Received 30 April 2023
Revised 16 July 2023
Accepted 13 August 2023

# Symmetry-breaking and trade in neoclassical economies with domestic policies having diminishing effect to production scale

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## Abstract

**Purpose** – The aim of this paper is to investigate whether a Nash equilibrium of a two-country trading economy is symmetry-breaking or not.

**Design/methodology/approach** – The approach to tackle this topic is a theoretical treatment by the general equilibrium trade theory and game theory.

**Findings** – If each government's domestic policy serving private production is diminishing to the private production scale, the Nash equilibrium is not symmetry-breaking.

**Originality/value** – In the existing study of Chatterjee (2017), a similar result is derived by focusing on the properties of each country's GDP function. The authors, however, consider an economy where each country's PPF is strictly concave and show that the Nash equilibrium uniquely exists and this equilibrium is symmetry.

**Keywords** Symmetry-breaking, Neoclassical economy, Production possibility frontier, Nash equilibrium

**Paper type** Research paper

## 1. Introduction

One of the resent issues of game theories is to introduce a new behavioral norm of players called as the Kantian behavior into game. The late Professor Ngo Van Long had an interest in games, where both Nashian and Kantian players coexist and examined the Kant-Nash equilibrium. And he explored the broad applicability of this equilibrium approach by the analyses of various interesting and realistic economic problems in industrial organization, global environment, public policies, *etc.* in Long (2016, 2017, 2018, 2020a, b, 2022) and Grafton, Kompas, and Long (2017) [1]. There is, however, another recent application of game theories as a powerful tool to investigate symmetry-breaking equilibrium. The present paper is concerned with this topic in general equilibrium trading economies with reciprocal reaction between governments.

It is often observed that similar countries do not necessarily adopt the same domestic policies. Matsuyama (2002) advocates the idea of a symmetry-breaking equilibrium for this



This work was supported by the Japan Society for the Promotion of Science Grant-in-Aid for Scientific Research No. 19K01679 and No. 20K01605. The authors thank two anonymous reviewers for their valuable comments and suggestions.

phenomenon and formalizes it properly by discussing the stability property of dynamic equilibria. In the context of international trade, Chatterjee (2017) focuses on this topic in the framework of Heckscher–Ohlin economies and inspects the condition for a symmetry-breaking equilibrium to emerge. She follows the work of Amit, Garcia, & Knaff (2010) treating this topic as a lattice programing and concentrates attention on the properties of the GDP function to ensure the symmetry-breaking to appear. On the other hand, Suga, Tawada, & Yanase (2022) deals with a Ricardian type of the trading economy, where the domestic policy is introduced as a source of increasing external economies to the private production and examines what kind of symmetry-breaking arises in a precise manner where they illustrate the graphs of each country's response function.

In the present paper, we consider a trading model of the Ricardian type in the sense that only one primary factor exists. And we show that the Nash equilibrium of the domestic policy game between governments is unique and symmetric between identical countries if the domestic policies of each country serve for private production but their effects are diminishing along with the production expansion. Although Chatterjee (2017) inspects under what circumstances the trading equilibrium is necessarily symmetry, the analysis is focused solely on the properties of the country's GDP function and abstract about a concrete image on what sort of economies to assure the equilibrium to be symmetry [2]. Our analysis will make full use of the country's production frontier as well as the world production frontier to distinguish the circumstances where a unique symmetry equilibrium exists from those where a symmetry-breaking equilibrium arises.

The paper is organized as follows: Section 2 proposes the model. The main analysis and its extension are carried out in Sections 3 and 4, respectively. The last section provides conclusion.

## 2. Model

Consider a trading world with two countries, two goods and one primary factor exist. Two countries are Home and Foreign, two goods are good 1 and good 2 and one primary factor is labor. Two countries are assumed to be identical with respect to preferences, technologies and the labor endowment. In this section, we describe mainly the economy of the Home country.

The production functions of two goods are expressed by

$$Q_i = F^i(L_i, R), i = 1, 2, \tag{1}$$

where $L_i$ is the labor input in the production of good $i$, $Q_i$ is the produced amount of good $i$ and $R$ is the level of the domestic policy executed by the government. The domestic policy is supposed to supply various infrastructures such as the communication and transportation systems, the facilities for skill formation of labor and institutions of research and development activities, which serve for private production but accompany congestion in their use as an expansion of the production scale [3].

So, the effect of this type of policies is diminishing as production scale expands. Hence $F^i(L_i, R)$ is assumed to be strictly quasi-concave and linearly homogeneous with respect to $L_i$ and $R$. It is also assumed that $\partial F^i/\partial L_i$ and $\partial F^i/\partial R$ are positive, $\partial^2 F^i/\partial L_i^2$ and $\partial^2 F^i/\partial R^2$ are negative and $F^i(L_i, R) > 0$ if and only if $(L_i, R) > 0$. .

The execution of the domestic policy of the level $R$ requires the labor input formulated by

$$R = L_R, \tag{2}$$

where $L_R$ is the labor input for the execution of the domestic policy of $R$.

Suppose that perfect competition prevails in the good and labor markets. Thus the private firms behave as a profit maximizer under perfect competition. In order to finance the policy cost to execute $R$, the government imposes a uniform tax on all private firms so as to keep production efficiency [4]. Labor is supposed to supply inelastically with respect to wage, so the following full employment condition is satisfied.

$$L_1 + L_2 + L_R = L, \tag{3}$$

where $L$ is the labor endowment which is assumed to be given and fixed.

The country's aggregate utility function is given by

$$U = U(C_1, C_2), \tag{4}$$

where $C_i$ is the consumption of good $i$, $U(C_1, C_2)$ is assumed to be linearly homogenous with respect to $C_1$ and $C_2$ and $U(C_1, C_2) > 0$ only if $(C_1, C_2) > 0$. Moreover, the indifference curves derived by (4) is assumed to be downward sloping and strictly convex to the origin. The equilibrium consumption levels of both goods are determined by the consumer behavior that is to maximize (4) subject to the budget constraint under given good prices and national income. Let good 2 be numeraire and $p$ be the price of good 1 relative to good 2.

Now suppose the government executes the domestic policy of level $R$. Then, on the one hand, the production equilibrium amount of good $i$ is determined by the profit maximizing condition under given $R$ and $p$, which is denoted as $Q_i = Q_i(p, R)$. On the other hand, the consumption equilibrium amount of good $i$ is determined by the utility maximizing condition under given $R$ and $p$, which is denoted as $C_i = C_i(p, R)$. This is because the national income $Y$ is determined as

$$Y = Y(p, R) \equiv pQ_1(p, R) + Q_2(p, R) = wL, \tag{5}$$

where $w$ is the wage rate in terms of good 2, by the production equilibrium condition. And thus, we have $C_i = C_i(p, Y(p, R)) \equiv C_i(p, R)$. Then, the autarkic equilibrium price $p$ denoted as $p^A$ is determined by

$$Q_i(p^A, R) = C_i(p^A, R), i = 1, 2. \tag{6}$$

Under the linear homogeneity assumption of $U(C_1, C_2)$, the indirect utility function is expressed as

$$U = V(p, R) \equiv v(p)Y(p, R). \tag{7}$$

Since $p^A$ is a function of $R$ by (6), it is defined as $p^A = p^A(R)$. Then, in view of (5) and (7), the autarkic national income is described as $Y^A = Y(p^A, R)$ and the national welfare level under autarky is $U^A = V(p^A, Y^A)$.

Consider the Foreign country which is assumed to be identical to the Home country. The assumption implies that equations (1) to (4) are supposed to hold in the Foreign country as well.

First of all, the Home and Foreign governments decide the levels of the domestic policies as $R$ and $R^*$, respectively, where we denote the variables with asterisk as those of the Foreign country in order to distinguish variables between countries. Under given $R$ and $R^*$, two countries trade two goods with each other. Then the trading equilibrium price denoted as $p^T$ is the price satisfying the following world good market equilibrium conditions:

$$Q_i(p^T, R) + Q_i^*(p^T, R^*) = C_i(p^T, R) + C_i^*(p^T, R^*), i = 1, 2. \tag{8}$$

Because of (8), $p^T$ can be expressed as a function of $R$ and $R^*$ such that $p^T = p^T(R, R^*)$.

Home government determines the level of $R$ so as to maximize their own country's welfare. The welfare maximization problem of the Home country under autarky is

$$\underset{R}{\text{Max}}\, V\left(p^A(R), Y\left(p^A(R), R\right)\right). \tag{9}$$

Under trade between countries, $R$ can be similarly determined as an optimal solution of the following welfare optimization problem:

$$\underset{R}{\text{Max}}\, V\left(p^T(R, R^*), Y\left(p^T(R, R^*), R\right)\right), \tag{10}$$

under a given level of $R^*$. Thus, in the trading economy, $R$ is determined by (10) once $R^*$ is given, which implies that the optimal level of Home $R$ is expressed by a function of $R^*$. We denote it as

$$R = r(R^*). \tag{11}$$

Likewise, the optimal level of Foreign $R^*$ is expressed as a function of $R$. That is

$$R^* = r^*(R), \tag{12}$$

where we should notice, by the symmetry assumption between two countries, that $r(R) = r^*(R)$.

We proceed to the simultaneous determination of $R$ and $R^*$. To see this we suppose the pure strategy noncooperative normal game with respect to $R$ and $R^*$ between the Home and Foreign governments. In other words, $R$ and $R^*$ are determined by the Nash equilibrium of this game, implying that $R$ and $R^*$ are given as the solutions of (11) and (12). Once $R$ and $R^*$ are given, each country determined the supply and demand of each good under trade. Then the trading equilibrium price $p^T$ is determined by (8) and thus gives the welfare level of each country.

## 3. Analysis

We begin with the examination of Home's production possibility frontier (PPF). The PPF is defined as the upper boundary of the production possibility set,

$$S \equiv \left\{(Q_1, Q_2) | Q_i = F^i(L_i, R), i = 1, 2, R = L_R \; and \; L_1 + L_2 + L_R \leq L\right\}.$$

Let the PPF under a given level of $R$ be denoted as PPF($R$). Then, the country's PPF is the envelope of PPF($R$) for $R$ taken as a parameter. Both the PPF and PPF($R$) are shown to be strictly concave [5]. We assume an increase in $R$ to yield a comparative advantage in good 1, implying that $Q_1$ rises and $Q_2$ falls along the PPF as $R$ increases [6].

The autarkic equilibrium of the Home is indicated by $E^A$ in Figure 1, where $R$ is given as $R^A$, the equilibrium price is $p^A$, the equilibrium quantity of $Q_i$ is $Q_i^A$, which is equal to the equilibrium consumption of $C^i$ and $U(R^A)$ is the indifference curve assured under $R = R^A$.

For the government maximizing the country's welfare by means of $R$, the effective range of $R$ is $\left[\underline{R}, \overline{R}\right]$ ,where $\underline{R}$ and $\overline{R}$ correspond production points $(0, \overline{Q}_2)$ and $(\overline{Q}_1, 0)$ on the PPF. Any $R$ in $\left[\underline{R}, \overline{R}\right]$ can attain a production point on the PPF. Any $R$ not in $\left[\underline{R}, \overline{R}\right]$ never attain a production point on the PPF, so that it cannot be effective. So each government necessarily chooses $R$ within the interval $\left[\underline{R}, \overline{R}\right]$ so as to maximize the country's welfare.

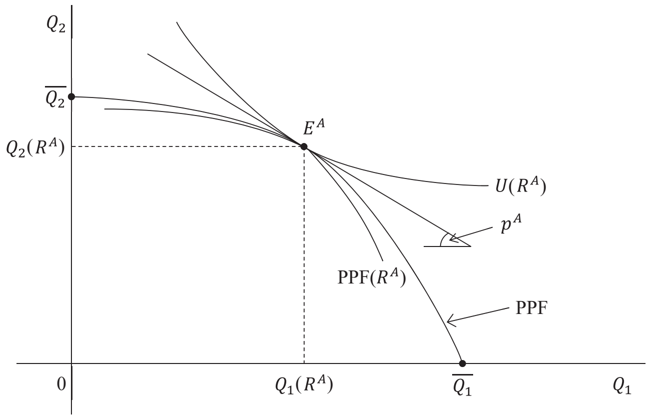



**Figure 1.**
Autarkic equilibrium

**Source(s):** Figure by authors

The assumption that the Home and Foreign are identical implies that the shape of the PPF is the same between these countries. Then the world PPF becomes strictly concave, where the world PPF is the upper boundary of the world production possibility set,

$$\left\{\left(Q_1^W, Q_2^W\right)\middle|\left(Q_1^W, Q_2^W\right) = \left(Q_1 + Q_1^*, Q_2 + Q_2^*\right) for\ (Q_1, Q_2) \in S\ and\ (Q_1^*, Q_2^*) \in S^*\right\}.$$

Because of the identical PPF between Home and Foreign, the world PPF becomes strictly concave and any point on the frontier corresponds a common $R$ for these countries. So, even in the trading world, there is no trade whenever the world production point is on the world PPF.

Next, we assume the world welfare under trade to be

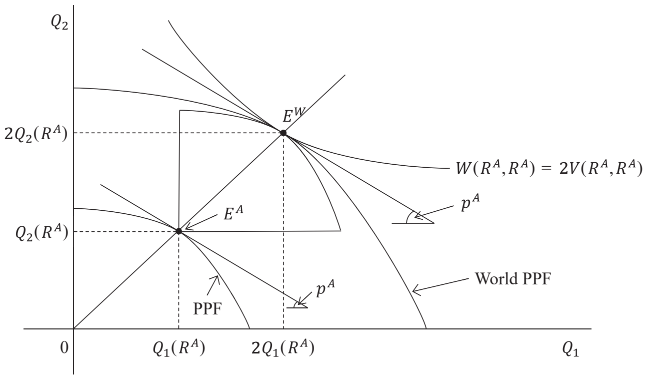
$$\begin{aligned} U^W &= W(R, R^*) \equiv V\left(p^T(R, R^*), Y\left(p^T(R, R^*), R\right)\right) + V\left(p^T(R, R^*), Y\left(p^T(R, R^*), R^*\right)\right) \\ &= V^H(R, R^*) + V^F(R^*, R), \end{aligned}$$

where $U^W$ is the level of the world welfare, $V^H(R, R^*) \equiv V(p^T(R, R^*), Y(p^T(R, R^*), R))$ and $V^F(R^*, R) \equiv V(p^T(R, R^*), Y(p^T(R, R^*), R^*))$.

With respect to $R$ and $R^*$ in $\left[\underline{R}, \overline{R}\right]$, the maximum world welfare is attained when $R = R^* = R^A$. This can be seen in Figure 2.

In this figure, the production point $E^W$ is the world welfare maximizing production point at which the world utility indifference curve is tangent to the world PPF. The level of $R$ for each country is $R^A$ and the equilibrium $p$ pertaining to this point under trade is $p^A$, so that there is no trade.

In what follows, it is shown that the Nash equilibrium $(R, R^*)$ of the policy game between governments becomes $(R^A, R^A)$, and thus the world production point is $E^W$ at the Nash equilibrium. For any $R \in \left[\underline{R}, \overline{R}\right]$, the strategic pair $(R, R)$ between Home and Foreign results in no trade even under the trading world, that is, both countries are under autarky. Since $R^A$ is the optimal level of $R$ under autarky,




**Source(s):** Figure by authors

**Figure 2.** World PPF

$$V^k\left(R^A,R^A\right) > V^k(R,R), \textit{for } k = H \textit{ and } F \textit{ and any } R \neq R^A. \tag{13}$$

Next, we suppose a strategic pair $(R,R^*)$ such that $R \neq R^*$. Under this asymmetric pair $(R,R^*)$, any possible world production point should be inside of the world PPF because any point on the frontier must be attained by a symmetric pair. Let us consider the case $(R,R^*) = (R^A,R')$ where $R' \neq R^A$. Then, we have

$$W\left(R^A,R'\right) = V^H\left(R^A,R'\right) + V^F\left(R',R^A\right) < W\left(R^A,R^A\right) = V^H\left(R^A,R^A\right) + V^F\left(R^A,R^A\right), \tag{14}$$

because the world PPF is strictly concave.

There is a positive amount of trade under $(R^A,R')$ for $R' \neq R^A$. The equilibrium trading price $p^T(R^A,R')$ must differ from the autarkic price $p^A$. Now suppose that Foreign government changes the level of $R^*$ from $R'$ to $R^A$. Then both countries become autarkic for $(R,R^*) = (R^A,R^A)$. Home country's PPF($R^A$) is not affected by this Foreign's change of $R^*$, so that the Home will lose a trade gain by giving up trade. This implies that

$$V^H\left(R^A,R^A\right) < V^H\left(R^A,R'\right). \tag{15}$$

In view of (14) and (15), we have

$$V^F\left(R^A,R^A\right) > V^F\left(R',R^A\right), \textit{for all } R' \neq R^A.$$

Since two countries are identical, it is also true that

$$V^H\left(R^A,R^A\right) > V^H\left(R',R^A\right), \textit{for all } R' \neq R^A.$$

Therefore, $(R,R^*) = (R^A,R^A)$ is a Nash equilibrium.

Finally, we confirm that $(R^A, R^A)$ is a unique Nash equilibrium pair. Suppose that $(R, R^*)$ is a Nash equilibrium pair and $(R, R^*) \neq (R^A, R^A)$. Following a similar manner to obtain (14), we can show

$$V^H(R,R^*) + V^F(R^*,R) < V^H(R^*,R^*) + V^F(R,R) < V^H\left(R^A,R^A\right) + V^F\left(R^A,R^A\right) < V^H\left(R^A,R^*\right) + V^F\left(R^A,R\right), \tag{16}$$

from (13), so that

$$\left[V^H\left(R^A,R^*\right) - V^H(R,R^*)\right] + \left[V^F\left(R^A,R\right) - V^F(R^*,R)\right] > 0.$$

Therefore we have

$$V^H\left(R^A,R^*\right) > V^H(R,R^*)\ or\ V^F\left(R^A,R\right) > V^F(R^*,R).$$

In the case where $V^H(R^A, R^*) > V^H(R, R^*)$, $R$ is not a best response of Home for Foreign's response $R^*$, implying that $(R, R^*)$ is not a Nash equilibrium. Likewise, $(R, R^*)$ is shown not to be a Nash in the other case. Therefore, only $(R^A, R^A)$ is Nash equilibrium.

## 4. Extension

In the previous sections, we treated the economy where there are two goods, one primary factor and one domestic policy in each country. Now we extend this framework by allowing arbitrary numbers of goods, factors and policies. So we assume that there are $n$ goods, $m$ primary factors and $k$ domestic policies in each country.

The production function of good $i$ is expressed as

$$Q_i = F^i(V_i, \mathbb{R}), i = 1, \ldots, n,$$

where $V_i \equiv (V_{i1}, \ldots, V_{im})$ is the input vector for the production of good $i$ and $\mathbb{R} \equiv (R_1, \ldots, R_k)$ is the vector of domestic policies available for domestic production activities. The function $F^i$ is assumed to be linearly homogenous as well as strictly quasi-concave with respect to $V_i$ and $\mathbb{R}$.

The execution of the domestic policy $j$ needs the use of primary factors, the functional relationship of which is displayed by

$$R_j \equiv G^j\left(V_j^R\right), j = 1, \ldots, k,$$

where $V_j^R \equiv (V_{j1}^R, \ldots, V_{jm}^R)$ is the vector of the primary factors used for the execution of domestic policy $j$. The function of $G^j$ is assumed to be linearly homogenous and strictly quasi-concave with respect to $V_j^R$.

The full employment conditions of primary factors are

$$\sum_{i=1}^{n} V_i + \sum_{j=1}^{m} V_j^R = V,$$

where $V \equiv (V_1, \ldots, V_m)$ is the endowment vector of the primary factors which are given and fixed.

Under these suppositions, the PPF(ℝ) is shown to be strictly concave. The country's PPF, which is an envelope of PPF(ℝ), is also strictly concave [7]. Therefore, we can apply the analytical logic of the previous section and show that the Nash equilibrium of the domestic policy game uniquely exists as a symmetry equilibrium between identical countries if each production point on the country's PPF corresponds a unique domestic policy bundle ℝ.

## 5. Conclusions

In this paper, we have considered a world economy where there are two identical countries, two goods and one primary factor and the government of each country executes a domestic policy in order to maximize the national welfare. In particular, each government is supposed to determine the level of the domestic policy strategically by taking the policy level of the other country into account under trade. The domestic policy of each country is supposed to serve for the private production in that country and the policy effect is diminishing according to the production expansion. Then it was shown that there is a unique Nash trading equilibrium of the policy game between two governments and the equilibrium level of policy is the same between countries. In order to derive this result, use is made of the world production frontier as well as each country's production frontier, which enabled us to capture the circumstances where the symmetry Nash equilibrium emerges. We should notice that, if the country PPF is concave, the general equilibrium under trade becomes Pareto optimum [8]. Moreover, there is no trade at that equilibrium if two trading countries are completely symmetric. Therefore, our analysis suggests that the essential source of symmetry-breaking is the inefficiency of the trading general equilibrium. We further generalized the framework by allowing arbitrary numbers of goods, factors and policies and asserted that the same result carries over in this generalized framework.

Concerning the topic of symmetry-breaking, Suga *et al.* (2022) employ another model, where the domestic policy is assumed to yield the increasing returns to scale effect to the private production. Then they derived that any Nash equilibrium becomes symmetry-breaking, that is, the policy level differs between countries. A main source of the difference in the result is the difference in the shape of the country's production possibility frontier and thus the world production frontier. In the case of Suga *et al.* (2022), the PPF is strictly convex to the origin while, in our case, it is strictly concave.

Chatterjee (2017) investigates the circumstances where the symmetry-breaking appears in the Heckscher and Ohlin trading framework with the assumption that the country's payoff function is twice continuously differentiable with respect to strategic policy variables. Since the Ricardian type of trading economies, specialization is general in equilibrium and the differentiability property is difficult to be assured. Thus Chatterjee's analysis is hardly applicable. On the other hand, there is an interesting model of Clarida and Findlay (1992), which might fit Chatterjee's case. They accommodated a government policy into the specific-factor model inheriting the Heckscher–Ohlin spirit and discussed comparative advantage in trade. Succeeding their analysis, Tawada, Suga, & Yanase (2022) pointed out a possibility that the country's PPF becomes concave-convex-concave. Chatterjee's analysis strongly suggests that a symmetry-breaking equilibrium appears in this case. Although the Nash equilibrium is necessarily symmetric between identical countries in the Heckscher–Ohlin model with decreasing returns to scale policies, the country's PPF has a convex portion in general in the case where the domestic policies are of the increasing returns to scale type [9]. Therefore, the symmetry-breaking Nash equilibrium might appear in the case of IRS policies. We need a detailed and precise analysis for those cases in future.

### Notes

1. In fact, Professor Long has been paid rigorous attention on the wide area of game theory including dynamic game theory, applications of game theory to natural resources, environmental issues and

industrial organization and performed the great academic contribution by vast publications over his research career. Among many of his publications, his most recent works in these fields are, for example, Long, Prieuer, Tidball, & Puzon (2017), Benchekroun & Long (2018), Yanase & Long (2021), Laussel, Long, & Resende (2022) and Colombo, Labrecciosa, & Long (2022).

2. See Proposition 1 of Chatterjee (2017) that, if aggregate income is concave in policy, no asymmetric Nash equilibrium exists and that, if aggregate income is sufficiently convex in own policy, any Nash equilibrium is asymmetric in the open economy.
3. Following Clarida and Findlay (1992), we interpret $R$ as a public policy by the government in a broader sense. However, it is possible to interpret $R$ as a public intermediate good or public infrastructure in a narrow sense.
4. We suppose that the model is game theoretic. So, once after the government sets the level of the public policy, the private production takes place under given level of $R$ and the resource constraint such that $L_1 + L_2 = \mathrm{L} - L_R$. For the firms there is no cost to use $R$, so that firms have a positive profit. Then, the government imposes the tax with tax rate $t$ on all firms so as to satisfy $t(p_1Q_1 + p_2Q_2) = wL_R$, where $w$ is the wage rate.
5. See Tawada (1980) for this fact.
6. See Tawada and Yanase (2021) for the condition to satisfy this assumption.
7. See Tawada (1980) for the fact that the country's PPF is strictly concave under arbitrary numbers of goods, factors and policies.
8. In order for the general equilibrium to be Pareto optimum, we need not only the concavity of the PPF but also the production efficiency condition that the price line is tangent to the PPF at the equilibrium. As shown in our model, this latter condition is satisfied as well. If there are market failures such as externalities or imperfect competitions, symmetry-breaking may emerge even under the concave PPF in the trading economy with two symmetry countries.
9. See Tawada and Abe (1984) and Okamoto (1985) for some exceptional cases, where the country's PPF is globally concave under the Heckscher–Ohlin framework.

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## Further reading

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