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Mathematics Creativity in Elementary Teacher Training

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Abstract

Creativity plays an important role in mathematics learning, so teachers must provide students with appropriate learning opportunities. This means using tasks, in particular those with multiple solutions and/or multiple resolutions, that usually require creative thinking and it could be a possible way to promote creativity in students. In this paper, we identify some traits of creativity in elementary pre-service teachers through tasks productions used during math classes.

Key words: Creativity; Tasks; Problem Solving and Posing; Elementary Pre-service Teachers.

Introduction

Students' mathematical literacy is usually determined by how they use their knowledge, skills and attitudes to solve problems. It is therefore necessary to offer them diverse experiences in order to develop their abilities for problem solving, so they can take advantage of lifelong learning mathematics. In order to deepen students' understanding of mathematics, we need to recognize that the mastery of rules, algorithms, and strategies is not the end goal of mathematics education, instead they should use these procedural tools to explore, test and defend their solutions to meaningful problems.

Creativity plays an important role in mathematics learning, however there is no single description of creativity, but it can be argued that it begins with curiosity and involves students in exploring and experimenting, raising their imagination and originality (e.g. Conway, 1999; Silver, 1997). Research has showed that what students learn is largely influenced by the tasks given to them (Doyle, 1988; Stein & Smith, 2009; Vale, 2009), in particular those with multiple solutions and/or multiple resolutions that usually require creative thinking (e.g. Leikin, 2009; Leikin & Pitta-Pantazi, 2013). Research also has shown that problem posing and problem solving in mathematics are closely related to creativity and can be characterized by three dimensions: fluency, flexibility and originality (e.g. Conway, 1999; Leikin, 2009; Silver, 1997). Thus, learning environments, where tasks give students opportunities to use several strategies to solve and formulate their own problems, may involve them in rich mathematical explorations, increase their motivation and encourage them to investigate, make decisions, generalize, look for patterns and connections, communicate, discuss ideas and identify alternative paths.

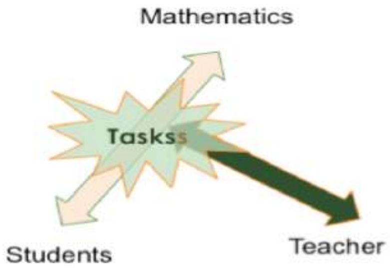
In this sense the initial training courses in education must propose tasks for future teachers to experience and deeply explore, so they can use them with their future students. Thus we developed an exploratory study with elementary pre-service teachers where our main question was: Does problem solving and posing tasks have the potential to identify the dimensions of creativity in future teachers through their written productions?

Mathematics teaching

Nowadays we are experiencing deep changes in different areas of society, in particular in mathematics education. So mathematics educators have a great challenge to face mainly in the development of higher order thinking skills in students, such as formulating and solving problems, reasoning and communication. We need as well to analyze what features of teaching and learning are associated with better performance in mathematics by improving the quality of the main agents of change: teachers. These factors justify innovative strategies to improve teaching and learning, in particular tasks and other instructional materials that call for independent and critical problem solvers.

The teacher unquestionably affects students learning, either with his knowledge and conceptions on teaching and learning of mathematics, or with the choices he makes and the actions he develops in his practice (Vale, 2009). In the context of mathematics classrooms, teacher, students, and (mathematical) content are linked in a system, which several researchers called “the didactical triangle” (Sträβer, 1994) or the “instructional triangle” (Lampert & Ghousseini, 2012), where teaching depends on the coordination of students’ active engagement in meaningful mathematical work, and on the materials that are used to represent the content to be learned in teacher-student interaction. In particular, the teacher’s role in this instructional system is to assure that students are engaged with content in a productive way, where we can highlight the role of the tasks (Figure 1).

Figure 1: The instructional triangle (adapted from Lampert & Ghousseini, 2012; & Sträβer, 1994)



Many of the fragilities that students have in learning mathematics are due to teachers' conceptions and attitudes, which influence their actions in the classroom, and the interactions with the students, but also to the gaps in mathematical knowledge and in innovative teaching strategies.

The tasks that teachers select for their classes are fundamental and characterize their work (Stein & Smith, 2009). The orientation of inquiry, discussion and reflection of ideas is critical to student learning and arises only when teachers have a good knowledge of the subject they teach, how they teach it and when to teach it. Thus, it is important that teachers develop certain skills, including creative ones, based on a deep mathematical knowledge and teaching of subjects, allowing them to build, adapt, and exploit good mathematical tasks for the classroom. On the one hand, teachers must propose tasks that help motivate their students to learn and develop their creative thinking, on the other hand teachers should themselves formulate creative tasks and teaching strategies to offer their students. It is therefore crucial that teachers can take advantage of all the potential contained in a task and, for that, they need to have opportunities to explore and solve them in the same way they think to explore with their own students. Among all the different tasks that we use in mathematics classes, problem solving plays an important role in the learners' competences, involving rich discussions that are cognitively challenging and are the primary mechanism for promoting conceptual understanding of mathematics (Stein & Smith, 2009). Problem solving tasks must be also revisited through new approaches, encouraging students to be persistent and look for creative ideas in order to raise a flow of mathematical ideas, flexibility of thought and originality in the responses.

An exploratory teaching in which the teacher promotes conditions for students to discover and construct their own knowledge is the key to achieve this objective. We pay special attention to the work in figurative contexts due to their importance in any mathematical activity, being a learning component with many possibilities and often neglected in the learning trajectory of all students. Such importance is due to the fact that the display is not only related with the mere illustration, but also recognized as a component of thinking, deeply involved in the conceptual and not just in the perceptual. Students without this visual capacity will have great difficulty in learning. *Seeing* is an important component to explore mainly the generalization ability and this can only be developed through experiences that require this type of thinking (e.g. Barbosa, 2011; Rivera & Becker, 2005; Stylianou & Silver, 2004; Vale & Pimentel, 2011).

Recognizing the central role of tasks in the teaching and learning process, it is necessary to involve teachers in their selection and preparation in order to acquire a deeper awareness of their effectiveness and educational value. So it is important to have rich tasks to help teachers develop creativity skills with their students.

Mathematics and creativity

Mathematics is naturally engaging, useful, and creative and challenging tasks usually require creative thinking. Challenge in mathematics requires prior knowledge and is related to solving high-level mathematical tasks where memorization is not enough, and among other things involves the use of procedures with connections. According to Liljedahl and Sriraman (2006) students' ability to present new ideas and/or (re)solutions to problems in mathematics is considered as an indicator of creativity.

Defining mathematical creativity is a very complex task. It is argued that it begins with curiosity and engages students in exploration and experimentation, involving imagination and originality. Several authors (e.g. Conway, 1999; Leikin, 2009; Meissner, 2005; Silver, 1997; Vale et al., 2012), consider that creativity involves divergent thinking contributing to higher order reasoning, which highlights three main dimensions: fluency, flexibility and originality (novelty). Fluency is the ability to generate a large number of different solutions obtained by the student for the same task. The flexibility is the ability to produce a variety of different ideas about the same problem, organized in various categories. Originality is the ability to create ideas that have been identified as unique as compared to students in the same group.

Environments where students have the opportunity to solve mathematical problems, using diverse strategies, and formulate their own problems, allows them to be engaged in their exploration, increasing motivation and encouraging them to investigate, to make decisions, to look for patterns and connections, to generalize, to communicate, to discuss ideas and identify alternatives. However, we can say that problem posing has been a rather neglected component of classroom mathematics but is as crucial in the learning of mathematics as problem solving. So, tasks that may contribute to the development of creative mathematical explorations should involve solving and formulating problems. Thus, teachers should provide tasks that raise multiple (re)solutions, triggering a flow of mathematical ideas, flexibility of thought and originality in the responses (e.g. Leikin, 2009; Vale, Pimentel, Cabrita, Barbosa & Fonseca, 2012).

Creativity is a topic that should be part of the mathematics programs at all educational levels, but it is still much neglected in math classes, because teachers are unaware and/or have not yet perceived its importance. For all these reasons it should be given special attention to teacher training, providing them experiences that will enable the acquirement of a deep understanding of mathematics teaching and of how to teach.

Methodology

We adopted a qualitative approach to understand in what way an experience, grounded on tasks that privilege figurative contexts, is a suitable way for promoting creativity through its three dimensions, and consequently foster mathematical knowledge. In particular we are interested in finding potentialities in the used tasks that allow the identification of traits of creativity in students, future elementary teachers (future teachers of children from 6 to 12 years old), through the written resolutions of the tasks. So our main question was: Does problem solving and posing tasks have the potential to identify the dimensions of creativity in future teachers through their written productions?

The didactical experience was grounded on theoretical references about creativity, focused on problem solving and posing, where we used several tasks which required multiple (re)solutions (e.g. Conway, 1999; Leikin, 2009; Silver, 1997). Each task was not intentionally designed to enhance one specific component of mathematical creativity, although, in some cases, one of the components was more relevant, but they had to be challenging for the students. Whenever possible, we privileged figurative contexts and patterning tasks (e.g. Vale & Pimentel, 2011; Vale et al., 2012). We used ten tasks with 21 future elementary teachers, which were applied during the didactics of mathematics classes, a two hours period two times a week, during three weeks. In these classes we intended to discuss the role of creativity in mathematics with students, assuming an exploratory dynamics where: (a) problem solving was the work context; (b) students were asked to analyze and discuss the tasks proposed; (c) communication was privileged, particularly the use of questioning and of different representations; and (d) a set of challenging tasks were used which raised multiple (re)solutions within problem solving and posing. We provided opportunities for students to share ideas and clarify their reasoning; develop convincing arguments regarding why and how things work; develop a language for expressing mathematical ideas; and learn to see things through other people's perspective. Students worked individually with the supervision of the teacher, followed by a collective discussion (Stein & Smith, 1998). Data was collected through classroom observations and written productions of the tasks. Data analysis consisted of measuring students' creativity through the three dimensions - fluency, flexibility, originality - for problem solving and problem posing tasks, following the basic ideas of Conway (1999) and Silver (1997). No score was assigned to the students concerning these dimensions, instead we made an overall analysis of the work presented, considering the frequency of the most common and the most original responses.

Some results

Several tasks of different kinds were used during the didactical experience. We present here three of those tasks (Figure 2) that require producing various and different responses. In this paper we shall highlight the responses of the group of students involved.

Figure 2: Examples of tasks

Task 1 - The Cherries

1. Find different ways to count the cherries.
2. Write the numerical expression that translates the way of counting them.

Tasks 2- The dots

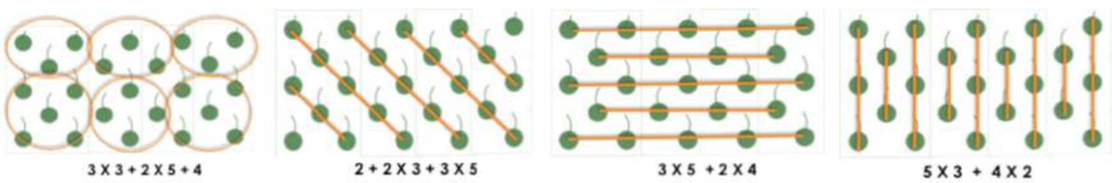
1. Imagine that the given figure is the 1st term of a growing sequence. Draw the next terms.
2. Write a numerical expression translating a way to calculate the nth number term of the sequence you've built.
3. Imagine that the growing sequence you constructed began with the 2nd term. Draw the 1st term. Construct as much sequences as you can.

Task 3 - The expression

Suppose that you are an elementary school teacher and you intended to propose to your students a problem that could be translated by the numerical expression 3x5+2. Invent problems that can be translated by the given expression.

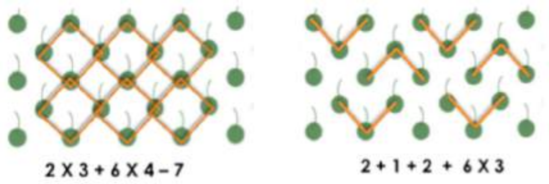
Task 1. This type of task requires students to see the arrangement in different ways, connecting previous knowledge about numeric relationships and their relations with basic geometric concepts. There are different ways to count the arrangement of the cherries and each counting can be respectively written through a numerical expression that translates the students' thinking and way of *seeing*. This task suggested students many and varied ways of visualizing the counting of the cherries. Figure 3 illustrates the summary of the most common resolutions, with the respective expressions corresponding to each way of *seeing*.

Figure 3: The most common resolutions



These expressions must be verbalized, for instance, like the example: "I see three groups of three cherries each, two groups of five cherries and one group of four cherries" or " I see three horizontal rows with five cherries each and two horizontal rows with four cherries". It is important to discover that each expression illustrates one way of seeing but they are all equivalent and correspond to the same number of cherries, that is 33. Figure 4 illustrates the most original responses.

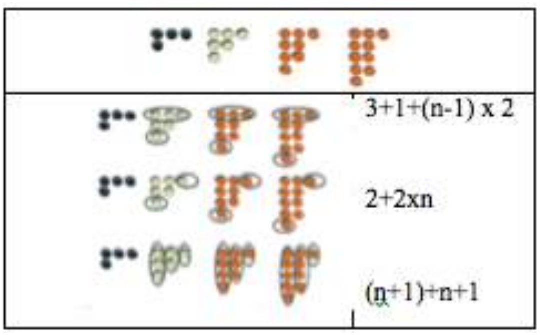
Figure 3: The most original resolutions



It is important that future teachers realize that they must encourage their students to look for many ways of seeing and explain what they visualize. Teachers should also highlight mathematical concepts involved in each of the tasks proposed to students. We can stress that most of the students revealed fluency and flexibility in their productions. Concerning fluency, almost all the students presented more than one correct solution. As for flexibility, it was revealed in the different approaches used to count the cherries that we analyzed through the categorization constructed, from the resolutions proposed by the students.

Task 2. We intended that students constructed as much growing pattern sequences as they could and, after looking for the pattern in the figurative sequence, described it, producing arguments to validate it and using different representations. The previous work with visual counting may help to see a visual arrangement that changes in a predictable form and write numerical expressions translating the way of seeing, in order to make possible the generalization to distant terms. They achieved a general rule through schemes, drawings or tables, but mainly used functional reasoning that allowed them to accomplish far generalization. We will regard only the different ways of seeing the pattern to achieve far generalization, as we are convinced that it is the most important aspect of solving these tasks in which students can be creative. Figure 5 illustrates some different ways of seeing one of the sequences that the students built and the respective algebraic expression that generalizes that rule. We can highlight that most of the students revealed fluency and flexibility. Concerning fluency almost all the them presented more than one correct solution (all the students used two or three from figure 5). Flexibility is revealed in the different approaches used to reach the nth term, that we analyzed through the categorization that we constructed, from the proposed resolutions by the students. Nevertheless, none of the students confirmed the equivalence of the expressions obtained.

Figure 5: Different ways of seeing



This task had one additional question that was to imagine, for each sequence constructed, that it began with the 2nd term and they had to draw the 1st term. This part of the task wasn't completely solved by all the students. Figure 6 includes only the most common answers, from the students who completed the task. It was difficult for them to work backwards and discover the new first term. The last solution (Figure7) was the most original since only one student presented it.

Figure 6: The most common answers

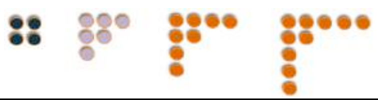
		
3+1+(n-1) x 2	4+(n-1) x 2	$n^2 + 2$

Figure 7: The most original answer


(n+1)(n+2)/2

Task 3. This task, despite not having a visual context, could lead to the formulation of a problem using visual contexts. However this situation only occurred with one of the students (Figure 8). It was a unique solution. The previous work on visual counting, previously developed with these students, could have influenced the choice of this student.

Figure 8: The most original answer

Write a numerical expression for counting the dots quickly.

The common solutions presented by the students were statements identified as closed problems using only the basic arithmetic operations (Figure 9).

Figure 9: The most common answer

António went to the bookstore and bought 3 books that costed 5 euros each and a pencil for 2 euros. How much did António spent?	Mary's aunt gave three euros to each of her three nephews and two euros to Maria. Write an expression that translates the money spent by Mary's aunt.

We can conclude that most of the students revealed fluency, since all of them presented more than one correct solution. Flexibility wasn't observed because, despite students having presented many resolutions, all the proposals had the same structure. They were almost all of the same type of the examples in figure 9, just varying the contexts (e.g. cakes, animals, fruits, shells, CDs, tables, chairs, chocolates, flowers, dots).

Main Conclusions

The tasks used involved problems to solving and pose and were selected to foster creativity of elementary students, beginning with their future teachers. They promote a flexible approach looking for multiple (re)solutions, using different representations. They also involve elementary mathematical concepts, can be applied in different contexts of the classroom and recontextualized for different students and other contents, allowing students to explore beyond mathematical concepts and processes. The counting and pattern tasks (tasks 1 & 2) were the ones that led to more resolutions. The problem posing task (task 3) followed the most evident and elementary approach and the proposals weren't different from those obtained by elementary students. Fluency and flexibility were largely identified mainly in the counting tasks.

Not all students have the same performance on tasks. The discussions raised in the classroom resulted in a better understanding of some of the more confusing aspects of the tasks. Our concern was not to categorize students but to identify potentialities in the tasks to develop creativity in students, detecting their mathematical strengths or weaknesses, and that patterns have potentialities. It can be said that the students were engaged and willing to overcome the barriers they experienced. We concluded that the used tasks have great potential to promote creativity in students, however not all the tasks provide the development of the three dimensions of creativity. Students recognized that both flexibility and originality encourage divergent thinking, which promotes higher-level thinking, so as future teachers they should seek out appropriate curricular materials to develop mathematical creativity with their own students.

Future teachers must become themselves creative thinkers and they must be aware to act in the same way with their own students, encouraging them to seek unusual and original responses. Different students have used these tasks in different academic years and the results were very similar, which leads us to infer that the results may depend more on the tasks than on students, but we need to further examine this aspect.

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