Study of Economic Inequality: Econophysics Perspective

Ana Pais da Silva
Mestrado em Engenharia Física
Departamento de Física e Astronomia
2023

Orientador
José Abílio de Oliveira Matos, Professor Auxiliar, Faculdade de Economia da Universidade do Porto

Coorientador
João Manuel Viana Parente Lopes, Professor Auxiliar, Faculdade de Ciências da Universidade do Porto

U.PORTO

FACULDADE DE CIÊNCIAS
UNIVERSIDADE DO PORTO

MSc

Contents

List of Figures

List of Tables

Acknowledgments

I want to begin by expressing my gratitude to Professor Pedro Jorge, our course director, for accepting the proposal of this thesis in the field of Econophysics. I extend my gratitude to both my supervisor, Professor José Matos, and my co-supervisor, Professor João Viana, for accepting the challenge of guiding me through this academic endeavor, thank you for all the advises and the useful conversations.

I would also like to thank my family, especially my parents and brother. Over the course of eight years, they unwaveringly believed in my ability to complete this academic journey, even during moments when I doubted it myself. This gratitude extends to my loyal and playful companion, Billy, my dog, who has been by my side throughout the past three years.

Finally, my most profound and significant thanks go to Dr. Ana Pinto. I am immensely grateful for her invaluable role in helping me gain a full awareness of my own limitations and how to deal with them. Without her support and guidance, the daunting thoughts I once had of never reaching the end of this journey would have likely become my reality today.

Summary

Historically, economic inequality received limited attention in economic discourse. However, recent revelations have underscored the potential negative repercussions of excessive inequality on both economic stability and societal well-being. This thesis aims to shed light on the dynamics of economic inequality by exploring the consequences of money redistribution among homogenous economic agents. To achieve this objective, we conducted extensive computer simulations employing diverse money exchange rules and initial money distributions. These distributions ranged from perfect equality (Dirac distribution) to extreme inequality (Power distribution). Key statistical metrics, including the Gini coefficient, percentiles, and kurtosis values, were gathered to facilitate comparative analyses. Our investigation revealed that the rules governing money exchange significantly influenced the level of inequality within the final money distribution. Surprisingly, the initial money distribution displayed a comparatively minor influence on the outcomes. While acknowledging the simplicity and limitations of this study, it provides valuable insights into the impact of money redistribution on economic inequality. Our findings highlight the critical role of money exchange mechanisms in shaping the distribution of money within an economy.

Keywords: Economic Inequality, Money Redistribution

Resumo

Historicamente, a desigualdade económica tem recebido pouca atenção no discurso e na teoria económica. No entanto, estudos mais recentes destacam as potenciais repercussões negativas da excessiva desigualdade tanto na estabilidade económica quanto no bem-estar social. Esta tese tem como objetivo obter um perceção sobre a dinâmica da desigualdade económica, explorando as consequências de diferentes redistribuições da moeda entre agentes económicos homogéneos. Para atingir esse objetivo, realizaram-se diversas simulações computacionais utilizando diferentes regras na troca de moeda e diferentes distribuições iniciais de moeda. Essas diferentes distribuições iniciais variaram desde a igualdade perfeita (distribuição Dirac) até a desigualdade extrema (distribuição de potência). Foram recolhidos valores estatísticos, como o coeficiente de Gini, percentis e valores de curtose, de forma a realizar uma análise comparativa. Neste trabalho foi possível verificar o impacto significativo que diferentes regras de troca de moeda têm na desigualdade da distribuição final da moeda. Surpreendentemente, a distribuição inicial da moeda exerceu uma influência comparativamente menor nos resultados. Reconhecendo a simplicidade e as limitações deste estudo, ainda assim é possível observar o impacto da redistribuição de moeda na desigualdade económica.

Palavras-chave: Desigualdade Económica, Redistribuição da Moeda

1 Introduction

While my academic journey has been centered around physical engineering, my curiosity spans through a vast spectrum of knowledge domains. So, this master's thesis served as a remarkable opportunity to indulge in subjects that have long captivated my interest: physics, economics, and history. Additionally, it provided a valuable platform for improving my coding skills. My motivation for undertaking this interdisciplinary work comes from a deeply held belief that comprehensive problem-solving necessitates a panoramic view. To truly grasp and evaluate complex issues, one must immerse themselves in the widest possible context, exploring both historical and socioeconomic dimensions. This holistic approach was a choice of my responsibility, and it is intended to reflect the way I enjoy looking for answers to any questions/problems I encounter. One of the main topics in economy that sparks my interest is economic inequality and its relation with monetary theory. Like any other macroeconomic phenomena this one as well is highly complex to study and understand. One might argue that economic equality is something unreachable however, my intention is to argue on how much inequality can be endured until it starts undermining the health of an economic system and its social fabric. So, when I came across with the works of Victor Yakovenko [2] [3], using statistical physics to debate and present a new perspective on monetary theory and economic inequality, I saw it as an acceptable excuse to bring economics to the realm of my master's degree thesis in physical engineering. As Econophysics is an area I am approaching for the first time I asked myself on the why these two seemingly different subjects, physics and economics, where brought together. Only to find out that Econophysics was not the first attempt in history to study economy through a physics lens.

For these reasons Chapter 2 of this thesis was entirely dedicated to the exploration of the longstanding interconnection between physics and economics: In the 18th century, the formal division of knowledge led to specialized approaches in various disciplines. Physics had long employed the scientific method, while economics encompassed politics, law, and philosophy. Physics advanced rapidly, becoming both a science of credible predictions and a foundation of the industrial revolution. Adam Smith, regarded as the father of modern economics, drew inspiration from the scientific method and the work of scientists like Isaac Newton. However, it was in the 19th and early 20th centuries that economics underwent a profound transformation. Economists such as Walras, Jevons, Edgeworth, and Fisher sought to elevate economics from a social science conglomerate to a natural science by rigorously applying the scientific method. They aspired to make economics a respected field with predictive power, much like physics. While physics evolved in the 19th century, economics remained tethered to earlier paradigms. This disparity sparked debates. In the late 20th century, some physicists ventured into economics with fresh perspectives, giving rise to Econophysics.

In Chapter 3 it is presented and discussed one of the most difficult concepts to characterize in economy and yet one of the most important to understand inequality, money. It is given an historical view of its origins and it is presented the different existing theories on how money is seen in economic theory. Moreover, it is introduced as well the Econophysics perspective on money theory provided by Yakovenko [4].

Chapter 4 is a computational exploration of based on Yakovenko 's work cited above. It is discussed how statistical physics could be associated with economics in relation to the money distribution. Moreover the computer simulations performed were based on a simple agent-based model featuring homogeneous economic agents engaging in random money exchange processes. It explores the impact of different rules governing money exchange and different initial money distributions in the final money distribution. The main focus in this chapter is the collective behavior and the statistical results of the final money distribution in order to assess its level of inequality.

Chapter 5 is a extension of the previous chapter work with agent-based models. However, in this chapter I made modifications to the code to enable individual-level tracking of agents. The aim is to conduct a comparative analysis focusing on three specific agents: the richest, the poorest, and an agent with a mid-level of money. Additionally, it was developed a mathematical deduction method to assess the average monetary fluctuations of an agent within an economic model comprising multiple levels, each associated with the amount of money owned by agents.

At last, in the final chapter 6 there was a final reflection on the results obtained, the limitations behind the assumptions of the model and the future possibilities of agent-based model methods.

2 Origins and History of Econophysics

The interconnection between physics and economics has a longstanding history that predates the modern era of specialized knowledge domains. In previous times, intellectuals possessed a broad spectrum of knowledge spanning fields such as law, science, philosophy, engineering, and the arts. However, access to such knowledge was limited to a select few. To comprehend the origins of economics as a distinct field of study and the role physics played in its development, it is essential to examine historical contexts. This chapter aims to explore significant historical events, notable individuals, and their contributions to the emergence and evolution of economics, paying particular attention to how physics influenced economic thought. Philip Mirowski, a historian and philosopher of economic thought, made a thorough analysis [5] of this connection and the overall unintended consequences it may have caused [6].

2.1 The rise of Classical Economy

The 18th century witnessed a critical period in history that contributed indirectly to the specialization and fragmentation of knowledge domains. However, this period did not occur suddenly but was rather the result of a three-century period of rediscovering classical texts and breaking the monopoly of knowledge held by the Church. The Enlightenment, a cultural and intellectual movement in the European Continent, marked a rupture from traditional sources of authority and a rejection of dogmatic and superstitious thought in favor of reason and empirical observation. This movement had a profound impact on the restructuring of political thought, giving rise to new ideals such as democracy, human rights, and the rule of law. However, this transformation was unevenly spread across Europe, as political, economic, military, cultural, and intellectual factors led to substantial divergence between the countries' development. Two nations, France and Great Britain, emerged as epicenters of the movement.

These two states have a history of enduring political and economic rivalry that usually culminated in direct or indirect wars. The 18th century was a particularly violent one for Europe and specially between this two nations. The aftermath of the ongoing conflict witnessed the emergence of Great Britain as the dominant world power.

The Enlightenment movement flourished in Europe during a time of significant political and economic instability. It was during this era that a highly effective public postal service was developed [7], and small publishers emerged in various regions. These publishers produced books, small magazines, and journals that were accessible to a broader audience beyond the aristocracy. This new era of print culture facilitated the rapid exchange of ideas and more efficient communication between intellectuals and thinkers across Europe.

The dissemination of ideas across Europe led to a growing awareness of one's place in the world, prompting the reconsideration of previously discarded ideas and the acceptance of new ones. The authority of knowledge shifted to philosophy and reason, and even in science, which had long been oppressed by the Church, a systematic approach to scientific discovery emerged, giving rise to fields such as physics and astronomy. It became increasingly apparent that human knowledge was vulnerable to bias and subjective interpretation. Therefore, a systematic approach to scientific discovery became essential to minimize interference and foster an objective perspective. This stance, coupled with significant advancements in mathematical theory, which served as a universal scientific language, facilitated the exploration of diverse areas of knowledge and enabled a deeper understanding of them.

The field of Economics emerged as a distinct discipline in the 18th century, following a period of significant transition in Europe, particularly in England. Prior to this period, the world economy was based on the economic policy of mercantilism, which prioritized the maximization of exports and the minimization of imports, with the ultimate goal of reducing account deficits. This required measures to be taken in order to secure the accumulation of precious metals like gold, which were used as monetary reserves at the time. The two main ways of securing positive money reserves were either to extract them from colonies or to ensure a positive trade balance. During the 17th century, Portugal and Spain were the biggest colonizers with rich land in precious metals, which meant that both countries' monetary reserves depended on the exploitation of their colonies since their trading balances were chronically deficient. However, France and England faced difficulties in accessing money reserves in this way, so they soon realized the importance of commerce in securing their geopolitical power. England's strategy relied on a strong navy and long-distance trade, whereas France explored the need for a positive trade balance at a higher level. This included controlling what farmers would plant, the quantity exported, and increasing taxes over farmer's crops, which raised significant opposition [8].

In the early 18th century, some prominent figures in France developed an alternative theory to mercantilist policies on how a nation's wealth was created. The physiocrats, led by François Quesnay, were the precursors of classical economic theory. Influenced by the ideas of the Enlightenment, Quesnay argued that labor, specifically agricultural labor, was the sole source of value. He advocated for less state intervention, landlords' rents, and not farmers' income, should be taxed, markets should work freely without price fixations or restrictions on exportations. "Laissez-faire la nature" was the main message of the physiocrats [8]. Upon closer examination of François Quesnay's work in political economy, one may discern a fundamental structure of economic thought that is comparable to the mechanics of the human body. Before entering the field of political economy, François Quesnay, who was already an esteemed physician, was influenced by rationalist philosophical thought that held knowledge to be derived primarily from logical reasoning. François Quesnay drew inspiration for his work in political economy from the laws of nature and conservation theories, which were proposed by René Descartes, a philosopher whom he greatly admired. This impact is particularly evident in Quesnay's seminal publication, the 1758 book "Tableau Économique", in which he presents his vision of the economy as a self-regulating system governed by natural laws. Despite the intricacies of François Quesnay's application of natural science concepts to economic concepts, his transfer of such concepts is partially demonstrated in his own words, wherein he asserts that "The natural laws of social order are the very same physical laws of the reproduction of goods necessary for subsistence, for conservation, and the comfort of men." [5] This represents one of the earliest attempts to conceive economics as an autonomous field of study that could be guided by the principles of natural science.

Later on this theory equally faced a lot o criticism. In France Jean-Baptiste Say argued that agriculture was not the sole productive sector and A.R.J. Turgot argued that money was not conserved and it was subjective to psychological factors. It seems that an European consensus did not exist around the problem of value [5], with different countries, perhaps influenced by distinct historical perspectives, having divergent views. In Great Britain the most prominent figure was Adam Smith, after completing his academic education he traveled extensively across Europe and in France he had the opportunity to contact with the physiocratic group. Although Smith shared some sympathy over their economic thought he had different perspectives in some key points.

Upon returning to Scotland, Adam Smith wrote one of the foundational texts of modern economics, "The Wealth of Nations," which was first published in 1776. In this seminal work, Smith advocated his vision for the most efficient economic system. Unlike Quesnay, Smith did not view agriculture as the sole source of value, likely due to the beginning of the industrial transformation he was witnessing in his own country. Smith emphasized the role of free markets, the division of labor, and the significance of individual self-interest in generating national wealth. He also contended that government intervention ought to be limited and utilized solely to ensure market competition, enforce property rights, and provide public goods such as infrastructure and education. In concurrence with the physiocrats, Smith believed that labor, both agricultural and industrial, was the origin of any commodity's value. Consequently, labor division led to augmented productivity and efficiency, leading to economic growth and the prosperity of a nation. [8]

In order to gain a comprehensive insight into Adam Smith's intellectual contributions, it is imperative to delve into his extensive body of work and his academic formation. Smith embarked on his academic journey at a tender age at the University of Glasgow, where he immersed himself into the study of a wide range of subjects including logic, metaphysics, mathematics, Newtonian physics, and moral philosophy. Smith's intellectual landscape was enriched by the profound influence of prominent figures from the Scottish enlightenment [9], such as David Hume, Adam Ferguson, and John Millar, with whom he maintained regular correspondence.

Displaying a remarkable breadth of intellectual interests, Smith's lesser-known essay, "History of Astronomy," engages in a discourse that explores the connection between advancements in the field of astronomy and the inherent human inclination towards seeking order and coherence [10].This work reflects Smith's contemplation on the progress of scientific inquiry, wherein he expresses his profound admiration for the groundbreaking contributions made by the eminent Sir Isaac Newton. Smith's admiration for Newton's achievements became a direct source of inspiration for him, as seen in his book 'Wealth of Nations' [11] where Smith establishes a connection between Newton's concept of gravitational forces and the determination of the natural price of commodities, writing : 'The natural price, therefore, is, as it were,the central price, to which the prices of all commodities are continually gravitating. Different accidents may sometimes keep them suspended a good deal above it, and sometimes force them down even somewhat below it. But whatever may be the obstacles which hinder them from settling in this center of repose and continuance, they are constantly tending towards it'. [11]

In another of his major works, 'Theory of Moral Sentiments' [12], Adam Smith presents a revealing perspective on the disciplines of mathematics and natural philosophy, elucidating Smith's respect for these fields and the individuals engaged in them, according to his own words "Mathematicians, on the contrary, who may have the most perfect assurance, both of the truth and of the importance of their discoveries, are frequently very indifferent about the reception which they may meet with from the public...The great work of Sir Isaac Newton, his Mathematical Principles of Natural Philosophy, I have been told, was for several years neglected by the public. The tranquility of that great man, it is probable, never suffered, upon that account, the interruption of a single quarter of an hour. Natural philosophers, in their independence upon the public opinion, approach nearly to mathematicians, and, in their judgments concerning the merit of their own discoveries and observations, enjoy some degree of the same security and tranquility. The morals of those different classes of men of letters are, perhaps, sometimes somewhat affected by this very great difference in their situation with regard to the public. Mathematicians and natural philosophers, from their independence upon the public opinion, have little temptation to form themselves into factions and cabals, either for the support of their own reputation, or for the depression of that of their rivals" [12]. Smith's insightful reflection sheds light on his perceptions, highlighting the reasons behind his reverence towards the scientific method. By examining his own words, it becomes evident that Smith holds mathematics and natural philosophy in high regard. Building upon his extensive background and the intellectual climate of his era, Adam Smith emerged as a prominent figure shaped by the prevailing philosophies and scientific advancements of his time. With a particular focus on scientific and mathematical disciplines, Smith sought to transpose their methodology into the realm of economics, while upholding the significance of ethics, moral philosophy, and human behavior [12]. His work laid the foundations for the classical theory of economics, which was further developed by Malthus and David Ricardo. For a classical economist the notion of value of a good was primarily determined by objective factors such as the amount of labor and production costs. They assumed economic utility, referring to the overall satisfaction or benefit derived from consuming a good, was a highly subjective and individualistic concept. They contended that the complexity of individual preferences and the variability of utility across different individuals made it challenging to generalize and incorporate utility as a definitive component of a good's value [8].

In the next section, we will observe that classical economics did not achieve unanimous agreement, and the 19th century witnessed the emergence of various new schools of economic thought. While some economists viewed this diversity of perspectives as beneficial, others perceived it as an indication that the goal of establishing economics as a "pure" science was faltering [8].

2.2 From Marginalist Revolution to Neoclassical Economy

The 19th century in Europe witnessed numerous social and political uprisings, leading to a period of considerable instability and transformation across the continent. The rise of industrial and capitalist society brought about various conflicts among those involved in shaping the new economic order. Consequently, a multitude of economic schools of thought emerged to address these diverse perspectives within the economic system.

Within the framework of British political economy, French utopian socialism, and German idealism, Karl Marx emerged as the founder of the Marxian school of economic thought, marked by the publication of his seminal work "Capital" [13] in 1867. While Marx agreed with Adam Smith's labor theory of value, he vehemently criticized Smith's perception of the economy as a fixed system governed by deductive laws. Marx delved deeper into the dynamics of class division within the capitalist system, offering a more comprehensive analysis. Germany also witnessed the rise of another influential economic school known as the Historical school of economics, which emerged around the 1870s. Founded by Gustav von Schmoller, this school, much like Marx, harshly criticized classical economics [14]. However, it adopted an inductive approach, focusing on the dynamic social behavior influenced by historical, political, cultural, and social contexts. Schmoller argued that the state played a vital role in societal development, challenging the notion that its influence should be minimized. The Historical school exerted significant influence on German economic policies in the late 19th and early 20th centuries, and its impact extended to the development of the American school of Institutional Economics in the early 20th century [8].

In the 18th century, a significant paradigm shift occurred in the pursuit of knowledge, marked by the growing adoption of the scientific method. This method proved highly successful in the natural sciences, particularly physics, enabling better predictions and a more objective understanding of natural phenomena. The field of economics also underwent a transformation in response to this shift, as economists sought to elevate economy from philosophy and social science to a 'pure' science [8]. Inspired by the achievements of the scientific method, these economists aimed to apply similar principles to the study of economic phenomena. Their goal was to enhance the rigor and precision of economic analysis, enabling greater predictive power and a more systematic understanding of economic processes. By establishing economy as a 'pure' science, they sought to bolster its credibility and provide it with a more objective and quantifiable framework. These endeavors represented a significant departure from the traditional philosophical and social science foundations of economics. This paradigm shift had a profound and disruptive impact on the development of economic theory, paving the way for the emergence of more formalized and mathematical approach to the understanding economic behavior.

In the 1870s, the Marginalist school of economics emerged as a highly influential and dominant force in economic theory. Independently developed by Stanley Jevons, Léon Walras, and Carl Menger, this school introduced a groundbreaking approach to economic analysis centered around the concept of marginal utility. This marked a departure from the classical labor theory of value. Inspired by Utilitarianism, the philosophical school of John Mill and Jeremy Bentham, the marginalists emphasized that value creation was no longer dependent on production and labor, but on individual consumption. As Jevons eloquently stated, ' Utility only exists when there is on the one side the person wanting, and on the other the thing wanted... Just as the gravitating force of a material body depends not only on its mass, but also on the masses, relative positions, and distances of the surrounding material bodies, so utility is an attraction between a wanting being and what is wanted ' [15]. The concept of marginal utility quantified the satisfaction gained from consuming an additional unit of goods or services.

Physics also experienced remarkable advancements in the 19th century with the emergence of thermodynamics and electromagnetism. These breakthroughs revolutionized our understanding of energy conservation and the concept of fields in the physical world. Physics not only thrived intellectually but also played a pivotal role in driving industrialization and societal progress. However, amidst this progress, conflicts arose within the field of physics. One notable dispute was between a deterministic and mechanical worldview and a probabilistic perspective. These conflicting views reflected ongoing debates on the fundamental nature of reality, with some scientists advocating for a deterministic, cause-and-effect understanding, while others embraced the inherent probabilistic nature of physical phenomena. Nonetheless the rational and systematic approach of physics gradually permeated other fields of knowledge, including economics, as ideas and methodologies were imported and applied to different disciplines.

The examination of the published works of the three founders of the Marginalist school of economics reveals a clear influence of intellectual inspiration on the development of new economic theories. Notably, Léon Walras [16] and Stanley Jevons [17] exhibited a strong inclination towards incorporating concepts from physics into the realm of economics. Jevons displayed a strong inclination toward employing physical metaphors to elucidate economic concepts. Despite his father's wishes for him to pursue engineering, Jevons ultimately opted to study chemistry and mathematics in London. During his time there, he had the opportunity to attend several of Michael Faraday's public lectures at the Royal Institution. Furthermore, Jevons demonstrated familiarity with the works of William Thomson (Lord Kelvin) and James Joule, which explored the fascinating interplay between heat and mechanical work. This familiarity with Thomson's and

Joule's research on the convertibility of heat and energy significantly influenced Jevons understanding of energy dynamics and later on the application of these ideas to economy. Additionally, Jevons corresponded with the physicist James Clerk Maxwell, engaging in insightful discussions on the theory of heat [15].

During a time of political instability and social upheaval in France, Léon Walras found himself deeply influenced by the turbulent environment. Both he and his father, Auguste Walras, who was also an economist, shared a keen interest in societal reforms and were driven by social concerns. Initially pursuing a career in mining engineering, Walras soon realized his passion lay in the intellectual debates surrounding the reforms needed in French society. Despite facing criticism and opposition for diverging from the mainstream views of the time, Walras remained steadfast in his convictions. While Walras initially contemplated a path in art and literature, his father convinced him to continue his work in economics. Both father and son faced significant challenges throughout their academic careers as they sought to challenge prevailing orthodoxies. For Walras, the use of mathematics was not only a means to establish the scientific nature of his ideas but also to lend authority to his proposals for social reform [18]. Léon Walras primary objective was to develop a theory of general market equilibrium. In his pursuit of equilibrium, Walras embraced a mechanistic perspective. He, along with Jevons, found classical thermodynamics to be a fitting metaphor for conveying their ideas. They drew inspiration from the inter-convertibility of heat and mechanical work, applying this concept to redefine the notion of utility, aligning it with the concept of energy. In one of his most famous work, ' Elements of Pure Economics ' [19], Walras argues that 'the pure theory of economics is a science which resembles the physic-mathematical sciences in every respect' [15]. Despite the insistence of Walras and Jevons on drawing parallels between economics and physics, many physicists were skeptical of applying mathematical principles to utilitarian social theories. They argued that the concept of utility, which played a central role in economic analysis, was inherently difficult to measure or quantify. Among the three founders of the Marginalist school of economics, Carl Menger stood as somewhat of an outsider. While Stanley Jevons never had direct interactions or correspondence, Walras did exchange letters with Menger. However, Menger's views diverged significantly from the mathematical approach embraced by Walras, leading to a lack of serious consideration of Menger's ideas by Walras. Menger emerged as a vocal critic of the Walrasian mathematical approach, and this critical stance set him apart. As a result, Menger embarked on a different path and developed what would become known as the Austrian school of economics.

Marginalist sought to establish economics as a mathematical science, driven by the desire to resolve internal conflicts and achieve greater objectivity and predictive power. Jevons, in particular criticized classical economic theory on the grounds that its assumptions were not sufficiently general and timeless. He argued that such theories could not be considered truly scientific. In his view, the inclusion of moral values and subjective value judgments posed significant challenges, as these factors were highly subjective and affected by historical variability. Jevons aimed to separate ethics from economics, striving to develop abstract economic theories that could operate independently of individual perspectives and historical and political contexts. By emphasizing mathematical and quantitative approaches, marginalists aimed to bring greater rigor and objectivity to economic analysis. This approach marked, not a continuation, but a significant departure from earlier economic traditions and laid the groundwork for the development of modern economic theories [8].

Following in the footsteps of Jevons and Walras, economists such as Francis Edgeworth, Vilfredo Pareto, and Irving Fisher shared a similar perspective on how the economy should be analyzed and developed. They embraced the physical-mathematical approach put forth by Jevons and Walras, and in some cases, expanded upon it. Their primary objective was to construct a general theory of equilibrium that could calculate prices of commodities in market equilibrium. Neoclassical economists viewed equilibrium as a concept borrowed from 19th-century thermodynamics, characterized by static equilibrium where a single price corresponds to that state of balance in the market [5].

Francis Edgeworth, an Irish economist born in 1845, had a significant influence on the development of economic thought. He maintained correspondence with Jevons, a prominent economist who was ten years his senior. Edgeworth had a broad range of interests and was particularly drawn to the field of physics. His knowledge encompassed various important works, including Fourier's theory of heat, Poisson's mechanics, Maxwell's contributions, and Laplace's work on the theory of probability. Notably, Edgeworth's mother had a close connection with the physicist William Hamilton. Inspired by Hamilton's approach to calculating the least action of physical systems, Edgeworth applied variational calculus to find minima and maxima for equilibrium in market exchange [5]. In his paper "Rational Exchange" (1884), he drew an analogy between the field of competition and two groups of particles in a plane, each particle seeking its own position of maximum kinetic energy or minimum potential energy. Edgeworth viewed equilibrium as becoming determinate in the limit, a concept reminiscent of physical systems. Furthermore, Edgeworth played a crucial role in promoting the adoption of statistical methods in economy. His work "Methods of Statistics" [20] made significant contributions to the dissemination of statistical methodology in the field. These works sparked debates and discussions on statistical methods, including exchanges with the Italian economist Vilfredo Pareto [21].

Pareto, who was only three years younger than Edgeworth, also made significant contributions to the field of economy. As statistics gained traction in the late 19th century in physics, it also found its way into the realm of economics, prompting debates and discussions. Pareto, like Edgeworth, approached economics with a positivist scientific philosophy, advocating for the exclusion of metaphysical or non-scientific elements from economic study. He believed that economics should be treated as a natural science, akin to physics or chemistry, and should remain independent of philosophical and psychological ideas. Vilfredo believed that through the use of mathematics, economic science could attain the same rigor as rational mechanics, deriving its results from empirical evidence without introducing metaphysical entities. In his own words, "Thanks to the use of mathematics... the theory of economic science thus acquires the rigor of rational mechanics; it deduces its results from experience without bringing in any metaphysical entity" [22]. However, both Pareto and Edgeworth recognized the challenge of establishing causal mechanisms based solely on statistical correlations. Their debates on scaling laws revolved around the role of statistics in interpreting empirical data [21]. The scaling laws are still present today in Econophysics. Pareto was among the pioneering economists who conducted empirical research on income distribution in society [23]. He collected wealth and income data from various countries spanning different time periods. To analyze this data, he created graphical representations that depicted the number of individuals at different income levels. Interestingly, he discovered a striking similarity in the patterns across the countries he studied. The distribution exhibited a substantial concentration of individuals at the lower end of the income scale, with a sharp decline in numbers as income levels increased. This observation led Pareto to recognize that the income distribution generally followed a power law distribution.

On the other side of the Atlantic, we encounter Irving Fisher, an American economist who holds the distinction of being the first to receive a PhD in economics from Yale University. Notably, Fisher's connections with physics are evident in his doctoral thesis [24], for which his supervisor was the renowned physicist Josiah Willard Gibbs. In 1891, the same year Fisher completed his dissertation, Francis Edgeworth, the founding editor of The Economic Journal, had the opportunity to review and positively assess Fisher's work. In his doctoral thesis, Fisher drew inspiration from fluid mechanics and developed a cistern model that simulated systems of differential equations for analyzing general equilibrium in markets. Fisher believed that economics should primarily concern itself with objective commodity relations, diverging from the prevailing methodology that combined empirical psychology with economics. Instead, he focused on understanding the human desires and motivations underlying market transactions rather than delving into the psychology of consumers.

Throughout his dissertation, Fisher consistently established analogies between physical and economic concepts. For instance, he presented a table that drew parallels between terms such as "particle," "space," "force," "work," and "energy," and their economic counterparts as "individual," "commodity," "marginal utility," "disutility," and "utility" [5]. Fisher's intent was to demonstrate the connections between physical and economic principles. In his own words, "The total work done by a particle in moving from the origin to a given position is the integral of the resisting forces along all space axes... multiplied by the distances moved along those axes." He then presented the economic equivalent: "The total disutility suffered by an individual in assuming a given position in the economic world is the integral of the marginal disutility along all commodity axes... multiplied by the distances moved along the axes". To provide a visual representation of how equilibrium prices adjust in response to changes in supply and demand, Fisher presented a conceptual hydraulic machine equipped with pumps and levers. This machine demonstrated how the water volumes of interconnected cisterns, symbolizing individual consumption of a commodity, changed as marginal utility was defined as the remaining space. Fisher's model operated on the assumption that consuming additional units of a commodity led to a decrease in marginal utility. By applying his equations, the volumes of the cisterns would be altered until equilibrium was reached, indicated by the cessation of fluid transfer and the attainment of a stable state. Fisher believed that since the total amount of water remains constant over time, it would be logical to apply basic principles of conservation to explain how utility and price interact in an economy. He was aware that real-world economies do not achieve perfect equilibrium, but he believed that just as physics can employ idealizations to derive meaningful results when studying the real world, economics should also strive to be a science with a similar capacity [21].

The economic theories put forth by classical economists had a tangible impact on the economy during the early 19th century. Europe, in particular, experienced a significant increase in free trade as liberal ideas gained prominence. Despite some political instability in certain countries, the prevailing influence of liberalism gradually led to the reduction or abolition of tariffs, marking a decline in protectionism. As a result of this international economic integration, price fluctuations became synchronized across national borders. They were no longer solely influenced by local or regional factors such as natural phenomena. The price fluctuations in this era became more closely tied to demand fluctuations in commerce, and they exhibited cyclical patterns that spread contagiously throughout international trade. Over the course of the 19th century, it became increasingly evident that financial crises occurred on average every decade. No one seemed to understand this cyclical problem and, the depression of 1873 being one of the most severe and widespread, provided the so wanted pretext for industrialists in each country to exert pressure on governments to protect their industries from international competition. In 1879, under the leadership of Otto von Bismarck, the German Empire took the first step toward protectionism by introducing a new tariff. As a response, in France, despite the strong liberal influence, protectionist factions gained a majority in the deputy chamber after the 1889 elections. Similarly, Italy and France engaged in a tariff war at the end of the century. This protectionist movement spread to various other European countries, as industrialists successfully lobbied political powers to secure protection from international competition. The United States experienced a similar shift. Prior to the Civil War (1861-1865), the Southern region held more influence in political decision-making and favored low tariffs, as they relied heavily on exports. However, after the victory of the Northeast and the West, industrial interests became dominant and influenced political powers to protect their industries. The United States maintained a highly protectionist stance, even after World War II. On the European continent, this economic trade war, combined with longstanding political and historical resentments, ultimately culminated in the outbreak of the First World War in 1914 [25]. The WWI had a devastating impact on the European continent, it resulted in immense human casualties, the destruction of entire industries and cities, and a significant loss of economic influence in the global trade arena. Throughout the war, all countries involved implemented centralized planned economies, with all efforts directed towards the war effort.

By the war's end in 1918, Europe not only lay in ruins but also faced the additional challenge of a flu epidemic known as the "Spanish flu". The continent had to contend with the loss of export markets, making the path to recovery even more challenging. Meanwhile, the United States emerged from the war with increased economic influence, further exacerbating Europe's decline in the world trade order. In the 1920s, while the United States pursued a policy of internal liberalization, Europe faced a more complex situation. Although a peace treaty was in place, it only served as a constrain for recovery. Germany was heavily punished by the Versailles Treaty, Russia underwent the Bolshevik Revolution in 1917, leading to the establishment of a communist regime, and the Austrian-Hungarian Empire dissolved into several rival countries [25]. Additionally, many nations returned to the gold standard, imposing austerity measures. Recovery was slow, and in 1929, the situation worsened when a financial crisis in the United States escalated into an economic depression. Foreign investment from the US declined significantly, leaving Europe in an unstable financial state. Economists of the time struggled to explain the causes of this catastrophic market downturn, and more importantly, why it did not naturally correct itself to equilibrium.

Around the same time, in the late 1920s and early 1930s, there was a notable influx of physicists, engineers, and mathematicians into the field of economics. This influx may have been an attempt to help understand where economic theory had gone astray. Physics itself was undergoing a profound transformation, with the development of the Copenhagen interpretation of quantum mechanics challenging the notion of determinism in nature and giving rise to a probabilistic view of the world. This paradigm shift in physics had an impact on economic theory as well. In 1930, the Econometric Society was founded by Ragnar Frisch, with the assistance of Irving Fisher and Joseph Schumpeter.

Some of the most prominent figures of Econometrics such as Ragnar Frisch, Harold T. Davis, Tjalling Koopmans, Jan Tinbergen and Charles Roos had by far more ties with physics and mathematics than with economics. Ragnar Frisch, Harold T. Davis and Charles Roos took a PhD in mathematics in 1926 and all of them were familiar with some physical concepts. Jan Tinbergen took in PhD in physics in the University of Leiden (1929) under the orientation of the renowned physicist, Paul Ehrenfest, which apparently was the one who encouraged Tinbergen to venture into economics. Tjalling Koopmans started his academic life in physics and later on he took a PhD in mathematical statistics. Koopman's adviser was the known quantum physicist, Hendrik Kramers and Koopman's first academic publication was in quantum physics [26]. It is noteworthy and somewhat surprising that despite their shared aim of developing a mathematical and statistical approach to economics, the prominent figures in Econometrics held diverse political opinions. Among the major contributors, Ragnar Frisch, Jan Tinbergen, and Tjalling Koopmans held contrasting views regarding the role of the market. Koopmans was known for actively advocating for the market, emphasizing its ability to address economic challenges. Frisch, on the other hand, rejected the notion that the market alone could effectively resolve crises, suggesting a need for direct state control in economic strategies. Tinbergen took a more moderate stance in this regard [8].

In the 19th century, neoclassical economists drew heavily from physics to develop their theories, adopting a deterministic and mechanical worldview. However, as physics advanced in the 20th century, it became apparent that the real world might not adhere strictly to determinism. This posed a challenge for neoclassical economists, as other economic schools had already criticized their approach. They could no longer claim superiority in scientific rigor when the prevailing scientific understanding pointed towards indeterminacy. Faced with this dilemma, a new generation of neoclassical economists emerged, still pursuing a general equilibrium theory [5]. They had to decide whether to construct a new theory based on the emerging physical reality or adapt the old one. Similar to certain physicists who were hesitant to abandon determinism, some economists clung to their deterministic framework. They incorporated limited indeterminacy into their models by assuming a deterministic underlying process and attributing economic fluctuations to stochastic shocks.

Ragnar Frisch in his early work [27], explicitly explored the analogies between neoclassical price theory and rational mechanics and to explain any errors and divergences, he proposed the use of least squares models. In quantum mechanics, one of Max Planck's early contributions was the black-body radiation problem, which involved simple models of oscillators. Likewise, Werner Heisenberg's first exercise with matrices focused on a simple oscillator. Econometricians like Jan Tinbergen and Ragnar Frisch were familiar with these models [26], which inspired them to apply similar principles to studying the dynamics of business cycles. They viewed the economy as a pendulum, with each addition of energy comparable to an external shock in economy. Later in their lives, both Jan and Ragnar would become the first Nobel laureates in economy for their contributions to the study of economic cycles using temporal series statistics.

On the other side of the Atlantic, in United States, Columbia University emerged as the focal point for the heterodox movement dedicated to constructing a stochastic approach to economics. Among its notable members were Wesley Mitchell, Frederick Mills, and Henry Moore. Wesley Mitchell, a prominent figure in the American Institutional school of economics, was a student of Thorstein Veblen and played a crucial role in shaping the field. He held a skeptical view of neoclassical economic theory and sought to promote a more empirical and scientifically grounded approach to economics. In 1920, Mitchell was instrumental in establishing the National Bureau of Economic Research (NBER), which aimed to foster the development of modern, evidence-based economic research. In a speech to the American Economic Association in 1925, Mitchell expressed his criticism of the resemblance between orthodox economics and older Lagrangian mechanics. He advocated for the reconstruction of economic theory along the lines of modern physics, embracing a more contemporary and dynamic approach.Wesley Mitchell and other institutional economists advocated for a more diverse approach to economic analysis that encompassed insights from disciplines such as statistical mechanics and evolutionary biology. They strongly opposed the neoclassical perspective that regarded stochastic phenomena as mere random errors imposed upon a deterministic structure. As early as 1915, Mitchell and his contemporaries raised concerns about the limitations of the neoclassical model in describing perturbations within a system using Gaussian distributions and central limit theorems [26]. In the 1920s, researchers at the NBER studying price changes observed that the tails of these distributions were much fatter than what could be accounted for by a Gaussian distribution. Many neoclassical economists appeared to overlook this aspect, but the discussion was later reignited by a very talented mathematician, Benoit Mandelbrot.

Benoit Mandelbrot was born in Warsaw in 1924, into a Jewish family during a tumultuous period. His upbringing was marked by challenges and upheaval. In 1936, his family relocated to Paris, but their peaceful existence was disrupted when France fell to Nazi invasion in 1940. Mandelbrot and his younger brother were compelled to move once again, seeking refuge in a small, discrete town in the French countryside. However, due to a misunderstanding with the police, they were forced to relocate yet again, seeking safety in a different place. It was during this turbulent time that Mandelbrot's remarkable talent for mathematics began to emerge. In 1944, while attending school in Lyon, Mandelbrot's aptitude for geometry became evident, and he would later go on to pioneer a new branch of mathematics known as fractal geometry, which he applied across a diverse range of disciplines. In the 1960s, Mandelbrot embarked on his first exploration in the field of economics. He was among the pioneers who delved into the study of power-law distributions, which would later become a significant area of research in Econophysics. Similar to Mitchell's observations in 1915 regarding price fluctuations distributions, Mandelbrot also recognized that the variation in prices exceeded the expectations of conventional models and exhibited fat-tailed distributions, he consistently emphasized that the stability of financial markets did not align with how orthodox financial theory depicted them [23]. His groundbreaking papers from the 1960s are now widely regarded as the foundational contributions to a growing interdisciplinary field known as Econophysics.

2.3 The birth of Econophysics

The historical connection between physics and economics is more intertwined than one might initially expect. From Nicolaus Copernicus's work ' Treatise on money ' in 1526, to Isaac Newton's influential role in advising Britain's treasury on monetary matters after becoming Master of the Mint in 1699 [28], there have been notable instances of physicists making contributions to economics and occasionally some economists making contributions to physics. One such figure is Louis Bachelier, known as the founder of mathematical finance. In 1900, Bachelier wrote his thesis, which analyzed price fluctuations in financial markets using random walks. This work can be seen as a precursor to Einstein's research on Brownian motion. Bachelier argued that short-term price fluctuations should be independent of the current price, assuming a lack of dependence on past behavior. By combining this idea with the central limit theorem, he deduced that the increments in this process are independent and normally distributed. Bachelier also introduced the concept of "lack of memory" in his thesis, a property that was later formalized by Andrey Markov in 1906. Despite being largely ignored by economists of his time, Bachelier's work [29]was later recognized and revived by Paul Samuelson. In the 1960s, it became influential in the development of the “Efficient Market Hypothesis” *[30]*.

The 1980s witnessed a technological revolution with the rapid expansion of computers, enabling more effective data collection. This advancement contributed to the development of complexity studies, and it was during this time that physicists began conducting research on economic systems. The Santa Fe Institute in the United States emerged as one of the pioneers in this field, leading the way in studying dynamic complexity and its association with economics [31].

The term "Econophysics" was coined by physicist H. Eugene Stanley in the 1990s. It represents a relatively new interdisciplinary approach that seeks to apply the methodologies of modern physics to understand complex dynamic systems in the field of economics. Initially regarded as a branch of physics, most of the research in this field found its place in physics journals. The birth of Econophysics is reportedly associated to a paper published in *Physica A* in 1991 entitled “Lévy walks and enhanced diffusion in Milan Stock-Exchange” [32] by Rosario Nunzio Mantegna, who was a student of H. Eugene Stanley. In his paper Mantegna discovered the break of of central limit theorem on the stock market.

Despite some progress today, Econophysics largely remained overlooked by mainstream economists. Even among econophysicists themselves, opinions vary on engaging in a dialogue with neoclassical economists. Some advocate for an open dialogue, hoping to challenge prevailing dogmas and influence change, while others believe that such discussions are premature until the assumptions made by mainstream economists undergo empirical testing and review [21]. While financial economics, including security markets and foreign exchange markets, has been the primary focus of Econophysics, a substantial body of research also examines macroeconomic phenomena such as business cycle fluctuations, factors influencing economic growth, income distribution, and issues related to economic equilibrium [33]. Methodologically, Econophysics can be divided into three distinct approaches. Statistical Econophysics, the first and most prevalent approach, takes a macroscopic perspective, relying on a vast number of observations to discern macro patterns. The bottom-up agent-based approach, which emerged later, seeks to study emergent macro-order through predefined micro-interactions. Lastly, the top-down agent-based Econophysics approach combines elements of the previous two methodologies, offering a comprehensive framework for analysis [31].

Mainstream economics and Econophysics represent two distinct approaches to understanding and modeling economic phenomena. Mainstream economics, rooted in neoclassical and Keynesian traditions, focuses on rational agents making decisions based on utility maximization and the efficient allocation of resources. It employs mathematical models and econometric techniques to analyze market behavior, macroeconomic variables, and policy outcomes. Prominent economists like Milton Friedman, Paul Krugman, and John Maynard Keynes have contributed to the development of mainstream economic theories. However, critics argue that these models often oversimplify complex real-world dynamics and struggle to capture the inherent uncertainty and non-linearity of financial markets. In contrast, Econophysics emerged from the physics community’s interest in applying statistical physics concepts to economic systems. Econophysicists employ methods from statistical mechanics, complex systems theory, and network science to describe economic phenomena. They emphasize empirical analysis, often employing large datasets and computer simulations to study market dynamics, wealth distribution, and financial crashes. Econophysics recognizes the presence of collective behavior, herding effects, and emergent properties in financial markets, challenging traditional economic assumptions. These differences in methodology and philosophy have led to valuable insights and critiques within both disciplines. While mainstream economics relies on equilibrium models and representative agents, Econophysics embraces heterogeneity, non-linearity, and the role of large-scale interactions in shaping economic outcomes. By comparing and contrasting these two approaches, researchers can gain a more comprehensive understanding of the intricacies of economic systems [33].

Econophysics has made significant strides in advancing our understanding of complex economic systems, but it also faces several challenges. One notable challenge is the integration of sociological and psychological aspects into Econophysics models. While Econophysics excels in modeling large-scale market behavior and statistical regularities, it often lacks the granularity to account for individual decision-making processes and behavioral biases. Bridging this gap requires interdisciplinary collaboration and the development of agent-based models that incorporate psychological factors. Additionally, the field faces the challenge of addressing criticisms of oversimplification, as critics argue that some Econophysics models may not capture the full complexity of real-world financial markets. Despite these challenges, Econophysics has achieved milestones in understanding various phenomena such as power-law distributions of wealth, market crashes, and network structures in financial systems. Milestone publications include "The Black Swan" by Nassim Nicholas Taleb [34] and the work of Eugene Stanley [35] and Jean-Philippe Bouchaud [36]. As Econophysics continues to evolve, addressing these challenges while building on its milestones is crucial for its ongoing contribution to the field of economics.

The primary objective of this dissertation is to examine the intricate relationship between money distribution and economic as well as social inequality. This study seeks to explore the multifaceted interactions between these two fundamental factors under varying circumstances. Specifically, it aims to elucidate how the manner in which money is distributed can impact economic inequalities. Furthermore, this research endeavors to investigate whether the mechanisms governing money distribution exert a similar influence on both economic growth and economic development.The foundational framework for much of this study draws inspiration from the work's of Victor Yakovenko and others in the field of agent-based modeling and classical Econophysics [37]. Yakovenko 's contributions serve as a significant source of guidance and insight, providing valuable perspectives for the investigation of money distribution and its repercussions. By employing agent-based modeling techniques, this dissertation aspires to contribute to a deeper understanding of the dynamics of money distribution and its effects on economic and social structures.

3 Money Creation in Modern Economy

' It is well enough that people of the nation do not understand our banking and monetary system, for if they did, I believe there would be a revolution before tomorrow morning '
Henry Ford

3.1 Short History of Money

3.1.1 What is money what is its purpose and different money standards through history

Money, a cornerstone of economic theory, has always presented a complex puzzle to unravel. At its core, money is widely understood as a commodity embraced by consensus, facilitating the exchange of economic values. It serves as a ubiquitous tool, streamlining transactions among various economic actors. Throughout history, money has taken various forms, from seashells to livestock and metals. In modern times, it has evolved into fiat currency, often seen as paper notes or digital transactions. In adherence to established economic principles, money fulfills fundamental roles, encompassing its functions as a versatile medium for facilitating trade across diverse transactions. It assumes the role of a universally recognized intermediary, streamlining exchanges in various contexts. Simultaneously, money operates as a yardstick for evaluating the worth of goods and services, enabling the comprehensive establishment of prices. It assumes a pivotal position as a conduit for settling debts and facilitating future dealings. Additionally, money assumes the responsibility of a reservoir for accumulated wealth, offering a storage mechanism for resources awaiting utilization [38].

One of the most compelling arguments for the adoption of money as medium of exchange revolves around its role in simplifying and expediting economic transactions. In earlier times, human interactions primarily relied on direct barter, yet as societies grew more intricate and interconnected, the need for a more efficient means of trade became evident. Consequently, a solution emerged wherein a specific commodity, endowed with attributes such as durability, divisibility, and portability, was designated as a universal measure against which other goods were evaluated. This choice enabled the facilitation of economic exchanges. Initially, metals like gold and silver emerged as prominent candidates for this role, their value serving as a benchmark. These metals were minted into coins by ruling authorities, imbuing them with legal validity. As the 20th century commenced, the utilization of physical metal coins gave way to paper notes. This transition circumvented the need to carry heavy metals while still upholding their value. Remarkably, these paper notes were anchored by a backing of gold, constituting the foundation of the gold standard system [39]. This transformation allowed a wider access to credit due to the possibility of banks to loan the gold to a person that someone else had previously deposited.

Following World War II, a concerted effort to avert a recurrence of the economic calamities that exacerbated the Great Depression led to the establishment of the International Monetary Fund (IMF). This was followed by the implementation of the Bretton Woods System, an orchestrated endeavor for international economic collaboration. Under this system, participating nations tethered the value of their currencies to the U.S. Dollar, while the United States itself anchored the Dollar to gold. However, the Bretton Woods framework met its demise towards the close of 1971, when the United States suspended the convertibility of the Dollar into gold, marking the system's collapse. In today's economic landscape, the prevailing currency, commonly referred to as fiat money, lacks backing from tangible resources. Concurrently, the dynamics of the banking system have also undergone transformation.The former limitations tied to money creation based on the quantity of gold holdings have dissipated [10].

3.1.2 Theories of Money

Schumpeter's examination of monetary theories reveals a classification into two overarching categories: the commodity theory and the claim theory. Alongside these, the neoclassical interest rate theory, also known as the loanable funds theory, emerges as a third perspective that shares certain parallels with the commodity theory. Among the classical economists, including notable figures like Smith, Ricardo, and Marx, a prevailing view treated money as a commodity intertwined with the authentic value of goods and services, rather than dictating their valuation. This classical stance rests upon the conviction that a commodity's genuine value stems from the labor invested in its production. Within this framework, money emerged as a practical intermediary for trade, boasting qualities such as durability, divisibility, portability, and uniformity. Importantly, the classical perspective demarcated two distinct economic realms: one encompassing production and consumption, where employment and output held sway, and the other representing the monetary or nominal sphere, where the labor-rooted real value of goods was translated into monetary terms [40].

In the classical era, economists regarded money as akin to other commodities, valuing it based on production costs [40]. These perspectives delineated two relatively distinct realms—monetary values exerted minimal influence on concrete production processes. Yet, the advent of the marginalist revolution sparked a pivotal shift. Value theories evolved from the classical focus on production and distribution to the neoclassical paradigm, which centered on the intricate dynamics of exchange. This transformative shift propelled attention from the production and availability of money towards a concentrated exploration of individual consumption behaviors. This reorientation facilitated a departure from an objective standpoint on value. Instead, value was seen as the price consumers were inclined to offer for additional units of commodities.

The transformative shift in economic thought was underscored by the subjective theory of value proposed by Jevons, Menger, and Walras [41]. At the heart of this paradigm change lay the assertion that the value or price of an exchange was contingent upon an individual's personal assessment of utility in relation to the prevailing supply of commodities. This neoclassical perspective spotlighted value evaluation through the prism of consumer preferences, diverging from the classical emphasis on value derived from labor-based calculations. This pivotal transition signified a departure from the established classical framework and laid the cornerstone for the contemporary neoclassical comprehension of value and exchange that permeates modern economic discourse [42]. However previous insights shed light on a limitation of conventional neoclassical general equilibrium models. These models struggled to explain the logical inconsistency of how money comes into play in the economy. While aiming to establish market equilibrium, Walras introduced the concept of a unit representing values in the economy. Nevertheless, these models didn't address how money enters the economic picture initially. In essence, the introduction of the equilibrium concept within these models parallels a scenario akin to barter exchange, where the focus is on establishing a state of balance between supply and demand. The distinction emerges in the form of a numerical unit that becomes associated with these transactions. Notably, this approach lacks a comprehensive explanation for the entry and role of money within the system, leaving a void in understanding the underlying mechanisms that bring money into circulation and drive monetary prices.

During the late 19th century, a notable concern among economists revolved around comprehending the repercussions of fluctuations in money demand. This ongoing inquiry later paved the way for the development of a quantitative theory of money, formulated by Irving Fisher. At the heart of this theory lay the "equation of exchange," which served as a cornerstone for understanding the intricate relationships within the monetary system.

The equation of exchange, expressed as $MV + M^1V^1 = \sum p.Q = P.T$, encapsulated Fisher's perspective. He delineated bank deposits M^1 and cash balances M as integral components of the money supply. Simultaneously, Fisher aimed to establish a unidirectional correlation between the quantity M^i and the velocity of money V^i in relation to the broader context of the general price level P, multiplied by the aggregate volume of transactions T transpiring in the economy.

However, it's important to note that Fisher's quantitative theory of money did have its limitations. While it provided a useful framework for understanding the relationships between money, its speed of circulation, prices, and economic transactions, it fell short when it came to explaining how bank deposits come into existence. As the late 20th century rolled around, a key assumption in their model came to light – one that assumed that the money supply was naturally controlled by a central authority outside of the regular market dynamics [42]. This essentially meant that they believed the flow of money supply was determined from external (exogenous) factors rather than being directly influenced by market forces(endogenous). Nonetheless neoclassical economist maintained the same view of money as a commodity.

Drawing inspiration from ideas that gained prominence in the 1970s, and also influenced by Fisher's theory, the monetarist theory of money supply emerged being supported by a well known economist Milton Friedman. According to this theory, in the short term, the money supply wields the ability to impact economic activity, consequently influencing the overall price level. Yet, in the long run, a shift occurs as real variables gradually gravitate towards their intrinsic equilibrium states. Consequently, the once-dominant impact of money supply starts to diminish. Within this framework, the concept of the natural interest rate takes center stage. This interest rate signifies the equilibrium point at which the supply and demand forces within real capital markets harmonize. However, it is proposed that this natural interest rate remains unaffected by the sway of short-term money interest rates, which are driven by market dynamics. This delineation highlights an intriguing dynamic where the alignment between these rates remains crucial for maintaining economic stability. Nonetheless, the theory posits that substantial disparities between these interest rates can set in motion cumulative processes, potentially leading to inflationary or deflationary pressures.

In the field of monetary theory, there has been an ongoing debate regarding the nature of money and its role in the economy. Initially, the monetarist perspective considered money as a commodity, viewing it primarily as a medium of exchange with limited power to disrupt economic equilibrium, except for its impact on inflation. In this view, money couldn't simply be conjured out of thin air, and investments were believed to be contingent on prior savings. Thus, money was considered an exogenous variable. However, as time passed, it became evident that measuring and controlling the money supply was a complex challenge. This led to the opportunity of credit money theorists to challenge the traditional monetarist viewpoint [42]. They argued that money could be created endogenously through the issuance of debt-backed claims. According to this perspective, money is not a passive medium of exchange; it actively influences social relationships and is an endogenous variable within the monetary system.

In summary, the classical monetarist theory treats money as a neutral commodity, while credit money theorists assert that money's creation through debt issuance plays a non-neutral role, impacting the dynamics of the economy and social interactions.

3.1.3 How is money seen in Modern Economy

The concept of money holds a central place in economic theory, influencing the way economies function and transactions are conducted. Both modern and orthodox economic perspectives offer distinct viewpoints on the nature of money, its origins, and its role in shaping economic outcomes. These perspectives, while providing valuable insights, also invite critical examination due to their implications for economic policy and societal well-being. In modern economic theory, money is often seen as a neutral medium of exchange that facilitates transactions and eliminates the need for barter systems. This view, known as the "neutrality of money," emphasizes that changes in the quantity of money primarily affect nominal variables such as prices and wages, while leaving real economic outcomes unchanged. Modern economists often advocate for monetary policy interventions to stabilize economies and promote optimal resource allocation. The monetarist school, championed by economists like Milton Friedman, emphasizes the importance of controlling the money supply to achieve stable price levels and economic growth [43]. Contrastingly, orthodox economic perspectives, influenced by classical and neoclassical theories, often treat money as a veil that obscures the underlying real economic forces. Orthodox economists emphasize the significance of real factors, such as productivity, technology, and resource allocation, in determining long-term economic growth. They argue that money's influence on the economy is limited and that real factors drive economic prosperity. This viewpoint downplays the potential role of monetary policy and money creation in influencing economic outcomes. However, both perspectives have faced criticism for their limitations and assumptions. The modern view's emphasis on the neutrality of money has been challenged by insights from behavioral economics and empirical evidence suggesting that money creation can have real effects on economic behavior, income distribution, and wealth accumulation. Furthermore, the orthodox perspective's treatment of money as a passive veil overlooks the potential for money creation, credit dynamics, and financial intermediation to shape economic cycles, income inequality, and financial stability. Moreover, modern monetary theory (MMT), a relatively recent perspective, challenges some aspects of both the modern and orthodox views. MMT highlights the role of government in creating money and emphasizes that in a sovereign fiat currency system, the government can spend money into existence. This perspective suggests that government spending, rather than being constrained by tax revenue or borrowing, can play a pivotal role in managing economic conditions and promoting full employment. While MMT introduces novel ideas about money creation and government spending, it has also sparked debates and discussions within the economic community [44]. In a rapidly evolving global economy, the conventional demarcation between money as a mere medium of exchange and its broader implications warrants reconsideration. A holistic understanding of money's role necessitates an interdisciplinary approach that encompasses insights from economics, sociology, and political economy. By critically examining both modern and orthodox perspectives, policymakers and researchers can develop a nuanced understanding of money's multifaceted nature and its potential to shape economic and societal outcomes.

3.2 Creation of Money

3.2.1 The banking system: Commercial Banks, Shadow Banking Institutions and Central Banks

In the modern economic landscape, the process of money creation is a multifaceted phenomenon involving various key players, including commercial banks, non-banking entities, and central banks. The traditional notion of central banks as the exclusive creators of money has evolved significantly. A important moment in this transformation occurred in 2014 when the Bank of England published a seminal quarterly bulletin [45], shedding light on the intricate dynamics of money creation.This report expounds that the majority of money in the modern economy is, in fact, generated by commercial banks through lending activities. While central banks remain the sole entities capable of producing physical currency (such as notes and coins), they are not the exclusive architects of money. Within the realm of Financial Monetary Institutions, comprising central banks and Other Financial Monetary Institutions (OFMI), the collective entity known as the Banking System possesses the unique ability to create deposits [46].

Contrary to conventional wisdom, this report highlights that banks do not merely function as intermediaries that lend out the deposits of savers. Instead, banks create new money by issuing loans and corresponding digital deposits. This process dispels the notion of banks merely multiplying central bank money to originate new loans and deposits.

It is essential to emphasize that, ultimately, the volume of money circulating in the economy is subject to the monetary policy decisions of the central bank [45]. Typically, this influence is exercised through setting interest rates. In extraordinary circumstances, central banks can directly impact the money supply by engaging in asset purchases, commonly referred to as quantitative easing. The overarching objective of most central banks across different economies is to ensure that the economic landscape remains aligned with the goals of low and stable inflation.

At the heart of this contemporary comprehension of money creation lies the acknowledgment of the substantial role played by commercial banks. While central banks were historically viewed as the primary creators of money, the reality is more nuanced. Commercial banks, employing the mechanism of fractional reserve banking, significantly contribute to the process. When these banks extend loans to borrowers, they effectively generate new money in the form of digital deposits, which, in turn, fuels economic transactions. Beyond commercial banks, non-banking entities constitute a critical component of the financial ecosystem and also partake in money creation [47]. These entities encompass a broad spectrum, including shadow banks, money market funds, and various financial intermediaries. They engage in activities that give rise to money-like instruments, such as short-term commercial paper, asset-backed securities, and repo agreements. These instruments function as near-money substitutes, facilitating transactions and enhancing overall economic liquidity. The intricate interplay between traditional banks and non-banking entities introduces complexity into the money creation process, as these entities generate money-like claims pivotal for maintaining financial stability and liquidity. Central banks, serving as the custodians of monetary policy and guardians of financial stability, wield substantial influence over the money creation process. They employ mechanisms like open market operations, reserve requirements, and direct monetary issuance to shape the money supply and guide economic outcomes. Central banks also fulfill the crucial role of lenders of last resort [47], stepping in during times of financial turbulence to provide essential liquidity. This role ensures the stability of the financial system and underscores the intricate relationship between money creation, financial intermediation, and macroeconomic stability.

3.2.2 Limits for Creation of Money and its consequences

While the process of money creation holds immense potential for fostering economic growth and facilitating transactions, it is not without its limits and potential consequences. The intricate relationship between money creation and the stability of the financial system underscores the need to understand the boundaries within which this process operates. The modern economy grapples with several limitations that can have far-reaching effects on economic dynamics and financial stability.

One crucial limit to money creation lies in the concept of fractional reserve banking. Commercial banks are required to maintain a fraction of their deposits as reserves, limiting the extent to which they can create new money through lending. This mechanism serves as a buffer against excessive money creation, preventing banks from extending loans beyond their capacity to manage risks [45]. However, these limits are not absolute, as the fractional reserve requirement can vary across jurisdictions and time periods, influencing the overall money supply and its potential impacts on inflation and economic activity.

The consequences of surpassing the prudent limits of money creation can be severe and disruptive to the economy. Excessive money creation, often driven by lax lending standards or speculative behavior, can lead to inflationary pressures that erode purchasing power and undermine the stability of the currency. Furthermore, the unchecked expansion of credit can contribute to asset bubbles and unsustainable levels of debt, exposing the financial system to vulnerabilities and precipitating crises. The global financial crisis of 2008 served as a stark reminder of the risks associated with unrestrained money creation, as the proliferation of complex financial instruments and excessive leverage led to a cascading series of events that had profound implications for the global economy.

In the modern economy, central banks play a critical role in monitoring and managing the limits of money creation. Through monetary policy tools such as interest rates, reserve requirements, and quantitative easing, central banks aim to balance the benefits of money creation with the potential risks of inflation, financial instability, and imbalances in the economy. Striking this delicate balance requires a deep understanding of the interplay between money creation, credit dynamics, and overall economic health [47].

In addition to the dynamics of money creation discussed earlier, the financial landscape is also influenced by regulatory frameworks that aim to maintain stability within the banking sector. One notable set of regulations is the Basel Accords, initiated by the Basel Committee on Banking Supervision. These international banking standards, including Basel I, Basel II, and Basel III, have been designed to enhance the resilience of the banking system and mitigate systemic risks. While the Basel Accords primarily focus on capital adequacy and risk management, they indirectly impact the creation of money within the economy. Under Basel III, for example, banks are required to maintain higher capital buffers to withstand economic shocks, reducing their capacity to create money through lending. This tighter regulatory framework is aimed at preventing excessive money creation that could lead to financial instability, as observed during the global financial crisis of 2008. However, it's essential to note that the Basel Accords primarily address capital adequacy, and their impact on money creation is indirect. Banks may adjust their lending practices in response to regulatory requirements, influencing the broader money supply dynamics. [48] Moreover, it's important to acknowledge that while the Basel Accords address traditional banking activities, they may not directly regulate the shadow banking sector. Shadow banking, comprising non-bank financial intermediaries, operates with a different set of rules and often remains outside the purview of traditional banking regulations. This dynamic adds complexity to the regulatory environment, as the shadow banking sector can contribute to money creation and systemic risks independently of traditional banks. Thus, understanding the interplay between regulatory measures like the Basel Accords and shadow banking activities is crucial for a comprehensive analysis of money creation and financial stability.

3.3 Monetary Economics from Econophysics perspective

3.3.1 Conservation of Money

Money, as a fundamental element in economic transactions, plays a crucial role in facilitating trade and preventing free riding within an economic ensemble. In the world of economics, it serves as a digital accounting tool that goes beyond the simplicity of barter systems. While barter can enable direct exchanges of goods between two parties, its limitations become evident in complex, multi-agent exchanges. This is where money, in the form of digital tokens, emerges as a solution. These tokens represent a numerical value, serving as a record of an agent's contributions to society and, in turn, their entitlement to benefits from the society.

The core principle that underpins the functionality of money is the conservation law of money [4]. This principle stipulates that money should be conserved, meaning that agents should not have the ability to arbitrarily create or destroy digital money tokens. Unlike physical goods, such as apples, which can be produced and consumed, money is a digital representation of value that must maintain its integrity for the economic system to function effectively. While this conservation law might seem obvious, it's a cornerstone concept that ensures the stability and fairness of economic systems. It is worth noting that this principle is not explicitly emphasized in economic textbooks, even though the three primary functions of money—medium of exchange, measure of value, and store of value—rely on the assumption of money conservation. If agents could create money at will, the entire framework of economic exchange would crumble, rendering the medium of exchange meaningless and disrupting the measure of value and store of value functions.

In practice, the conservation of money finds its expression in various economic systems. One illustrative example is the Local Exchange Trading System (LETS) [49], which embodies a bottom-up approach to creating a monetary system. In LETS, participants initiate with zero initial money balances. When one agent provides a service or a good to another, their balances are adjusted accordingly—increasing for the service provider and decreasing for the recipient. Importantly, this adjustment ensures that the conservation law is met algebraically, as the sum of balances across agents remains zero.

However, the conservation law alone is not sufficient to guarantee fairness in economic exchanges. Some agents may exploit the absence of an upper limit and accumulate unlimited negative balances, essentially engaging in free riding by consistently receiving services without contributing. To counter this, boundary conditions are imposed, primarily on the negative side. These conditions dictate that an agent's negative balance should not drop below a certain threshold ($m_{min} < m_i$). An agent who reaches this minimum balance is then required to contribute before receiving further services, effectively curbing free riding behavior.

To simplify such asymmetric systems, the author suggests shifting all money balances to introduce new money ($m_i^{'} = m_i - m_{min}$). This approach effectively starts all agents with positive initial balances, with the boundary condition now shifting to zero. This adjustment ensures that the new money must be positive ($m_i > 0$), with an average representing the money's temperature in the system.

In practice, the distribution of money in various economic systems deviates from perfect equality, even when agents start with equal initial money balances [4]. Agent-based computer simulations demonstrate that perfect equality is an unstable state, and as trading commences, the system naturally evolves towards unequal states with higher entropy. This underscores the significance of boundary conditions, particularly the lower limit, in achieving a stationary probability distribution of money.

3.3.2 State creation of Money (Exogenous Money)

In alignment with Yakovenko's line of reasoning, let us delve into his perspectives on the concept of exogenous money within the realm of economics. One of the intriguing facets of modern economics lies in the sovereign state's unique authority to create money. This authority allows the government to issue fiat money, essentially bringing new money into circulation. Unlike the conservation principles we often associate with physics, money creation in economics involves a different dynamic—one where new money flows into an economic system from an external source.

To draw a parallel with physics, he asks us to consider the Earth as a closed system. Within this system, the total energy remains constant, subject only to changes due to energy transfer across its boundaries—such as the energy received from the Sun or radiated into space. Similarly, in economics, we can define an economic system comprising private agents, with the state positioned outside of this system. Transactions among these private agents within the system, often referred to as "horizontal" transactions, do not alter the total money supply. However, when we introduce transactions between the economic system and the state, referred to as "vertical" transactions, new money flows across the boundary into the economic system.

Yakovenko then proceeds to ask: Why would a state need to inject new money into its economy? [4] It is argued that one key reason is population growth. When a population increases, but the money supply remains constant, the money per capita—referred to as the money temperature—decreases. This can lead to deflation, a situation where prices decline. When people anticipate falling prices, they tend to hoard money rather than spending it. This hoarding behavior further reduces the amount of money circulating in the economy, exacerbating deflationary pressures and potentially stifling economic activity.

To prevent such adverse effects, the state may find it necessary to increase the money supply, ideally in proportion to the population growth. This concept aligns with the idea of a monetary rule for injecting money into the economy at a steady rate, which can help stimulate spending and deter money hoarding. One compelling approach for the state to introduce new money is by investing in public infrastructure projects that benefit society as a whole. These projects, financed through a combination of taxes and new money creation, have the dual advantage of addressing crucial infrastructure needs and maintaining economic stability.

In practice, many countries segregate their governments from their central banks. While the government can generate revenue through taxes, fees, or borrowing via treasury bonds, these actions do not directly alter the total money supply within the economic system. Instead, money creation occurs when the central bank, often independently, purchases treasury bonds from commercial banks. This process introduces new money into the system, albeit through an indirect route. The term "monetization of government debt" arises in this context. When the Federal Reserve acquires treasury bonds, it effectively creates new money, facilitating government spending. It's important to note that the interest paid on these bonds by the Treasury to the Fed—though recorded as "profit" by the Fed—is eventually returned to the Treasury, leading to a net effect of an interest-free loan from the Fed to the Treasury. This debt between the Treasury and the Fed can be considered as an accounting artifact rather than a true debt burden. Understanding the mechanisms of exogenous money creation by the state is essential for comprehending the dynamics of modern economies, their stability, and the globalized nature of monetary flows [2].

3.3.3 Debt (Endogenous Money)

In his paper, Yakovenko delves into the intricate relationship between money and debt, shedding light on the crucial distinctions that often elude common understanding. His primary objective is to offer a comprehensive analysis of debt and its interaction with money, particularly in the realm of peer-to-peer lending. Through his meticulous examination, he aims to dissipate prevalent misconceptions and provide a new perspective on these fundamental concepts. His central argument revolves around the assertion that the total amount of money only experiences genuine growth when it is created by the state or central bank. In all other scenarios, what may appear as an expansion of money supply is, in fact, the creation of debt.

Yakovenko starts his exploration by scrutinizing peer-to-peer lending, a practice where "wealthier" individuals extend loans to "less affluent" counterparts in exchange for an IOU, encompassing a commitment to repay the principal along with interest. A critical clarification emerges at this juncture: the act of lending within this framework does not precipitate the generation of new money. The pivotal realization is that the overall quantity of money circulating within the system remains static. Instead, what unfolds is the transfer of financial assets from one participant to another. In this process, the lender's money balance diminishes, while the borrower's balance escalates, maintaining equilibrium within the monetary system. Interest constitutes an essential aspect of lending transactions. When borrowers discharge their obligations by repaying the principal alongside the accumulated interest, it is imperative to acknowledge that this supplementary money does not materialize out of thin air. Rather, it originates from within the system itself, typically involving other agents who contribute to this monetary redistribution. Hence, it is important to emphasize that interest linked to debt transactions does not instigate the creation of fresh currency units; instead, it orchestrates a reallocation of money among various agents. This recalibration aligns with the fundamental principle of money conservation.

One of Yakovenko's most surprising contributions is the distinction he draws between money and debt [4]. While superficially, debt and negative balances in an individual's financial account may bear semblances to money, it is of paramount importance to delineate these concepts accurately. Money transactions maintain the virtues of anonymity, finality, and temporal independence, contrasting starkly with debt transactions, which encompass personal commitments and specified repayment timelines. Furthermore, debt carries the potential for legal ramifications in the event of non-payment, a dimension that remains absent in the realm of money transactions. Notably, while debt may incorporate interest, money transactions intrinsically lack this interest-bearing element.

Yakovenko's work establishes a thought-provoking analogy between debt and particle-antiparticle pair creation in physics. To delve deeper into the intricacies of debt, the concept of net worth, denoted as w, is introduced. Net worth is defined as the summation of an agent's money and financial obligations, including debt. Drawing a parallel from the realm of particle physics, borrowing money can be likened to the creation of a particle-antiparticle pair, where an agent's net worth remains unchanged. Essentially, borrowing enables agents to temporarily exist in a "negative" net worth state, effectively eliminating a boundary condition that would otherwise restrict their spending capacity. Contrary to some prevailing notions, Yakovenko asserts that debt does not naturally stabilize itself without external constraints. Through an agent-based simulation, it becomes evident that in the absence of limitations on debt, the system's entropy experiences unbounded growth. Negative balances accumulate alongside positive balances, perpetuating a state of ongoing instability. This dynamic bears a resemblance to diffusion processes, underscoring the absence of an equilibrium state for debt without the imposition of external restrictions. Moreover, introducing interest into the debt equation fails to stabilize the system; rather, it exacerbates its inherent instability. Negative balances become increasingly negative, while positive balances grow more positive, leading to a further widening divergence within the system. It is important to acknowledge that, in isolation, interest does not contribute to stability. Only when debt restrictions are enforced can the system achieve the desired equilibrium [2].

Furthermore the author highlights a common fallacy in economic models: the assumption that all debt will be repaid as promised. This fallacy overlooks the statistical reality that not all agents will be able to fulfill their debt obligations. When agents default on their debts, it has significant consequences. Bankruptcy may be declared, erasing the debt but also affecting the asset held by another agent, resulting in a kind of debt annihilation. Alternatively, agents may resort to taking on new debt to repay old debts, delaying repayment and potentially leading to a critical point where a wave of bankruptcies occurs, akin to what economist Hyman Minsky referred to as a "Minsky moment" [50]. The author also explores the concept of collective behavior and synchronization in debt dynamics. Debt expansion and contraction often occur in a highly correlated and synchronized manner among agents, leading to the emergence of strong collective modes within the economic system. Mathematically and conceptually understanding this transition between debt expansion and contraction poses a challenging problem in the field of Econophysics. Furthermore, the author discusses how inflation of collateral prices can synchronize debt. Lenders often require borrowers to pledge collateral, and paradoxically, lending for asset purchases can inflate their market prices, contributing to the creation of credit bubbles. It’s noted that banking regulations, such as Basel regulations, failed to account for the circular logic in "mark-to-market" accounting, exacerbating the instability caused by lending [4].

4 Boltzmann-Gibbs Distribution and Agent-Based Simulations

In the preceding chapters, we explored the intertwined history of physics and economics and discussed the fundamental role of money in our economic systems. Now, we continue our exploration by building upon the perspective of Yakovenko regarding the unequal distribution of money. To gain deeper insights into the extent of money inequality under various conditions, we employ computational simulations using agent-based models.

Our computational model is based on a straightforward premise: a group of individuals, represented as agents, starts with some initial amount of money, denoted as m_i, and they exchange money with each other, referred to as m_e. The primary goal of our study is to observe how money inequality evolves over time under different circumstances. To ensure the accuracy and reliability of our analysis, we conduct the computer simulation over a sufficient number of rounds until we reached a dynamical equilibrium. Additionally, we repeat the simulation multiple times to validate our results.

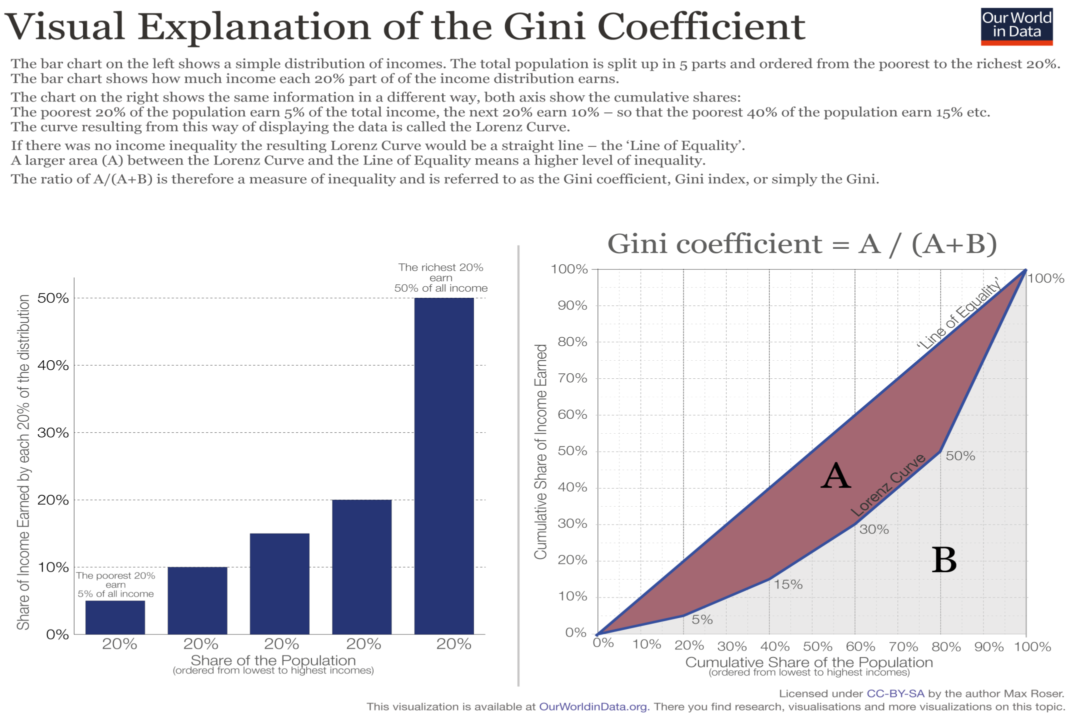
To assess changes in money distribution and inequality, we employ various statistical measures. These include the Gini index, which quantifies the level of inequality; percentiles, which offer insights into specific sections of the distribution; and moments of the probability distribution, which capture different characteristics like the average, spread, skewness, and kurtosis.

In essence, we aim to use computational models and statistical analysis to gain a deeper understanding of how money inequality develops over time in different scenarios.

4.1 Statistical Measurements

4.1.1 Gini Coefficient

The Gini Coefficient is a widely used measure that helps us understand how unequal a distribution is, whether it's income, wealth, or money. It provides a numerical representation of inequality, ranging from 0 to 1. Values closer to 0 suggest lower inequality, while values closer to 1 indicate higher inequality. There are two methods to calculate the Gini Coefficient. The first method looks at the expected absolute difference between individual incomes or wealth compared to the average of the entire population. This difference is measured relative to twice the mean income or wealth. The second method involves the Lorenz curve, providing a visual perspective on coefficient calculation [1].


Visual Explanation of the Gini Coefficient

The bar chart on the left shows a simple distribution of incomes. The total population is split up in 5 parts and ordered from the poorest to the richest 20%.
The bar chart shows how much income each 20% part of of the income distribution earns.
The chart on the right shows the same information in a different way, both axis show the cumulative shares:
The poorest 20% of the population earn 5% of the total income, the next 20% earn 10% – so that the poorest 40% of the population earn 15% etc.
The curve resulting from this way of displaying the data is called the Lorenz Curve.
If there was no income inequality the resulting Lorenz Curve would be a straight line – the 'Line of Equality'.
A larger area (A) between the Lorenz Curve and the Line of Equality means a higher level of inequality.
The ratio of A/(A+B) is therefore a measure of inequality and is referred to as the Gini coefficient, Gini index, or simply the Gini.

Gini coefficient = A / (A+B)

Licensed under CC-BY-SA by the author Max Roser.
This visualization is available at OurWorldinData.org. There you find research, visualisations and more visualizations on this topic.

Figure 4.1: Gini Coefficient (Taken from: [1])

In essence, the Gini Coefficient quantifies the disparity or uniformity in a distribution, helping us understand how income, wealth, or money is shared among a group of people.

The computer program employs a specific formula to calculate the Gini Coefficient:

$$G = \frac{\sum_{i=1}^{n}(2i - n - 1).x_i}{n\sum_{i=1}^{n} x_i} \tag{4.1}$$

4.1.2 Percentiles and Moments of Probabilities distributions

While the Gini Coefficient is valuable, it has limitations in capturing all aspects of a distribution. In this agent-based model, we use additional statistical indicators to gain a more comprehensive understanding of probability distributions. These indicators include percentiles and moments of the probability distribution.

Percentiles are a statistical measure that express a specific position or value within a data set. They indicate the percentage of data points that fall below or equal to a particular value. In our analysis, we focus on specific percentiles like 50, 75, 90, and 99, which are significant in economic literature [51].

The mean is the first central moment of a probability distribution, is also known as the expected value, represents the average value that you would expect to obtain if you repeatedly sample from the distribution. Mathematically the mean of a discrete probability distribution with outcomes $x_1, x_2, ..., x_n$ and the corresponding probabilities $p_1, p_2, ..., p_n$ is given by:

$$\mu = E[X] = \sum_{i=1}^{n} x_i . p_i \tag{4.2}$$

The variance, almost similarly as the Gini coefficient, it is the second central moment of a distribution and a value that measures the spread or dispersion of the data in relation to the mean of the distribution. A higher variance indicates greater variability in relation to the mean, a lower variance on the other hand suggests consistency of the outcomes in relation to the mean of the probability distribution. Mathematically is given by:

$$\sigma^2 = E[(X - \mu)^2] \tag{4.3}$$

Skewness is the third standardized moment of a probability distribution. It measures the asymmetry of the distribution's tail around its mean. It indicates whether the distribution is skewed to the left (negatively skewed) or to the right (positively skewed). The skewness γ_1 is directly calculated by the third central moment of the distribution:

$$\gamma_1 = \frac{E[(X - \mu)^3]}{\sigma^3} \tag{4.4}$$

The last indicator that will be presented is the Kurtosis, the fourth standardized moment of a probability distribution, it measures the tailedness or the degree of peakedness of a distribution's shape compared to the normal distribution. It provides information about the distribution's tails and the concentration of data near the mean. It is defined as:

$$k = \frac{E[(X - \mu)^4]}{\sigma^4} \tag{4.5}$$

The probability distributions can be characterized in relation to the excess kurtosis. Three distinct regimes are obtained: the mesokurtic distribution where the excess kurtosis is near zero, like the normal distribution; the leptokurtic distribution, which is a distribution with positive excess kurtosis when compared to the normal distributions, these one's have fatter tails. Then we have platykurtic distributions where the excess kurtosis is negative and so they are characterized by thinner tails.

4.2 Statistical Physics of Money Distribution

In statistical physics, the focus is on understanding the macroscopic behavior of nature by studying the collective behavior of large assemblies of microscopic entities. Yakovenko applies this perspective to economics, considering economic agents as the microscopic entities whose interactions give rise to macroeconomic phenomena. By studying the behavior of these agents and their interactions, he aims to provide insights into how macroeconomic properties, such as income distribution, emerge from the underlying micro-level dynamics [2].

He primarily focused on studying random variables, such as energy, that adhere to conservation laws. For instance, when a system with energy E is divided into two subsystems, the total energy of the subsystems is given by $E_1 + E_2 = E$, however the probability distribution, denoted as $P(E)$, can be expressed as the product of the probability distributions of the two subsystems: $P(E) = P(E_1) * P(E_2)$. This relationship yields the exponential function as the general solution for these equations:

$$P(E) \propto e^{-\frac{E}{T}} \tag{4.6}$$

known as the Boltzmann-Gibbs distribution, where T is the temperature and in this equation k_B, Boltzmann constant is unitary, so T has the same dimension of E.

Yakovenko believed that this equation had broader applicability beyond physics. He aimed to apply this equation to a wider range of systems, as long as they could be characterized as statistical ensembles with a conserved variable. The field of economics appeared to be a promising candidate for applying this equation, given that it represents a large-scale statistical ensemble. However, a key challenge remained: determining the specific variable that could be considered as conserved within the economic system.

The main purpose of economy is the production and exchange of goods and services but has they are used/consumed it seems there is no conservation. However, he argues, in a monetary economy this goods and services are exchanged for money and ordinary agents can only give and receive money not create it. In the following example Yakovenko tries to show that money satisfies a local conservation law.

Let us imagine two agents, agent 1 has initial money $m_1$and transfers money m to agent 2, which has initial money m_2,in exchange of a good g agent 2 can provide. Then agent 1 consumes that good making it disappear. However the new money balance (m'_1, m'_2) of the agents remains:

$$\begin{cases} m'_1 = m_1 - m \\ m'_2 = m_2 + m \end{cases} \tag{4.7}$$

He argues that while in physics conservation laws are derived from fundamental space-time symmetries, conservation of money is the 'law' of accounting. Following the previous analogy Yakovenko concluded that when applying this to a agents in an economic system in statistical equilibrium the probability distribution of money $P(m)$ ought to be given by the exponential function in the form:

$$P(m) \propto e^{-\frac{m}{T_m}} \tag{4.8}$$

where T_m is the money temperature and it is given by the average amount of money per agent.

In order to explain the evolution of $P(m)$ in time t and its stable result he uses the Boltzmann kinetic equation:

$$\frac{dP(m)}{dt} = \int\int\{-f_{[m,m']\to[m-m_e,m'+m_e]}P(m)P(m')+f_{[m-m_e,m'+m_e]\to[m,m']}P(m-m_e)P(m'+m_e)\}dm'dm_e \tag{4.9}$$

where the term $f_{[m,m']\to[m-m_e,m'+m_e]}$ represents the probability of transferring a sum of money m_e from an agent with money m to an agent with money m' per unit of time. And the terms $P(m)$ and $P(m')$ represent the occupation numbers, i.e., the probability of finding an agent with money m and another agent with money m' in the system, respectively.

As it can be seen from the previous equation when the rates of direct transitions (money transfers) and reversed transitions (money returns) are equal (resulting of time-reversal symmetry), the probability distribution reaches a state of equilibrium, where $\frac{dP(m)}{dt} = 0$.This is the principle of detailed balance and it ensures that the system is not continually changing and has reached a stable state.

In this circumstances equation 4.9 is reduced to:

$$\int\int\{P(m)P(m') + P(m - m_e)P(m' + m_e)\}dm'dm_e = 0 \tag{4.10}$$

and so the general solution for this equation is4.8 which is the Boltzmann-Gibbs distribution.

Also, when discussing the nature of economic transactions, Yakovenko distinguishes between additive and multiplicative processes. In additive processes, the amount of money transferred (m_e) does not depend on the initial amount (m_i), while in multiplicative processes, m_e is a percentage of the initial amount. These distinctions can affect whether the time-reversal symmetry condition holds.

For small transaction amounts m_e, the Boltzmann Kinetic Equation can be approximated by the Fokker-Planck equation:

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial m}[AP + \frac{\partial(DP)}{\partial m}] \tag{4.11}$$

This equation, also known as the Kolmogorov forward equation, describes how the probability distribution evolves over time and is linear with respect to $P(m)$. The coefficients A (drift) and D (diffusion) in this equation represent the first and second moments of money balance changes per time increment Δt and are given by [4]:

$$\begin{cases} A = -\frac{<m_e>}{\Delta t} \\ D = \frac{<m_e^2>}{2\Delta t} \end{cases} \tag{4.12}$$

However Yakovenko shares a similar perspective of some classical economists [4]. He makes a distinction between the physical layer and the monetary layer of the economy and the way they interact. Within the physical layer of our economy, we witness the tangible essence of economic activity—the production, exchange, and utilization of physical objects, encompassing goods and services alike. Governed by the unwavering laws of physics and bound by constraints such as energy availability, natural resource limits, and environmental impacts, this layer is subject to the flow of reality. Here, objects can be both brought into existence and subjected to consumption or depletion, a dynamic interplay that shapes the physical realm of our economic landscape. Now, shifting the focus to the monetary realm—the dynamic stage where money flows between economic actors in exchange for an array of goods and services. In this dimension, money takes on a somewhat surreal form, essentially a collection of digital data bits. It operates as an informational undercurrent within the economic landscape, guided by its unique set of principles, including the nuanced laws of accounting. Money, in this context, assumes a role akin to a knowledgeable authority, communicating valuable insights to economic participants. These insights serve as guiding lights, directing their decisions towards paths that ideally resonate with the broader needs of society. In this utopian scenario, economic actors would naturally gravitate toward income-generating avenues that align with society's most pressing demands. The intrinsic constraint of money—a guard against free-riding—instills a sense of discipline, compelling these agents to embrace roles dictated by societal necessities, even if they may differ from their personal preferences. Although this two layers are interconnected in economic transactions between agents, physical goods and digital money flow in opposite directions [2]. However, it's crucial to understand that objects in these different layers cannot be directly converted into each other. They are fundamentally separate.

So the purpose of the computer simulations in the next section is the exploration of the monetary layer in economy. Although we are aware this represents a somewhat limiting view of economy in the real world it still feels like something worth being explored.

4.3 Computer Simulations of Economic agent's model

4.3.1 Two Level Model

In this model, economic agents exhibit homogeneous behavior. Initially, each agent receives an equal amount of initial money denoted as m_i. Over time, they engage in exchanges involving a fixed and uniform quantity of money units, represented as m_e. To ensure that no agent incurs debt, a constraint is enforced: agents with money units m less than the number of units m_e to be exchanged are only allowed to receive money but not give it. Agents falling into the category of $m < m_e$ are considered in level 0, while those with $m \geq m_e$ are in level 1. Importantly, this simulation conserves the total amount of money within the economic system. To gain comprehensive insights into the behavior of the probability distribution of money units, we conducted simulations and examined their correlations with various factors. These factors encompass the number of agents, the initial amount of money units, the exchanged money units, and the duration of the simulation. The following graphics present the results of our analysis.

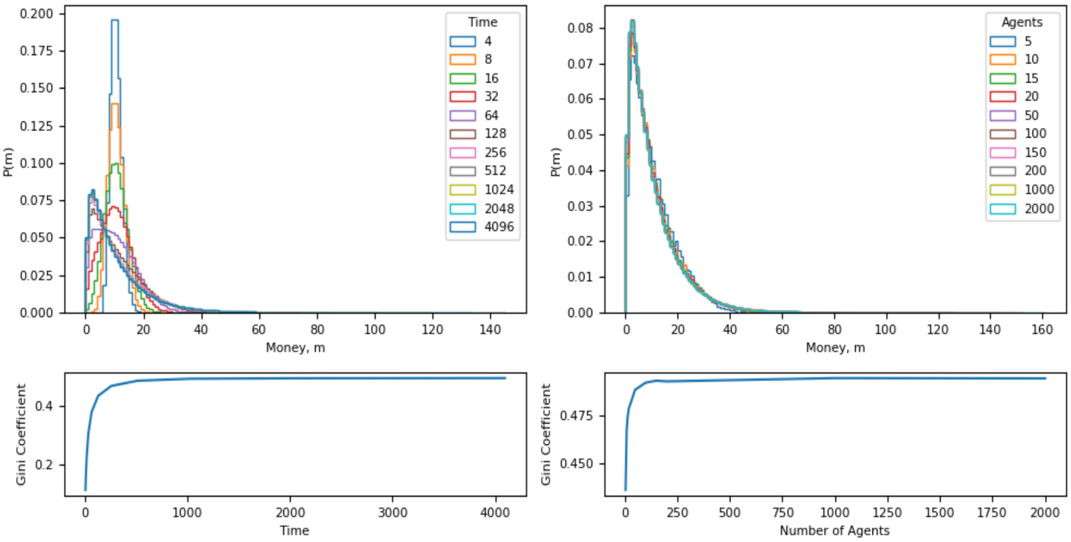


Figure 4.2: Evolution of money probability distribution in time and with number of agents

In Figure 4.2, all agents initially possess 10 units of money ($m_i = 10$), and each round involves an exchange of $m_e = 1$ unit. This figure illustrates the evolution of the money distribution over time. Notably, as indicated by the corresponding Gini coefficient graph, after approximately 2000 rounds, the money distribution stabilizes, resulting in higher inequality, despite all agents having started with equal initial amounts. Furthermore, we explored the impact of the number of participating agents on the money distribution and made a surprising discovery: the number of participating agents has a minimal effect on the money distribution's characteristics.

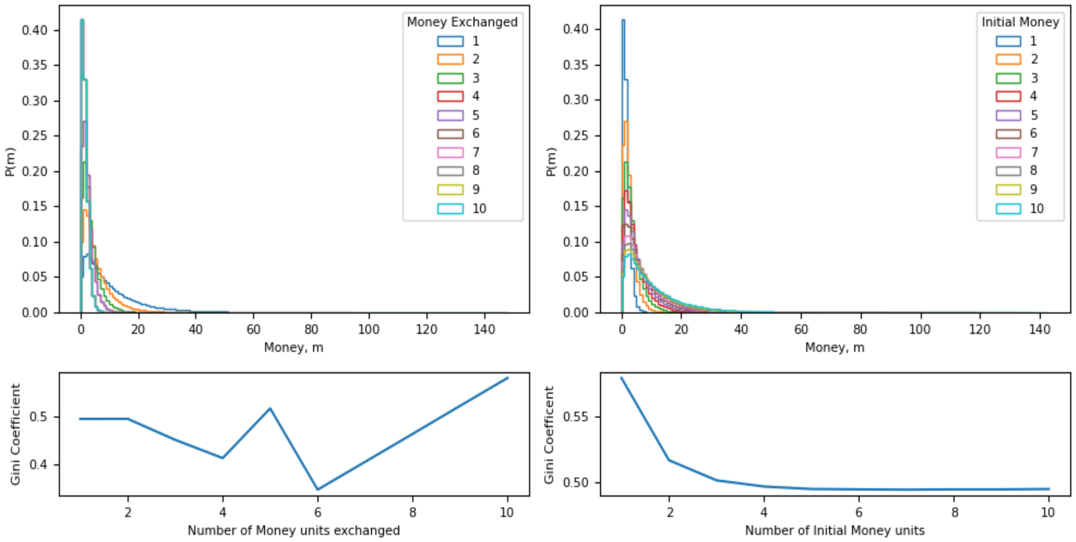


Figure 4.3: Evolution of money probability distribution with number of money exchanged and with number of initial money

Figure 4.3 delves into the interplay between money distribution, the quantity of money units exchanged (m_e), and initial money units. In the first graph, all agents commenced with $m_i = 10$, and we incrementally raised the exchanged units by one in each simulation. After 4000 rounds, we gauged the degree of economic inequality. Strikingly, we observed that when the number of money units exchanged closely matched the initial endowment, the convergence towards stable economic inequality occurred more swiftly. Conversely, in the second graph, we held the number of units exchanged at $m_e = 1$ for all agents. Across various simulations, we incrementally augmented the initial money units by one. Subsequently, after 4000 rounds, we assessed inequality levels. The corresponding Gini coefficient graph unveiled a consistent pattern: for initial money units equivalent to or exceeding 4, with only one unit exchanged per round, inequality remained stable.

In summation, our analysis underscores that the final money distribution is influenced by a multitude of factors, encompassing time duration, exchanged money units, and the initial money units.

4.3.2 Diffusion behavior

In the simulation's initial phase, when all agents possess an equal amount of money and before any agent encounters a situation where they cannot engage in exchanges, the system's behavior can be effectively approximated using the 1D diffusion equation,

$$\frac{\partial p(m,t)}{\partial t} = D\frac{\partial^2 p(m,t)}{\partial^2 m} \tag{4.13}$$

where $p(m,t)$ represents the probability distribution of money m at time t and D represents the diffusion coefficient. The initial condition in this model is the delta Dirac probability distribution $\delta(m - m_i)$ where m_i are the initial money units. However, as no agent is allowed to go into debt there is a constrain $p(m < 0, t) = 0$

When running the simulation, we notice that the initial Dirac money distribution transforms into a Gaussian money distribution after several rounds of agents engaging in exchanges. In the figure below, we can visually observe the diffusion behavior during this transformation.

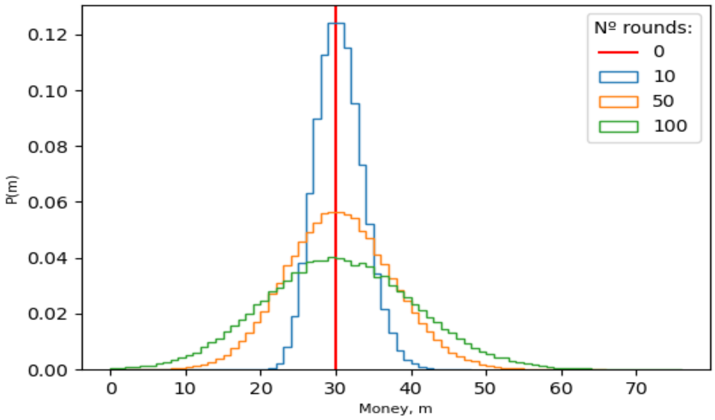


Figure 4.4: Initial Dirac Money Distribution evolving to Gaussian Money Distribution

The solution to equation 4.13, prior to reaching the lower boundary of zero, follows the Gaussian distribution equation:

$$p(m,t) = \frac{1}{\sqrt{4\pi D t}} e^{-\frac{(m-m_i)^2}{4Dt}} \tag{4.14}$$

Here, with an initial value of $m_i = 30$ and using equation 4.12, we can calculate the diffusion coefficient D. Throughout the simulation, the exchanged value remains constant at $m_e = 1$, and the time interval, represented by Δt, corresponds to the smallest time increment, which, in this simulation, is equivalent to 1 round.

$$D = \frac{< m_e^2 >}{2\Delta t} = \frac{1}{2} \tag{4.15}$$

Substituting the values above in the equation 4.14 and for $t = 100$ it can be observed, in the Fig. 4.5, the similarity between the analytical solution and the numerical result for a low number of rounds:

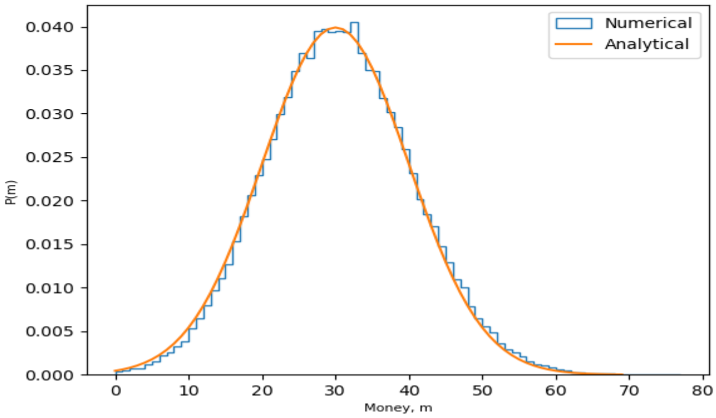


Figure 4.5: Analytical solution and numerical result for Gaussian distribution for 100 rounds

As certain agents reach the lower boundary ($m = 0$), the distribution undergoes an asymmetrical transformation, shifting towards an exponential distribution. This change in behavior corresponds to the solution of the diffusion equation under the condition that no agents are allowed to go into debt. In Figure 4.6, after 10,000 rounds and the stabilization of the final distribution, we can observe the similarity between the analytical result and the numerical simulation. In accordance with equation 4.8, where T_m represents the average amount of money per agent, the analytical solution takes the following form:

$$p(m) = \frac{1}{m_i} e^{-\frac{m}{m_i}} \tag{4.16}$$

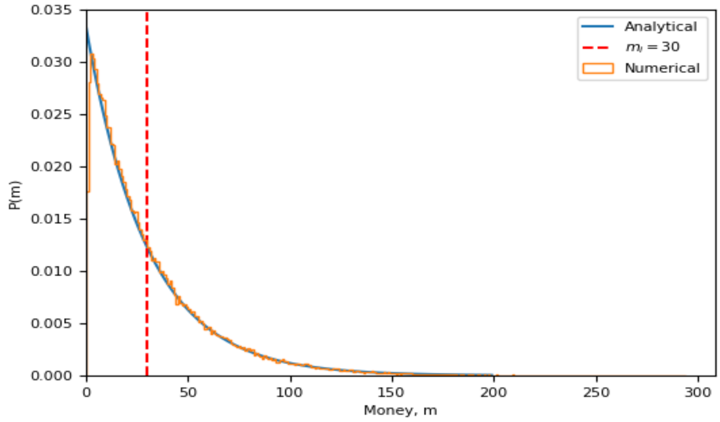


Figure 4.6: Analytical solution and numerical result for final exponential distribution for 10 000 rounds

In this straightforward simulation, where all agents commence on equal footing (Gini coefficient = 0) and exchange an identical number of money units, the final distribution unexpectedly demonstrates a trend towards inequality, specifically following an exponential distribution. The concluding Gini coefficient stabilizes at 0.5. In the below table some statistical values such as: initial and final Gini coefficients, percentiles 50,75,90,99 (respectively P_{50}, P_{75}, P_{90}, P_{99}) and Kurtosis, are presented so it can be observed that it does not matter which initial money distribution we choose the final Gini coefficient converges to a common value. All values of percentiles are displayed in money units and the Kurtosis values were obtained using the fisher's definition, where the reference value of kurtosis for a normal distribution is 0.

Initial Distribution	Initial	Final	P_{50}	P_{75}	P_{90}	P_{99}	Kurtosis
Dirac	0	0.50	21	42	69	137	5.64
Gaussian	0.19	0.50	21	41	68	134	5.46
Exponential	0.50	0.50	21	41	69	138	5.60
Power	0.60	0.50	20	41	69	139	11.2

Table 4.1: Statistical values of the Gini coefficient for: 2 Level Model for t = 50 000 and $m_e = 1$

4.3.3 Three and Four Level Model

To introduce some complexity into the simulations, we opted to explore a similar scenario where agents engage in exchanges without yet incurring in debt. However, we varied the rules governing the exchange of money units. In a model with three levels, we have agents in level 0 ($m = 0$), agents in level 1 ($m \geq m_e$), and agents in level 2. To determine which agents belong to level 2, we set a threshold (m_1) value for m, which must be greater than or equal to m_i. Agents whose money exceeds or equals this threshold are classified as level 2. Furthermore, agents in level 1 continue to exchange $m_e = 1$, while agents in level 2 (who possess more money than level 1) exchange $m_e = 2$.

In a model with four levels, we extend this framework to include agents in level 0 ($m = 0$), agents in level 1 ($m_e \leq m < m_1$), and agents in level 2 and level 3. To distinguish between level 2 and level 3, we establish two distinct thresholds for m, with the threshold (m_2) for level 3 being higher than that of level 2. In addition agents in level 3 will exchange $m_e = 4$ units. In general, individuals with higher money units contribute more funds.

Moreover in the subsequent simulations, we will initiate the experiments with varying initial money distributions, including Gaussian, exponential, and power distributions. To observe the outcomes when agents begin under different circumstances. While money conservation might lead one to expect that all initial distributions would eventually converge to the same final distribution over time, altering the rules of money exchange at different levels introduces variations in the final distribution.

We will begin by examining the three-level model, at first we will analyze this model with money exchanged defined by $m_e = [1, 2]$, which means that agents with more money contribute with more money, for different initial money distribution. The threshold decided was for $m_1 = 60$. Agents in level 1 ($1 < m < 60$) will give one money unit and agents in level 2 ($m \geq 60$) will give 2 money units. Moreover in the different initial distributions the average money of each agent is $m_i = 30$.

Next, within the same model, we alter the rules for money exchange to $m_e = [2, 1]$, where agents with less money contribute more. Tables 4.2 and 4.3 display the statistical values obtained after 50,000 rounds.

Initial Distribution	Initial	Final	P_{50}	P_{75}	P_{90}	P_{99}	Kurtosis
Dirac	0	0.33	30	45	54	59	-1,21
Gaussian	0.19	0.33	30	45	54	59	-1,19
Exponential	0.50	0.33	30	45	54	59	-1,19
Power	0.60	0.33	30	45	54	59	-1,19

Table 4.2: Statistical values of the Gini coefficient for: 3 Level Model for t = 50 000 and $m_e = [1, 2]$

Initial Distribution	Initial	Final	P_{50}	P_{75}	P_{90}	P_{99}	Kurtosis
Dirac	0	0.94	1	2	5	709	24.77
Gaussian	0.19	0.94	1	2	5	698	24.99
Exponential	0.50	0.94	1	2	5	689	25.41
Power	0.60	0.96	1	2	5	728	34.75

Table 4.3: Statistical values of the Gini coefficient for: 3 Level Model for t = 50 000 and $m_e = [2, 1]$

By comparing the two tables showcasing different money exchange rules, we gain insights into the substantial impact of our initial money distribution choices on the final wealth distribution. However, as expected, both scenarios, $m_e = [1, 2]$ and $m_e = [2, 1]$, converge to final distributions with nearly identical Gini coefficients and closely resembling percentiles. It is possible to observe as well, by kurtosis values, that with an 'unfair' money exchange rule it is much more probable to find extreme values in the final money distribution.

In the four-level model, the same analysis will be conducted, initially starting with $m_e = [1, 2, 4]$, which corresponds to a scenario where agents with higher money contribute with more money. This analysis is conducted for different initial money distributions. Specifically, we set thresholds at $m_1 = 30$ and $m_2 = 60$. Agents in level 1 ($1 \leq m < 30$) contribute one money unit, level 2 agents ($30 \leq m < 60$) contribute two money units, and level 3 agents ($m \geq 60$) contribute with four units. Subsequently, within the same model, we alter the rule of money exchange to $m_e = [1, 2, 1]$ and $m_e = [4, 2, 1]$, where agents with lower amounts of money contribute with more money. Tables 4.4, 4.5, 4.6 present the statistical findings obtained after 50,000 rounds.

Initial Distribution	Initial	Final	P_{50}	P_{75}	P_{90}	P_{99}	Kurtosis
Dirac	0	0.07	30	32	35	42	2.51
Gaussian	0.19	0.07	29	32	34	41	2.64
Exponential	0.50	0.08	30	33	37	45	2.78
Power	0.60	0.07	30	32	35	46	3.25

Table 4.4: Statistical values of the Gini coefficient for: 4 Level Model for t = 50 000 and $m_e = [1, 2, 4]$

Initial Distribution	Initial	Final	P_{50}	P_{75}	P_{90}	P_{99}	Kurtosis
Dirac	0	0.51	21	26	28	31	438
Gaussian	0.19	0.55	21	26	28	31	366
Exponential	0.50	0.65	16	24	28	490	33
Power	0.60	0.65	15	23	28	522	74

Table 4.5: Statistical values of the Gini coefficient for: 4 Level Model for t = 50 000 and $m_e = [1, 2, 1]$

Initial Distribution	Initial	Final	P_{50}	P_{75}	P_{90}	P_{99}	Kurtosis
Dirac	0	0.97	0	1	4	861	33
Gaussian	0.19	0.96	0	1	4	867	34
Exponential	0.50	0.96	0	1	4	879	33
Power	0.60	0.97	0	1	4	894	42

Table 4.6: Statistical values of the Gini coefficient for: 4 Level Model for t = 50 000 and $m_e = [4, 2, 1]$

When comparing the four-level model to the two-level and three-level models, we observe a significant reduction in the Gini coefficient when regular money exchange rules are applied. However, this may be a result of insufficient simulation rounds, as we would anticipate eventual convergence between the four-level and three-level models toward a distribution similar to that of the two-level model. It appears that the introduced thresholds in the various models have acted as delaying factors in achieving inequality in the final distribution.

Nevertheless, it's crucial to highlight the distinct behavioral patterns within the four-level model when comparing the cases of $m_e = [4, 2, 1]$ and $m_e = [1, 2, 1]$. As expected, in the former scenario, the initial money converges to an extremely unequal final distribution. In the latter case, the burden of contributing money units is heavier in the middle level, resulting in both lower and higher money holders contributing equally. Interestingly, this case exhibits variations in the final outcomes depending on the initial distribution. The Dirac and Gaussian initial distributions, after 50,000 rounds, yield similar Gini coefficients but remarkably higher kurtosis values. Conversely, the Exponential and Power distributions have higher Gini coefficients but lower kurtosis levels. Another significant difference is observed in the 99th-percentile, which, for the distributions with higher kurtosis, has considerably lower values compared to the other two distributions. Consequently, with Dirac and Gaussian initial distributions, most agents possess money units below 31, but there is a much higher likelihood of agents having significantly higher amounts of money.

4.4 Model with debt (endogenous money)

In this section, all agents continue to engage in money exchanges as described in previous sections. However, a new element is introduced: each agent now has the ability to accumulate individual levels of debt, up to a predetermined maximum threshold. Various initial money distributions are considered, including Dirac Initial Money Distribution, Gaussian Initial Money Distribution, Exponential Initial Distribution, and Power Initial Distribution. Although there is a slight difference in the exchange process due to the introduction of debt, it's important to note that money remains conserved within the system, thanks to the imposition of a maximum allowable debt level. In this scenario, agents with zero money are permitted to incur debt to facilitate ongoing exchanges with other agents. The primary objective is to explore how this debt factor influences the level of money inequality within the money distribution. It is anticipated that, given the closed-system nature of the simulation and the conservation of money, the final distribution will converge to the same outcome, given a sufficient amount of time, regardless of the initial distribution. In this context, debt becomes synonymous with the generation of money within the system, without any involvement from a state or central bank to inject money (exogenous money).

Initial Distribution	Gini Coefficient : Debt = 0	Gini Coefficient : Debt = -10
Dirac	0.50	0.61
Gaussian	0.50	0.61
Exponential	0.50	0.61
Power	0.50	0.64

Table 4.7: 2 Level: Inequality of final distributions with different levels of debt for $m_e = 1$

Initial Distribution	Gini Coefficient : Debt = 0	Gini Coefficient : Debt = -10
Dirac	0.33	0.36
Gaussian	0.33	0.36
Exponential	0.33	0.36
Power	0.33	0.36

Table 4.8: 3 Level: Inequality of final distributions with different levels of debt for $m_e = [1, 2]$

Initial Distribution	Gini Coefficient : Debt = 0	Gini Coefficient : Debt = -10
Dirac	0.07	0.07
Gaussian	0.07	0.07
Exponential	0.08	0.07
Power	0.07	0.07

Table 4.9: 4 Level: Inequality of final distributions with different levels of debt for $m_e = [1, 2, 4]$

The more levels the model has the lower is the change of the Gini Coefficient. As we are evaluating these three models in the same period of time (50 000 rounds) it is expected that in the two-level model more agents reach the maximum debt level thus giving a higher contribution for the inequality of money distribution. Both the other levels have thresholds that delay the amount of agents going into debt.

In Tables 4.10, 4.11 and 4.12 the case where money exchange rules are more unbalanced is presented:

Initial Distribution	Gini Coefficient : Debt = 0	Gini Coefficient : Debt = -10
Dirac	0.94	0.94
Gaussian	0.94	0.94
Exponential	0.94	0.94
Power	0.96	0.95

Table 4.10: 3 Level: Inequality of final distributions with different levels of debt for $m_e = [2, 1]$

Initial Distribution	Gini Coefficient : Debt = 0	Gini Coefficient : Debt = -10
Dirac	0.51	0.53
Gaussian	0.52	0.55
Exponential	0.64	0.77
Power	0.65	0.79

Table 4.11: 4 Level: Inequality of final distributions with different levels of debt for $m_e = [1, 2, 1]$

Initial Distribution	Gini Coefficient : Debt = 0	Gini Coefficient : Debt = -10
Dirac	0.97	0.96
Gaussian	0.96	0.95
Exponential	0.96	0.95
Power	0.97	0.96

Table 4.12: 4 Level: Inequality of final distributions with different levels of debt for $m_e = [4, 2, 1]$

Both the three-level model and the four-level model with the most unfair distribution of money ($m_e = [2, 1]$ and $m_e = [4, 2, 1]$) have similar final results with and without debt. However, in the case of $m_e = [1, 2, 1]$, as we observed earlier, the outcomes vary. Even though all four initial money distributions lead to distributions with greater inequality when debt is introduced compared to scenarios without debt, the exponential and power initial money distributions exhibit more pronounced changes than the previous two. This is because in both of these distributions, a larger number of agents begin with lower amounts of money units, increasing the likelihood of them going into debt and consequently making a more substantial contribution to the Gini coefficient.

4.4.1 Maximum Level of Debt and Gini Coefficient

In this concluding section, we will explore the relationship between various maximum levels of debt and their corresponding Gini coefficients across all levels and for the different money exchange rules examined in the previous section.$m_e = [1, 2]$, $m_e = [1, 2, 4]$

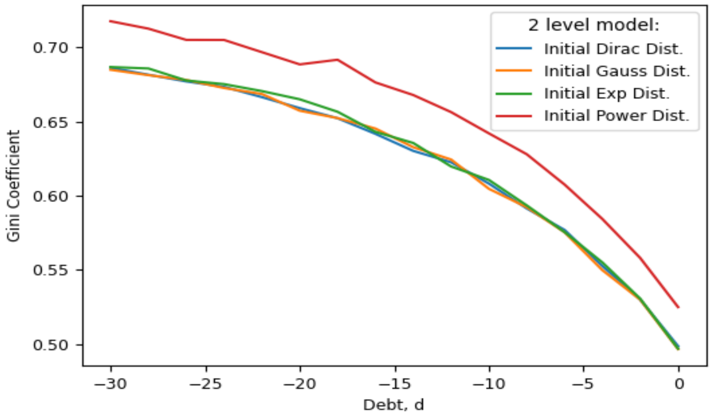


Figure 4.7: Gini coefficient dependency on the level of debt for $m_e = 1$

Starting with the two-level model and so, with $m_e = 1$, we observed that the higher the maximum level of debt the higher the inequality of the final distribution be. With an emphasis for the difference between the initial power distribution and the other three distributions, as seen previously as a first example in Table 4.7.

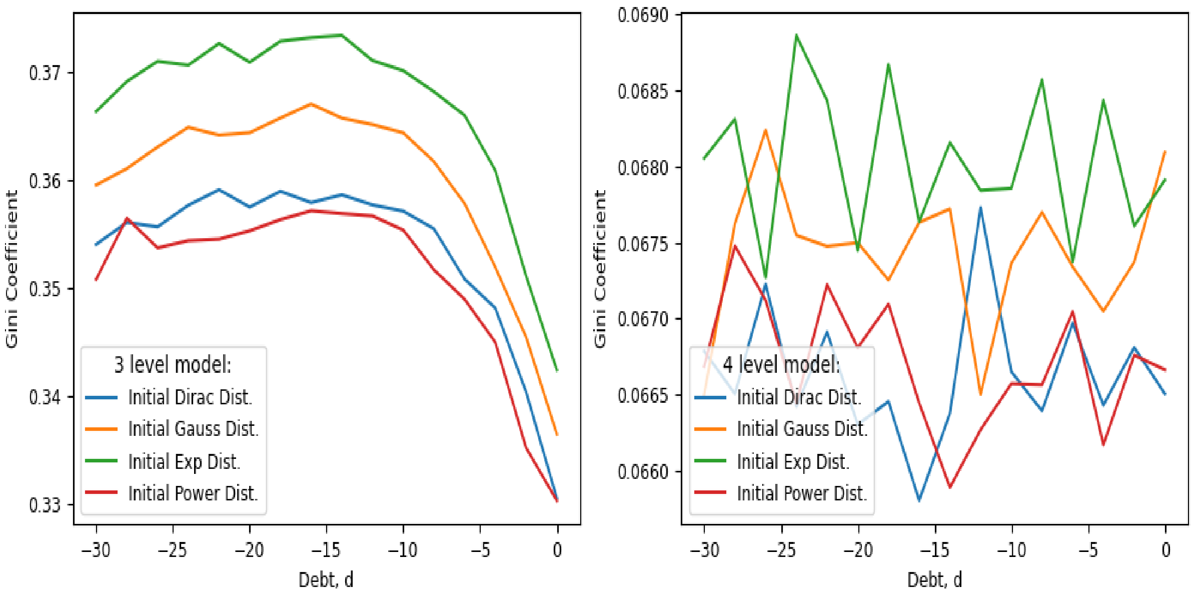


Figure 4.8: Gini coefficient dependency on the level of debt for $m_e = [1, 2]$ and $m_e = [1, 2, 4]$

As anticipated based on the initial findings presented in Tables 4.8 and 4.9, we observe minimal fluctuations in the Gini coefficient as we increment the maximum debt level. This phenomenon likely occurs due to the limited number of rounds in the simulation, which may not provide sufficient time for a substantial number of agents to accumulate debt.

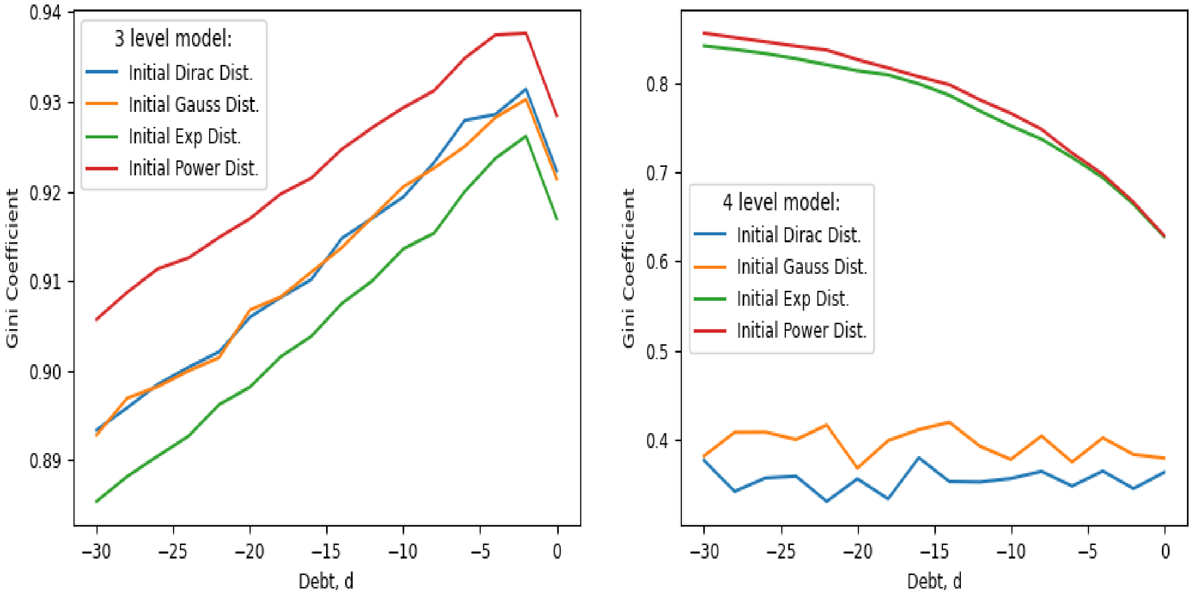


Figure 4.9: Gini coefficient dependency on the level of debt for $m_e = [2, 1]$ and $m_e = [1, 2, 1]$

In the final two sets of graphics, we observe distinct behaviors compared to the previous scenarios. In the three-level model with $m_e = [2, 1]$, there is a subtle decrease in the Gini coefficient as the level of debt increases. Conversely, in the four-level model with $m_e = [1, 2, 1]$, we discern a bifurcated pattern. The Dirac and Gaussian initial money distributions exhibit relatively stable Gini coefficient values as the level of debt rises. However, for the Exponential and Power initial money distributions, which start with higher inequality levels, the introduction of increased maximum debt amplifies the existing inequality even further.

4.5 Summary on the chapter

In this chapter we discussed the approach used by Yakovenko, on the use of statistical physics to understand the concept of money conservation in economy.

After, we performed some computer simulations and started by noting the diffusion behavior of the money distribution in the first rounds (before reaching the limit of $m = 0$) associated with the random exchange of money between agents. As it was seen in the graphics there is a good match between the analytical and numerical results.

Also it was possible to observe as well how the Boltzmann-Gibbs distribution fitted well the final money distribution associated with the two-level model money exchange process.

Then different money exchange rules and different initial money distributions were used to observe and compare the various statistical results displayed by the final money distributions in order to assess their different levels of inequality. It was shown that there is a clear impact depending if we chose a more or less equitable way of redistributing the money in the exchange process however, it seems the choice of initial money distribution does not have a significant impact.

Moreover, it was analyzed how different amounts debt in the system, with various level models, impacted the overall Gini coefficient of the final money distribution. The biggest impact of this variation was seen in the two-level model were clearly there was an increase in the Gini coefficient if the level of debt allowed in the system increased.

5 Individual Agent Paths and Money Unit Exchange

This chapter serves as an extension of the previous one; however, it takes a different approach. Instead of examining phenomena at a collective scale, this chapter explores the individual level. It should be noted that the results in this Chapter were developed specifically for this thesis and, as far as we know, are not present in any other bibliographic source.

5.1 Economic Agents' Model with l levels

The objective in this next step is to analyze how the money of agents in different levels changes on average. In this model, agents are categorized into multiple levels based on their amount of money, with each level denoted by l. Each level corresponds to a certain amount of money units exchanged (represented as m_l). As the level increases, agents contribute with more money units, except for the lowest level, which is labeled as the 'broke' level (N_0). In this level, agents cannot contribute. The primary aim of this model is to explore the average change of money units for agents belonging to a specific level.

5.1.1 Average money variation of an agent

The total money M of an agent a at time t, given in the below equation, will be a result of the initial money of the agent (m_i) together with the sum of all money variations of agent a in each round of the game

$$M_a(t) = m_i + \sum_t \delta m_a(t) \tag{5.1}$$

The change in money of an agent in each round is influenced by three main factors: their current level, the amount of money units exchanged in that level, and the contributions received from agents in all levels:

$$\delta m_a(t) = -m_{l(a)} + m_{l(a)}.n_{l(a)}(t) + \sum_{l=1;l\neq l(a)}^{L} m_l.n_l(t) \tag{5.2}$$

where $l(a)$ represents the level on which agent a is,$m_{l(a)}$ represents the amount of money units exchanged on agent a level ,$n_{l(a)}(t)$ represents the number of agents in agent a's level that give money to agent a, and $n_l(t)$ is the number of agents in a given level l, different from agent a level, in time t that contribute to agent a's amount of money .

For agents (N_0) who can not contribute:

$$\delta m_a(t) = \sum_{l=1}^{L} m_l.n_l(t) \tag{5.3}$$

And so the average money variation of an agent in level l is described by:

$$\overline{\delta m_a(t)} = \begin{cases} -m_{l(a)} + m_{l(a)} \cdot \overline{n_{l(a)}(t)} + \sum_{l=1;l\neq l(a)}^{L} m_l \cdot \overline{n_l(t)} & \text{for agents in } l \text{ level, } l = \{1, 2, ..., L\} \\ \sum_{l=1}^{L} m_l.\overline{n_l(t)} & \text{for agents in level } l = 0 \end{cases} \tag{5.4}$$

The number of agents that contribute to agent a's amount of money ($n_{l(a)}$ and $n_l(t)$), at time t in level l, are random variables that can be described by a binomial probability distribution.

So the probability of n agents, in the same level of agent a,to give money to agent i is:

$$P(n_{l(a)}) = \binom{N_{l(a)}-1}{n_{l(a)}} p^{n_{l(a)}}.(1-p)^{N_{l(a)}-1-n_{l(a)}} \tag{5.5}$$

$$P(n_{l(a)}) = \frac{(N_{l(a)}-1)!}{n_{l(a)}!(N_{l(a)}-1-n_{l(a)})!}.\left(\frac{1}{N-1}\right)^{n_{l(a)}}.\left(1-\frac{1}{N-1}\right)^{N_{l(a)}-1-n_{l(a)}} \tag{5.6}$$

where $N_{l(a)}$ is total number of agents in the same level of agent a and N is the total number of agents in all levels.

Other agents in different levels of agent a can contribute to its amount of money as well, so in the next equation we will calculate the probability of n of those agents to give money to agent a:

$$P(n_l) = \frac{(N_l)!}{n_l!(N_l-n_l)!}.\left(\frac{1}{N-1}\right)^{n_l}.\left(1-\frac{1}{N-1}\right)^{N_l-n_l} \tag{5.7}$$

In order to evaluate the average money variation of an agent in level l we need to calculate the average values of $n_{l(a)}$ and n_l.

So was its known the average value of a binomial variable can be deduced from the moment generating function of binomial distribution given by:

$$M_x(t) = \left(q + p.e^t\right)^n \tag{5.8}$$

where the expected value of the n-th moment of the binomial distribution is:

$$E\left(X^n\right) = \frac{d^n}{dt^n} M_x(t)\,|_{t=0} \tag{5.9}$$

in this particular case we want to calculate the first moment $(n = 1)$ and so the average value of a random variable X is

$$\overline{X}{=}np \tag{5.10}$$

The variance of a random variable X is given by:

$$Var(X) = E\left(X^2\right) - \overline{X}^2 \tag{5.11}$$

in order to calculate the variance of a binomial variable we need to calculate the second moment $(n = 2)$:

$$\left\langle X^2 \right\rangle = np\,(1-p) + n^2p^2 \tag{5.12}$$

substituting the terms in the previous variance equation we get

$$Var(X) = E\left(X^2\right) - E\,(X)^2 = np\,(1-p) + n^2p^2 - (np)^2 = np\,(1-p) \tag{5.13}$$

where n is number of independent trials and p is the probability of success. Substituting n and p by the model's values:

The average number of agents, from the same level of agent a, that will give agent a money is :

$$\overline{n_{l(a)}}{=}\frac{N_{l(a)} - 1}{N - 1} \tag{5.14}$$

and the correspondent variance is

$$Var(n_{l(a)}) = \left(\frac{N_{l(a)} - 1}{N - 1}\right) . \left(1 - \frac{1}{N - 1}\right) \tag{5.15}$$

The average number of agents, from a different level of agent a, that will give agent a money is:

$$\overline{n_l} = \frac{N_l}{N - 1} \tag{5.16}$$

and the correspondent variance is

$$Var(n_l) = \left(\frac{N_l}{N-1}\right) \cdot \left(1 - \frac{1}{N-1}\right) \tag{5.17}$$

Now if we substitute this values in the average money variation we have:

$$\overline{\delta m_a(t)} = \begin{cases} -m_{l(a)} + m_{l(a)} \cdot \frac{N_{l(a)}-1}{N-1} + \sum_{l=1;l\neq l(a)}^{L} m_l \cdot \frac{N_l}{N-1} & \text{for agents in } l \text{ level, } l = \{1, 2, ..., L\} \\ \sum_{l=1}^{L} m_l \cdot \frac{N_l}{N-1} & \text{for agents in level } l = 0 \end{cases} \tag{5.18}$$

5.1.1.1 Two-Level Model

The two-level model has a threshold in $m = m_e$, this represents the simplest model where agents with money below them_e exchange money units do not give but can receive and agents with money units above or equal the threshold can exchange m_e money units. All agents start with the same amount of initial money units and the average money variation of an agent in level 0 and an agent in level 1 is

$$\overline{\delta m_a(t)} = \begin{cases} -m_1 + m_1 \cdot \frac{N_1-1}{N-1} & \text{for agents in level 1} \\ m_1 \cdot \frac{N_1}{N-1} & \text{for agents in level 0} \end{cases} \tag{5.19}$$

The computer simulations in this section were not conducted due to the anticipated lengthy time required for obtaining results.

5.2 Agent Individual Path

To gain insight into the individual agent's dynamics throughout the simulation, we developed a computer program capable of tracking the number of money units for a selected agent in each round. We conducted this analysis for various money exchange rules and initial money distributions, focusing on three distinct types of agents. In all simulations, we began with a total of 1000 economic agents. To select these agents, we ranked them by their initial money units and chose representatives from different money levels: the poorest agent (agent 0), an agent with a median level of money (agent 500), and the wealthiest agent (agent 1000). The only exception to this selection process was for the initial Dirac money distribution, where all agents started with the same amount of initial money units and so any agent can be chosen.

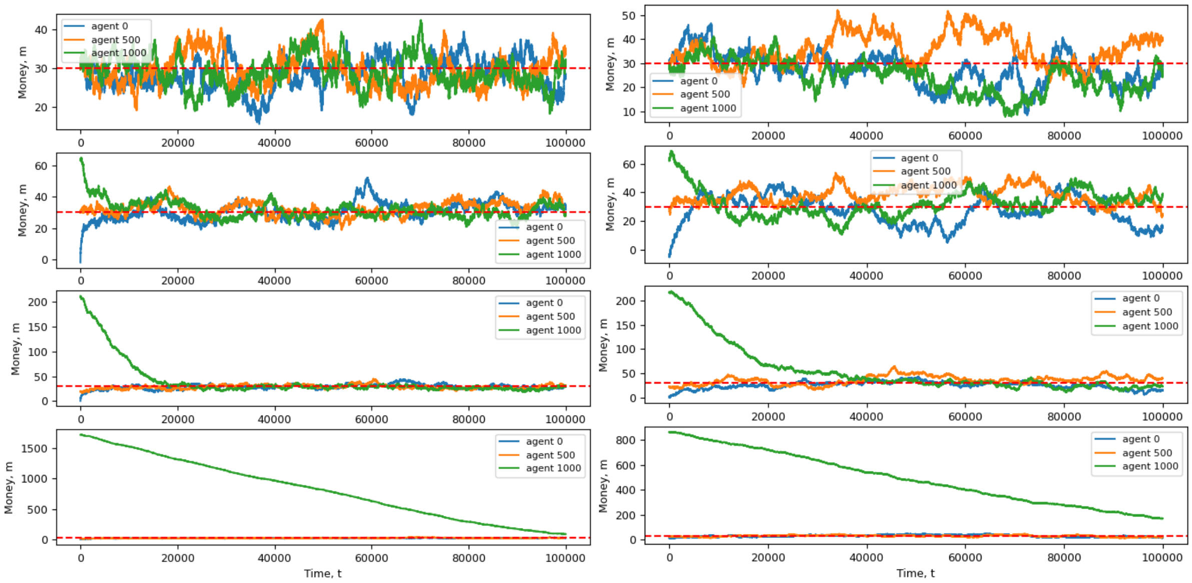


Figure 5.1: (Left) 2 level-model: No debt (Initial distributions respectively: Dirac,Gaussian, Exponential and Power) and (Right) 2 level-model: Debt=-30 (Initial distributions respectively: Dirac,Gaussian, Exponential and Power)

In Figure 5.1 We can analyze how individual agents' wealth behaves under different initial distributions and varying levels of debt. When comparing Dirac initial money distributions, we observe a greater deviation from the average of 30 money units when debt is introduced. In the case of Gaussian initial money distribution, we observe a gradual convergence of the wealthiest (agent 1000) and the poorest (agent 0) agents toward the average value, whether debt is present or not.

For both Exponential and Power initial money distributions, we notice that the richest agent eventually converges to the average wealth of other agents. However, it's worth noting that when agents are allowed to have a maximum debt of -30, the convergence to the average is slower compared to scenarios without debt. This suggests that in the latter two initial money distributions, the richest agent benefits from a scenario where debt is allowed because it takes longer for the them to lose their money.

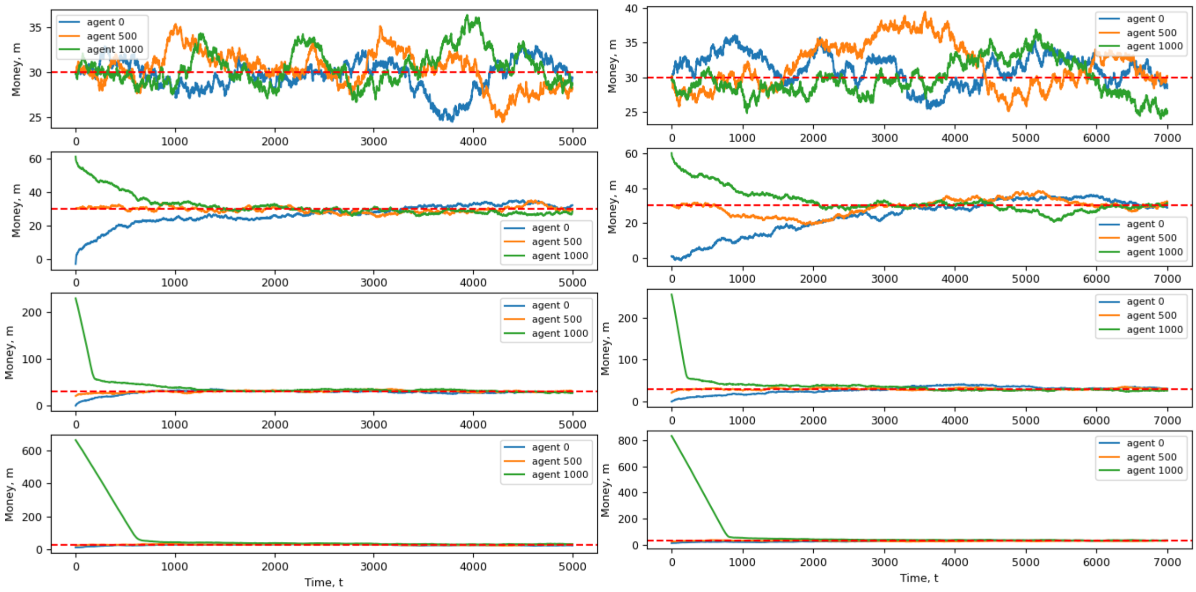


Figure 5.2: (Left) 3 level-model with $m_e = [1,2]$: No debt (Initial distributions respectively: Dirac,Gaussian, Exponential and Power) and (Right) 3 level-model with $m_e = [1,2]$: Debt=-30 (Initial distributions respectively: Dirac,Gaussian, Exponential and Power)

Figure 5.2 in relation to the previous figure shows us that the introduction of an extra level in the model makes all agents, richer or poorer converge faster to the average value of 30 money units. It is not noticeable a substantial different between the model with and the model without debt.

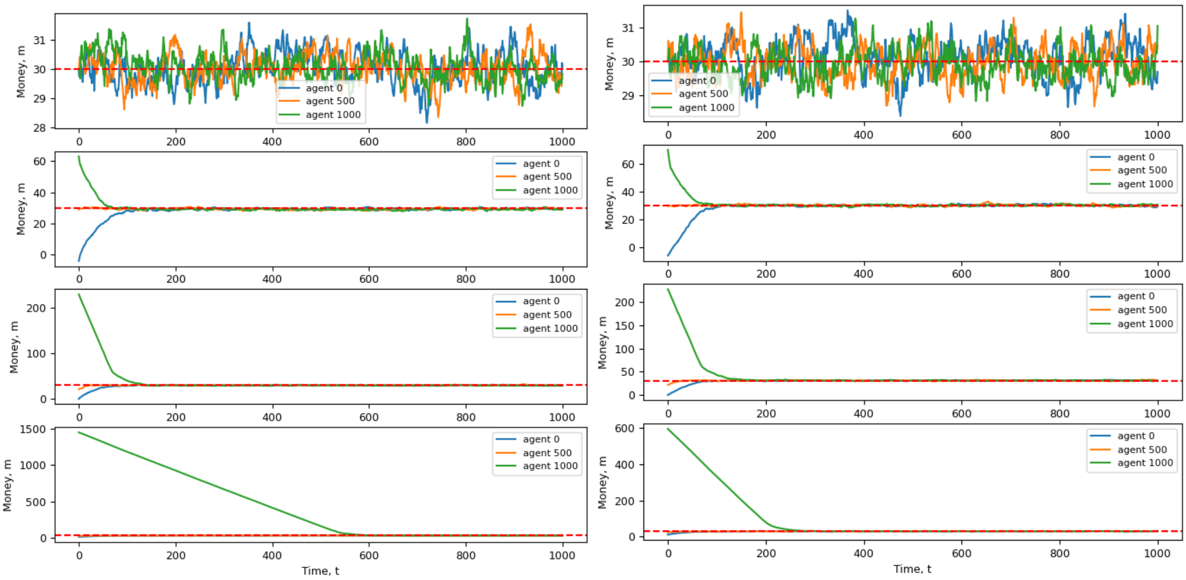


Figure 5.3: (Left) 4 level-model with $m_e = [1, 2, 4]$: No debt (Initial distributions respectively: Dirac,Gaussian, Exponential and Power) and (Right) 4 level-model with $m_e = [1, 2, 4]$: Debt=-30 (Initial distributions respectively: Dirac,Gaussian, Exponential and Power)

In Figure 5.3 it is shown again the the addition of one more level makes all agents convergence even more fast to the average value, just like in 3 level-model the existence of debt does not seem to make a difference. As seen in previous chapter this might happen because not enough agents have time to go into debt.

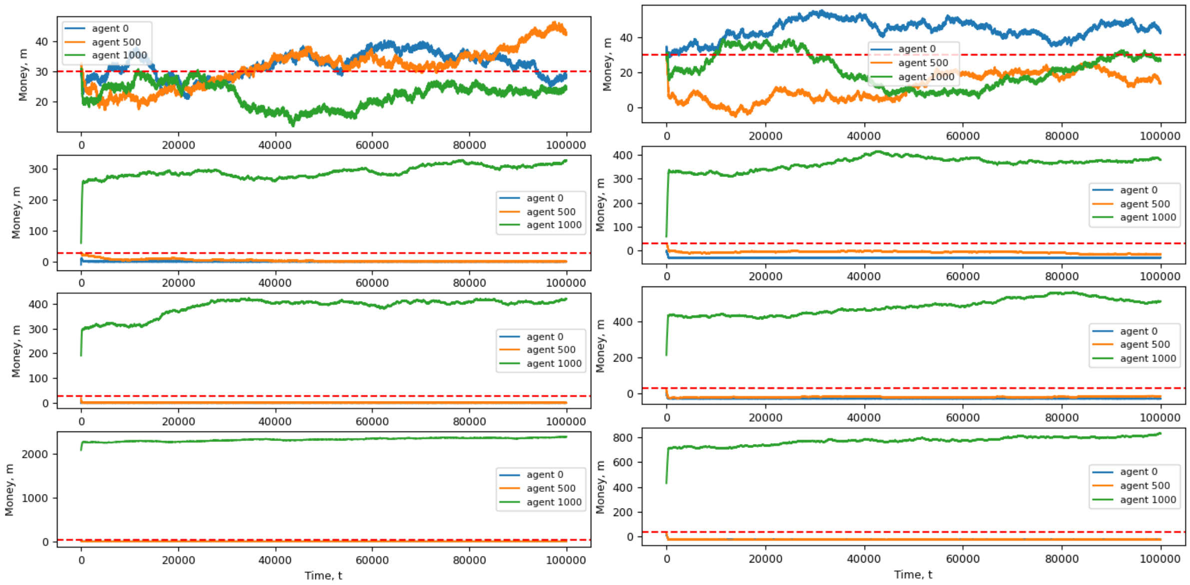


Figure 5.4: (Left) 3 level-model with $m_e = [2, 1]$: No debt (Initial distributions respectively: Dirac,Gaussian, Exponential and Power) and (Right) 3 level-model with $m_e = [2, 1]$: Debt=-30 (Initial distributions respectively: Dirac,Gaussian, Exponential and Power)

In Figure 5.4 we can gain insights into the impact of varying money exchange rules. In these scenarios, agents with more money units contribute less, while those with fewer units contribute more. When we consider the last three initial money distributions (Gaussian, Exponential, and Power), an interesting pattern emerges. The richest agent never converges to the average money, while the poorest and middle agents consistently remain below the average money line. Surprisingly, the level of allowed debt does not appear to influence this outcome significantly.

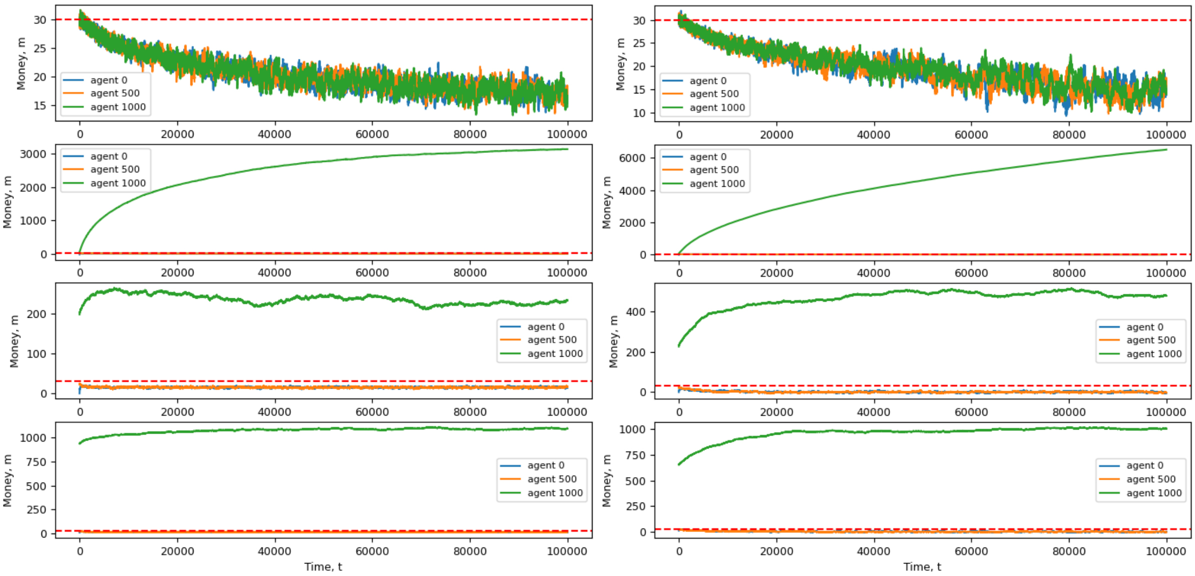


Figure 5.5: (Left) 4 level-model with $m_e = [1, 2, 1]$: No debt (Initial distributions respectively: Dirac,Gaussian, Exponential and Power) and (Right) 4 level-model with $m_e = [1, 2, 1]$: Debt=-30 (Initial distributions respectively: Dirac,Gaussian, Exponential and Power)

Finally, in this last scenario, we examine the four-level model with wealth distribution given by $m_e = [1, 2, 1]$. In this scenario, agents with lower wealth contribute as much as the richest agents, while those in the middle contribute a higher amount of money. One remarkable observation comes from the second line of plots, where a Gaussian initial money distribution is used. In this case, the richest agent begins near the average money but quickly diverges to very high levels, a trend that intensifies when agents are allowed to go into debt.

In the last two cases, using Exponential and Power distributions, the wealth of the richest agent appears very high, with or without debt, but remains relatively constant over time. Meanwhile, both the poorest agent and the one in the middle consistently hold less money than the average.

5.3 Summary on the chapter

This chapter started with a mathematical deduction of the average money variation each agent would have depending on the level they are in. After, it was conducted a comparative analysis between three agents in order to assess the behavior of the amount of money units they owned, in a dynamic equilibrium, under different circumstances.

Furthermore using different money exchange rules we could assess the impact they had in the convergent of divergent behavior of the three agents towards the average money per agent in the system. A more equitable distribution showed that eventually, given enough time, all agents amount of money units eventually converge to the average value. In the scenarios were debt was allowed the time convergence was slightly bigger.

In a less equitable redistribution, we could observe graphically that the amount of money between different agents has a divergent behavior. In certain cases, as seen with Gaussian distribution in the three-level model (with $m_e = [1, 2, 1]$) allowing debt in the system only made the divergence even more pronounced.

6 Conclusion

In this study, we delved into the intricate dynamics of money distribution, examining how it varies based on the rules governing money exchange, the allowance of debt within the system, and the degree of separation between agents with varying levels of money. We explored these dynamics by employing diverse initial money distributions, each characterized by a different level of inequality, to observe their impact on the final money distribution among agents.

In scenarios where the money exchange rules favored a more equitable approach, where agents with lower amounts of money contributed proportionally less and those with greater amounts contributed more, we noted that the final money distribution tended to converge, irrespective of the initial money distribution. Interestingly, the introduction of debt into the system in such scenarios either maintained or exacerbated money inequality.

In the three and four-level models, we observed that the introduction of debt had a limited impact on the overall money inequality. The predefined thresholds in these models acted as barriers, limiting the number of agents falling into debt. However, when the money exchange rules favored those with higher number of money units, we witnessed a significant escalation in the level of inequality in the final distributions.

Nevertheless, it's important to acknowledge the limitations of this study and its applicability to the real world. Our economic agents exhibited homogeneous behavior, and the money exchange process relied on randomness. We contemplated the idea of developing a model where agents' behavior would be influenced by their neighboring agents, leading to a less random money exchange process. Furthermore, in our exploration of the impact of debt (endogenous money) on wealth inequality, we considered the addition of an oscillatory term to represent the introduction of exogenous money (via institutions like the state or Central Bank) to understand its effects on agents at different wealth levels. Notably, our simulations only factored in the influence of debt, without considering the cumulative impact of interest rates associated with those debts.

Agent-based modeling presents a vast field, ready for the exploration of novel concepts. The aim is to bridge the gap between computer simulations and real-world data, with the aspiration of contributing to a deeper comprehension of the contemporary economic system that governs our lives.

Appendix

The following section presents the code employed in our computer simulations.

When computing the Gini coefficient for a distribution, all negative values were substituted with zero.

In establishing the different initial probability money distributions, parameters were meticulously defined to maintain the same average money units across all distributions. Furthermore, these parameters were adjusted based on the chosen Gini coefficient values for the initial distributions.

Throughout the code, the variable IMU signifies initial money units, and EMU denotes exchanged money units. The variable level is a list whose size depends on the model used (2, 3, or 4-level models).

While executing simulations, one significant challenge lies in the considerable time required for data acquisition. To ensure robust statistical analysis, a high number of trials (in the order of thousands) is necessary. Additionally, achieving equilibrium in the final distribution demands a substantial number of rounds (hundreds of thousands). Consequently, each simulation, depending on the chosen parameters, may span several hours.

[ ]:

```
import random as random
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

1 Gini Coefficient

[1]:

```
def gini_coef(data):
    data[data < 0] = 0
    norm = np.linalg.norm(data)
    normalized_data = data / norm
    sorted_data = np.sort(normalized_data)
    n = data.size
    coef = np.sum((2 * np.arange(1, n + 1) - n - 1) * sorted_data) / (n * np.
 ↪sum(sorted_data))
    return coef
```

2 Initial Probability Distributions

[2]:

```
def gaussian_dist(agents, average_wealth):
    mean = average_wealth
    std = average_wealth / 3  # Adjust the standard deviation as needed
    gaussian_wealth = np.round(np.random.normal(mean, std, size=agents)).
 ↪astype(int)
    return gaussian_wealth

def exponential_dist(agents, average_wealth):
    scale = average_wealth
    exp_values = np.random.exponential(scale, size=agents)
    exponential_wealth = np.round(exp_values).astype(int)
    return exponential_wealth

def power_dist(agents, average_wealth, alpha):
    rand = np.random.uniform(size=agents)
    power_law_values = np.round((1.0 - rand) ** (-1.0 / (alpha - 1.0))).
 ↪astype(int)
    scale_factor = average_wealth / np.mean(power_law_values) #make sure the␣
 ↪average value is average_wealth
    power_law_values = np.round(power_law_values * scale_factor).astype(int)
    return power_law_values
```

3 Money Distribution with 2,3 or 4 levels with (Limited DEBT)

[3]:

```
def money_dist_levels_debt_osc(agents,trials,rounds,IMU,EMU,level,debt):
    if len(level) == 0: #---------------------------------- There is 1␣
↪thresholds = EMU (level 0, level1) -------------#
        trial_data = np.zeros(shape=(agents, trials))
        for trial in range(trials):
            list_agents = np.arange(agents)
            money = np.zeros(agents) + IMU

            for r in range(rounds):
                partners = (list_agents + 1 + np.random.randint(agents - 1, size␣
↪= agents)) % agents
                indexB = np.where((money[list_agents] <= debt))[0]
                index0 = np.where((money[list_agents] > debt))[0]

                money[index0] -= EMU

                money_receiver0, n0  = np.unique(partners[index0],␣
↪return_counts=True)
                money[money_receiver0] += EMU * n0

            trial_data[:,trial] =  money
        return trial_data.flatten()

    if len(level) == 1: #----------------------------------- There are 2␣
↪threshold: level 0, and level 1, level 2
        trial_data = np.zeros(shape=(agents, trials))
        for trial in range(trials):
            list_agents = np.arange(agents)
            money = np.zeros(agents) + IMU

            for r in range(rounds):
                partners = (list_agents + 1 + np.random.randint(agents - 1, size␣
↪= agents)) % agents
                indexB = np.where((money[list_agents] <= debt))[0]
                index0 = np.where((money[list_agents] > debt) &␣
↪(money[list_agents] < level) )[0]
                index1 = np.where((money[list_agents] >= level))[0]

                money[index0] -= EMU[0]
                money[index1] -= EMU[1]

                money_receiver0, n0  = np.unique(partners[index0],␣
↪return_counts=True)
                money[money_receiver0] += EMU[0] * n0
```

```
                money_receiver1, n1 = np.unique(partners[index1],␣
↪return_counts=True)
                money[money_receiver1] += EMU[1] * n1

            trial_data[:,trial] = money
        return trial_data.flatten()

    if len(level) == 2: #----------------------there are 3 thresholds: level 0,␣
↪level 1, level 2, level 3
        trial_data = np.zeros(shape=(agents, trials))
        for trial in range(trials):
            list_agents = np.arange(agents)
            money = np.zeros(agents) + IMU

            for r in range(rounds):
                partners = (list_agents + 1 + np.random.randint(agents - 1, size␣
↪= agents)) % agents
                indexB = np.where((money[list_agents] <= debt))[0]
                index0 = np.where((money[list_agents] > debt) &␣
↪(money[list_agents] < level[0]))[0]
                index1 = np.where((money[list_agents] >= level[0]) &␣
↪(money[list_agents] < level[1]))[0]
                index2 = np.where((money[list_agents] >= level[1]))[0]

                money[index0] -= EMU[0]
                money[index1] -= EMU[1]
                money[index2] -= EMU[2]

                money_receiver0, n0  = np.unique(partners[index0],␣
↪return_counts=True)
                money[money_receiver0] += EMU[0] * n0
                money_receiver1, n1 = np.unique(partners[index1],␣
↪return_counts=True)
                money[money_receiver1] += EMU[1] * n1
                money_receiver2, n2 = np.unique(partners[index2],␣
↪return_counts=True)
                money[money_receiver2] += EMU[2] * n2

            trial_data[:,trial] = money
        return trial_data.flatten()
```

4 Individual Agent Path

[ ]:

```
def agent_path(agents, agent_index, trials, rounds, IMU, EMU, level, debt):
#--------------------------------------------------------------- 2 levels␣
 ↪----------------------------------------------------------------------#
    if len(level) == 0:
        trial_data = np.zeros(shape=(agents, trials))

        for trial in range(trials):
            list_agents = np.arange(agents)
            money = np.zeros(agents) + IMU
            sorted_money = np.argsort(money)  # Get the indexes that sort wealth␣
 ↪in ascending order
            chosen_agent_index = sorted_money[agent_index]  # Choose the␣
 ↪wealthiest agent to follow
            trial_data[trial,0] = money[chosen_agent_index]

            for r in range(1,rounds):
                partners = (list_agents + 1 + np.random.randint(agents - 1, size␣
 ↪= agents)) % agents
                levelB = np.where((money[list_agents] <= debt))[0]
                level0 = np.where((money[list_agents] > debt))[0]

                money[level0] -= EMU

                money_receiver0, n0  = np.unique(partners[level0],␣
 ↪return_counts=True)
                money[money_receiver0] += EMU * n0

            trial_data[trial,r] = money[chosen_agent_index]

        return trial_data.flatten()

#--------------------------------------------------------------- 3 levels␣
 ↪-----------------------------------------------------------------------#
    if len(level) == 1:
        trial_data = np.zeros(shape=(trials, rounds))

        for trial in range(trials):
            list_agents = np.arange(agents)
            money = np.zeros(agents) + IMU
            sorted_money = np.argsort(money)  # Get the indexes that sort wealth␣
 ↪in ascending order
            chosen_agent_index = sorted_money[agent_index]  # Choose the␣
 ↪wealthiest agent to follow
            trial_data[trial,0] = money[chosen_agent_index]
```

```
            for r in range(1,rounds):
                partners = (list_agents + 1 + np.random.randint(agents - 1, size␣
 ↪= agents)) % agents
                levelB = np.where((money[list_agents] <= debt))[0]
                level0 = np.where((money[list_agents] > debt) &␣
 ↪(money[list_agents] < level[0]))[0]
                level1 = np.where((money[list_agents] >= level[0]))[0]

                money[level0] -= EMU[0]
                money[level1] -= EMU[1]

                money_receiver0, n0 = np.unique(partners[level0],␣
 ↪return_counts=True) # receiver of money from agents in level 0
                money[money_receiver0] += EMU[0] * n0
                money_receiver1, n1 = np.unique(partners[level1],␣
 ↪return_counts=True) # receiver of money from agents in level 1
                money[money_receiver1] += EMU[1] * n1

                trial_data[trial,r] = money[chosen_agent_index]

        return trial_data.flatten()

#---------------------------------------------------------------- 4 levels␣
 ↪----------------------------------------------------------------------#
    if len(level) == 2:
        trial_data = np.zeros(shape=(trials, rounds))

        for trial in range(trials):
            list_agents = np.arange(agents)
            money = np.zeros(agents) + IMU
            sorted_money = np.argsort(money)  # Get the indexes that sort wealth␣
 ↪in ascending order
            chosen_agent_index = sorted_money[agent_index]  # Choose the␣
 ↪wealthiest agent to follow
            trial_data[trial,0] = money[chosen_agent_index]

            for r in range(1,rounds):
                partners = (list_agents + 1 + np.random.randint(agents - 1, size␣
 ↪= agents)) % agents
                levelB = np.where((money[list_agents] <= debt))[0]
                level0 = np.where((money[list_agents] > debt) &␣
 ↪(money[list_agents] < level[0]))[0]
                level1 = np.where((money[list_agents] >= level[0]) &␣
 ↪(money[list_agents] < level[1]))[0]
                level2 = np.where((money[list_agents] >= level[1]))[0]
```

```
                money[level0] -= EMU[0]
                money[level1] -= EMU[1]
                money[level2] -= EMU[2]

                money_receiver0, n0  = np.unique(partners[level0],␣
↪return_counts=True)
                money[money_receiver0] += EMU[0] * n0
                money_receiver1, n1 = np.unique(partners[level1],␣
↪return_counts=True)
                money[money_receiver1] += EMU[1] * n1
                money_receiver2, n2 = np.unique(partners[level2],␣
↪return_counts=True)
                money[money_receiver2] += EMU[2] * n2

                trial_data[trial,r] = money[chosen_agent_index]

        return trial_data.flatten()
```

Bibliography

[1] Joe Hasell. Measuring inequality: What is the gini coefficient? *Our World in Data*, 2023. https://ourworldindata.org/what-is-the-gini-coefficient.

[2] Allin F Cottrell, Paul Cockshott, Gregory John Michaelson, Ian P Wright, and Victor Yakovenko. *Classical econophysics*. Routledge, 2009.

[3] Adrian Dragulescu and Victor M Yakovenko. Statistical mechanics of money. *The European Physical Journal B-Condensed Matter and Complex Systems*, 17:723–729, 2000.

[4] Victor M Yakovenko. Monetary economics from econophysics perspective. *The European Physical Journal Special Topics*, 225(17-18):3313–3335, 2016.

[5] Philip Mirowski. *More Heat than light: Economics as social physics, Physics as nature's economics*. Cambridge University Press, 1989.

[6] Philip Mirowski. *Against mechanism: protecting economics from science*. Rowman & Littlefield Publishers, 1992.

[7] Fernando Savater. *História da Filosofia sem medo nem pavor*. Planeta, 2011.

[8] Francisco Louçã ; Mariana Mortágua. *Manual de Economia Política*. Bertrand Editora, 2021.

[9] Craig Smith. The scottish enlightenment, unintend consequences and the science of man. *Journal of Scottish Philosophy*, 7:9–28, 2009.

[10] IMF. The end of the bretton woods system (1972-81). Accessed on 2023-09-12.

[11] Adam Smith. *An Inquiry into the Nature and Causes of the Wealth of Nations*. MetaLibri, 2007.

[12] Adam Smith. *Theory of Moral Sentiments*. Liberty Fund, 1984.

[13] Friedrich Engels and Karl Marx. *Das Kapital*. 1968.

[14] Gustav Von Schmoller. *The Mercantile System and Its Historical Significance, Illustrated Chiefly from Prussian History*. Macmillan, 1895.

[15] Philip Mirowski. Physics and the marginalist revolution. *Cambridge journal of Economics*, 8(4):361–379, 1984.

[16] Michael H Turk. The fault line of axiomatization: Walras' linkage of physics with economics. *The European Journal of the History of Economic Thought*, 13(2):195–212, 2006.

[17] Margaret Schabas. The" worldly philosophy" of william stanley jevons. *Victorian Studies*, 28(1):129–147, 1984.

[18] Jan Van Daal ; Albert Jolink. *The Equilibrium Economics of Léon Walras*. Routledge, 2006.

[19] Leon Walras. *Elements of pure economics*. Routledge, 2013.

[20] Francis Ysidro Edgeworth. Methods of statistics. *Journal of the Statistical Society of London*, pages 181–217, 1885.

[21] Adrian K Yee. Econophysics: making sense of a chimera. *European Journal for Philosophy of Science*, 11(4):100, 2021.

[22] Stavros Drakopoulos and Ioannis Katselidis. From edgeworth to econophysics: a methodological perspective. *Journal of Economic Methodology*, 22(1):77–95, 2015.

[23] Benoit Mandelbrot and Richard L. Hudson. *The (Mis)behaviour of Markets: A fractal view of risk,ruin and reward*. Profile Books, 2008.

[24] Irving Fisher. *Mathematical investigations in the theory of value and prices, and appreciation and interest*. Cosimo, Inc., 2007.

[25] Larry Neal and Rondo Cameron. *História Económica do Mundo*. Escolar Editora, 2019.

[26] Philip Mirowski. The probabilistic counter revolution, or how stochastic concepts came to neoclassical economic theory. *Oxford Economic Papers*, 41(1):217–235, 1989.

[27] Ragnar Frisch. *Sur un problème d'économie pure*. Grøndahl & søns boktrykkeri, 1926.

[28] Henryk Fukś. Mathematics on coins, part i: The tale of two queens and two towering figures. *arXiv preprint arXiv:1601.07394*, 2016.

[29] Louis Bachelier. Théorie de la spéculation. In *Annales scientifiques de l'École normale supérieure*, volume 17, pages 21–86, 1900.

[30] Eugene F Fama. Efficient market hypothesis. *Diss. PhD Thesis, Ph. D. dissertation*, 1960.

[31] Christophe Schinckus. 1996–2016: Two decades of econophysics: Between methodological diversification and conceptual coherence. *The European Physical Journal Special Topics*, 225:3299–3311, 2016.

[32] Rosario Nunzio Mantegna. Lévy walks and enhanced diffusion in milan stock exchange. *Physica A: Statistical Mechanics and its Applications*, 179(2):232–242, 1991.

[33] Aleksander Jakimowicz. Econophysics as a new school of economic thought: Twenty years of research. *Acta Physica Polonica A*, 129(5):897–907, 2016.

[34] Nassim Nicholas Taleb. *The black swan: The impact of the highly improbable*, volume 2. Random house, 2007.

[35] H Eugene Stanley and Rosario N Mantegna. *An introduction to econophysics.* Cambridge University Press, Cambridge, 2000.

[36] Jean-Philippe Bouchaud and Marc Potters. *Theory of financial risk and derivative pricing: from statistical physics to risk management.* Cambridge university press, 2003.

[37] W Paul Cockshott, Allin Cottrell, Gregory J Michaelson, Ian P Wright, and Victor M Yakovenko. *Classical econophysics*, volume 2. Routledge, 2009.

[38] Anirban Chakraborti, Ioane Muni Toke, Marco Patriarca, and Frédéric Abergel. Econophysics review: Ii. agent-based models. *Quantitative Finance*, 11(7):1013–1041, 2011.

[39] Mariana Rodrigues Mortágua. *No Sonho Selvagem do Alquimista.* Tinta da China, 2021.

[40] Pierangelo Garegnani. Value and distribution in the classical economists and marx. *Oxford economic papers*, 36(2):291–325, 1984.

[41] Ivan Moscati. *Measuring utility: From the marginal revolution to behavioral economics.* Oxford Studies in History of E, 2018.

[42] Mariana Rodrigues Mortágua. *Capital accumulation and stagnation: a Veblenian-Kaleckian approach to the portuguese lost decade.* PhD thesis, SOAS University of London, 2019.

[43] Milton Friedman. The counter-revolution in monetary theory. In *Explorations in Economic Liberalism: The Wincott Lectures*, pages 3–21. Springer, 1970.

[44] L. Randall Wray. *Modern Money Theory: A Primer on Macroeconomics for Sovereign Monetary Systems.* Palgrave Macmillan UK, 2012.

[45] Michael McLeay, Amar Radia, and Ryland Thomas. Money creation in the modern economy. *Bank of England Quarterly Bulletin*, page Q1, 2014.

[46] Emanuel Reis Leão, Pedro Reis Leão, and Sérgio Chilra Lagoa. Política monetária e mercados financeiros. *Edições Sílabo, Lisboa*, 2019.

[47] Deutsche Bundesbank. The role of banks, non-banks and the central bank in the money creation process. *Monthly Report*, 69(4):13–34, 2017.

[48] Ioana Raluca Sbarcea. International concerns for evaluating and preventing the bank risks–basel i versus basel ii versus basel iii. *Procedia Economics and Finance*, 16:336–341, 2014.

[49] Michael Pacione. Local exchange trading systems as a response to the globalisation of capitalism. *Urban Studies*, 34(8):1179–1199, 1997.

[50] Hyman P Minsky. The financial instability hypothesis: An interpretation of keynes and an alternative to "standard" theory. *Challenge*, 20(1):20–27, 1977.

[51] World inequality database.