent, our proposal is that abductive conclusionality must be related to the existence of a hypothetical winning strategy. In this section, we begin with deductive dialogues[5], on the basis of which we give additional rules for abductive plays thereafter.

3.1 Basic Definitions

Let L be a propositional language, defined as follows:

$\varphi := \varphi | \varphi \wedge \varphi | \varphi \vee \varphi | \varphi \rightarrow \varphi | \neg \varphi$

Lower case letters p, q, r, ... refer to atomic formulas in L. We use lower case Greek letters φ, ψ, χ, ... to refer to L-formulas, and upper case Greek letters Γ, Σ, Δ, ... to refer to finite sets of L-formulas. To define the structural rules,

[4] For an application of full abduction in ancient medical diagnosis, see [1].

[5] Our definitions are inspired in the notations of Clerbout [5]. See also Rahman & Keiff [17] and Redmond & Fontaine [19] for other ways of presenting dialogical logic.

we will make use of two labels, **P** and **O**, standing for the players of the games, the *Proponent* and the *Opponent* respectively. The identities of **P** and **O** are not relevant at the local level.[6] That is why to define the particle rules we will make use of player variables **X** and **Y** (with $\mathbf{X} \neq \mathbf{Y}$). We will use force symbols '!' for assertions and '?' for requests. A move is an expression of the form $\mathbf{X}-e$ where **X** is a player variable and e is either an assertion or a request.

We use $n := r_i$ and $m := r_j$ with r_i, $r_j \in \mathbb{N}*$ for the utterance of the rank the players choose according to the rule **[SR0]** given in Section 2.3. Ranks are positive integers bounding the number of attacks and defences the players can perform in a play.

A *play* is a sequence of moves performed in accordance with the game rules. Since we want to study how a thesis is drawn from a set of premises, the initial thesis will be either a formula φ or an argument of the form $\psi[\varphi_1, ..., \varphi_n]$ which amounts to the claim that there is a winning strategy for the conclusion ψ given the concession of $\varphi_1, ..., \varphi_n$.[7] The premises $\varphi_1, ..., \varphi_n$ are referred to as the initial concessions. In case the premise set is empty, the initial thesis is simply ψ. The dialogical game for a claim $\psi[\varphi_1, ..., \varphi_n]$ (respectively ψ) is the set $\mathcal{D}(\psi[\varphi_1, ..., \varphi_n])$ (respectively $\mathcal{D}(\psi)$) of all the plays with $\psi[\varphi_1, ..., \varphi_n]$ (respectively ψ) as the initial thesis.[8]

For every move M in a given sequence $\mathcal{S}$ of moves, $p_{\mathcal{S}}(M)$ denotes the position of M in $\mathcal{S}$. Positions are counted starting with 0. We will also use a function F such that the intended interpretation of $F_{\mathcal{S}}(M) = [m', Z]$ is that in the sequence $\mathcal{S}$, the move M is an attack (if $Z = A$) or a defence (if $Z = D$) against the move of previous position m'.

3.2 Particle Rules

Dialogues are governed by two kinds of definitory rules: the particle rules and the structural rules. The particle rules of dialogical logic are given in the following table:

[6] The identities of **P** and **O** will be defined by means of the structural rule **[SR0]** given in Sect.2.3.

[7] In other words, the fact that **P** claims that he is able to draw ψ on the basis of φ_1, ..., φ_n by stating the tesis $\psi[\varphi_1, ..., \varphi_n]$ amounts to say something like "I can defend ψ under the concessions of φ_1, ..., φ_n".

[8] Where $\Sigma = \{\varphi_1, ..., \varphi_n\}$, we will sometimes write $[\Sigma]$ instead of $[\varphi_1, ..., \varphi_n]$, for the sake of presentation.

Assertion	Attack	Defence
$\mathbf{X}$-$!\varphi \wedge \psi$	$\mathbf{Y}$ - $?\wedge_L$ or $\mathbf{Y}$ - $?\wedge_R$	$\mathbf{X}$ - $!\varphi$ or $\mathbf{X}$ - $!\psi$ respectively
$\mathbf{X}$ - $!\varphi \vee \psi$	$\mathbf{Y}$ - $?\vee$	$\mathbf{X}$ - $!\varphi$ or $\mathbf{X}$ - $!\psi$
$\mathbf{X}$ - $!\neg\varphi$	$\mathbf{Y}$ - $!\varphi$	$- - -$ No Defence
$\mathbf{X}$ - $!\varphi \rightarrow \psi$	$\mathbf{Y}$ - $!\varphi$	$\mathbf{X}$ - $!\psi$
$\mathbf{X}$ - $!\forall x\varphi$	$\mathbf{Y}$ - $?x/k_i$	$\mathbf{X}$ - $!\varphi[x/k_i]$
$\mathbf{X}$ - $!\exists x\varphi$	$\mathbf{Y}$ - $?\exists$	$\mathbf{X}$ - $!\varphi[x/k_i]$
$\mathbf{X}$ - $!\psi[\varphi_1, ..., \varphi_n]$	$\mathbf{Y}$ - $!\varphi_1$ ... $\mathbf{Y}$ - $!\varphi_n$	$\mathbf{X}$ - $!\psi$

Particle rules are abstract descriptions consisting of sequences of moves such that the first member is an assertion, the second is an attack and the third is a defence (except in the case of negation, for which there is no possible defence). They are abstract because they are defined independently of any specific context of argumentation and independently of the players' identities. When a player **X** asserts a conjunction, he is committed to give a justification for both of the conjuncts. That is why the attacker (**Y**) requests the conjunct of his choice (left or right). In the case of a disjunction, it is the defender (**X**) who chooses. A universal quantifier is challenged by requesting an instantiation of **Y**'choice, whereas the choice is for **X** in the case of an existential quantifier. An attack may be a request or an assertion (in the case of the negation) or even a composite speech act (in the case of the conditional or an argument of the form $\psi[\varphi_1, ..., \varphi_n]$).

3.3 Structural Rules

The structural rules provide the global level of semantics:

[SR0][Starting Rule]

(i) If the initial thesis is of the form $\psi[\varphi_1, ..., \varphi_n]$, then for any play $\mathcal{P} \in \mathcal{D}(\psi[\varphi_1, ..., \varphi_n])$ we have:

- (ia) $p_{\mathcal{P}}(\mathbf{P}{-}!\psi[\varphi_1, ..., \varphi_n]) = 0$,
- (ib) $p_{\mathcal{P}}(\mathbf{O}{-}n := r_1) = 1$ and $p_{\mathcal{P}}(\mathbf{P}{-}n := r_2) = 2$.

(ii) If the initial thesis is of the form ψ, then for any play $\mathcal{P} \in \mathcal{D}(\psi)$ we have:

- (iia) $p_{\mathcal{P}}(\mathbf{P}{-}!\psi) = 0$,
- (iib) $p_{\mathcal{P}}(\mathbf{O}{-}n := r_1) = 1$ and $p_{\mathcal{P}}(\mathbf{P}{-}n := r_2) = 2$.

Clause (ia) (respectively (iia)) warrants that every play in $\mathcal{D}(\psi[\varphi_1, ..., \varphi_n]$

(respectively $\mathcal{D}(\psi)$) starts with **P** asserting the thesis $\psi[\varphi_1, ..., \varphi_n]$ (respectively ψ). In clause (ib) (respectively (iib)) the players choose their respective repetition ranks among the positive integers. We recall that a rank is a positive integer bounding the number of attacks and defences the players can perform in a play.[9] Clerbout [5] (p. 791) showed that there is a **P**-winning strategy when **O** chooses rank 1 if and only if there is a **P**-winning strategy for any other choice of **O**. Moreover, if **O** chooses rank 1 then if there is a **P**-winning strategy, **P** never has to choose a rank higher than 2 in order to win a dialogue. In some cases, **P** needs rank 2 because he needs the concession of both conjuncts of a conjunction asserted by **O** in order to win. By contrast, if there is an **O**-winning strategy, then **O** can win already by picking rank 1. (This is linked to the fact that **O** always chooses his rank first and that she does not play under the formal restriction.)

[SR1c][Classical Development Rule] For any move M in $\mathcal{P}$ such that $p_{\mathcal{P}}(M) > 2$ we have $F_{\mathcal{P}}(M) = [m', Z]$ where $Z \in \{A, D\}$ and $m' < p_{\mathcal{P}}(M)$. Let r be the repetition rank of Player **X** and $\mathcal{P} \in \mathcal{D}\psi[\varphi_1, ..., \varphi_n]$ (respectively $\mathcal{D}(\psi)$) such that:

- the last member of $\mathcal{P}$ is a **Y**-move,
- $M_0 \in \mathcal{P}$ is a **Y**-move of position m_0,
- there are n moves M_1, ..., M_n of player X in $\mathcal{P}$ such that $F_{\mathcal{P}}(M_1) = F_{\mathcal{P}}(M_2) = ... = F_{\mathcal{P}}(M_n) = [m_0, Z]$ with $Z \in \{A, D\}$.

Let N be an **X**-move such that $F_{\mathcal{P}\frown N}(N) = [m_0, Z]$. We have $\mathcal{P} \frown N \in \mathcal{D}(\varphi)$ if and only if $n < r$.[a]

[a] "$\mathcal{P} \frown N$" denotes the extension of $\mathcal{P}$ with N.

[SR1c] ensures that after the repetition ranks have been chosen, every move either is an attack or a defence against a previous move made by the other player; players move alternately, and the number of attacks and defences they can perform in reaction to a same move is bounded by their repetition ranks. Intuitionistic dialogical games are defined with a rule **[SR1i]**, by modifying **[SR1c]** so that the repetition ranks bound only the number of challenges, and players can defend only once against the last non-answered challenge. This illustrates how different logics can be distinguished at the structural level, without having to change the local level.

[SR2][Formal rule] The sequence $\mathcal{S}$ is a play only if the following condition is fulfilled: if $N =$**P** $-!\psi$ is a member of $\mathcal{S}$, for any atomic sentence ψ, then there is a move $M =$**O**$-!\psi$ in $\mathcal{S}$ such that $p_{\mathcal{S}}(M) < p_{\mathcal{S}}(N)$.

[9] A move M' performed by **X** in a dialogue is a repetition of a previous move M if (i) M' and M are two attacks performed by **X** against the same move N performed by **Y**, or (ii) M' and M are two defences performed by **X** in response to the same attack N performed by **Y**. The ranks guarantee the finiteness of plays by limiting the repetitions allowed in a dialogue.

This rule means that **P** can assert an atomic sentence ψ only if **O** previously asserted the same atomic sentence ψ. Then, we define the notion of **X**-terminal:

[D1][X-terminal] Let $\mathcal{P}$ be a play in $\mathcal{D}(\psi[\varphi_1, ..., \varphi_n])$ (respectively $\mathcal{D}(\psi)$) the last member of which is an **X**-move. If there is no **Y**-move N such that $\mathcal{P} \frown N \in \mathcal{D}(\psi[\varphi_1, ..., \varphi_n])$ (respectively $\mathcal{D}(\psi)$) then $\mathcal{P}$ is said to be **X**-terminal.

And the winning rule for plays:

[SR3][Winning Rule for Plays] Player **X** wins a play $\mathcal{P} \in \mathcal{D}(\psi[\varphi_1, ..., \varphi_n])$ (respectively $\mathcal{D}(\psi)$) if and only if $\mathcal{P}$ is **X**-terminal.

According to **[SR3]**, **X** wins a play if it is**Y**'s turn to play and no move is available to**Y**. The rules of the game do not say anything about validity or how to play. Dialogical validity is grasped at the strategic level. The thesis of **P** is valid if and only if **P** has a winning strategy according to the following definition:

[D2][Winning]

1. A strategy of a player **X** in $\mathcal{D}(\psi[\varphi_1, ..., \varphi_n])$ (respectively $\mathcal{D}(\psi)$) is a function s_x which assigns a legal **X**-move to every non terminal play $\mathcal{P} \in \mathcal{D}(\psi[\varphi_1, ..., \varphi_n])$ (respectively $\mathcal{D}(\psi)$) the last member of which is a **Y**-move.
2. A **X**-strategy is winning if it leads to **X**'s win no matter how **Y** plays.

On the basis of the definition of winning strategy, we can define the notion of consequence for dialogical **CL** (classical logic); that is, a dialogical logic played with **[SR0]**-**[SR3]**, the so-called **CL**-rules:

[D3][CL-Consequence] $\Sigma \vdash_{CL} \psi$ (respectively $\vdash_{CL} \psi$) iff according to the **CL**-rules, there is a **P**-winning strategy for the thesis $\psi[\varphi_1, ..., \varphi_n]$ (respectively ψ).

A similar definition of consequence for dialogical logic **IL** (intuitionistic logic) is obtained by substituting the **IL**-rules to the **CL**-rules; i.e. by substituting **[SR1i]** to **[SR1c]**.

In dialogical logic, the existence of a proof is determined by the existence of a winning strategy. Consequence-having and consequence-drawing cannot be approached independently of one another. That is why dialogical logic appears to be a good candidate to understand the relation of conclusionality at stake in abduction, thought of as an agent-based inference.

4 Abductive Dialogues

Abductive dialogues are *triggered* by a concession-problem. That is, the Proponent has not the required concessions to find a winning strategy according to the standard rules of deductive dialogical logic. The Proponent's target is thus to search what would be a winning strategy, despite this lack of concessions. An ampliative structural rule allows the Proponent to introduce, or to *guess*, a formula whose conjecture serves as a basis for new moves that hypothetically lead him to victory. Such a hypothetical play is a dialogical "scant-resource adjustment strategy", to put it in Woods's terms ([20] (p. 371)), where the resources are the concessions of the Opponent. As stressed by Barés and Fontaine [2], the illocutionary force of a hypothetical move is different from the assertions of standard deductive dialogues. Being hypothetical, the *commitment* they carry is weaker. Indeed, the process must be unconceded-preserving; that is, what is conjectured by the Proponent remains unconceded and might be defeated. In what follows, we present three kinds of abductive dialogues, based upon different kinds of hypotheses. First, we define a sentential abductive dialogical logic, namely ADA^r (section 4.1); second we present a formal abductive dialogical logic, IAD (section 4.2); third, we have a general look at frame-based dialogues, SSD (section 4.3).

4.1 Sentential Hypotheses: the Case of ADA^r

Sentential abduction, as Magnani ([14], p. 2016) defines it, is related to logic and to verbal or symbolic inferences. A hypothesis is formed by relying to the sentential aspects of natural or artificial languages, like in the case of logic. ADA^r, the Adaptive Dialogic for Singular Fact Abduction, is based upon a dialogical form of affirming the consequent. That is, a player is allowed to ask for the antecedent of a conditional by asserting the consequent only if he commits himself to additional conditions, which can be defeated. The main idea is that dialogically affirming the consequent triggers a hypothetical dialogue in which the Proponent looks for an alternative (non-deductive) strategy. The conjectural aspect of the hypothesis is handled by means of concepts that have their roots in Batens and Meheus's adaptive logic for abduction.[10] Actually, an adaptive dialogical logic can be seen as the dynamic articulation of two logics: a lower limit dialogical logic (**LLD**) and an upper limit dialogical logic (**ULD**), which in some sense ampliates the range of possible moves allowed for a player. Moves applied in accordance with the **ULD**-rules are subject to additional conditions; we call them the "conditional moves". The condition can be challenged in accordance with the relevant notions of abnormality and adaptive strategy, depending on the adaptive logic considered.

More concretely, ADA^r is defined by the following triple:

1. Lower Limit Dialogical Logic (**LLD**) = classical rules (**[SR0]**, **[SR1]**, **[SR2.1]**, **[SR2.2]**, **[SR3]**)

[10] See also [9] for a quick presentation. Most of the rules are inspired in [3].

2. Set of abnormalities $= \Omega = \{(\forall\alpha)(A(\alpha) \to B(\alpha)) \wedge B(\beta) \wedge \neg A(\beta))|$no predicate occurring in B occurs in $A\}$

3. Adaptive Strategy = Reliability

In addition to the **LLD**-rules of standard classical deductive logic, a rule will be added to perform conditional moves; namely **[SR4.1.1]**, an ampliative rule that allows affirming the consequent. By applying this rule, a player commits himself to its reliability; that is, affirming the consequent should not yield an abnormality pertaining to the set Ω. This condition will be explicitly indicated in the dialogue. Moves of ADA^r are now sequences of the form $\mathbf{X}{-}e - C - d$, where $\mathbf{X}$ and e are as before, and C is the corresponding condition. Moreover, when challenging the condition, the burden of proof may change, and the Opponent may become subject to formal restriction. This dynamic in the application of the formal rule can be handled by distinguishing subdialogues: d is either the main dialogue - in which case we write d_1 - or a subdialogue - in which case we write $d_{1.i}$ for the i-th subdialogue.

The structural rules of the previous section can be generalized in that way; that is, a dialogue begins with an empty condition in the subdialogue d_1. Then, the introduction of conditions and the passage to subdialogues are governed by the other structural rules. The classical development rule **[SR1]** is likewise generalized to conditional moves by replacing moves of the form $\mathbf{X}{-}e$ with moves of the form $\mathbf{X}{-}e - C - d$. The formal rule **[SR2]** will be substituted by **[SR2.1]** and **[SR2.2]**. The other rules of ADA^r are defined as follows: in order to define the ampliative rule **[SR4.1.1]** of ADA^r, we must define the notion of A-Move by means of which the affirmation of the consequent is restricted to universally quantified formulas[11]:

[D4][A-Move] The Move M is an A-Move if the following condition is satisfied: M is a move $M = \mathbf{X}{-}!\varphi(x/k) \to \psi(x/k) - \emptyset - d$ such that:

(i) $p_{\mathcal{S}}(M) = m$,

(ii) $F_{\mathcal{S}}(M) = [n, D]$,

(iii) $n = p_{\mathcal{S}}(N)$ such that $N = \mathbf{Y}{-}?k - \emptyset - d$ and $F_{\mathcal{S}}(N) = [m_1, A]$, and

(iv) $p_{\mathcal{S}}(M_1) = m_1$ such that $M_1 = \mathrm{X}{-}(\forall x)(\varphi \to \psi) - \emptyset - d$.

We also need to define the following notion of reliability for ADA^r:

[11] Such a restriction is motivated by the fact that singular fact abductions are performed on the basis of general laws, but also in order to avoid paradoxes of implication.

[D5][Reliability] Let $\varphi[\Sigma]$ the thesis of the **P**. A challenge $\psi(x/k)$ on an A-move $\varphi(x/k) \to \psi(x/k)$ is reliable with respect to Σ iff there is no $Dab(\Theta)$ such that:

(i) $(\forall x)(\varphi \to \psi) \wedge (\psi(x/k) \wedge \neg\varphi(x/k)) \in \Theta$, and

(ii) $\Sigma \vdash_{LLD} Dab(\Theta)$, and

(iii) $\Sigma \nvdash_{LLD} Dab(\Theta/(\forall x)(\varphi \to \psi) \wedge (\psi(x/k) \wedge \neg\varphi(x/k))\}$.

Conditional moves commit to reliability as defined in **[D5]**. The condition C they carry has the form of an abnormality and the player who utters it is committed to show that this abnormality cannot be derived unconditionally (i.e. within the **LLD**) from he premise-set Σ. In ADA^r, a simple strategy is not sufficient. That is why we consider the move as being unreliable if C pertains to a disjunction of abnormalities ($Dab(\Theta)$) that can be derived unconditionally.[12] A player who challenges the reliability of the condition must take the burden of the proof of $Dab(\Theta)[\Sigma]$ (i.e., to draw the $Dab(\Theta)$ from $[\Sigma]$) in a subdialogue in which he plays under formal restriction with the **LLD**-rules. The $Dab(\Theta)$ must be minimal, as stated by clause (iii), otherwise, it would always be possible to introduce a Dab containing the condition.

[SR4.1.1][Abductive Rule] The sequence $\mathcal{S}$ is a play only if the following condition is fulfilled: If there is amove $N = \mathbf{X}-!\psi(x/k) - C - d$ in $\mathcal{S}$ such that:

(ia) $n = p_{\mathcal{S}}(N)$

(iia) $F_{\mathcal{S}}(N) = [(m_1, ..., m_2), A]$, and

(iiia) $m_1 = p_{\mathcal{S}}(M_1), ..., m_n = p_{\mathcal{S}}(M_n)$ such that $M_1 = \mathbf{Y}-!\varphi_1(x/k) \to \psi(x/k) - \emptyset - d$, ..., $M_n = \mathbf{Y}-!\varphi_n(x/k) \to \psi(x/k) - \emptyset - d$,

then the two following conditions hold:

(ib) $M_1, ..., M_n$ are A-Moves, and

(iib) $N = \mathbf{X}-!\psi(x/k) - \mathfrak{R}^{\Sigma}_{\psi(x/k_1)|\neg\varphi_1(x/k),...,\varphi_n(x/k)} - d$,

and there is an extension $S \frown M'$ with M'such that:

(ic) $F_{\mathcal{S}}(M') = [n, D]$ and $M' = \mathbf{Y}$-$\varphi_1(x/k) \vee ... \vee \varphi_n(x/k)\ - \emptyset|n - d$-, or

(iic) $F_{\mathcal{S}}(M') = [n, A]$ and $M' = \mathbf{Y} - ?\mathfrak{R}^{\Sigma}_{\psi(x/k_1)|\neg\varphi_1(x/k),...,\varphi_n(x/k)} Dab(\Theta) - \emptyset - d$.

[12] Where $Dab(\Theta)$ is a disjunction of abnormalities on Ω, a simple strategy can be used only if the following condition holds: $\Sigma \vdash_{LLD} Dab(\Theta)$ only if there is an $A \in \Theta$ such that $\Sigma \vdash_{LLD} A$.

It is worth noting that according to **[SR4.1.1]**, a player **X** is allowed to affirm the consequent to challenge a set of several A-Moves sharing the same consequent by a unique conditional move. This is because ADA^r relies on what Gauderis ([9], p. 256) calls "practical abduction". That is, let Σ be such that $\Sigma = \{(\forall x)(Px \to Qx), (\forall x)(Rx \to Qx), Qk\}$. A player could introduce two singular hypotheses, namely Pk or Rk, or he could opt for a more cautious attitude by introducing a disjunctive hypothesis of the form $Pk \vee Rk$. This he does by means of move $X{-}!\psi(x/k) - \mathfrak{R}^{\Sigma}_{\psi(x/k_1)|\neg\varphi_1(x/k_1),\ldots,\neg\varphi_n(x/k_1)} - d$. The intended meaning of $\mathfrak{R}^{\Sigma}_{\psi(x/k_1)|\neg\varphi_1(x/k_1),\ldots,\neg\varphi_n(x/k_1)}$ is that $!\psi(x/k)$ is reliable in view of Σ ; more specifically that $(\forall x)((\varphi_1 \vee \ldots \vee \varphi_n) \to \psi) \wedge (\psi(x/k) \wedge \neg(\varphi_1(x/k) \vee \ldots \vee \varphi_n(x/k))$ is not unconditionally derivable from Σ. The reliability of the conditional move, as defined in **[D5]**, can be challenged in accordance with the following particle rule for the reliability operator $\mathfrak{R}$.

Partice rule for the reliability operator $\mathfrak{R}$		
Assertion	Attack	Defence
$\mathbf{X}{-}!\varphi - \mathfrak{R}^{\Sigma}_{\psi} - d_1$	$\mathbf{Y}{-}?\mathfrak{R}Dab(\Theta)$ where $\psi \in \Theta$	$\mathbf{X}{-}!\mathfrak{F}_{\Sigma}(Dab(\Theta)) - \emptyset - d_1$
		Or X counter-attacks $\mathbf{X}{-}\mathfrak{I}_{\Sigma}(Dab(\Theta/\psi) - \emptyset - d_1$ (where $(Dab(\Theta/\psi) \neq \emptyset$

The main idea of the particle rule is that **Y** introduces a minimal disjunction of abnormality $Dab(\Theta)$ such that $\psi \in \Theta$. Then, **X** faces two possibilities. Either he claims that $Dab(\Theta)$ cannot be **LLD**-drawn from Σ; this he does by making use of the failure operator $\mathfrak{F}$ whose meaning is given by another particle rule. Or he claims that $Dab(\Theta)$ is not minimal (i.e. a smaller disjunction without γ is **LLD**-derivable); this he does by making use of the indispensability operator $\mathfrak{I}$ whose meaning is also given by another particle rule.

Particle rule for the failure operator $\mathfrak{F}$		
Assertion	Attack	Defence
$\mathbf{X}{-}!\mathfrak{F}_{\Sigma}\varphi - \emptyset - d_1$	$\mathbf{Y}{-}?\varphi[\Sigma] - \emptyset - d_{1.i}$ **Y** opens a subdialogue $d_{1.i}$	$---$ No defence

The meaning of the $\mathfrak{I}$-operator is given by the following rule:

Particle rule for the indispensability operator $\mathfrak{I}$		
Assertion	Attack	Defence
$\mathbf{X}{-}!\mathfrak{I}_{\Sigma}\varphi - \emptyset - d_i$	$\mathbf{Y}{-}?\mathfrak{I}_{\Sigma}\varphi - \emptyset - d_i$	$\mathbf{X}{-}\varphi[\Sigma] - \emptyset - d_{i.j}$ **X** opens a subdialogue $d_{i.j}$

That is, **X** must show that a $Dab(\Theta)$ shorter than the one introduced by **Y** can be unconditionally derived from Σ.

When **Y** challenge the failure operator, he takes the burden of the proof of $\varphi[\Sigma]$ and must play under formal restriction, even if $\mathbf{Y} = \mathbf{O}$. That is why **Y** opens a subdialogue, in which he commits himself to defend $\varphi[\Sigma]$ by means of the **LLD**-rules.

We thus replace **[SR2]** by **[SR2.1]** and **[SR2.2]**, and we add **[SR4.2]**:

[SR2.1][Formal Restriction for Adaptive Dialogues] If **X** *plays under formal restriction*, then the sequence Δ is a play only if the following condition is fulfilled: if $N = \mathbf{X}{-}!\psi - C_j - d$ is a member of Δ, for any atomic sentence ψ, then there is a move $M = \mathbf{Y}{-}!\psi - C_i - d$ in Δ such that $p_\Delta(M) < p_\Delta(N)$.

[SR2.2][Application of Formal Restriction] The application of the formal restriction is regulated by the following conditions:

(i) In the main dialogue d_1, if $\mathbf{X} = \mathbf{P}$, then **X** plays under the formal restriction.

(ii) If **X** opens a subdialogue $d_{1.i}$, then **X** plays under the formal restriction.

[SR4.2][Adaptive LLD-Rule] In a subdialogue $d_{1.i}$, only **LLD**-rules apply. (I.e., in ADA^r, **[SR0]-[SR3]**.)

We now to illustrate the particule rule for the reliability operator$\mathfrak{R}$ and the failure operator $\mathfrak{F}$: Let Σ be $\Sigma = \{(\forall x)(Px \to Rx), (\forall x)(Qx \to Rx), Rk_1\}$. For the sake of lisibility, let $Dab(\Theta)$ introduced by **O** at move 9 be a short for $((\forall x)(Px \to Rx) \wedge (Rk_1 \wedge \neg Pk_1)) \vee ((\forall x)((Qx \wedge \neg Px) \to Rx) \wedge (Rk_1 \wedge \neg(Qk_1 \wedge \neg Pk_1)))$:

Dialogue 1							
		O			**P**		
d_1							
					$Pk_1[\Sigma]$	$\emptyset$	0
1	$\emptyset$	$r = 2$			$r = 2$	$\emptyset$	2
3.1	$\emptyset$	$\forall x(Px \rightarrow Rx)$					
3.2	$\emptyset$	$\forall x(Qx \rightarrow Rx)$	0				
3.3	$\emptyset$	Rk_1					
5	$\emptyset$	$Pk_1 \rightarrow Rk_1$		3.1	$?x/k_1$	$\emptyset$	4
7	$\emptyset$	$Qk_1 \rightarrow Rk_1$		3.2	$?x/k_1$	$\emptyset$	6
				5	Rk_1	$\mathfrak{R}^{\Sigma}_{Rk_1 \mid \neg Pk_1}$	8
9	$\emptyset \mid 8$	$?\mathfrak{R}Dab(\Theta)$	8		$\mathfrak{F}_{\Sigma}Dab(\Theta)$	$\emptyset \mid 8$	10
$d_{1.1}$							
11	$\emptyset$	$Dab(\Theta)[\Sigma]$	10		$---$		
					$\forall x(Px \rightarrow Rx)$	$\emptyset$	12.1
13	$\emptyset$	$Dab(\Theta)$		11	$\forall x(Qx \rightarrow Rx)$	$\emptyset$	12.2
					Rk_1	$\emptyset$	12.3
15	$\emptyset$	$(\forall x)(Px \rightarrow Rx)$ $\wedge(Rk_1 \wedge \neg Pk_1)$		13	$?\vee$	$\emptyset$	14
17	$\emptyset$	$Rk_1 \wedge \neg Pk_1$		15	$?\wedge_R$	$\emptyset$	16
19	$\emptyset$	$\neg Pk_1$		17	$?\wedge_R$	$\emptyset$	18
		$---$		19	Pk_1	$\emptyset$	20
21	$\emptyset$	$(\forall x)((Qx \wedge \neg Px) \rightarrow Rx)\wedge$ $(Rk_1 \wedge \neg(Qk_1 \wedge \neg Pk_1))$					
23	$\emptyset$	$(\forall x)((Qx \wedge \neg Px) \rightarrow Rx)$		21	$?\wedge_L$	$\emptyset$	22
25	$\emptyset$	$(Qk_2 \wedge \neg Pk_2) \rightarrow Rk_2$		23	$?x/k_2$	$\emptyset$	24
33	$\emptyset$	Rk_2		25	$Qk_2 \wedge \neg Pk_2$	$\emptyset$	26
27	$\emptyset$	$?\wedge_L$			Qk_2	$\emptyset$	28
29	$\emptyset$	$?x/k_2$	12.2		$Qk_2 \rightarrow Rk_2$	$\emptyset$	30
31	$\emptyset$	Qk_2	30		Rk_2	$\emptyset$	32
35	$\emptyset$	$Rk_1 \wedge \neg(Qk_1 \wedge \neg Pk_1)$		21	$?\wedge_R$	$\emptyset$	34
37	$\emptyset$	Rk_1		35	$?\wedge_L$	$\emptyset$	36
39	$\emptyset$	$\neg(Qk_1 \wedge \neg Pk_1)$		35	$?\wedge_R$	$\emptyset$	38
		$---$			$Qk_1 \wedge \neg Pk_1$	$\emptyset$	40
41	$\emptyset$	$?\wedge_R$	40		$\neg Pk_1$	$\emptyset$	42
43	$\emptyset$	Pk_1	42		$---$		

Explanation: In standard deductive dialogues, it is sufficient for **P** and **O** to choose ranks 2 and 1 respectively. However, given that **O** plays under formal restriction in a subdialogue, he must choose rank 2 as well.[13] A standard deductive dialogue would end in move 7. In ADA^r, **P** is allowed to perform an

[13] This might yield unecessary repetitions that will be omittted in the next dialogues for the sake of presentation.

ampliative move, what he does in move 8 by applying **[SR4.1.1]**. **O** counter-attacks the condition (move 9) and introduces a disjunction of abnormalities $Dab(\Theta)$. **P** defends himself in accordance with the particle rule for the $\mathfrak{R}$-operator (move 10). By challenging the $\mathfrak{F}$-operator, **O** opens a subdialogue, in which he takes the burden of proof of $Dab(\Theta)[\Sigma]$ (move 11). In the subdialogue, **LLD**-rules apply and **O** must play under formal restriction. **P** challenges **O**'s thesis by conceding the premises (move 12.1, 12.2, 12.3), and **O** answers with $Dab(\Theta)$ (move 13). Both players will then play alternately, untill move 43, the final move played by **O**, who thus wins the subdialogue. Then, we come back to the main dialogue and it is still **P**'s turn. Since there is no more move available to **P**, he loses. In some sense, after the subdialogue, the main dialogue continues as if the move 8 had been defeated and had not been played. In a dialogue in which other moves would be possible, **P** could try to continue otherwise, but here he cannot.

Defeasibility is in some sense handled by means of a winning rule for subdialogue, which says when and how we come back to the main dialogue:

[SR3.1][Winning rule for subdialogues] A subdialogue $d_{1.i}$ is won by **X** if it is **Y**'s turn and there are no more moves available to **Y**. If **X** wins the subdialogue, we return to the main dialogue d_1 in which it is (still) **Y**'s turn.

Finally, we define a notion of consequence for ADA^r:

[D6][ADA^r-consequence]$\Sigma \vdash_{ADA^r} \varphi$ iff according to theADA^r-rules there is a **P**-winning strategy for the thesis $\varphi[\Sigma]$.

We conclude this section with another illustration. In the GW-m a distinction is made between full abduction and partial abduction. In a full abduction, the hypothesis is conjectured as a basis for new moves:

Dialogue 2

		O			**P**		
			d_1				
					$Sk_1[\Sigma]$	$\emptyset$	0
1	$\emptyset$	$r = 2$			$r = 2$	$\emptyset$	2
3.1	$\emptyset$	$\forall x(Px \to Rx)$					
3.2	$\emptyset$	$\forall x(Px \to Sx)$	0		Sk_1	$\emptyset \mid 8$	12
3.3	$\emptyset$	Rk_1					
5	$\emptyset$	$Pk_1 \to Rk_1$		3.1	$?x/k_1$	$\emptyset$	4
7	$\emptyset$	$Pk_1 \to Sk_1$		3.2	$?x/k_1$	$\emptyset$	6
9	$\emptyset \mid 8$	Pk_1		5	Rk_1	$\mathfrak{R}^{\Sigma}_{Rk_1 \mid \neg Pk_1}$	8
11	$\emptyset \mid 8$	Sk_1		7	Pk_1	$\emptyset \mid 8$	10

Explanation: In this dialogue, the conditional move 8 triggers a hypothetical play in which **O** hypothetically concedes Pk_1(move 9). Then, Pk_1 is conjectured and serves as a basis for new moves. Finally, **P** wins by stating Sk_1on the basis of the conjecture of Pk_1. Notice that **O** might challenge the condition after move 8, that we omit here since this would not yield any change in the final victory of **P**. This dialogue provides a justification of step 11 in the GW-m; that is, **P** has a hypothetical winning strategy for an initial thesis of the form $Sk_1[\Sigma]$ on the basis of the conjecture of Rk_1. A partial abduction would be represented by a dialogue in which an initial thesis $Sk_1[\Sigma]$ would be defended by means of the introduction of Sk_1 itself. This would provide a justification of the step 10 in the GW-m.

In ADA^r, conditional moves trigger hypothetical plays. Abductive dialogues are unconceded-preserving, the concession-problem is not solved and what has not been conceded remains unconceded. The winning strategy put forward by **P** is only hypothetical; i.e. deductive validity has not been proven. Although we might consider that an abductive winning strategy is weaker than deductive ones, conditional moves exhibit some kind of dialogical virtue by ampliating the range of possible plays in an argumentative interaction. ADA^r displays other interesting features, such as the impossibility to justify the introduction of random hypotheses for tautological formulas or to introduce inconsistent hypotheses to justify random claims, but we cannot discuss it in more details here.

4.2 Formal Hypotheses: the Case of IAD

Abduction has not always the form of affirming the consequent. Sometimes, hypotheses concern the meaning of connectives or the context of argumentation itself, and the rules of the dialogue. This assumes a certain dynamic in the relation of consequence that can be handled thanks to the pluralism underlying the dialogical framework. In this section, we illustrate the idea with the case of IAD, the Inconsistency-Adaptive Dialogical Logic of Beirlaen and Fontaine [3].

Classical dialogues are explosive: from an inconsistent set of premises, we can derive anything. This reflects the validity of the well-known *Ex Falso Sequitur Quodlibet* (EFSQ). But it is not uncommon that inconsistencies occur in the course of an argumentative interaction. And usually, this is not a reason to infer random statements or to stop the interactive process. That is why we may agree to begin with a paraconsistent logic. **P** is not allowed to assume the normal behaviour of a negation unless **O** made that concession for another occurrence of the same negative formula previously. This is enough to block the EFSQ. But it has the undesirable effect that perfectly acceptable inferences like the disjunctive syllogism are now invalid. In IAD, **P** may conjecture the normal behaviour of the negation by means of conditional moves. If it can be shown by **O** that **P** relies on an abnormality, an inconsistency, then the conditional move is not reliable. IAD is defined according to the following triple:

1. **LLD** = Paraconsistent Dialogical Logic (**[SR0]-[SR3]** + **[SR5]**),
2. $\Omega =_{DF} \{\varphi \wedge \neg\varphi | \varphi \text{ is a formula}\}$,

3. Strategy = Reliability

The **LLD** of IAD is defined by adding the negation rule **[SR5]** to the rules of standard deductive dialogical logic:

[SR5][Negation Rule] The sequence $\mathcal{S}$ is a play only if the following condition is fulfilled: If there is a move $N = \mathbf{P}-!\psi - C - d$ in the sequence $\mathcal{S}$ such that:

(i) $n = p_{\mathcal{S}}(N_1) = n_1$,

(ii) $F_{\mathcal{S}}(N_1) = [m_1, A]$, and

(iii) $m_1 = p_{\mathcal{S}}(M_1)$ such that $M_1 = \mathrm{O}-!\neg\psi - C - d$.

Then, there is a move $M_2 = \mathrm{O}-!\psi$ in $\mathcal{S}$ such that:

(i) $p_{\mathcal{S}}(M_2) = m_2$ and $m_2 < n_1$,

(ii) $F_{\mathcal{S}}(M_2) = [n_2, A]$, and

(iii) $n_2 = p_{\mathcal{S}}(N_2)$ such that $N_2 = \mathrm{O}-!\neg\psi - C - d$.

Intuitively, this rule means that **P** is allowed to challenge a negated formula $\neg\psi$ only if **O** has already challenged an occurrence of the same negated formula before.

Then, let Ω the set of abnormalities be $\Omega =_{DF} \{\varphi \wedge \neg\varphi | \varphi \text{ is a formula}\}$. Reliability can be defined as follows:

[D7][Reliability] Let $\varphi[\Sigma]$ be the thesis of the Proponent. A formula ψ behaves reliably with respect to Σ iff there is no formula $Dab(\Theta)$ such that:

(i) $\psi \wedge \neg\psi \in \Theta$,

(ii) $\Sigma \vdash_{LLD} Dab\Theta$, and

(iii) $\Sigma \nvdash_{LLD} Dab(\Theta / \{\psi \wedge \neg\psi\})$.

Finally, we add the following rule:

[SR5.1][IAD Negation Rule] The sequence $\mathcal{S}$ is a play only if the following condition is fulfilled: If there is a move $N = \mathbf{P}-!\psi - C - d$ in the sequence $\mathcal{S}$ such that:

(i) $n = p_{\mathcal{S}}(N)$

(ii) $F_{\mathcal{S}}(N) = [m, A]$, and

(iii) $m = p_{\mathcal{S}}(M)$ such that $M = \mathbf{O}-!\neg\psi - \emptyset - d$

Then one of the following two conditions holds:

(i) N is performed by **P** in accordance with the **LLD** negation rule **[SR5]**, or

(ii) $N = \mathbf{P}-!\psi - \mathfrak{R}^{\Sigma}_{\psi} - d$ where $\mathfrak{R}^{\Sigma}_{\psi}$ abbreviates that ψ behaves reliably in view of the premise set Σ.

The rules for the $\mathfrak{R}$-operator, and consequently the rules for the failure and the indispensability operators ($\mathfrak{F}$ and $\mathfrak{I}$ respectively) are defined as before (applied in accordance with the relevant sets of abnormalities).

Dialogue 3							
		O			**P**		
d_1							
					$q[p \vee q,\, \neg p,\, p]$	$\emptyset$	0
1	$\emptyset$	$r = 2$			$r = 2$	$\emptyset$	2
3.1	$\emptyset$	$p \vee q$					
3.2	$\emptyset$	p	0				
3.3	$\emptyset$	$\neg p$					
5	$\emptyset$	p		3.2	$?\vee$	$\emptyset$	4
		$- - -$		3.3	p	$\mathfrak{R}^{\Sigma}_{p}$	6
7	$\emptyset \mid 6$	$?\mathfrak{R}p \wedge \neg p$	6		$\mathfrak{F}_{\Sigma}(p \wedge \neg p)$	$\emptyset \mid 6$	8
...	...	...	...	...	...	...	...

Explanation: **P** is allowed to challenge 3.3 by means of a conditional move; i.e. by assuming that the negation in $\neg p$ has a normal behaviour. Then, **O** challenges the reliability operator by claiming that $p \wedge \neg p$ can be derived from Σ. The subdialogue is obviously won by **O**, we do not give the details here. By contrast, if **P**'s initial thesis had been $q[p \vee q,\, \neg p]$, he would have had a winning strategy in IAD.

Although it had not been originally oriented towards the study of abduction, considering IAD as an abductive process highlights the originality and the scope of applications of a dialogical understanding of abduction. From a wider perspective, we can see adaptive rules as various devices to articulate different logics in a pluralist framework. Since the purpose of the rules we are looking for is to increase the range of possibilities for a player, dialogical hypotheses are not intrinsically explanationist. If looking for an explanation to a fact P consists in looking for another fact A such that the union $T \cup A$ of the background theory T and A allows deriving P[14], the kind of formal hypothesis displayed by IAD cannot be explanatory, at least in such a consequentialist view. Indeed, the conjecture impacts the relation of entailment between the premises and the conclusions, not the set of premises.[15] Such a hypothesis brings into the context of the dialogue information external to the argument itself.

How defeasibility and non-monotony are represented in these dialogues deserves further comments. Given that we have focused on the strategic level, defeasibility remains difficult to understand. Indeed, strategic level is concerned with the existence of winning strategies. At the strategic level, we only recapitulate optimal moves in order to display the existence of a winning strategy. Therefore, moves are never strictly speaking defeated, otherwise this would mean that one player has not made optimal moves. Another way to grasp defeasibility would be by considering a dynamic introduction of new information. In that case, players would play optimally wioth respect to a given state of information but their moves might be revised in the light of new information. Albeit a possible option, this goes beyond the scope of this paper. This feature of our dialogues seems to meet Dutilh Novaes's argument according to which monotonicity and non-defeasibility are consequences of how dialogues are defined. By considering natural deduction as having internalized the opponent, thereby motivating the inferential steps in reasoning process, Duthil Novaes [7] argues that the Opponent is a stubborn ideal interlocutor, which allows the Proponent proving validity by always performing optimal moves. This is what she called the Built-In Opponent. Defeasibiltiy would assume, in some sense, suboptimality. Recently, Rahman et al. [18] have shown that Dutilh Novaes's argument relied on a confusion between the play level and the strategic level. In the context of the dialogical reconstruction of the Constructive Type Theory, meaning is primarily given at the play level, by the definitory rules, independently from the strategic level. Therefore, defeasibility of moves is probably to understand at the play level. At the play level, if the condition associated with a conditional move was successfully challenged by the Opponent, then we might consider that the move is defeated. At the strategic level, what is defeated is the argument; i.e. defeasibility is understood in terms of non-monotony. It means that the existence of a P-winning strategy for $\varphi[\Sigma]$ does not warrant the existence of a P-winning strategy for $\varphi[\Sigma \cup \psi]$, where ψ does not pertain to Σ. Although the details of this discussion go beyond the scope of the present paper, our dialogue sheds a light non only on abductive reasoning, but also on how to understand defeasibility in dialogues - e.g. through the distinction between defeasibility of a move and defeasibility of the initial thesis.

[14]As suggested by Hintikka ([11], p. 507), while criticising the explanatory nature of abductive hypotheses.

[15]Gabbay and Woods [8] (p. 41) also think that abduction is not necessarily explanationist and suggest a similar example in which abduction consists in looking for different logical rules.

4.3 Frame-Based Hypotheses: the case of *SSD*

We finally briefly explain the main idea behind the Structure Seeking Dialogues (*SSD*) of Rahman and Keiff [17] and Keiff [12], which illustrate what we call a frame-based abduction. The *SSD* also involve a dynamic of rules related to hypotheses about the underlying modal framework. The *SSD* are grounded on modal dialogical logics, in which moves are sequence $\mathbf{X}{-}e - c$, where $\mathbf{X}$ and e are like before and where c is an assignment of context (possible world) to a formula. The language is enriched by means of the two modal operators, $\Box$ and $\Diamond$, for the necessity and the possibility, respectively. Their local meaning is given by the following particle rules:

Assertion	Attack	Defence
$\mathbf{X}$-$!\Box\varphi - i$	$\mathbf{Y}$ - $?\Box/j - i$	$\mathbf{X}$ - $!\varphi - j$
$\mathbf{X}$-$!\Diamond\varphi - i$	$\mathbf{Y}$ - $?\Diamond - i$	$\mathbf{X}$ - $!\varphi - j$

For the sake of clarity, we first illustrate the use of these operators in the following dialogue, without structural restriction:

Dialogue 4

		c	**O**			**P**	c		
						$\Box p \to \Box\Box p$	0		0
1		0	$\Box p$	0		$\Box\Box p$	0		2
3	$R(0,1)$	0	$?\Box/1$	2		$\Box p$	1		4
5	$R(1,2)$	1	$?\Box/2$	4		p	2		8
7		2	p		1	$?\Box/2$	0	$R(0,2)$	6

In this dialogue, when **O** challenges move 2, he also concedes that the context 1 is accessible from the context 0. Similarly, when he challenges move 4, he concedes that context 2 is accessible from context 1. Then, when **P** challenges move 1, he assumes that context 2 is accessible from context 0. These concessions and assumptions are made explicit in an additional column and noted by means of first-order relation, $R(0,1)$, $R(1,2)$ and $R(0,2)$ respectively. The crucial move is that assumption made by **P** in move 6. This move is not allowed if the underlying modal logic system is K, in which case **P** is not allowed to introduce a context, and its accessibility, by challenging a box or by defending a diamond unless **O** has previously conceded that the same context was accessible. This means that the use of modal contexts in K are also subject to **O**'s concessions. Now, the leading idea of *SSD* is that **P** is allowed to introduce a hypothesis as a basis for new actions. That is, we begin without assumptions concerning the underlying modal framework. Then, as it is the case in Dialogue 7, **P** is allowed to introduce the hypothesis that the modal frame is transitive. After move 6, the dialogue follows *as if* the underlying modal frame was transitive. In other words, **P** assumes that the following rule for S4 applies:

[SR-S4][S4 Context Rule] Let $\Delta \in \mathcal{D}(\varphi)$ whose last member is an O-move.

(1) Let us assume that $M_0 \in \Delta$ with $M_0 = O{-}!\Box\psi{-}i$ and $p_\Delta(M) = m_0$. Let $\Delta \frown N$ the sequence such that $F_{\Delta \frown N}(N) = [m_0, A]$: $\Delta \frown N \in \mathcal{D}(\varphi)$ iff [RS-K] is observed and:

- (i) O has chosen c_j in c_i in Δ, or
- (ii) $c_i = c_j$, or
- (iii) There is a c_k such that c_k is available in c_i and c_j is available for P in c_k in Δ.

(2) Let us assume that $M_0 \in \Delta$ with $M_0 = O{-}?\Diamond\psi{-}i$ and $p_\Delta(M) = m_0$. Let $\Delta \frown N$ the sequence such that $F_{\Delta \frown N}(N) = [m_0, D]$: $\Delta \frown N \in \mathcal{D}(\varphi)$ iff [RS-K] is observed and:

- (i) O has chosen c_j in c_i in Δ, or
- (ii) $c_i = c_j$, or
- – There is a c_k such that c_k is available in c_i and c_j is available for P in c_k in Δ.

And the dialogue runs with that ruel instead of the K context rule.

5 Conclusions

Dialogical logic sheds light on the strategic dimension of abduction. Different kinds of hypotheses can be conjectured, at different levels, and set as bases for hypothetical plays in which the Proponent looks for alternative (non-deductive) winning strategies. This emphasizes the instrumental, rather than explanatory, nature of abductive hypotheses.

We are not rejecting the explanatory nature of hypotheses in general. We only consider that it does not characterize abductive inference in its specificity. Actually, if explanation is understood in consequentialist terms, the explanatory nature of sentential hypotheses of ADA^r could be recognized. However, what is fundamental is the strategy looked for by the Proponent in the course of hypothetical plays, a hypothesis is conjectured as a basis for a new course of action. The hypotheses conjectured in IAD and SSD cannot be understood as explanatory in consequentialist terms, since it is the consequence relation itself that is the object of the conjecture. At best, such conjectures could be hypotheses that might explain what could make valid the initial thesis of the Proponent. But, since the initial thesis is not valid, they cannot be actual explanations. Here we meet the subjunctive attainment relation of the GW-m (step 8) and the fact that abductive dialogues are unconceded-preserving in the sense that what has not been conceded by the Opponent remains unconceded at the end of the dialogue, or better said only hypothetically conceded rather than assertorically conceded.

Interestingly, IAD and SSD might be conceived as formal pendents of model-based abduction. The term “model-based reasoning”, as introduced by Nersessian [15], is used to indicate the construction and manipulation of various kinds of representations, not mainly sentential and/or formal, but mental (visual imagistic, analogical, etc.) and/or related to external mediators. According to Magnani ([14], p. 213), a considerable part of the abductive processes is model-based. That is, a considerable part of hypothesis creation and selection is occurring in the middle of a relationship between brains and model-based aspects of external objects and tools that have received cognitive and/or epistemological delegations. The point of abductive dialogue is not only to complete a set of premises in order to derive the conclusion, but also to look for hypotheses of other levels and external to the argument under consideration. In the case of IAD we look for another set of rules, another logic, or another meaning of a connective. The SSD involve considerations relative to the modal framework and the rules for the use of contexts in dialogues. Abductive dialogues lie between sentential and model-based abductions.

Finally, another way we have not explored yet is the manipulative nature of dialogical abduction. Magnani ([13], p. 213) defines manipulative abduction as a process in which a hypothesis is formed and evaluated resorting to a basically extra-theoretical and extrasentential behavior that aims at creating communicable accounts of new experiences to integrate them into previously existing systems of experimental and linguistic (theoretical) practices. For example, humans make use of the construction of external diagrams in geometrical reasoning, useful to make observations and “experiments” to transform one cognitive state into another for example to discover new properties and theorems. Although we have limited ourselves to define the rules for abductive dialogues at the play level (the level of the definitory rules), we could study the manipulative aspect of dialogical abduction by moving to the strategic level and the different ways of winning a game. By manipulating various possible course of the game, through hypothetical plays, manipulation would occur at the strategic level, by looking for an alternative strategy which may recommend a course of actions.

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